Generalized Perturbations in Neutrino Mixing

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Outline

I. Why perturbations?

II. Real perturbations (charged lepton basis)

III. Complex perturbations (charged lepton basis)

IV. Perturbations to charged lepton mass matrix

J. Liao, D. Marfatia, KW, PRD 87, 013003 (2013) [arXiv:1205.6860 [hep-ph]]
J. Liao, D. Marfatia, KW, PRD 92, 073004 (2015) [arXiv:1506.03013 [hep-ph]]

- Current ν data (solar, atmospheric, reactor, long-baseline) indicate :
  - ✧ One small angle (θ_{13}), two large angles (θ_{12},θ_{23})
  - ✧ Two distinct mass-squared differences ($\delta m^2_{21} << |\delta m^2_{31}|$)
- In a 3-ν scenario, a global fit to PMNS matrix gives Gonzalez-Garcia et al.

$$\begin{aligned} \delta m^2 = \delta m^2_{21} &= (7.50^{+0.19}_{-0.17}) \times 10^{-5}\ \mathrm{eV}^2 & & \\ \theta_{12} &= (33.48^{+0.78}_{-0.75})^\circ & \theta_{13} &= (8.5^{+0.2}_{-0.2})^\circ \end{aligned}$$

$$\begin{aligned} \Delta m^2 = \frac{\delta m^2_{32} + \delta m^2_{31}}{2} &= (2.457^{+0.047}_{-0.047}) \times 10^{-3}\ \mathrm{eV}^2 &&= (-2.449^{+0.048}_{-0.047}) \times 10^{-3}\ \mathrm{eV}^2 \\ \theta_{23} &= (42.3^{+3.0}_{-1.6})^\circ &&= (49.5^{+1.5}_{-2.2})^\circ \end{aligned}$$

Normal ordering (NO) Inverted ordering (IO)

- Currently unknown
  - ✧ Sign of Δm^2 (NO or IO)
  - ✧ CP phases (Dirac phase δ and Majorana phases ϕ_2, ϕ_3)
  - ✧ Absolute masses (parametrized as the lightest mass, m_1 for NO and m_3 for IO)

- Tri-bimaximal (TBM) mixing gives $\theta_{23} = 45^o$, $\theta_{12} \approx 35^o$, but $\theta_{13} = 0$

- Use perturbations to bring models in agreement with data
  - ✧ Renormalization group corrections
  - ✧ Radiative corrections
  - ✧ Vacuum misalignment corrections

QUESTIONS

- What models can give experimentally acceptable results after perturbations?

- How are the CP violating phases affected by perturbations?

Perturbation formalism (charged lepton basis)

$$M_\nu = M_0 + E = U_0^* \overline{M_0} U_0^\dagger + \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}$$

Original mass matrix

Perturbation

Diagonalizes M_0

$$\overline{M_0} = \begin{pmatrix} m_1^0 & 0 & 0 \\ 0 & m_2^0 & 0 \\ 0 & 0 & m_3^0 \end{pmatrix}$$

unperturbed masses

New masses

$$M_\nu = U^* \overline{M} U^\dagger$$

New mixing

Real Perturbations (Real Mass Matrix)

- Assume ε_{ij}, $\delta m^0_{21} << |\delta m^0_{31}|$; then to leading order

$$\delta\theta_{13} = \delta'_{13} = \frac{\epsilon'_{13}}{\delta m^0_{31}} \, ,$$

$$\delta\theta_{23} = \frac{\delta'_{23}}{c^0_{13}} = \frac{\epsilon'_{23}}{c^0_{13}\delta m^0_{31}} \, ,$$

$$\delta\theta_{12} = \xi' = \frac{1}{2}\arctan\frac{2\epsilon'_{12}c^0_{2\times12} - (\epsilon'_{22} - \epsilon'_{11})s^0_{2\times12}}{(\epsilon'_{22} - \epsilon'_{11})c^0_{2\times12} + 2\epsilon'_{12}s^0_{2\times12} + \delta m^0_{21}}$$

small due to near degeneracy

- where

$$\epsilon'_{11} = \epsilon_1(c^0_{13})^2 + \epsilon_6(s^0_{13})^2 - \epsilon_3 s^0_{2\times13} \, , \quad \epsilon'_{12} = \epsilon_2 c^0_{13} - \epsilon_5 s^0_{13}$$

$$\epsilon'_{13} = \epsilon_3 c^0_{2\times13} + \frac{1}{2}(\epsilon_1 - \epsilon_6)s^0_{2\times13} \, , \quad \epsilon'_{22} = \epsilon_4 \, ,$$

$$\epsilon'_{23} = \epsilon_2 s^0_{13} + \epsilon_5 c^0_{13} \, , \quad \epsilon'_{33} = \epsilon_1(s^0_{13})^2 + \epsilon_6(c^0_{13})^2 + \epsilon_3 s^0_{2\times13}$$

- and

$$\epsilon_1 = \epsilon_{11} \, , \quad \epsilon_2 = \epsilon_{12}c^0_{23} - \epsilon_{13}s^0_{23} \, , \quad \epsilon_3 = \epsilon_{12}s^0_{23} + \epsilon_{13}c^0_{23} \, , \epsilon_5 = \epsilon_{23}c^0_{2\times23} + \frac{1}{2}(\epsilon_{22} - \epsilon_{33})s^0_{2\times23} \, ,$$

$$\epsilon_4 = \epsilon_{22}(c^0_{23})^2 + \epsilon_{33}(s^0_{23})^2 - \epsilon_{23}s^0_{2\times23} \, ,$$

$$\epsilon_6 = \epsilon_{22}(s^0_{23})^2 + \epsilon_{33}(c^0_{23})^2 + \epsilon_{23}s^0_{2\times23}$$

- Corrections to θ_{23} and θ_{13} are small, but $\delta\theta_{12}$ can be large
- Initial θ_{12} need not be close to experimental value!
- Example: μ-τ symmetry ($|U_{\mu i}| = |U_{\tau i}|\,,\ \text{for } i = 1, 2, 3$)
  - a) $\theta_{23} = 45^o$, $\theta_{13} = 0$
    - ✧ TBM ($\theta_{12} \approx 35.3^o$) Harrison, Perkins, Scott
    - ✧ Bimaximal ($\theta_{12} = 45^0$) Barger, Pakvasa, Weiler, KW; Vissani; Baltz, Goldhaber, Goldhaber
    - ✧ Hexagonal ($\theta_{12} = 30^o$) Albright, Dueck, Rodejohann
    - ✧ Golden ratio (A_5, $\theta_{12} \approx 31.7^o$) Kajiyama, Raidal, Strumia; Everett, Stuart
  - b) $\theta_{23} = 45^o$, $\theta_{12} = 0$
  - c) $\theta_{23} = 45^o$, $\theta_{12} = 90^0$
  - d) $\theta_{23} = 45^o$, $\delta = \pm 90^0$
    - ✧ Tetramaximal ($\theta_{12} \approx 30.3^o$, $\theta_{13} \approx 8.3^o$) Xing
- Bimaximal, Hexagonal, or even models on the “dark side” ($\theta_{12} > 45^o$) are now possible!

Class a ($\theta^0_{13} = 0$)

$$\delta\theta_{23} \simeq \frac{\epsilon_{22} - \epsilon_{33}}{2\delta m^0_{31}}\,, \quad \delta\theta_{13} \simeq \sqrt{2}\frac{\epsilon_{12} + \epsilon_{13}}{2\delta m^0_{31}} \longleftarrow \text{small}$$

$$\delta\theta_{12} \simeq \frac{1}{2}\arctan\frac{2\sqrt{2}\epsilon_3\cos 2\theta^0_{12} - \epsilon_6\sin\theta^0_{12}}{2\sqrt{2}\epsilon_3\sin 2\theta^0_{12} + \epsilon_6\cos\theta^0_{12} + 2\delta m^0_{21}}$$

$$\epsilon_3 = \epsilon_{12} - \epsilon_{13}\,, \quad \epsilon_6 = \epsilon_{22} + \epsilon_{33} - 2\epsilon_{23} - 2\epsilon_{11}$$

- Ratio for $\delta\theta_{12}$ involves only small quantities, can be O(1)

- Perturbations can be relatively small for *any* θ^o_{12}
- Smallest $\epsilon_{RMS} = \sqrt{\frac{\sum_{i,j=1}^{3} |M_{ij} - M_{0ij}|^2}{9}}$ in meV (m^0_3 = 60 meV):

$\theta^0_{12}(^\circ)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
60	-3.05	-3.50	-5.99	-2.72	-1.52	5.77	4.10
45 (BM)	-1.32	-4.74	-4.74	-3.58	-0.66	4.90	3.79
35.3 (TBM)	0.32	-4.66	-4.82	-4.40	0.16	4.08	3.74
30 (HM)	1.07	-4.31	-5.18	-4.78	0.54	3.71	3.79
0	0.00	-1.38	-8.11	-4.24	0.00	4.24	4.36

- Possibilities with all ε_{ij} having same order of magnitude:

60	5.41	-4.17	-4.52	-5.00	-9.94	3.36	6.14
45 (BM)	6.76	-4.43	-4.26	-5.67	-9.27	2.69	6.08
35.3 (TBM)	7.66	-4.32	-4.37	-6.12	-8.82	2.24	6.08
30 (HM)	8.11	-4.17	-4.52	-6.35	-8.59	2.01	6.09
0	9.46	-2.52	-6.17	-7.02	-7.92	1.34	6.28

Class b (θ^0_{12} = 0)

- θ^o_{13} must be relatively close to experimental value
- Smallest $\epsilon_{RMS} = \sqrt{\frac{\sum_{i,j=1}^{3} |M_{ij} - M_{0ij}|^2}{9}}$ in meV (m^0_3 = 60 meV):

$\theta^0_{13}(^\circ)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
0	0.00	-1.38	-8.11	-4.24	0.00	4.24	4.36
5	0.48	1.44	-5.28	-4.48	-0.24	4.00	3.27
10	-0.44	4.21	-2.52	-4.02	0.22	4.46	3.06
15	-2.64	6.59	-0.14	-2.92	1.32	5.56	3.90
20	-5.85	8.30	1.57	-1.32	2.93	7.17	5.24

- Possibilities with all ε_{ij} having same order of magnitude:

0	9.46	-2.52	-6.17	-7.02	-7.92	1.34	6.28
5	9.01	1.13	-2.52	-6.80	-7.69	1.56	5.41
10	7.66	4.67	1.02	-6.12	-7.02	2.24	5.22
15	5.47	8.00	4.35	-5.03	-5.93	3.33	5.80
20	2.50	11.00	7.35	-3.54	-4.44	4.82	6.92

Complex perturbations (charged lepton basis)

$$M = U_0^* \overline{M_0} U_0^\dagger + E = U_0^* \left[\overline{M_0} + \tilde{E} \right] U_0^\dagger \ = U^* \overline{M} U^\dagger$$

$$\tilde{E} = U_0^T E U_0 = \begin{pmatrix} a & b e^{i\phi_2^0/2} & d e^{i\phi_3^0/2} \\ b e^{i\phi_2^0/2} & c e^{i\phi_2^0} & f e^{i(\phi_2^0+\phi_3^0)/2} \\ d e^{i\phi_3^0/2} & f e^{i(\phi_2^0+\phi_3^0)/2} & g e^{i\phi_3^0} \end{pmatrix}$$

$$a = \epsilon_4 (s_{12}^0)^2 + [\epsilon_1 (c_{13}^0)^2 - \epsilon_3 s_{2\times13}^0 e^{i\delta^0} + \epsilon_6 (s_{13}^0)^2 e^{2i\delta^0}](c_{12}^0)^2 + (\epsilon_5 s_{13}^0 e^{i\delta^0} - \epsilon_2 c_{13}^0) s_{2\times12}^0 \,,$$

$$b = \epsilon_2 c_{13}^0 c_{2\times12}^0 + [\epsilon_1 (c_{13}^0)^2 - \epsilon_4 + \epsilon_6 (s_{13}^0)^2 e^{2i\delta^0}] c_{12}^0 s_{12}^0 - [\epsilon_3 s_{2\times12}^0 s_{13}^0 c_{13}^0 + \epsilon_5 c_{2\times12}^0 s_{13}^0] e^{i\delta^0} \,,$$

$$c = \epsilon_4 (c_{12}^0)^2 + [\epsilon_1 (c_{13}^0)^2 - \epsilon_3 s_{2\times13}^0 e^{i\delta^0} + \epsilon_6 (s_{13}^0)^2 e^{2i\delta^0}](s_{12}^0)^2 - (\epsilon_5 s_{13}^0 e^{i\delta^0} - \epsilon_2 c_{13}^0) s_{2\times12}^0 \,,$$

$$d = (\epsilon_1 c_{12}^0 c_{13}^0 - \epsilon_2 s_{12}^0) s_{13}^0 e^{-i\delta^0} - \epsilon_6 c_{12}^0 c_{13}^0 s_{13}^0 e^{i\delta^0} + \epsilon_3 c_{12}^0 c_{2\times13}^0 - \epsilon_5 c_{13}^0 s_{12}^0 \,,$$

$$f = (\epsilon_1 s_{12}^0 c_{13}^0 + \epsilon_2 c_{12}^0) s_{13}^0 e^{-i\delta^0} - \epsilon_6 s_{12}^0 c_{13}^0 s_{13}^0 e^{i\delta^0} + \epsilon_3 s_{12}^0 c_{2\times13}^0 + \epsilon_5 c_{13}^0 c_{12}^0 \,,$$

$$g = \epsilon_6 (c_{13}^0)^2 + \epsilon_3 s_{2\times13}^0 e^{-i\delta^0} + \epsilon_1 (s_{13}^0)^2 e^{-2i\delta^0} \,,$$

- Block diagonalize $N \equiv \overline{M_0} + \tilde{E}$ (set 1-3 and 2-3 elements to 0) using:

$$U_\delta = \begin{pmatrix} 1 & 0 & \delta_{13} \\ 0 & 1 & \delta_{23} \\ -\delta_{13}^* & -\delta_{23}^* & 1 \end{pmatrix}$$

$$\delta_{13} \approx \frac{|d|e^{-i\phi_{13}}}{|m_3^0 - m_1^0 e^{-2i\phi_{13}}|} \qquad \tan\phi_{13} = \tfrac{m_3^0+m_1^0}{m_3^0-m_1^0} \tan\left[\arg(d) + \phi_3^0/2\right]$$

$$\delta_{23} \approx \frac{|f|e^{-i\phi_{23}}}{|m_3^0 - m_1^0 e^{-2i\phi_{23}}|} \qquad \tan\phi_{23} = \tfrac{m_3^0+m_1^0}{m_3^0-m_1^0} \tan\left[\arg(f) + \tfrac{\phi_2^0+\phi_3^0}{2}\right]$$

δ_{13}, δ_{23} are small

- Diagonalize 1-2 submatrix of N using:

$$U_{12}(\xi,\phi) = \begin{pmatrix} c_\xi & s_\xi e^{-i\phi} & 0 \\ -s_\xi e^{i\phi} & c_\xi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\phi = \arctan \frac{|a+m_1^0|\sin(\phi_a-\phi_b) - |ce^{i\phi_2^0}+m_2^0|\sin(\phi_c-\phi_b)}{|a+m_1^0|\cos(\phi_a-\phi_b) + |ce^{i\phi_2^0}+m_2^0|\cos(\phi_c-\phi_b)}$$

$$\xi = \frac{1}{2}\arctan \frac{2|b|}{|ce^{i\phi_2^0}+m_2^0|\cos(\phi_c+\phi-\phi_b) - |a+m_1^0|\cos(\phi_a-\phi-\phi_b)}$$

$$\phi_a = \arg(N_{11})\,,\ \phi_b = \arg(N_{12})\,,\ \phi_c = \arg(N_{22})$$

- Putting it together we have:

Block diagonalizes N

Makes m$_i$ ≥ 0, real

$$U = U_0 U_\delta U_{12}(\xi,\phi) P$$

Unperturbed U

Diagonalizes 2X2

$$P = \begin{pmatrix} e^{i\omega_1/2} & 0 & 0 \\ 0 & e^{i\omega_2/2} & 0 \\ 0 & 0 & e^{i\omega_3/2} \end{pmatrix} \quad \begin{aligned} \omega_1 &= -\arg\left[(a+m_1^0)c_\xi^2 + (ce^{i\phi_2^0} + m_2^0)s_\xi^2 e^{2i\phi} - 2bs_\xi c_\xi e^{i\phi}\right] , \\ \omega_2 &= -\arg\left[(a+m_1^0)s_\xi^2 e^{-2i\phi} + (ce^{i\phi_2^0} + m_2^0)c_\xi^2 + 2bs_\xi c_\xi e^{-i\phi}\right] , \\ \omega_3 &= -\arg\left(m_3^0 + ge^{i\phi_3^0}\right) . \end{aligned}$$

- LO corrections to θ_{13},θ_{23} come from U_δ (small)
- LO corrections to θ_{12} and δ come from U_{12} (can be large)
- LO corrections to ϕ_2 and ϕ_3 come from U_{12} and P (can be large)

- Result (to LO):

$$\delta\theta_{13} = \frac{|d| c^0_{12} \cos(\delta^0 - \frac{\phi^0_3}{2} - \phi_{13})}{|m^0_3 - m^0_1 e^{-2i\phi_{13}}|} + \frac{|f| s^0_{12} \cos(\delta^0 + \frac{\phi^0_2 - \phi^0_3}{2} - \phi_{23})}{|m^0_3 - m^0_1 e^{-2i\phi_{23}}|}$$

$$\delta\theta_{23} = -\frac{|d| s^0_{12} \cos(\frac{\phi^0_3}{2} + \phi_{13})}{|m^0_3 - m^0_1 e^{-2i\phi_{13}}|} + \frac{|f| c^0_{12} \cos(\frac{\phi^0_2 - \phi^0_3}{2} - \phi_{23})}{|m^0_3 - m^0_1 e^{-2i\phi_{23}}|}$$

$$\delta\theta_{12} = \arcsin\sqrt{\sin^2(\theta^0_{12} + \xi) - \sin(2\theta^0_{12}) \sin(2\xi) \sin^2 \frac{\phi^0_2 + 2\phi}{4}} - \theta^0_{12}$$

- LO corrections to θ_{13},θ_{23} are small (~ $\varepsilon_{ij}/\delta m^0{}_{31}$)
- LO corrections to θ_{13} can be large ($-\xi \leq \delta\theta_{12} \leq \xi$)

- LO corrections to phases (can be large):

$$\Delta\delta = \alpha - \beta\,,$$

$$\Delta\phi_2 = -2(\alpha+\beta) + \omega_2 - \omega_1\,,$$

$$\Delta\phi_3 = -2\beta + \omega_3 - \omega_1\,.$$

- where

$$\alpha = -\arctan\frac{\tan\theta_{12}^0\tan\xi\sin(\phi_2^0/2+\phi)}{1-\tan\theta_{12}^0\tan\xi\cos(\phi_2^0/2+\phi)}\,,$$

$$\beta = \arctan\frac{\tan\xi\sin(\phi_2^0/2+\phi)}{\tan\theta_{12}^0+\tan\xi\cos(\phi_2^0/2+\phi)}\,.$$

Example: Perturbations to μ-τ symmetry

- Complex perturbations on Class a can give Class d (shown for particular case by Babu, Ma, Valle)
- Can show more generally
- General forms of mass matrix in Classes a and d: Grimus, Lavoura

$$M_a = \begin{pmatrix} x & y & -y \\ y & z & -w \\ -y & -w & z \end{pmatrix} \qquad M_d = \begin{pmatrix} u & r & -r^* \\ r & s & -v \\ -r^* & -v & s^* \end{pmatrix}$$

- Then

$$E = \begin{pmatrix} \epsilon_{11} - i\mathrm{Im}(x) & \epsilon_{12} & -\epsilon_{12}^* + 2i\mathrm{Im}(y) \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} + i\mathrm{Im}(w) \\ -\epsilon_{12}^* + 2i\mathrm{Im}(y) & \epsilon_{23} + i\mathrm{Im}(w) & \epsilon_{22}^* - 2i\mathrm{Im}(z) \end{pmatrix}$$

turns type M_a into type M_d

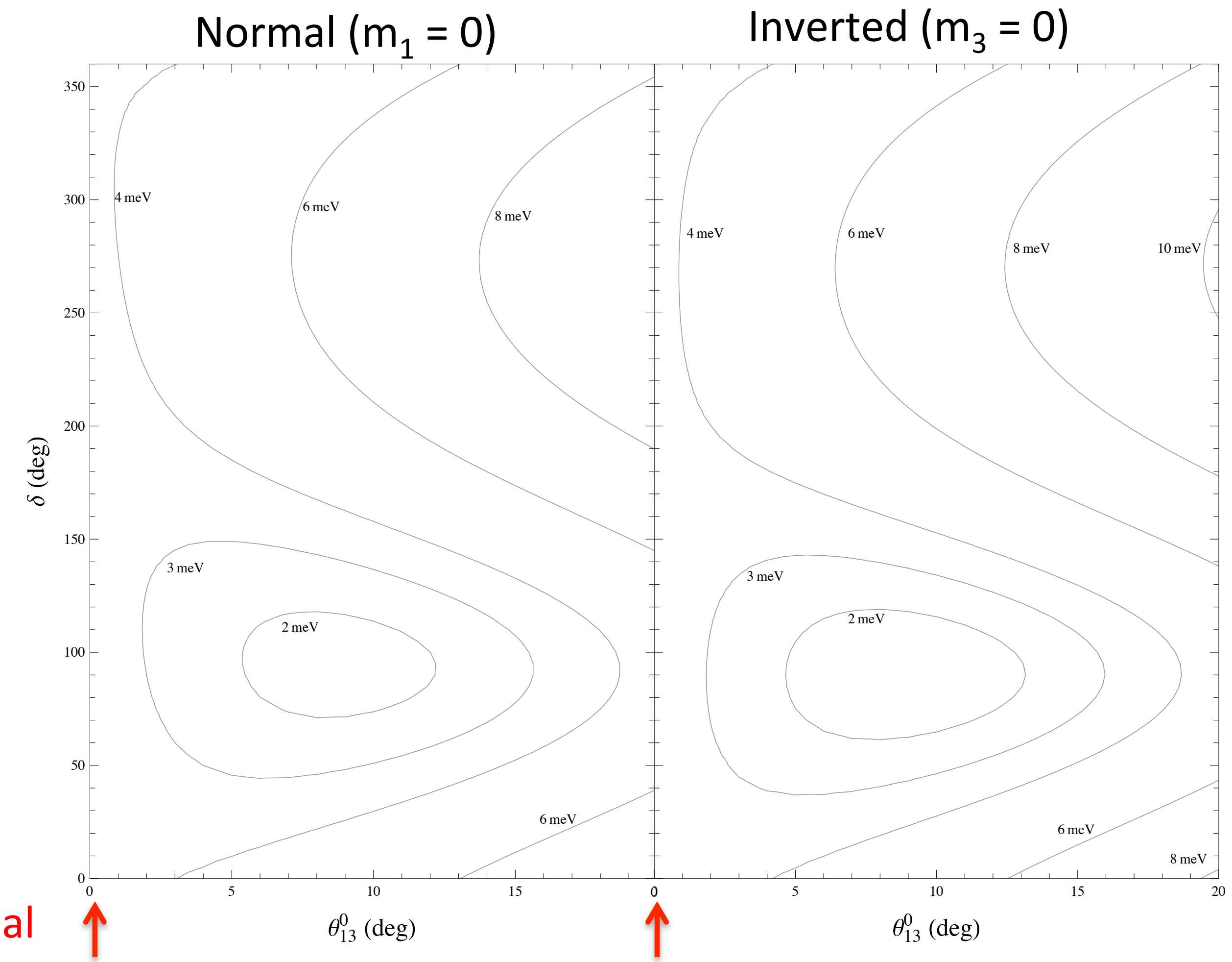
- For general M_a the required perturbations are large, but are small for real M_a:

$$E = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & -\epsilon_{12}^* \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ -\epsilon_{12}^* & \epsilon_{23} & \epsilon_{22}^* \end{pmatrix}$$

$$\begin{aligned} \epsilon_{13} &= -\epsilon_{12}^* \\ \epsilon_{33} &= \epsilon_{22}^* \end{aligned}$$

Another example with μ-τ symmetry

- Class d with $\theta^0_{23} = 45^o$, $\theta^0_{12} = 45^o$, $\delta^0 = 90^0$
  (bimaximal mixing is a special case when $\theta^0_{13} = 0$)
- Any δ can occur after small perturbation


Normal ($m_1 = 0$)

Inverted ($m_3 = 0$)

Contours of ε_{RMS}

Bimaximal

Perturbations in the charged lepton sector

- General PMNS matrix is $U_{PMNS} = U_l^\dagger U_\nu$, where

$$M_\ell = V_\ell \overline{M_\ell} U_\ell^\dagger \qquad\qquad M_\nu = U_\nu^* \overline{M_\nu} U_\nu^\dagger$$

RH charged lepton mixing

LH charged lepton mixing

Neutrino mixing

- Can find U_l by diagonalizing $(M_l)^\dagger M_l$:

$$(M_l)^\dagger M_l = U_l \overline{M_l}^2 (U_l)^\dagger$$

- Let perturbed charged lepton mass matrix be $M_l = M_l^0 + E_l$, where

$$(E_l)_{ij} \equiv \epsilon^l_{ij} \qquad\qquad |\epsilon^l_{ij}| \ll m_\tau$$

- If the unperturbed mixing is U_l^0 with $(M_l^0)^\dagger M_l^0 = U_l^0(\overline{M_l^0})^2(U_l^0)^\dagger$, then to LO

$$\begin{aligned}(M_l)^\dagger M_l &\approx U_l^0(\overline{M_l^0})^2(U_l^0)^\dagger + (M_l^0)^\dagger E_l + E^\dagger M_l^0 \\ &= U_l^0\left[\overline{M_l^0}^2 + N^l\right](U_l^0)^\dagger\,,\end{aligned}$$

where

$$N^l = (U_l^0)^\dagger\left[(M_l^0)^\dagger E_l + E^\dagger M_l^0\right]U_l^0$$

- Let U_δ^l diagonalize $(\overline{M_l^0})^2 + N^l$

$$\overline{M_l^0}^2 + N^l = \begin{pmatrix} (m_e^0)^2 + N_{11}^l & N_{12}^l & N_{13}^l \\ (N_{11}^l)^* & (m_\mu^0)^2 + N_{22}^l & N_{23}^l \\ (N_{13}^l)^* & (N_{23}^l)^* & (m_\tau^0)^2 + N_{33}^l \end{pmatrix} = U_\delta^l \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} (U_\delta^l)^\dagger$$

- Then

$$U_{PMNS} = (U_l^0 U_\delta^l)^\dagger U_\nu^0 = (U_\delta^l)^\dagger U_0$$

Charged lepton correction

Unperturbed PMNS mixing

- Similar to neutrinos, charged leptons have near-degeneracy between m_e and m_μ ; to LO:

$$U^l_\delta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta^l_{23} e^{-i\phi^l_{23}} \\ 0 & -\delta^l_{23} e^{i\phi^l_{23}} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \delta^l_{13} e^{-i\phi^l_{13}} \\ 0 & 1 & 0 \\ -\delta^l_{13} e^{i\phi^l_{13}} & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta^l_{12} & \sin\theta^l_{12} e^{-i\phi^l_{12}} & 0 \\ -\sin\theta^l_{12} e^{i\phi^l_{12}} & \cos\theta^l_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where

$$\delta^l_{13} \approx \frac{|N^l_{13}|}{m^2_\tau}\,, \quad \delta^l_{23} \approx \frac{|N^l_{23}|}{m^2_\tau}\,, \qquad \phi_{ij} \approx -\arg N^l_{ij}$$

$$\theta^l_{12} \approx \frac{1}{2}\arctan\frac{2|N^l_{12}|}{m^2_\mu + N^l_{22} - N^l_{11}}$$

- Similar to neutrinos, δ^l_{13} and δ^l_{23} are small, but θ^l_{12} can be large; hence correction to U_{PMNS} can be large

Example with large charged lepton correction

- Assume

$$U^l_\delta = \begin{pmatrix} \cos\theta^l_{12} & \sin\theta^l_{12} & 0 \\ -\sin\theta^l_{12} & \cos\theta^l_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

i.e., small corrections are vanishingly small

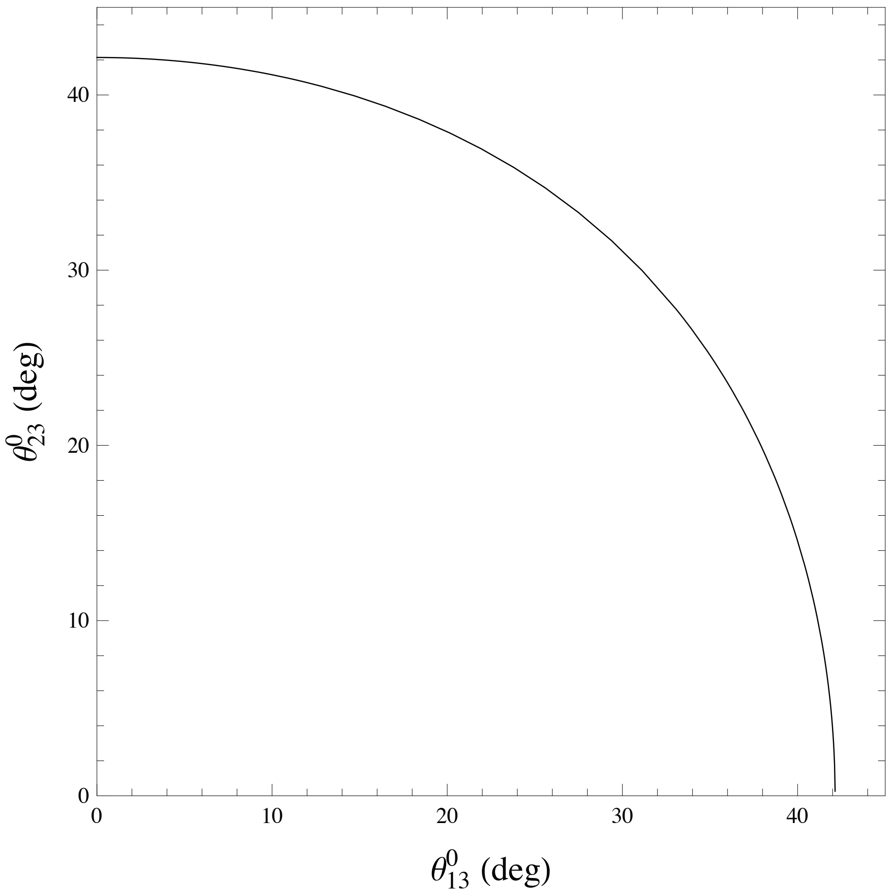
then final PMNS mixing angles satisfy

$$c_{13}c_{23} = c^0_{13}c^0_{23} \,,$$

$$s^2_{13} = (s^0_{13})^2(c^l_{12})^2 + (s^0_{23})^2(c^0_{13})^2(s^l_{12})^2 - 2s^0_{13}c^0_{13}s^0_{23}s^l_{12}c^l_{12}\cos\delta^0 \,,$$

$$c^2_{13}s^2_{12} = \left[(c^l_{12}c^0_{13}s^0_{12} - s^l_{12}c^0_{12}c^0_{23})^2 + (s^l_{12})^2(s^0_{12})^2(s^0_{13})^2(s^0_{23})^2 \right.$$
$$\left. + \, 2s^l_{12}s^0_{12}s^0_{13}s^0_{23}(c^l_{12}c^0_{13}s^0_{12} - s^l_{12}c^0_{12}c^0_{23})\cos\delta^0\right] \,,$$

- E.g.: if θ^l_{12} = Cabibbo angle and the initial PMNS mixing is bimaximal, then the final PMNS agrees with data to 2σ
- Another example; possible initial angles that are consistent with best-fit data after perturbation:



- All three initial PMNS angles can be different from their experimental values with charged lepton corrections!

Neutrino oscillations with nonstandard interactions

- Standard Model Hamiltonian:

$$H = \frac{1}{2E_\nu} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} \sqrt{2} G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Vacuum oscillations

Matter effect
from V-A
interactions

- Nonstandard vector interaction

$$\mathcal{L}_\mathrm{V} = \frac{G_F}{\sqrt{2}} \epsilon^V_{\alpha\beta} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma^5) \nu_\beta\right] \left[\bar{f} \gamma_\rho (1 \pm \gamma^5) f\right] + \mathrm{h.c.}$$

adds a term $\sqrt{2} G_F N_f \epsilon^V_{\alpha\beta}$ to H

- Nonstandard scalar interactions $\mathcal{L}_S = \lambda_\nu^{\alpha\beta} \bar{\nu}_\alpha \nu_\beta \phi + \lambda_f \bar{f} f \phi$ in mean-field approximation give effective contribution to M_ν:

$$\epsilon_{\alpha\beta} \approx \frac{\lambda_\nu^{\alpha\beta}}{m_\phi^2} \lambda_f N_f$$

- Then the effective Hamiltonian for neutrino propagation is

$$H_{eff} = \frac{1}{2E_\nu} M_{eff}^\dagger M_{eff} + \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \sqrt{2} G_F N_f \begin{pmatrix} \epsilon_{ee}^V & \epsilon_{e\mu}^V & \epsilon_{e\tau}^V \\ \epsilon_{e\mu}^{V*} & \epsilon_{\mu\mu}^V & \epsilon_{\mu\tau}^V \\ \epsilon_{e\tau}^{V*} & \epsilon_{\mu\tau}^{V*} & \epsilon_{\tau\tau}^V \end{pmatrix}$$

$$M_{eff} = U^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger + \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}$$

- E.g., oscillation probability due with V-A + matter effects:

Lindner, Ota, Sato;
Kikuchi, Minakata, Uchinami

$$P_{\mu\mu} \simeq 1 - s^2_{2\times 23}\left[\sin^2\frac{\Delta m^2_{31}L}{4E}\right]$$

$$- |\epsilon^V_{\mu\tau}|\cos\phi^V_{\mu\tau}s_{2\times 23}\left[s^2_{2\times 23}(\sqrt{2}G_F N_e L)\sin\frac{\Delta m^2_{31}L}{2E} + 4c^2_{2\times 23}\frac{2\sqrt{2}G_F N_e E}{\Delta m^2_{31}}\sin^2\frac{\Delta m^2_{31}L}{4E}\right]$$

$$+ (|\epsilon^V_{\mu\mu}| - |\epsilon^V_{\tau\tau}|)s^2_{2\times 23}c_{2\times 23}\left[\frac{\sqrt{2}G_F N_e L}{2}\sin\frac{\Delta m^2_{31}L}{2E} - 2\frac{2\sqrt{2}G_F N_e E}{\Delta m^2_{31}}\sin^2\frac{\Delta m^2_{31}L}{4E}\right]$$

- Let θ_{23} ➡ $\theta_{23} + \delta\theta_{23}$ using ε_{ij} from perturbation of the scalar interactions (plus correction to masses)

$$P_{\mu\mu} \simeq 1 - s^2_{2\times 23}\left[\sin^2\frac{\Delta m^2_{31}L}{4E}\right]$$

$$-2\delta\theta_{23}\sin 4\theta_{23}\sin^2\frac{\Delta m^2_{31}L}{4E} - \frac{(m_3\delta m_3 - m_1\delta m_1)L}{2E}s^2_{2\times 23}\sin\frac{\Delta m^2_{31}L}{2E}$$

$$- |\epsilon^V_{\mu\tau}|\cos\phi^V_{\mu\tau}s_{2\times 23}\left[s^2_{2\times 23}(\sqrt{2}G_F N_e L)\sin\frac{\Delta m^2_{31}L}{2E} + 4c^2_{2\times 23}\frac{2\sqrt{2}G_F N_e E}{\Delta m^2_{31}}\sin^2\frac{\Delta m^2_{31}L}{4E}\right]$$

$$+ (|\epsilon^V_{\mu\mu}| - |\epsilon^V_{\tau\tau}|)s^2_{2\times 23}c_{2\times 23}\left[\frac{\sqrt{2}G_F N_e L}{2}\sin\frac{\Delta m^2_{31}L}{2E} - 2\frac{2\sqrt{2}G_F N_e E}{\Delta m^2_{31}}\sin^2\frac{\Delta m^2_{31}L}{4E}\right]$$

Summary

- Due to near degeneracy of m_1 and m_2, large corrections to θ_{12} are possible with small perturbations on M_ν
- Underlying (unperturbed) theory need not have θ_{12} close to the experimental value
- Bimaximal mixing or models on the dark side are possible
- Complex perturbations can give any value for CP phases
- Small perturbations to charged lepton mass matrix can give large corrections to all three PMNS angles.