

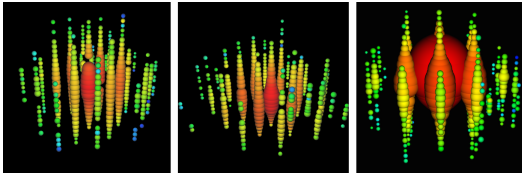
Flavor Ratios of Extragalactic Neutrinos and Neutrino
Shortcuts in Extra Dimensions
arXiv:1410.0408

Content

- 1 Introduction
- 2 Model
- 3 Constant Effective Potential
- 4 Adiabatic Limit
- 5 Resulting Constraints
- 6 Conclusion & Outlook

High Energy Neutrinos at IceCube

36 neutrino events detected by IceCube in the high energy region 30 TeV up to 2 PeV



Ernie: 1.0 PeV

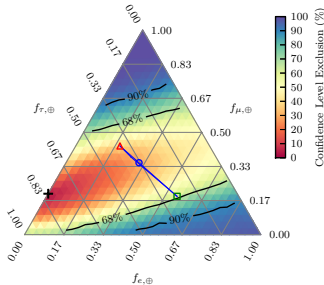
Bert: 1.1 PeV

Big Bird: 2.0 PeV

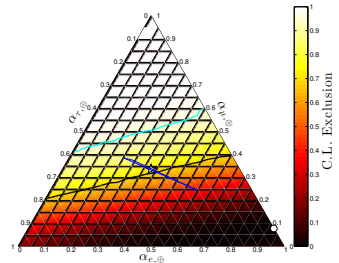


Flavor Composition ($\Phi_e : \Phi_\mu : \Phi_\tau$)

Deviation from canonical (1:1:1) ratio?!



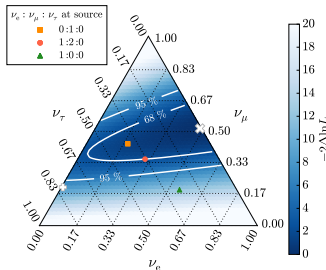
[IceCube, 2015]
35 TeV - 1.9 PeV, Best Fit: (0:0.2:0.8)



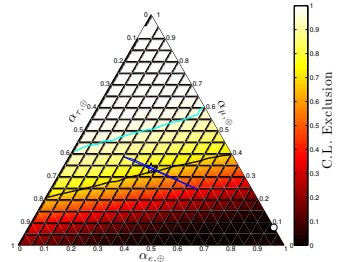
[Palomares-Ruiz, Vincent, Mena, 2015]
28 TeV - 3 PeV, Best Fit: (0.92:0.08:0)

Flavor Composition ($\Phi_e : \Phi_\mu : \Phi_\tau$)

Deviation from canonical (1:1:1) ratio?!



[IceCube, 2015]
25 TeV - 2.8 PeV, Best Fit: (0.49:0.51:0)



[Palomares-Ruiz, Vincent, Mena, 2015]
28 TeV - 3 PeV, Best Fit: (0.92:0.08:0)

Possible Explanation

- Results still compatible with expected canonical ratio!
- Wait for higher statistic to either confirm canonical ratio, or open the door to new physics
- no Glashow events at 6.3 PeV?
(resonant production of $\bar{\nu}_e e \rightarrow W$)
- several new physics models proposed
- idea in this talk: modification due to altered dispersion relation
→ shortcuts in extradimensions

How to change the expected flavor ratio?

- Astrophysical Origin of neutrino $\Rightarrow$ oscillations will be averaged out:

$$P_{\alpha\beta} = P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \delta_{\alpha\beta} - 4 \sum_{k>j} (U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*) \sin^2 \left(\frac{\delta m_{kj}^2 L}{4E} \right)$$

$$\rightarrow \sum_j |U_{\alpha k}|^2 |U_{\beta k}|^2 \quad \text{for } L \gg \frac{\delta m^2}{E}$$

- lead to canonical Flavor ratio of (1:1:1) in case of initial pion source
- change effective mixing angles in $U_{\alpha k}$ for change of flavor ratio

$$\Phi_\alpha = \sum_\beta P_{\alpha\beta} \Phi_\beta^{init} \quad \text{with initial flavor composition } \Phi_\beta^{init}$$

How to change the expected flavor ratio?

- Vacuum:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^0 |\nu_i\rangle \quad H_{mass}^0 = \frac{m_i}{2E} \quad H_{flavor}^0 = U^0 H_{mass} (U^0)^\dagger$$

- U^0 diagonalizes the Hamiltonian in flavor space (rotates from flavor space to mass eigenspace)
- Introduce flavor-specific potential

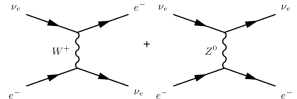
$$H_{flavor}^0 = U^0 H_{mass} (U^0)^\dagger + \textcolor{red}{V}$$

- leads in general to different effective rotation matrix $U \Rightarrow$ rotates from flavor space to propagation eigenspace

$\Rightarrow$ Different effective masses and different effective mixing angles!

Example: Neutrinos propagating through 1st generation matter (electrons)

$$V = 2 \frac{1}{\sqrt{2}} G_F n_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Problem: No Standard-Model Mechanism for neutrinos in the interstellar spacetime

- No matter
- neutrinos uncharged
- gravity: not flavor specific

⇒ Idea: Introducing **sterile** neutrino with potential caused by **spacetime**

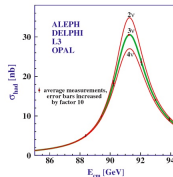
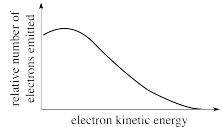
Side Note: What is a sterile neutrino?

- postulation of neutrino to explain spectrum of beta decay

"I have done a terrible thing, I have postulated a particle that cannot be detected" - Wolfgang Pauli, 1930

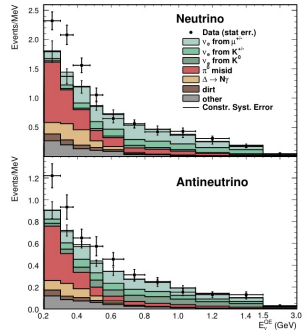
- neutrinos are detectable due to weak interaction

- number of neutrino species determined by Z-resonance



Side Note: What is a sterile neutrino?

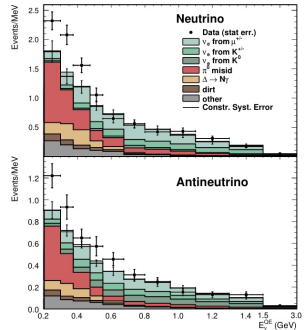
- frequency in neutrino oscillation is determined by $\delta m^2 \rightarrow$ only 2 oscillation expected
- MiniBoone, LSND anomalies: new oscillation frequency with $\delta m^2 \sim 1 \text{ eV}^2$ discovered?
- Gallium and Reactor anomalies: reduced active neutrino flux
- new neutrino can not interact weakly $\rightarrow$ sterile neutrino
- sterile neutrino: fermion, SM-singlet
- but mixes with SM neutrinos
- Jason Kumar: "half of the spectrum in String Theory is a sterile neutrino"



[MiniBooNE Collaboration, 2012]

Side Note: What is a sterile neutrino?

- frequency in neutrino oscillation is determined by $\delta m^2 \rightarrow$ only 2 oscillation expected
- MiniBoone, LSND anomalies: new oscillation frequency with $\delta m^2 \sim 1 \text{ eV}^2$ discovered?
- Gallium and Reactor anomalies: reduced active neutrino flux
- new neutrino can not interact weakly $\rightarrow$ sterile neutrino
- sterile neutrino: fermion, SM-singlet
- **but mixes with SM neutrinos**
- Jason Kumar: "half of the spectrum in String Theory is a sterile neutrino"



[MiniBooNE Collaboration, 2012]

Model

- Introducing a sterile neutrino
- vacuum mixing angle θ to mix flavor and mass eigenstates
- expand space time to an extra spatial dimension u with a non-trivial warping $\eta(u)$

$$ds^2 = dt^2 - \sum_{i=1}^3 \eta^2(u) (dx^i)^2 - du^2$$

- motivated by brane-world scenarios: Let sterile neutrino propagate in 4+1 dimensions
- original motivation: super luminal neutrinos at opera η can lead to shortcuts for the sterile neutrino

New Mixing

- Shortcut through extra dimension $\rightarrow$ time shift $\delta t \rightarrow$ difference in the action

$$\Delta S \sim \mathbf{H} \delta t \Rightarrow \Delta \mathbf{H}_{\text{eff}} = \mathbf{H} \frac{\delta t}{T} \sim \epsilon E$$

- shortcut-parameter [Päs, Pakvasa, Weiler, 2005]: $\epsilon = 1 - \eta(u)$
- new effective Hamiltonian:

$$\mathbf{H} = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & \epsilon E \end{pmatrix}$$

- New effective Mixing-Angle:

$$\cos 2\tilde{\theta} = \frac{\cos 2\theta - \frac{2E^2}{\delta m^2} \epsilon}{\sqrt{\left(\frac{2E^2}{\delta m^2} \epsilon - \cos 2\theta\right)^2 + \sin^2(2\theta)}}$$

Modification due to constant Potential

- introducing the resonance energy

$$E_{Res} = \sqrt{\frac{\delta m^2 \cos 2\theta}{2\epsilon}}$$

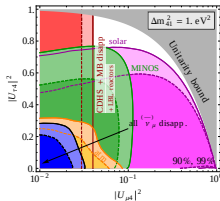
- new effective mixing angle

$$\sin^2 2\tilde{\theta} = \frac{\sin^2 2\theta}{\sin^2 2\theta + \cos^2 2\theta \left[1 - \left(\frac{E}{E_{Res}} \right)^2 \right]^2},$$

- leading to now energy depending mixing matrix $U(E)$ and therefore energy depending flavor Ratios

$$\Phi_\alpha(E) = \sum_\beta P_{\alpha\beta}(E) \Phi_\beta^{init} \neq \Phi_\alpha^0$$

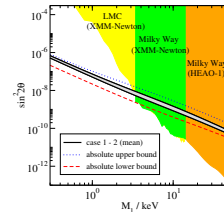
Two models of sterile neutrino [Kopp et al., 2013]



Light Sterile Neutrino (LSN)

- motivated by LSND, MiniBoone and Gallium anomalies
- $\delta m^2 \simeq 1 \text{ eV}^2$
- $\sin^2(2\theta) \simeq 0.12$, $(\sin^2(\theta) \simeq 0.03)$

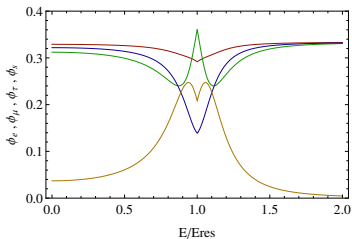
[Asaka, et al., 2007]



Neutrino Minimal Standard Model (ν MSM)

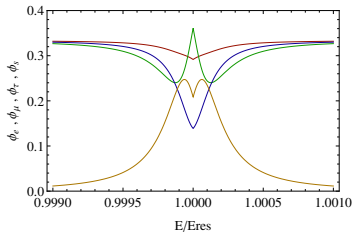
- sterile neutrino as warm dark matter
- $\delta m^2 \simeq 1(\text{keV})^2$
- $\sin^2(2\theta) \simeq 10^{-7}$, $(\sin^2(\theta) \simeq 10^{-8})$

$\Phi_e, \Phi_\mu, \Phi_\tau, \Phi_s$



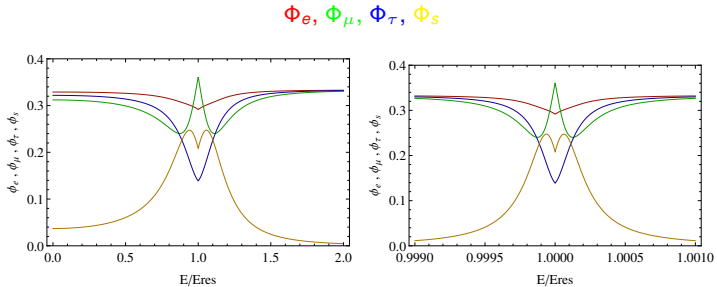
LSN

$$(\delta m^2 \simeq 1 \text{eV}^2, \quad \sin^2(2\theta) \simeq 0.12)$$



ν MSM

$$(\delta m^2 \simeq 1(\text{keV})^2, \quad \sin^2(2\theta) \simeq 10^{-7})$$



$$\frac{\delta E(\text{FWHM})}{E_{\text{Res}}} \simeq 2\theta$$

$$\frac{\delta E(\text{FWHM})}{E_{\text{Res}}} \simeq 35\%$$

$$\frac{\delta E(\text{FWHM})}{E_{\text{Res}}} \simeq 3 \cdot 10^{-2}\%$$

Flavor Ratios at Resonance

source	Φ_β^0	mixing	$\Phi_\beta(\theta^{\alpha 4})$	R	S	T	G
Pion	1:2:0:0	none ($\theta^{\alpha 4} = 0$)	1:1:1:0	1/2	1	1/3	1/9
		$\nu_e - \nu_4$	4:11:11:6	11/19	8/11	11/30	4/45
		$\nu_\mu - \nu_4$	5:5:5:3	1/2	1	1/3	1/9
		$(\nu_e, \nu_\mu) - \nu_4$	32:41:41:30	41/73	32/41	41/114	16/171
		$(\nu_\mu, \nu_\tau) - \nu_4$	21:26:10:15	26/31	21/10	26/57	7/57
Damped Muon	0:1:0:0	none ($\theta^{\alpha 4} = 0$)	4:7:7:0	7/11	4/7	7/18	2/27
		$\nu_e - \nu_4$	4:9:9:2	9/13	4/9	9/22	2/33
		$\nu_\mu - \nu_4$	1:2:2:1	2/3	1/2	2/5	1/15
		$(\nu_e, \nu_\mu) - \nu_4$	16:115:115:42	115/131	16/115	115/246	8/369
		$(\nu_\mu, \nu_\tau) - \nu_4$	7:16:4:9	16/11	7/4	16/27	7/81
Neutron Beam	1:0:0:0	none ($\theta^{\alpha 4} = 0$)	5:2:2:0	2/7	5/2	2/9	5/27
		$\nu_e - \nu_4$	2:1:1:2	1/3	2	1/4	1/2
		$\nu_\mu - \nu_4$	3:1:1:1	1/4	3	1/5	3/5
		$(\nu_e, \nu_\mu) - \nu_4$	10:1:1:6	1/11	10	1/12	5/12
		$(\nu_\mu, \nu_\tau) - \nu_4$	35:14:14:9	2/7	5/2	2/9	5/9

Constant Effective Potential - Summary

- substantial changes in flavor ratio possible due to shortcuts in extradimension
- significant effect at resonance Energy $E_{Res} = \sqrt{\frac{\delta m^2 \cos 2\theta}{2\epsilon}}$
- resonance width $\propto \theta$, therefore “large” vacuum mixing needed
- resonant behavior should be seen in future spectral analysis

Varying Effective Potential

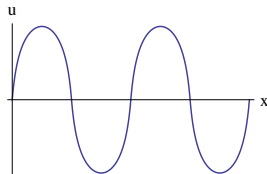
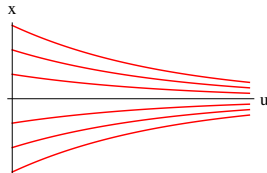
- Geodesics with non constant ϵ possible
- As a proof of concept we use an asymmetrically warped space time

$$ds^2 = dt^2 - e^{-2k|u|} dx^2 - du^2$$

- leads to geodesic for sterile neutrino [Hollenberg, Micu, Päs, 2009]

$$u(x) = \pm \frac{1}{2k} \ln[1 + k^2 x(l - x)]$$

- two free parameter: warp factor k , periodic length l



Varying Effective Potential

- new effective masses

$$\tilde{m}_i^2 = -\frac{A}{2} \pm \frac{\delta m^2}{2} \sqrt{\left(\frac{A}{\delta m^2} - \cos 2\theta\right)^2 + \sin^2(2\theta)}$$

with $A = 2E^2 \left(1 - \frac{1}{\sqrt{1+k^2 x(l-x)}}\right)$

- new effective mixing angles $\tilde{\theta}$

$$\cos 2\tilde{\theta} = \frac{\cos 2\theta - \frac{2E^2}{\delta m^2} \left(1 - \frac{1}{\sqrt{1+k^2 x(l-x)}}\right)}{\sqrt{\left(\frac{2E^2}{\delta m^2} \left(1 - \frac{1}{\sqrt{1+k^2 x(l-x)}}\right) - \cos 2\theta\right)^2 + \sin^2(2\theta)}}$$

⇒ Effective masses and mixing changing during propagation process x
comparable to MSW-Effect

A Side Note: The MSW-Effect [Wolfenstein, 1978], [Barger, Whisnant, Pakvasa, 1980], [Mikheyev, Smirnov, 1986]

- Problem: For Solar neutrinos P_{ee} is different for low and high energies.
- Explanation: Matter effects become dominant for higher energies:

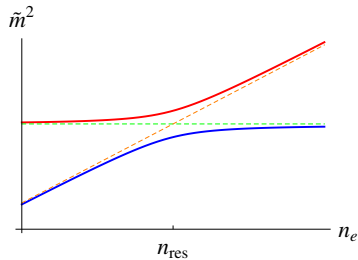
$$H_{\text{flavor}}^0 = U^0 \frac{1}{2E} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} (U^0)^\dagger + 2 \frac{1}{\sqrt{2}} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

- due to charged current interaction possible with $\bar{\nu}_e$.
- Note: Electron density n_e changes (slowly!) during propagation in the sun from core to surface

⇒ Varying effective potential in the adiabatic regime

A Side Note: The MSW-Effect [Wolfenstein, 1978], [Barger, Whisnant, Pakvasa, 1980], [Mikheyev, Smirnov, 1986]

- $\bar{\nu}_e$ is produced in high density-region $\rightarrow$ electron neutrino is nearly complete in heavier effective mass eigenstate $\bar{\nu}_e \approx \bar{\nu}_2$
- adiabatic limit: potential changes slowly enough to have no transitions between effective mass eigenstates
- stays in heavier mass eigenstates until surface
- in vacuum the fraction of $\bar{\nu}_e$ in $\bar{\nu}_2$ is significantly lower ($\theta \simeq 34^\circ$)



Adiabatic Limit

- Want to apply MSW effect for our extradimensional scenario
- condition for adiabatic behaviour (changing slow enough): $\tau_{\text{system}} \ll \tau_{\text{interaction}}$ leads to

$$\gamma_{\text{res}} = \frac{4E^3}{(\delta m^2)^2 \sin^2(2\theta)} \left. \frac{d\epsilon}{dx} \right|_{\text{res}} \ll 1$$

- if condition is satisfied at resonance, it is satisfied everywhere
- approximative condition for geometric parameters $k^2 l$:

$$\frac{2E^3}{(\delta m^2)^2 \sin^2(2\theta)} k^2 l \ll 1$$

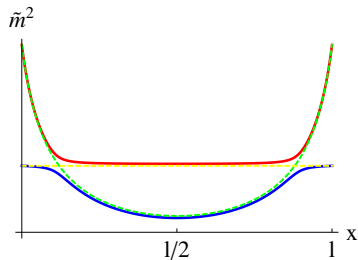
- lower bound for $k^2 l$

Effective Masses

$$\tilde{m}_i^2 = -\frac{A}{2} \pm \frac{\delta m^2}{2} \sqrt{\left(\frac{A}{\delta m^2} - \cos 2\theta\right)^2 + \sin^2(2\theta)}$$

with $A = 2E^2 \left(1 - \frac{1}{\sqrt{1 + k^2 x(l-x)}}\right)$

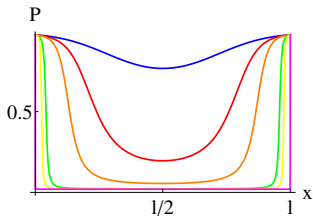
- produced active neutrino propagates as lighter neutrino
- converges to sterile neutrino at $x = \frac{l}{2}$ and back to active neutrino
- repetition after $x > l$



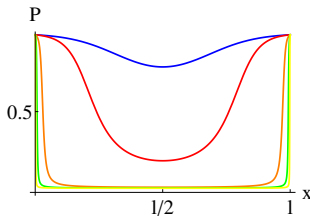
Survival Probability

$$P_{\nu_a \rightarrow \nu_a}(x) = \cos^2 \theta \cdot \cos^2 \tilde{\theta} + \sin^2 \theta \cdot \sin^2 \tilde{\theta}$$

$$P_{\nu_a \rightarrow \nu_s}(x) = \sin^2 \theta \cdot \cos^2 \tilde{\theta} + \cos^2 \theta \cdot \sin^2 \tilde{\theta}$$



Increasing E from blue to purple



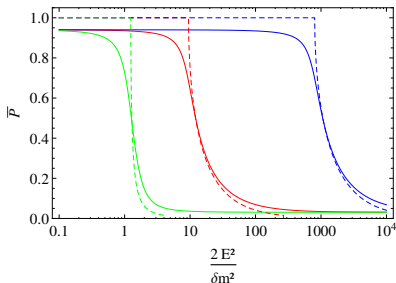
Increasing k from blue to yellow

Survival Probability

- average over periodic length

$$\bar{P} = \frac{1}{l} \int_0^l P_{\nu_a \rightarrow \nu_a}(x) dx$$

- depending on kl
- to lower transition probability, a larger kl is needed $\Rightarrow$ upper bound for kl
- blue: $kl = 0.1$, red: $kl = 1$, green: $kl = 10$



since $\bar{P} \rightarrow 0$ the neutrino will be undetectable (sterile state)

Expanding to Three Active Generations

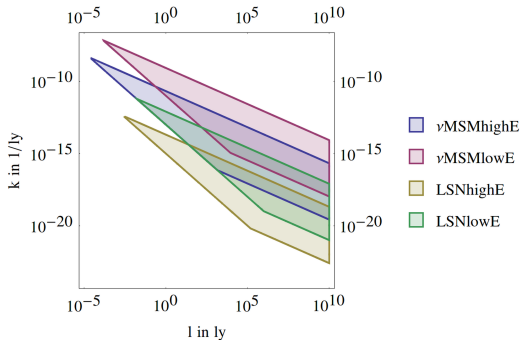
- mechanism provides transformation of mass eigenstates into (almost) purely sterile states
- coupling 2 sterile states each to two neutrino mass eigenstates and not to ν_i
 $\Rightarrow$ flavor ratio will be $(U_{ei}^2 : U_{\mu i}^2 : U_{\tau i}^2)$
- possible flavor ratios $\sim (4:1:1), (0:1:1), (1:1:1)$

Occurrence of this effect

- adiabatic condition $\rightarrow$ small $k^2 l$
- substantial decrease admixture of corresponding mass eigenstates:
 $\bar{P} < 0.25 \rightarrow$ big $k l$
- solar neutrinos should not be affected $\rightarrow$ position of resonance occur at distances higher than earth-sun
- apply again to sterile neutrino models ν MSM and light sterile neutrino
- leading to viable parameter space for geometric parameters k and l

Constraints for geometric parameters

Model	δm^2	$\sin^2(2\theta)$	E	kl	$k^2 l/\text{eV}$
LSN	$(1\text{eV})^2$	0.12	30 TeV	$\gtrsim 10^{-13}$	$\ll 2.2 \cdot 10^{-42}$
LSN	$(1\text{eV})^2$	0.12	1.2 PeV	$\gtrsim 10^{-15}$	$\ll 3.5 \cdot 10^{-47}$
νMSM	$(1\text{keV})^2$	10^{-7}	30 TeV	$\gtrsim 10^{-11}$	$\ll 5.5 \cdot 10^{-29}$
νMSM	$(1\text{keV})^2$	10^{-7}	1.2 PeV	$\gtrsim 10^{-13}$	$\ll 3.5 \cdot 10^{-32}$



Conclusion & Outlook

- measurement of high energy flavor ratios can be a probe for non-standard neutrino properties
- new effective potentials due to new sterile neutrino physics
- provide non standard energy dependence
- adiabatic conversion in asymmetric warped space time could explain a ratio of 4:1:1
- further experimental results needed

Conclusion & Outlook

- measurement of high energy flavor ratios can be a probe for non-standard neutrino properties
- new effective potentials due to new sterile neutrino physics
- provide non standard energy dependence
- adiabatic conversion in asymmetric warped space time could explain a ratio of 4:1:1
- further experimental results needed

⇒ Look at exotic phenomenology,
the time has come

Backup

Model	δm^2	$\sin^2(2\theta)$	E	kl	$k^2 l/\text{eV}$	$\frac{\delta m^2 \cos 2\theta}{E^2}$
LSN	$(1\text{eV})^2$	0.12	30 TeV	$\gtrsim 10^{-13}$	$\ll 2.2 \cdot 10^{-42}$	10^{-27}
LSN	$(1\text{eV})^2$	0.12	1.2 PeV	$\gtrsim 10^{-15}$	$\ll 3.5 \cdot 10^{-47}$	$6.5 \cdot 10^{-31}$
νMSM	$(1\text{keV})^2$	10^{-7}	30 TeV	$\gtrsim 10^{-11}$	$\ll 5.5 \cdot 10^{-29}$	$1.1 \cdot 10^{-21}$
νMSM	$(1\text{keV})^2$	10^{-7}	1.2 PeV	$\gtrsim 10^{-13}$	$\ll 3.5 \cdot 10^{-32}$	$6.9 \cdot 10^{-25}$

Backup

$$10^6 \text{ly} = 5 \cdot 10^{25} \frac{1}{\text{eV}} > x_{\text{res}} > 8 \cdot 10^{17} \frac{1}{\text{eV}} = 1.6 \cdot 10^{-5} \text{ly} \quad (1)$$

leads to

$$\frac{1}{64} 10^{-17} \frac{1}{\text{eV}} \frac{\delta m^2 \cos 2\theta}{2E^2} > k^2 l > \frac{1}{40} 10^{-25} \frac{1}{\text{eV}} \frac{\delta m^2 \cos 2\theta}{.} 2E^2 \quad (2)$$

Backup

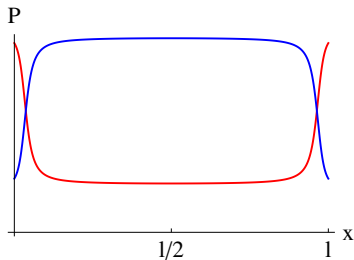


Figure : Appearance- and disappearance-probability

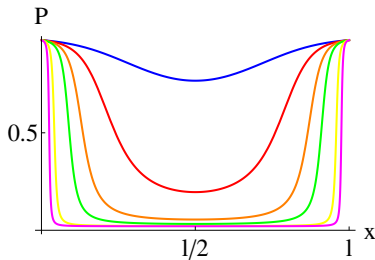


Figure : Appearance-probability for different energies

Backup

Effective masses

$$\tilde{m}_i^2 = -\frac{A}{2} \pm \frac{\delta m^2}{2} \sqrt{\left(\frac{A}{\delta m^2} - \cos 2\theta\right)^2 + \sin^2(2\theta)}$$

with $A=2E^2\left(1 - \frac{1}{\sqrt{1+k^2x(1-x)}}\right)$

Backup

