Strange Nonchaotic Stars

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Collaboration

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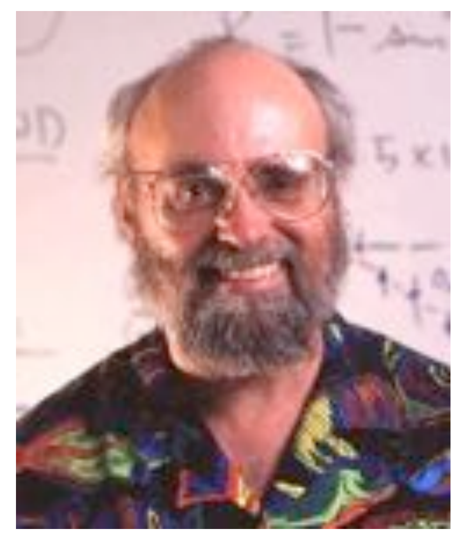

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The Elevator Talk

Chaotic road to stellar conclusions

We’ve applied
the techniques of nonlinear dynamics
to Kepler space telescope data
to find evidence of a kind of dynamics
between order and disorder
— fractal but not chaotic —
in the fine features of pulsating stars

Outline

Dynamical Attractors

Spectral Scaling

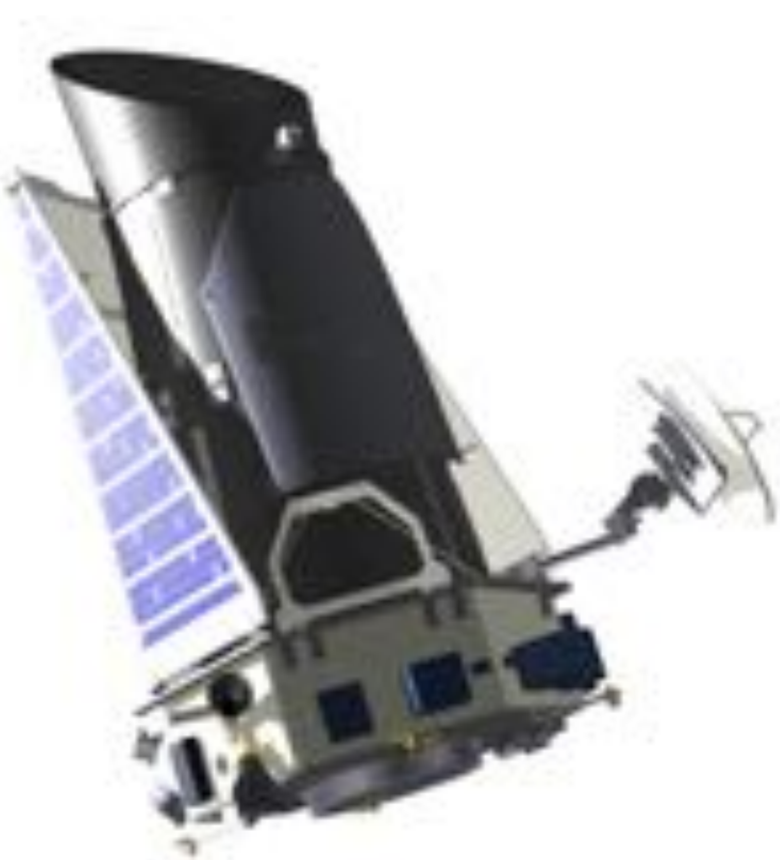
Golden Ratio

Variable Stars

Analysis

Toy Models

Conclusions



Outline

Dynamical Attractors

Spectral Scaling

Golden Ratio

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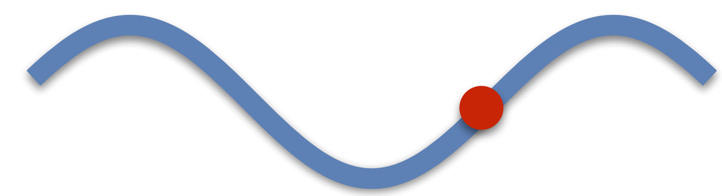
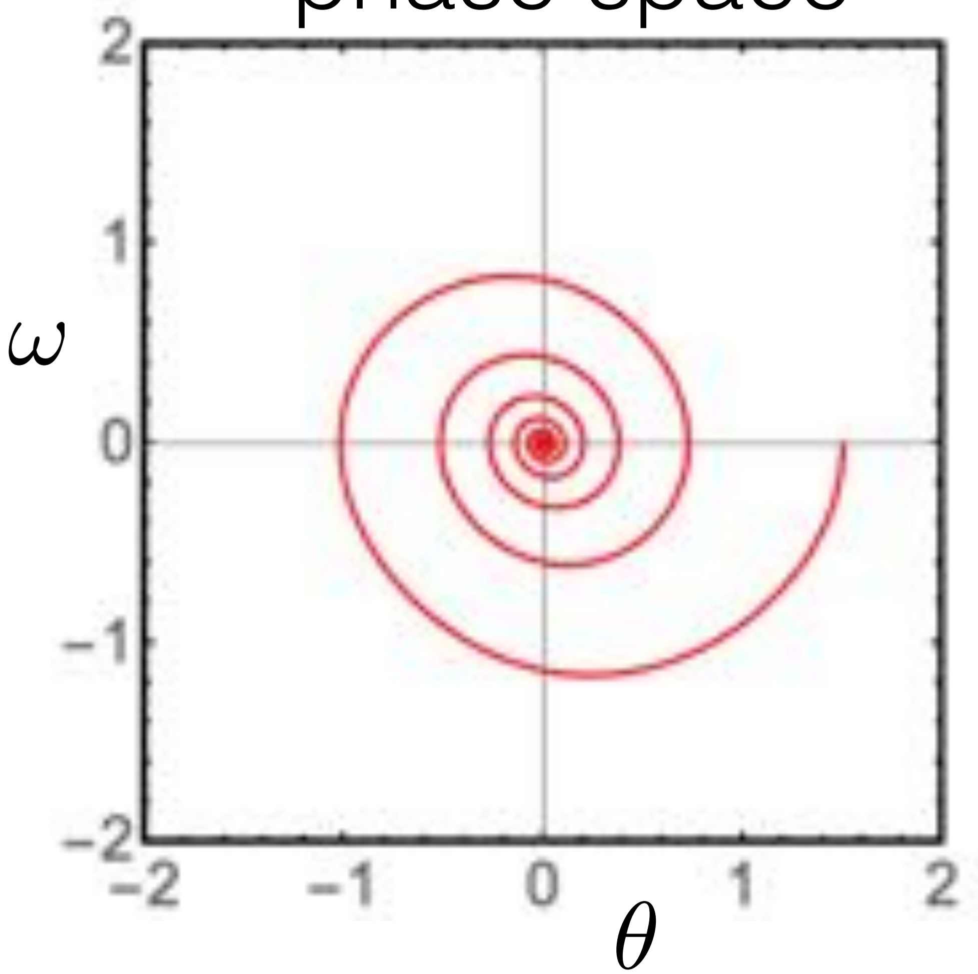
Point Attractor

Damped Pendulum

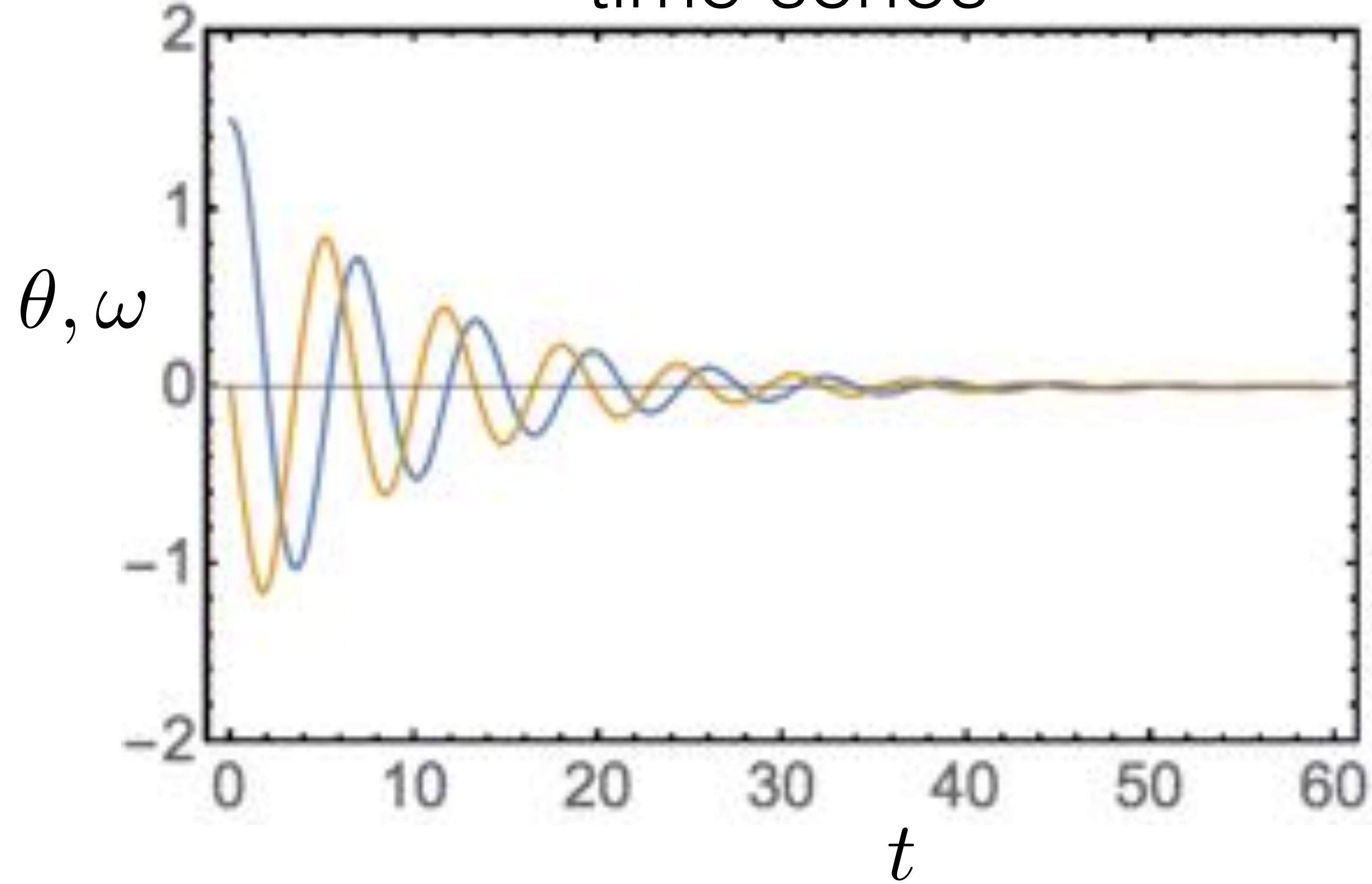
$$\dot{\theta} = \omega$$
$$\dot{\omega} + \gamma\omega = A \sin\theta$$



phase space



time series



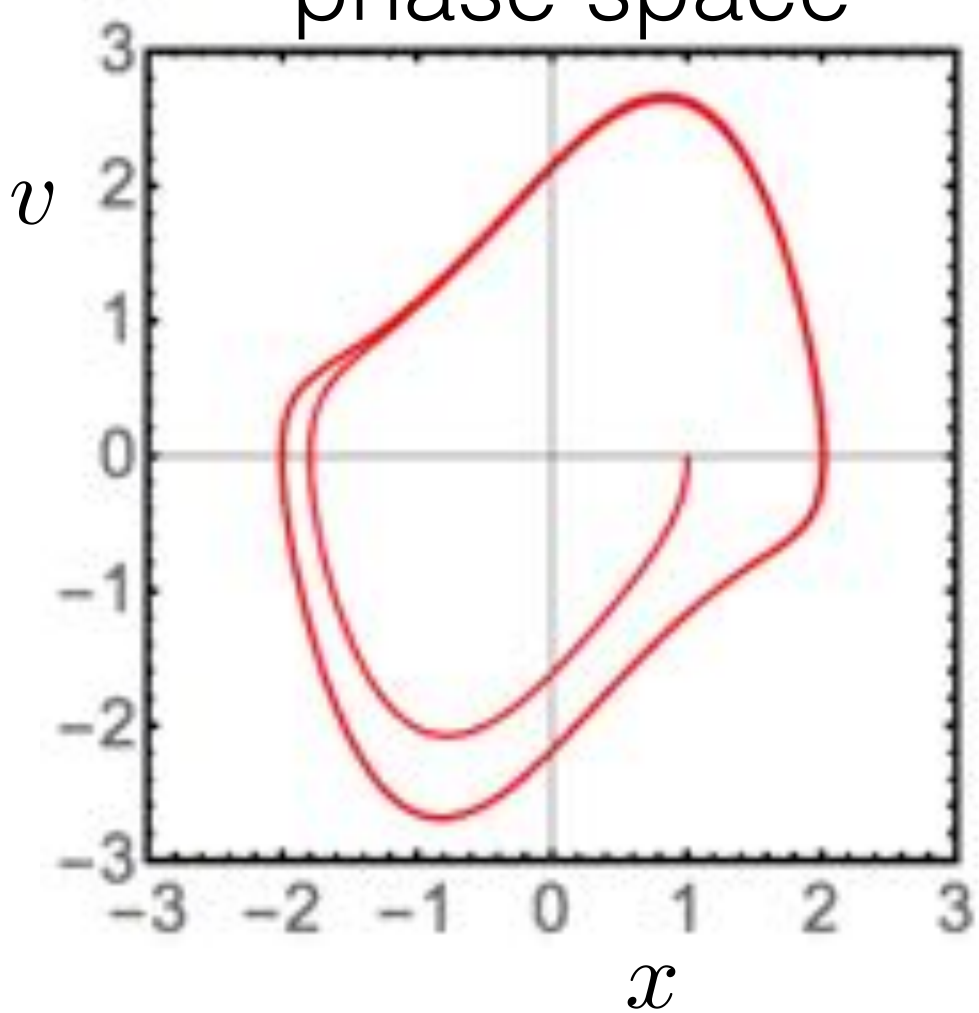
Limit Cycle Attractor

Van der Pol Oscillator has nonlinear viscosity

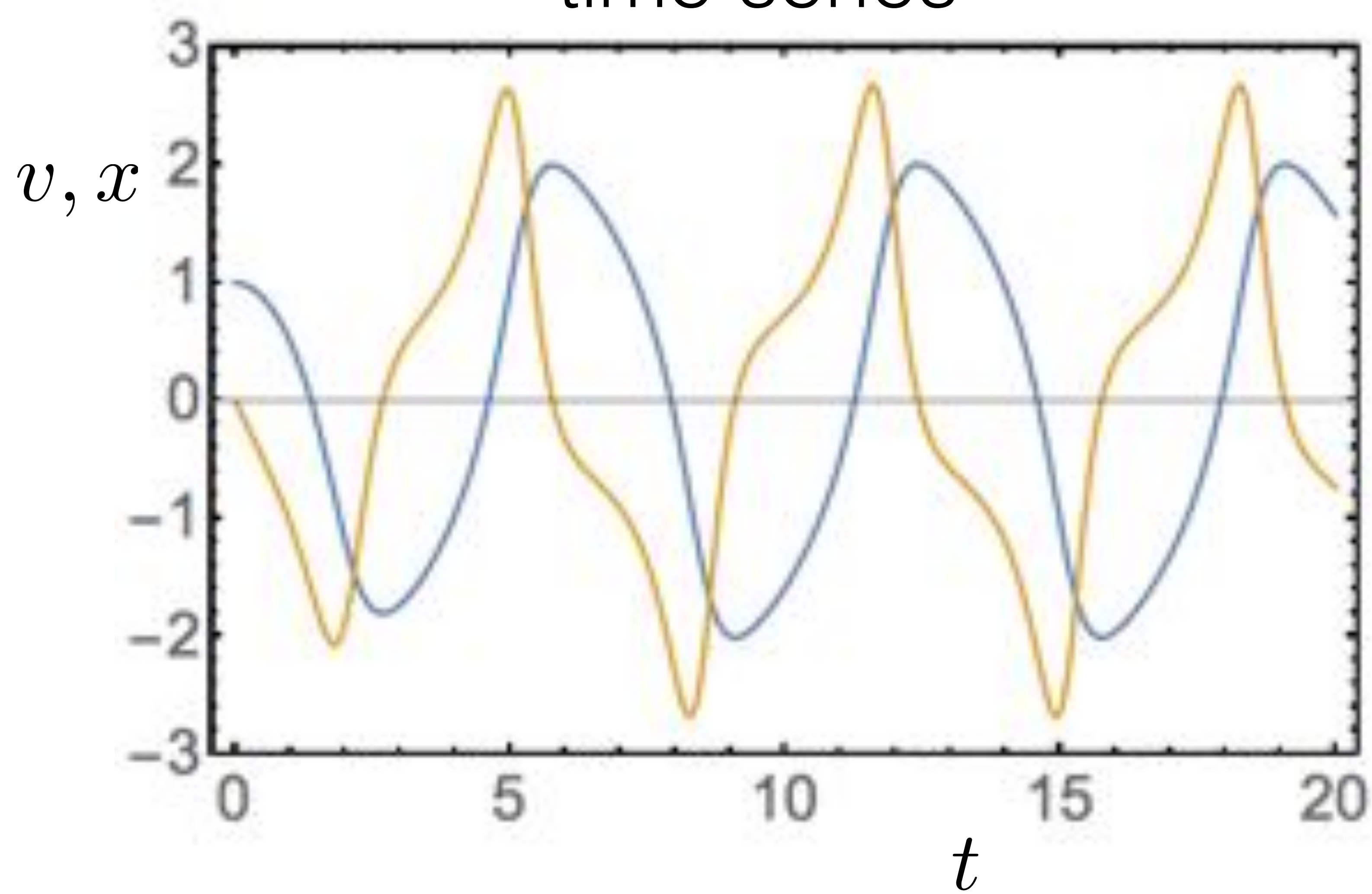
$$\dot{x} = v$$

$$\dot{v} + \gamma(x^2 - 1)v = -x$$

phase space



time series

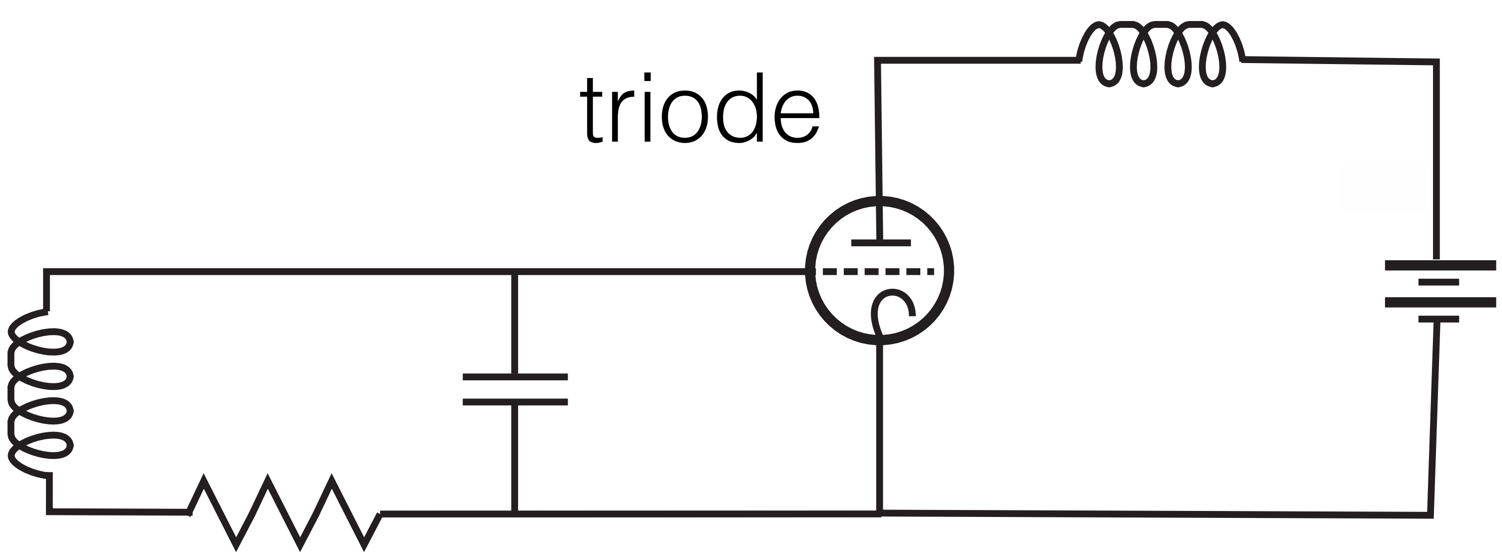


Limit Cycle Attractor

Van der Pol Oscillator models heart

$$\dot{Q} = I$$

$$\dot{I} + \gamma(Q^2 - 1)I = -Q$$



Barbara Van Riper

Limit Cycle Attractor

Van der Pol Oscillator models heart

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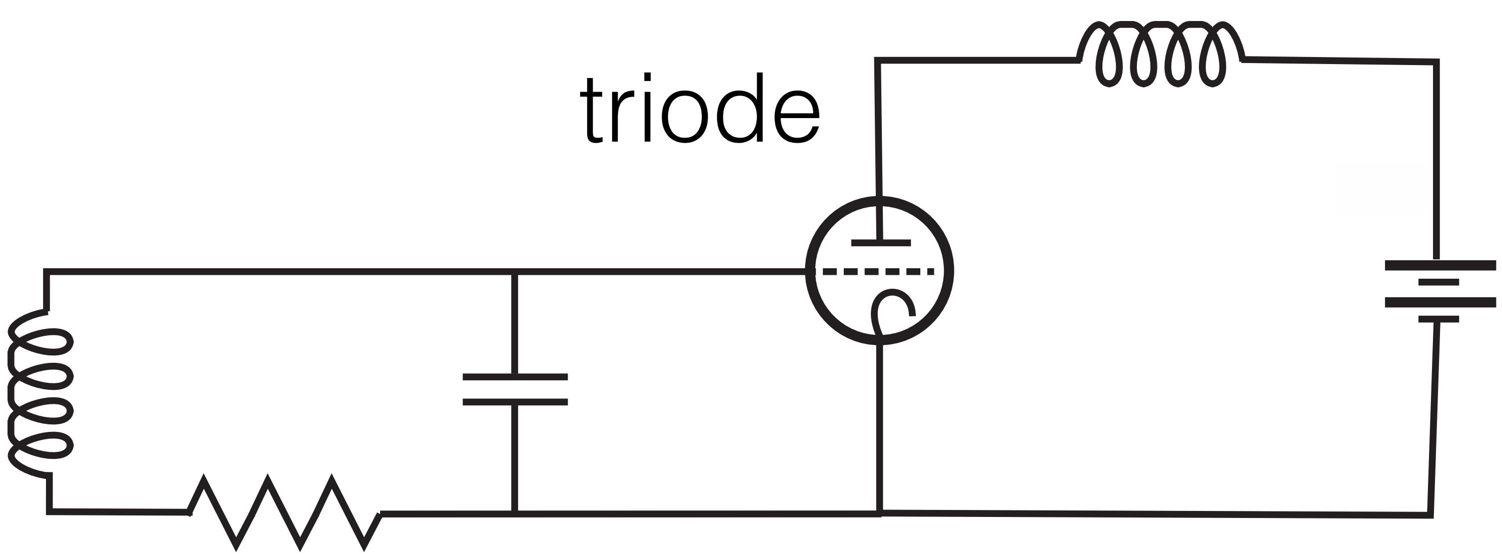
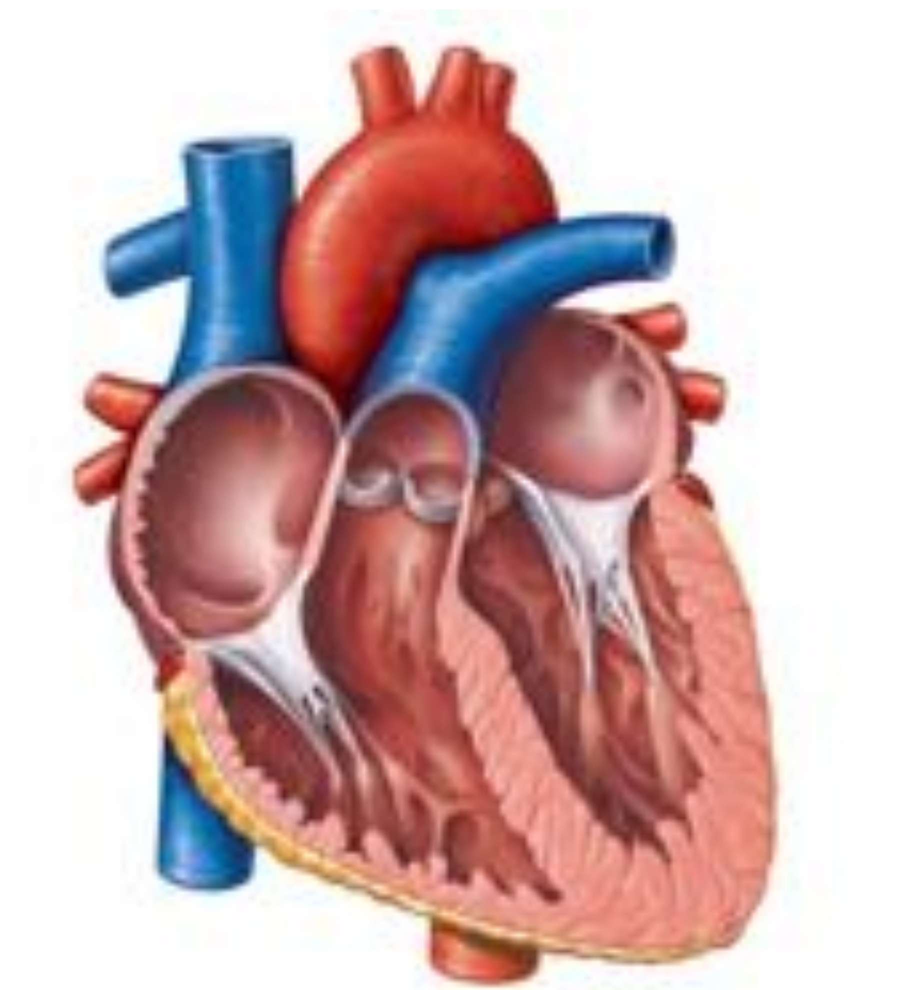




Barbara Van Riper

+ sun spot cycle + MSW neutrino oscillations in matter

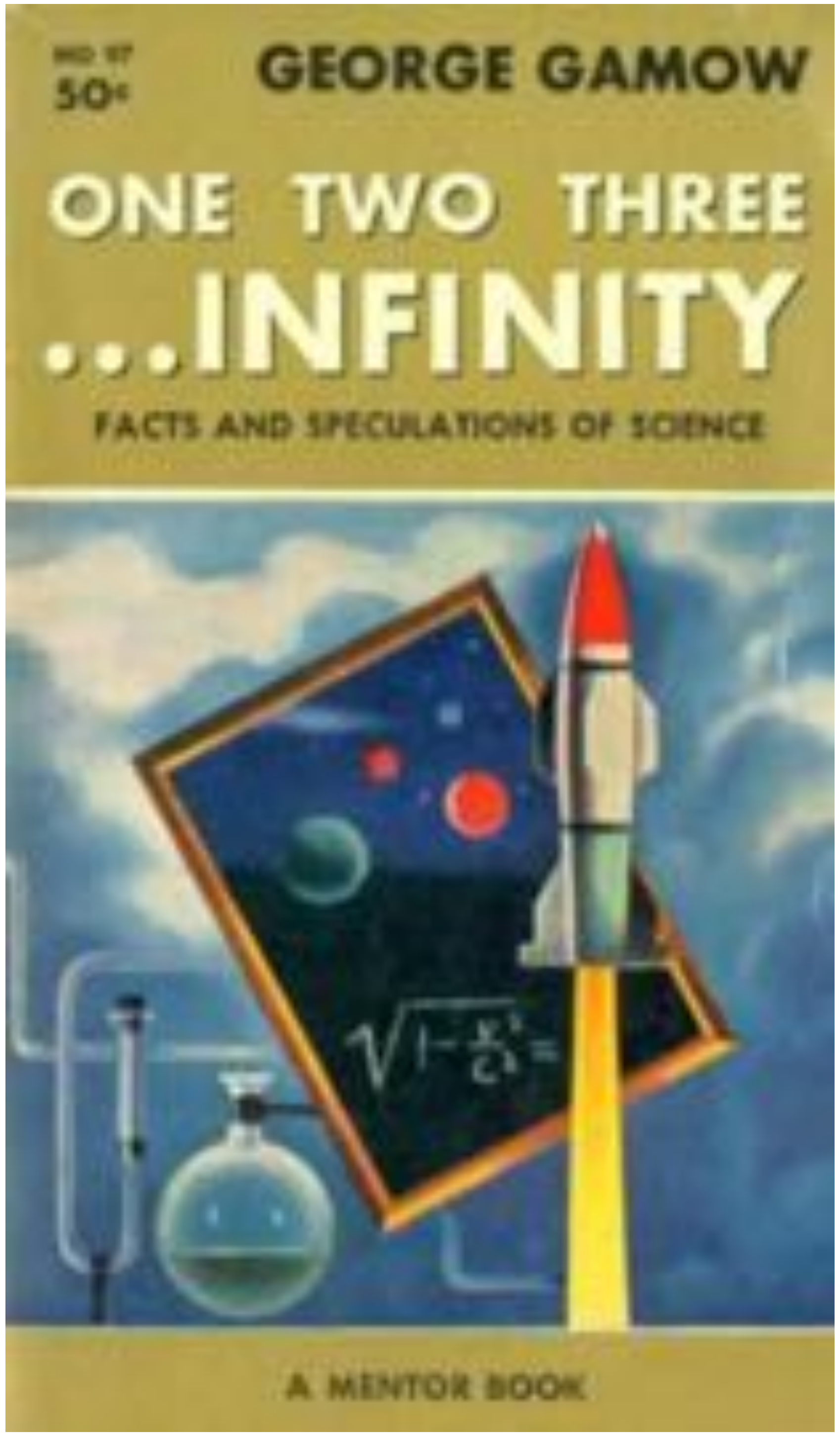
Poincaré-Bendixson

dim ≤ 2, attractor is either fixed point or cycle

+ combinations

Strange = Fractal Attractors

dim ≥ 3, enables infinitely complicated attractors



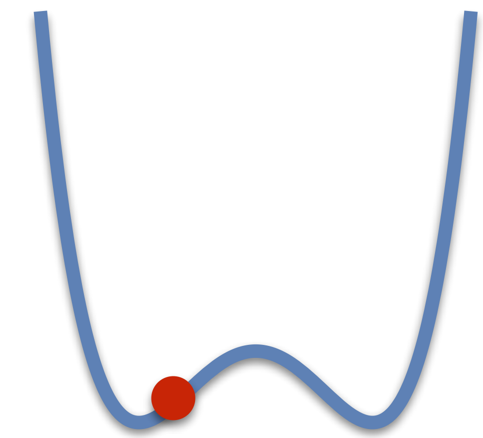
Strange Chaotic Attractor

Forced damped Duffing nonlinear spring

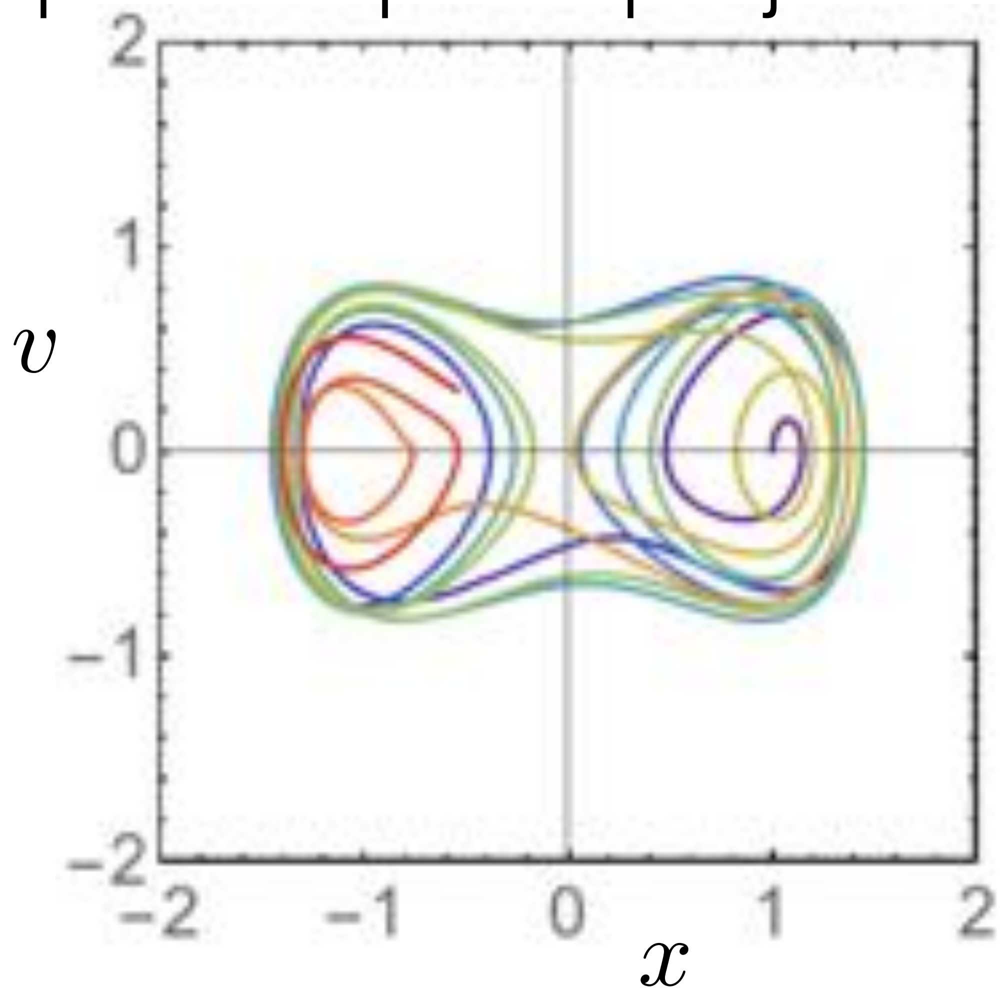
$$\dot{x} = v$$

$$\dot{v} + \gamma v = x - x^3 + A\cos\theta$$

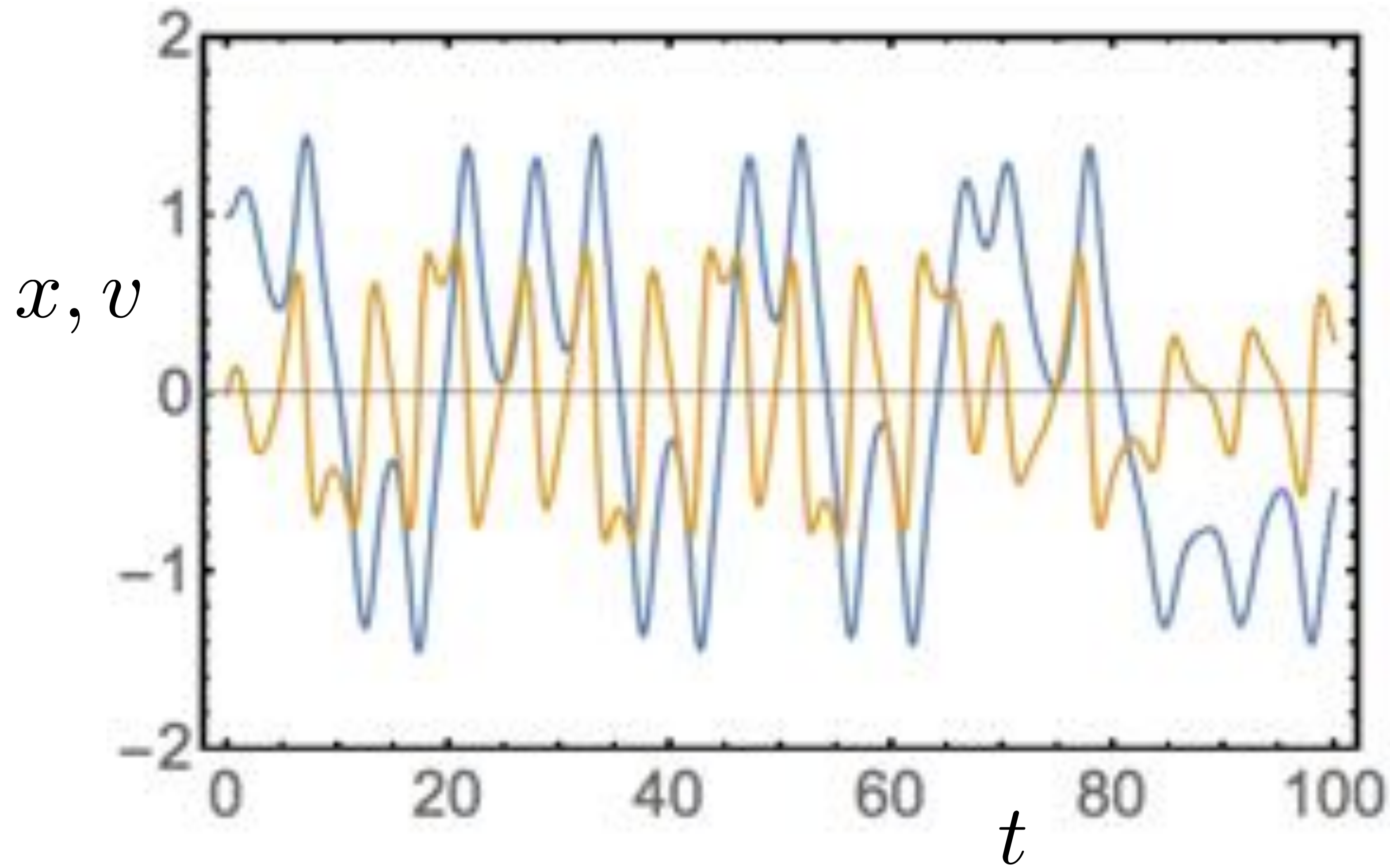
$$\dot{\theta} = \omega$$



phase space projection



time series



Strange Chaotic Attractor

Forced damped Duffing nonlinear spring

$\{x \sin \theta, x \cos \theta, v\}$

$\{x, v\}$ at θ

Poincaré section

Strange Chaotic Attractor

Spacetime Characterization

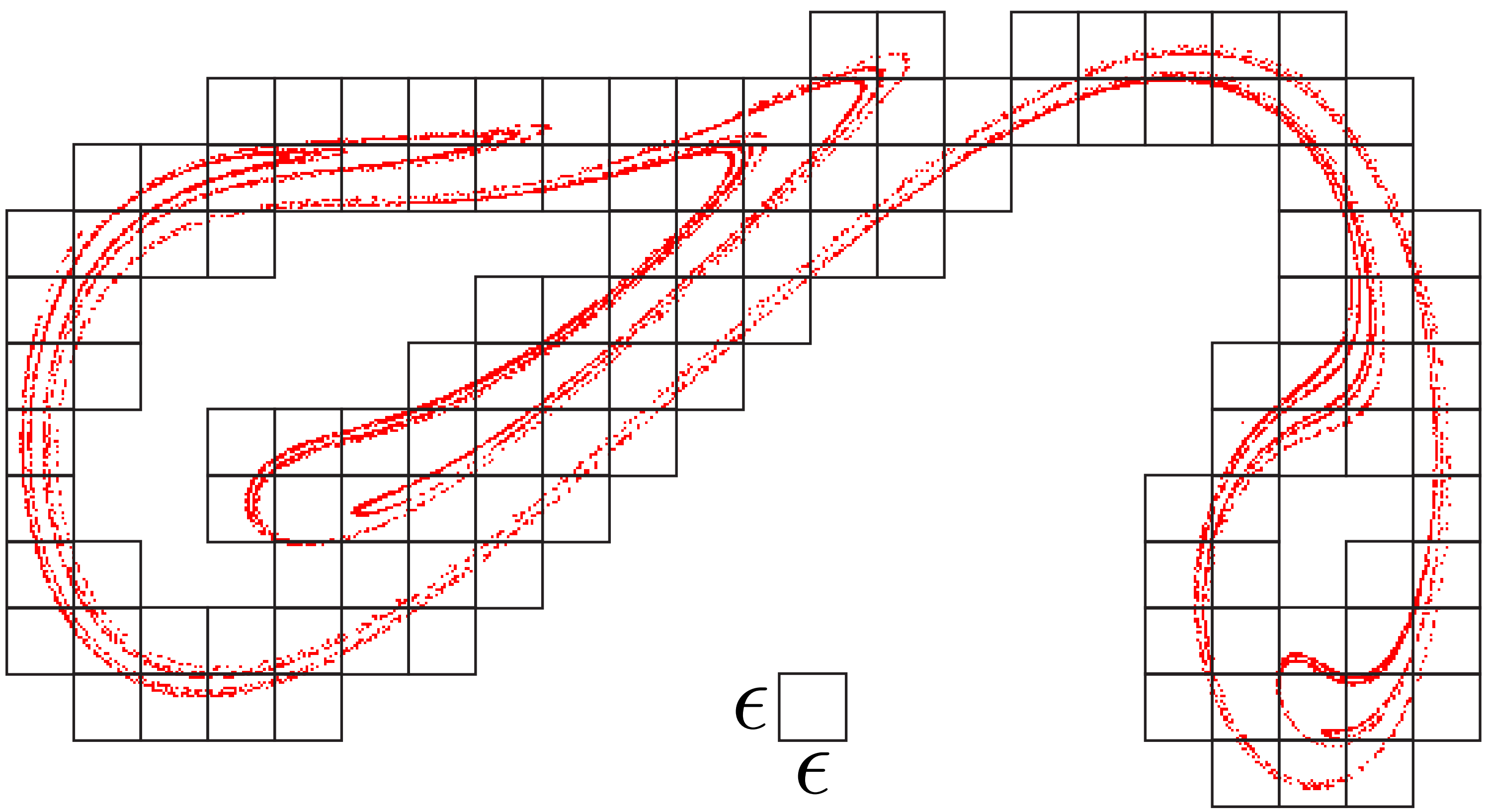
strange refers to
fractal geometry **of** the attractor

chaotic refers to
exponential divergence of orbits **on** the attractor

Fractal Dimension

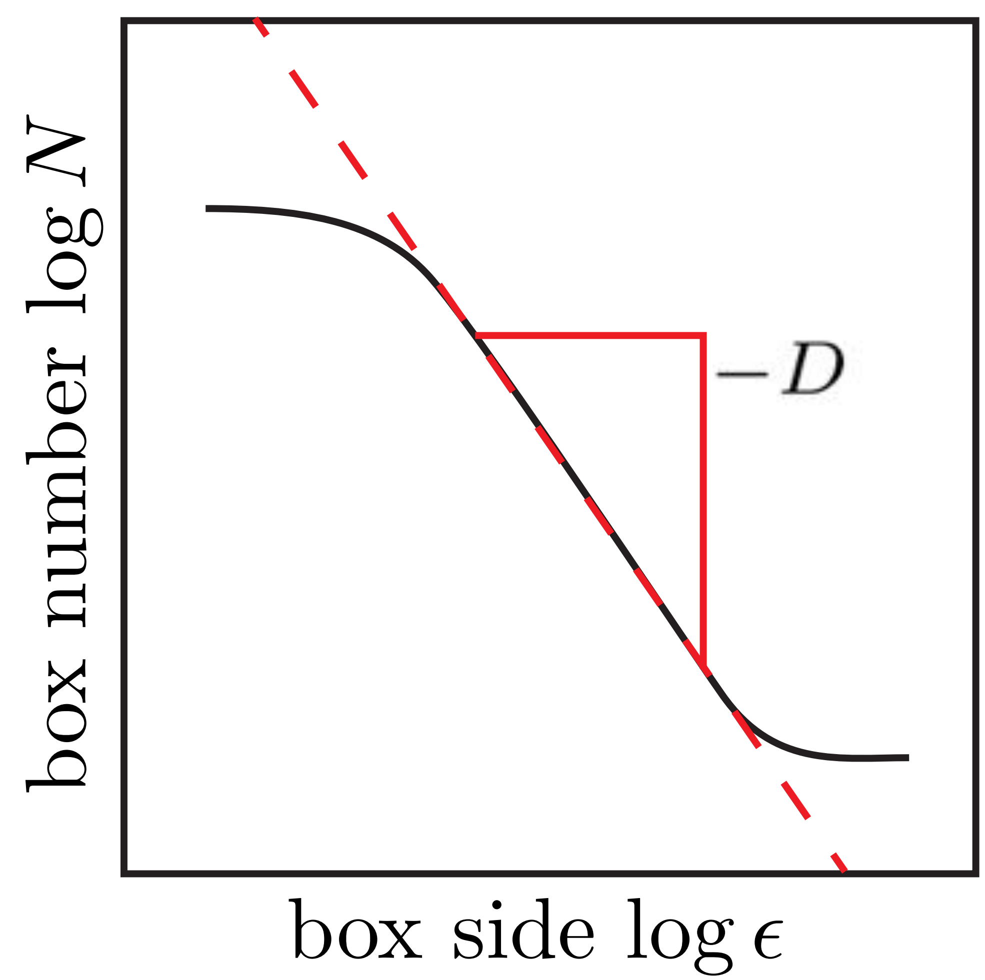
Quantifying self similarity

$\{x, v\}$ at θ



$$N \sim N_0 \epsilon^{-D}$$

$$\log N = \log N_0 - D \log \epsilon$$



Lyapunov Exponents

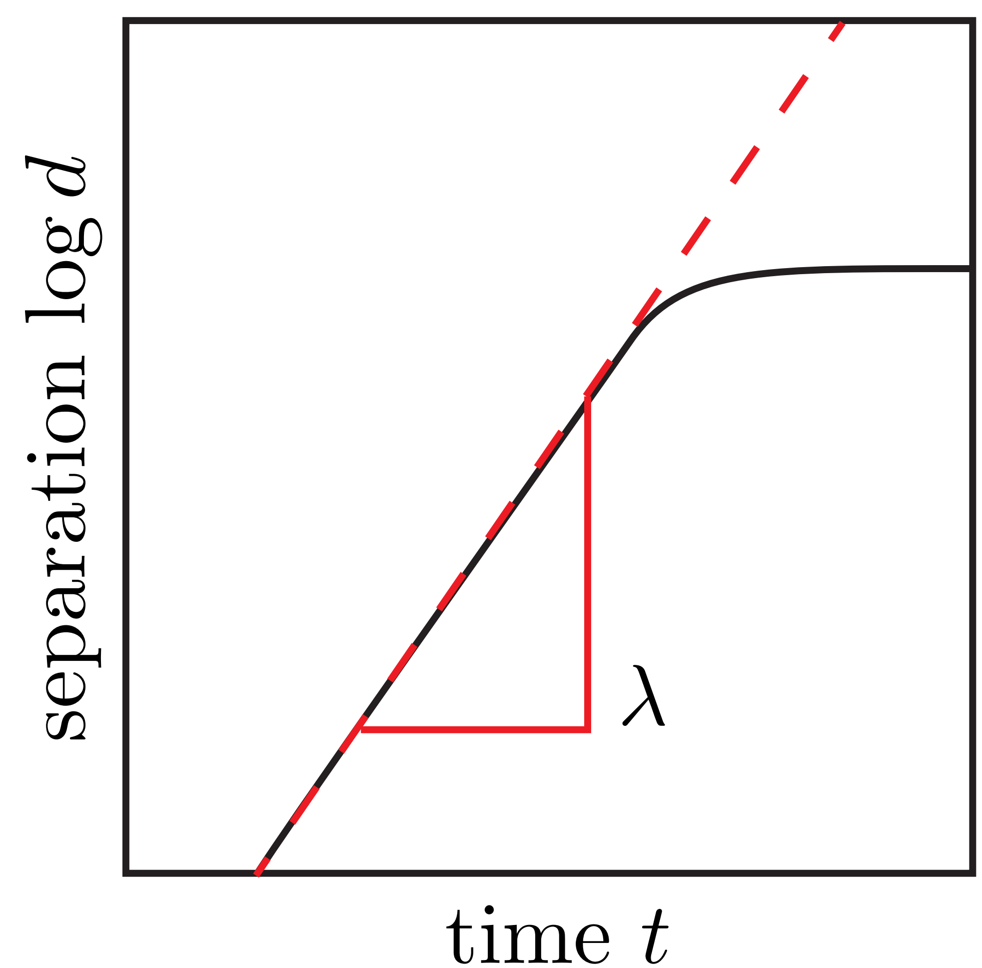
Exponential divergence of nearby orbits

$$\lambda \equiv \max\{\lambda_x, \lambda_v, \lambda_\theta\}$$

$$d \sim d_0 e^{\lambda t}$$

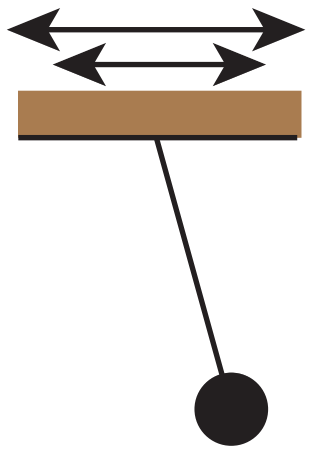
$$\log d = \log d_0 + \lambda t$$

$$\{x[t], v[t], \theta[t]\}$$



Strange Nonchaotic Attractor

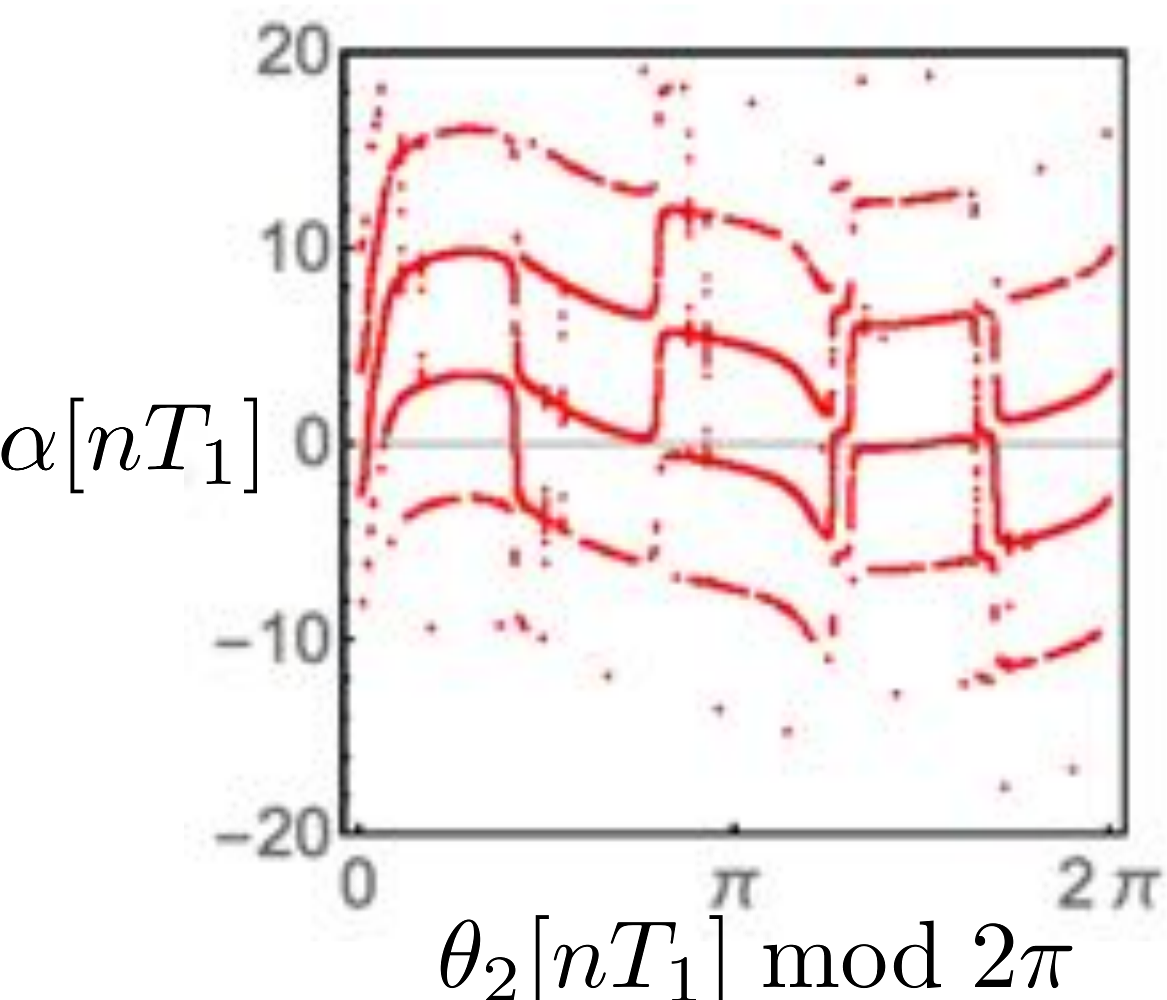
Quasiperiodically forced overdamped pendulum

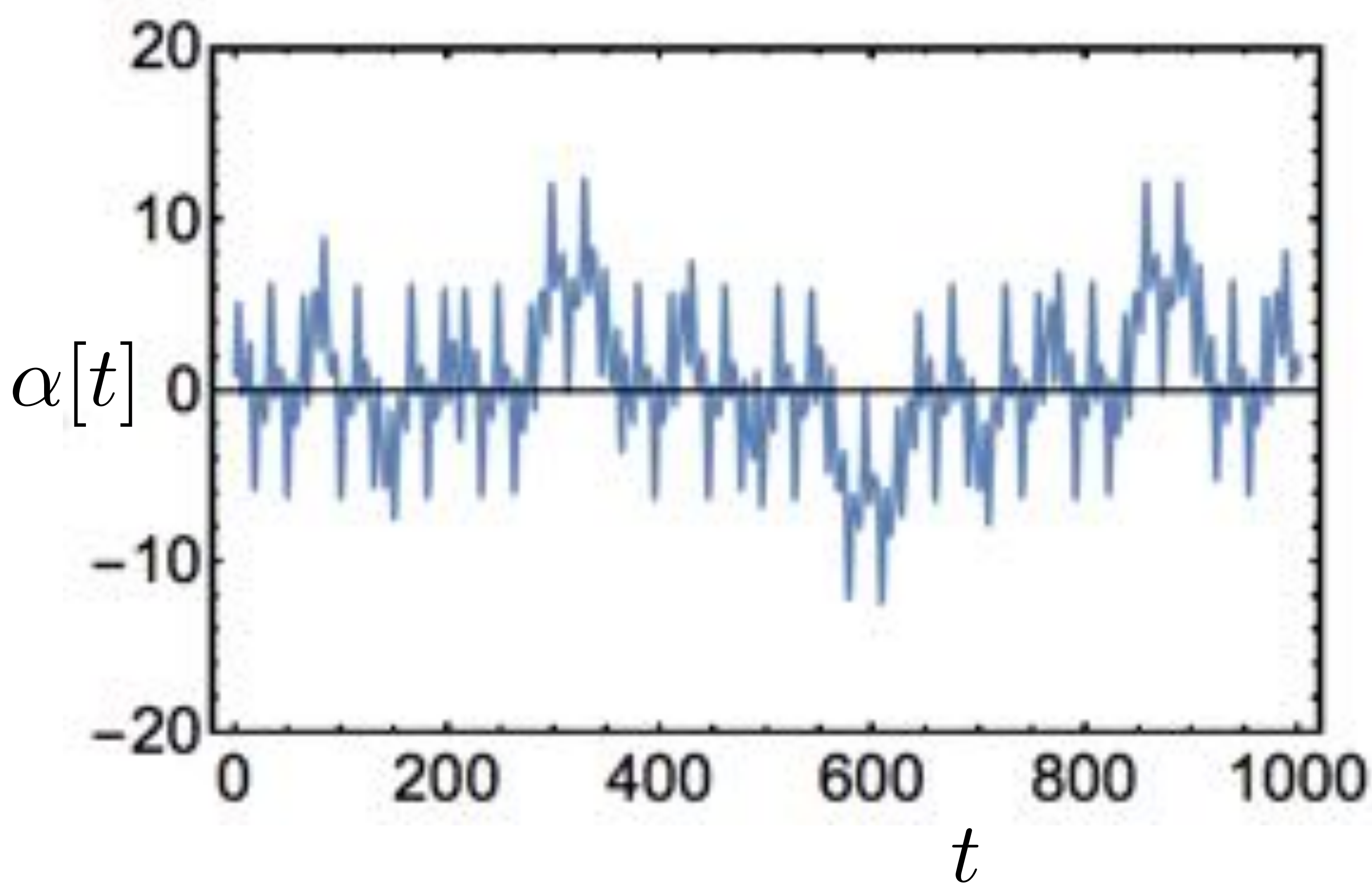


$$\dot{\alpha} = -\sin\alpha + A_1\cos\theta_1 + A_2\cos\theta_2$$

$$\dot{\theta}_1 = \omega_1 = 1$$

$$\dot{\theta}_2 = \omega_2 = 1/\varphi$$





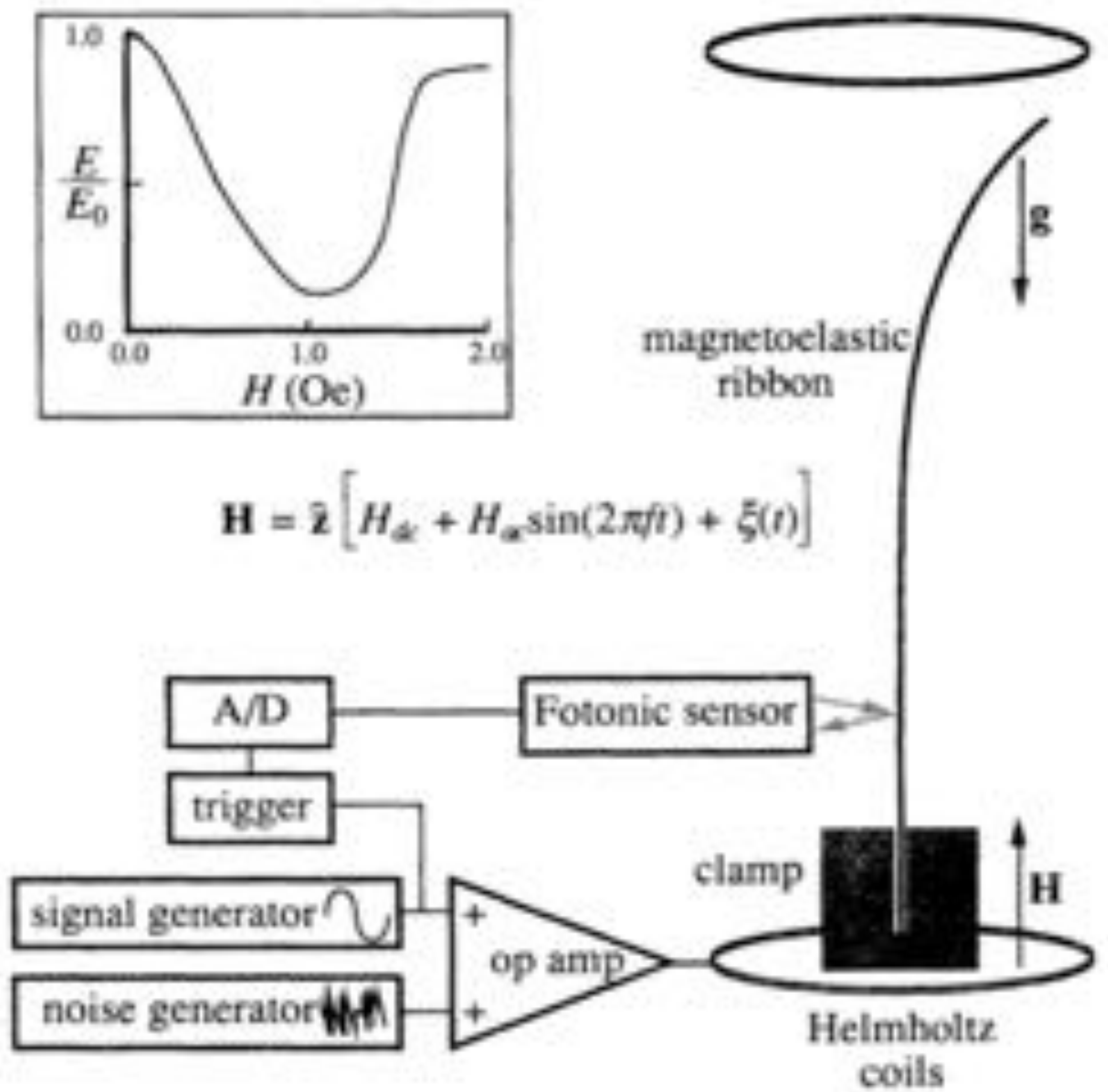
Experiments

Magnetoelastic Ribbon

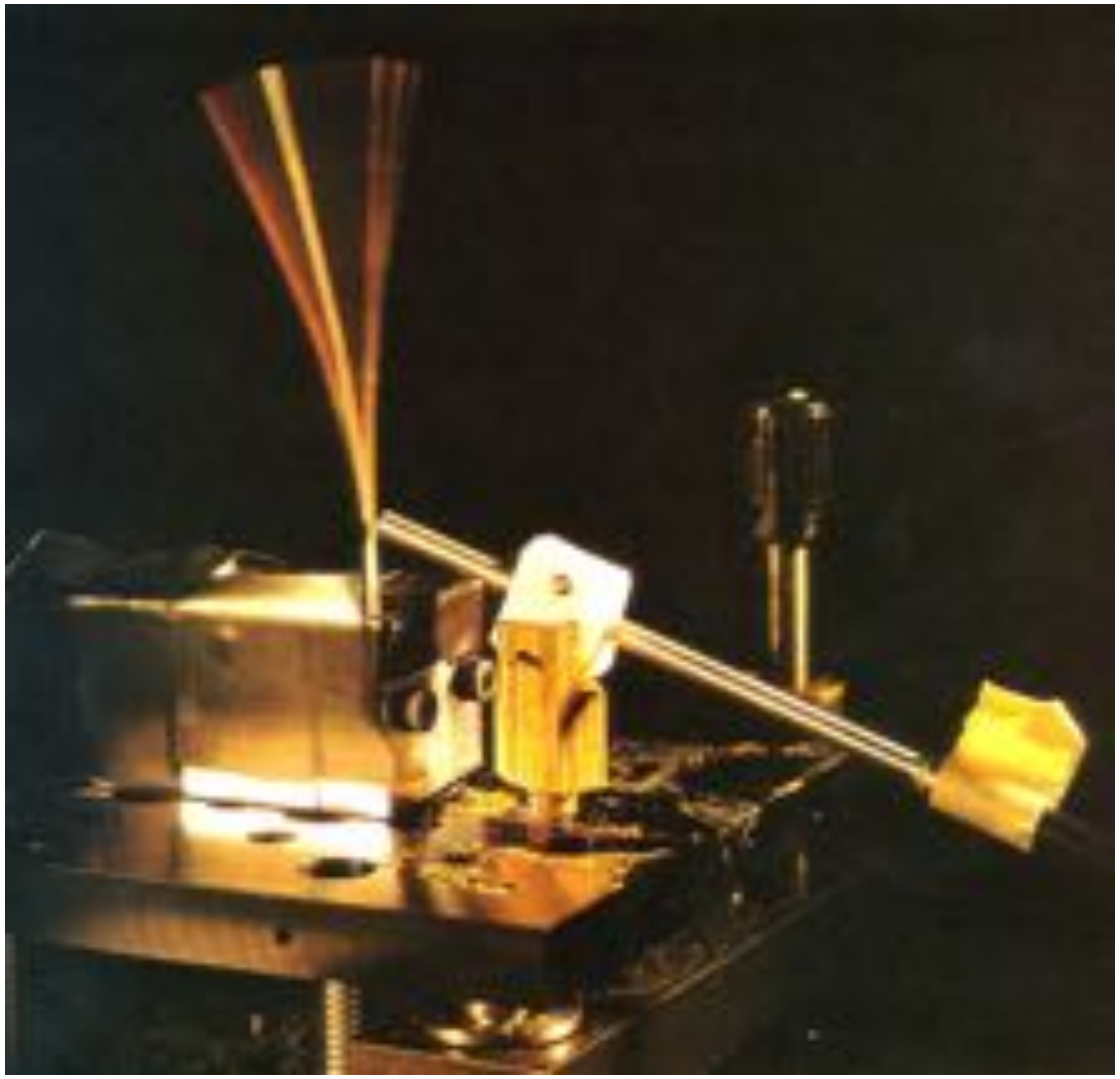
Ditto et al., Phys. Rev. Lett. **65**, 553 (1990)

Experiments

Magnetoelastic Ribbon



$f_2/f_1 = 1/\varphi$



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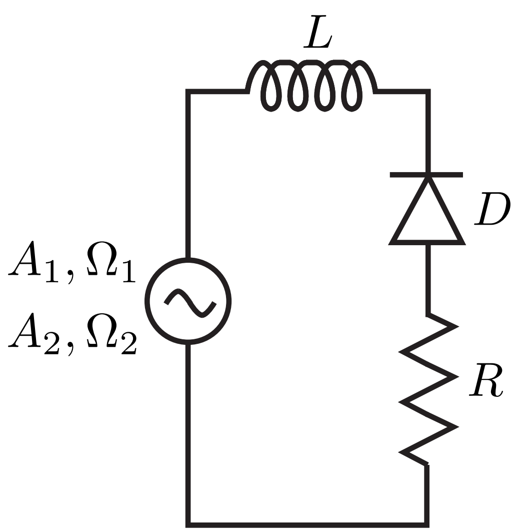
Experiments

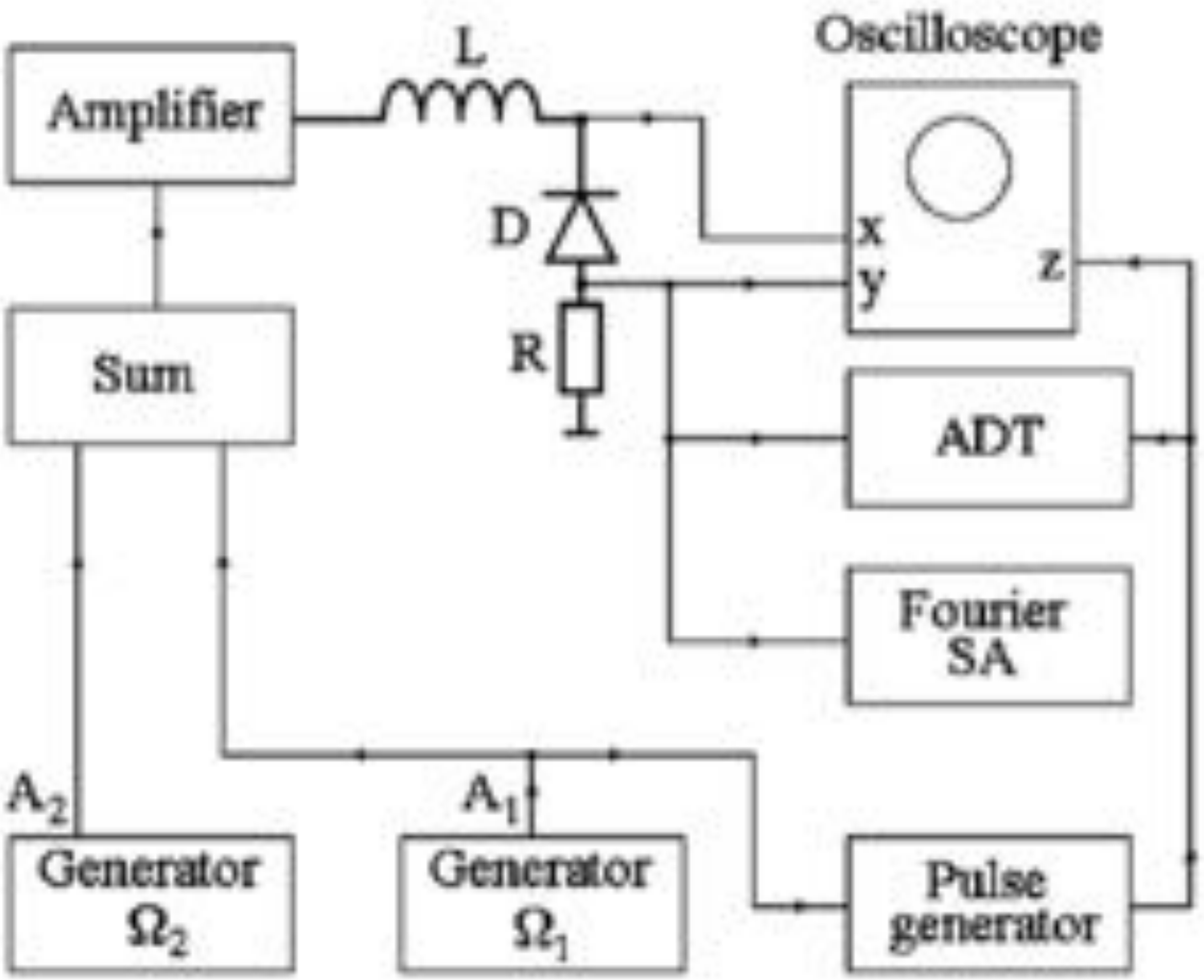
LRD Circuit



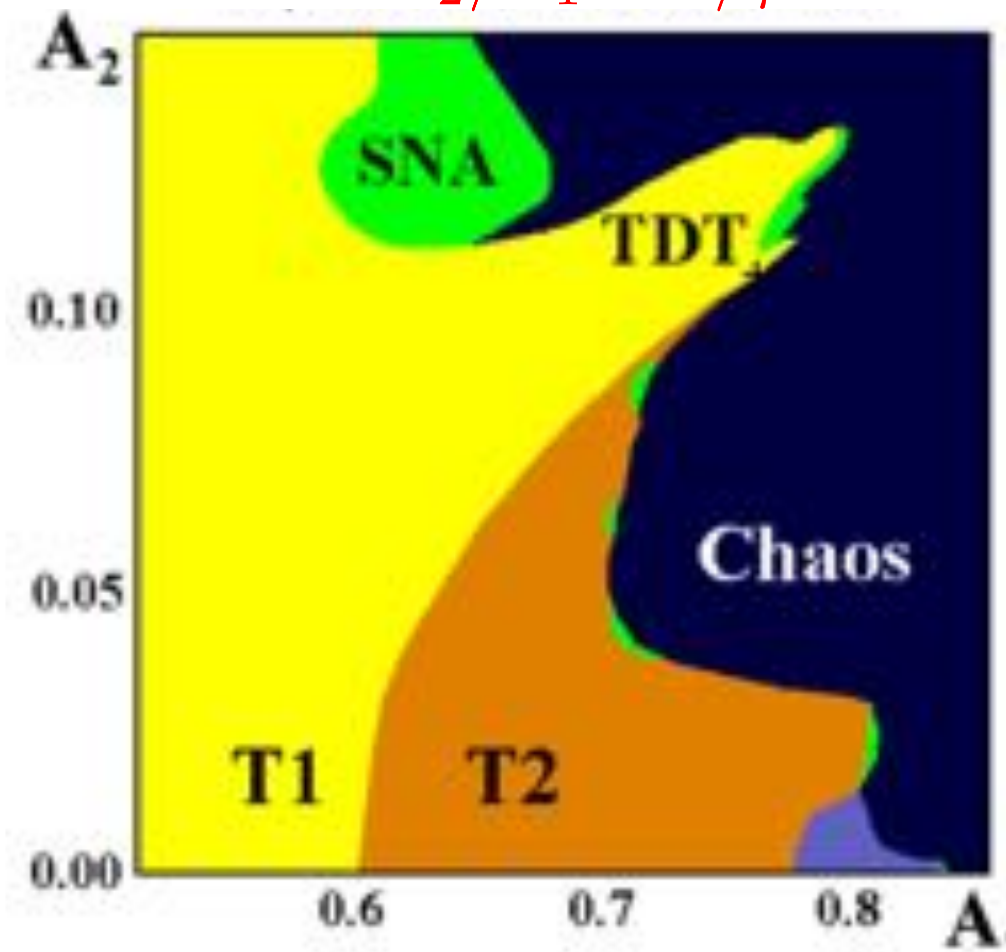
Bezruchko et al., Phys. Rev. E **62**, 7828 (2000)

Experiments

LRD Circuit



$\Omega_2/\Omega_1 = 1/\varphi$



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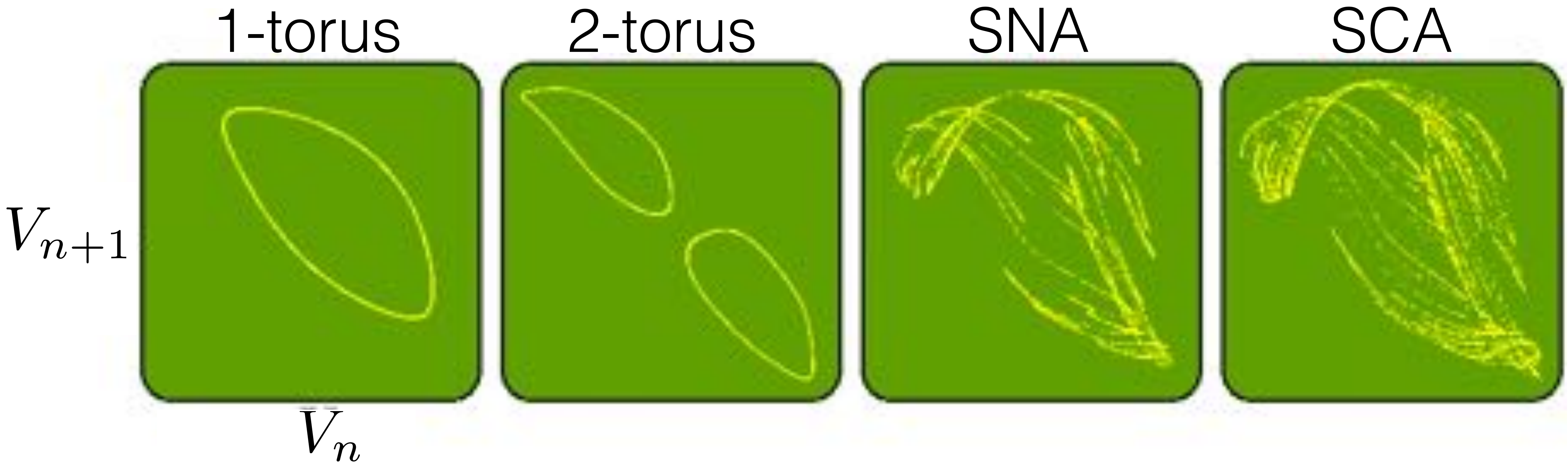
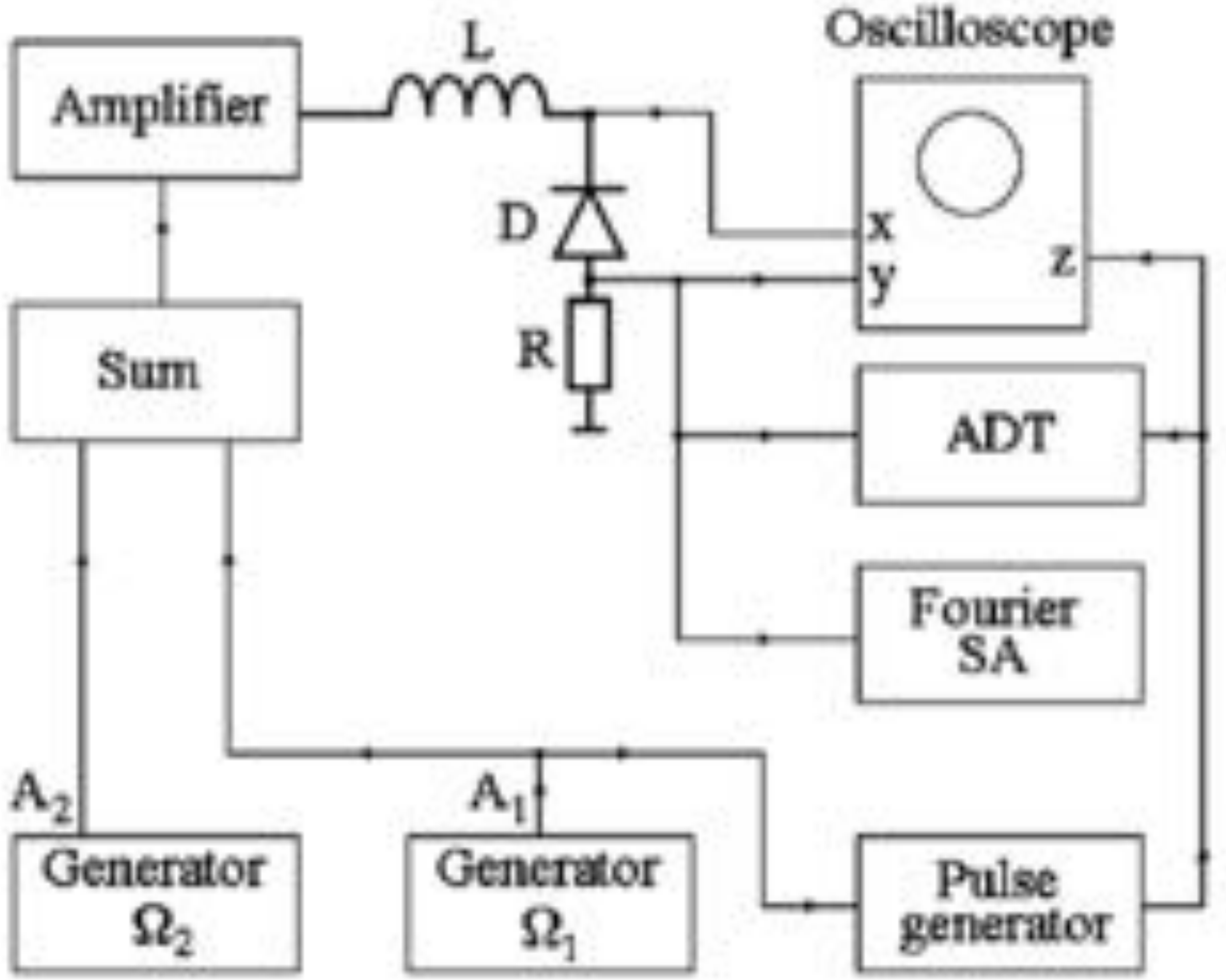
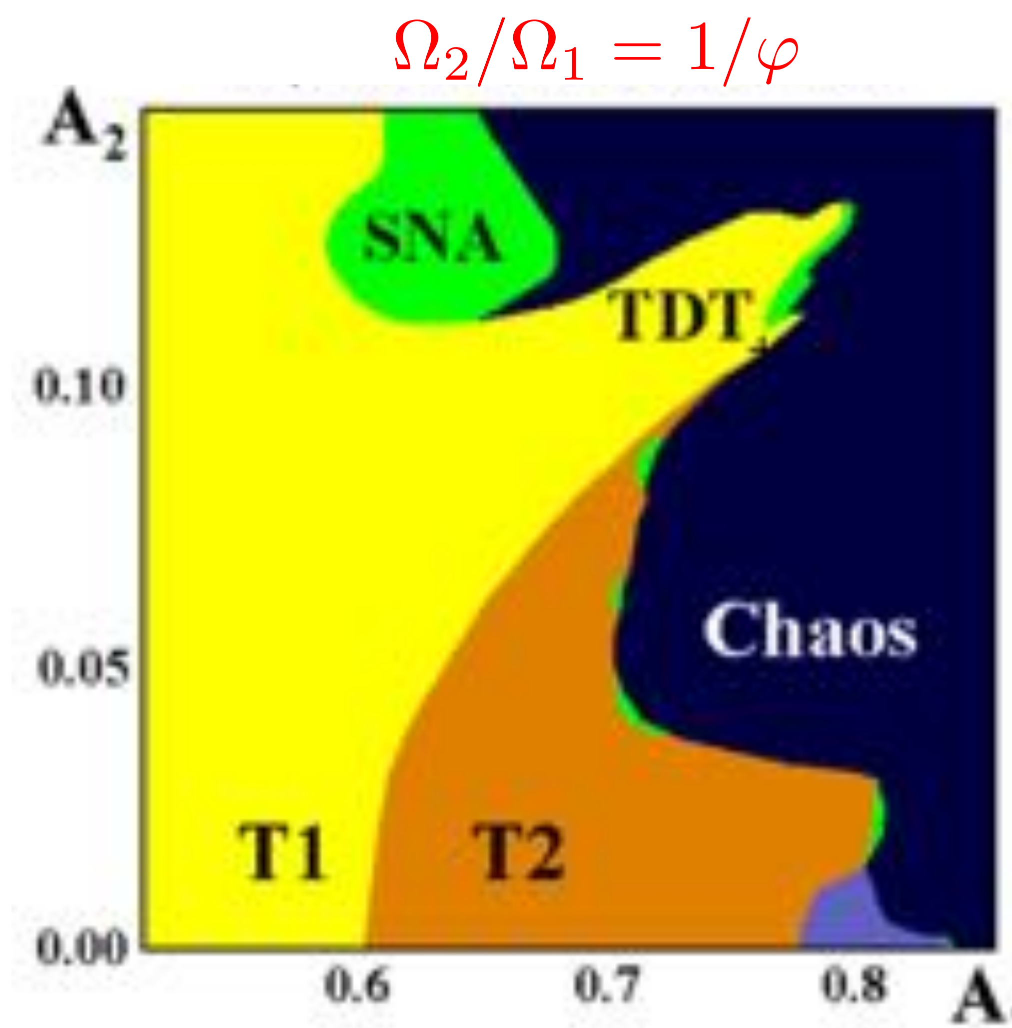






Bezruchko et al., Phys. Rev. E **62**, 7828 (2000)

Climate

http://www.petplus.rs/wp-content/uploads/2014/07/piceds-3205734.jpg

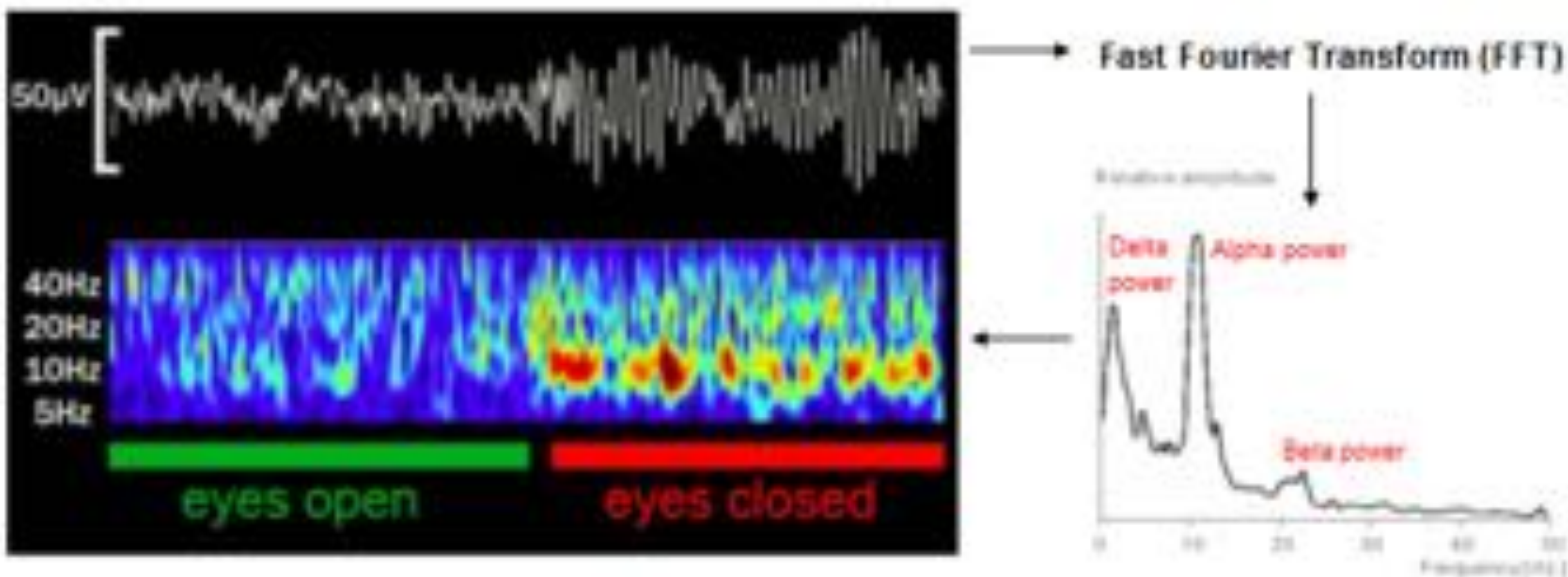
SCA describes weather

Apollo 17

SNA describes climate?

Brain Activity

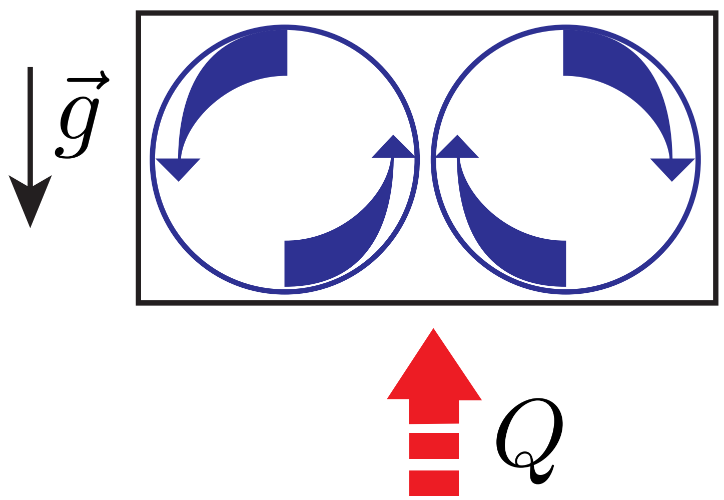
SNA → flexibility of fractals without penalty of chaos?



http://produceconsumerobot.com/pickyourbrain/

Lorenz Weather

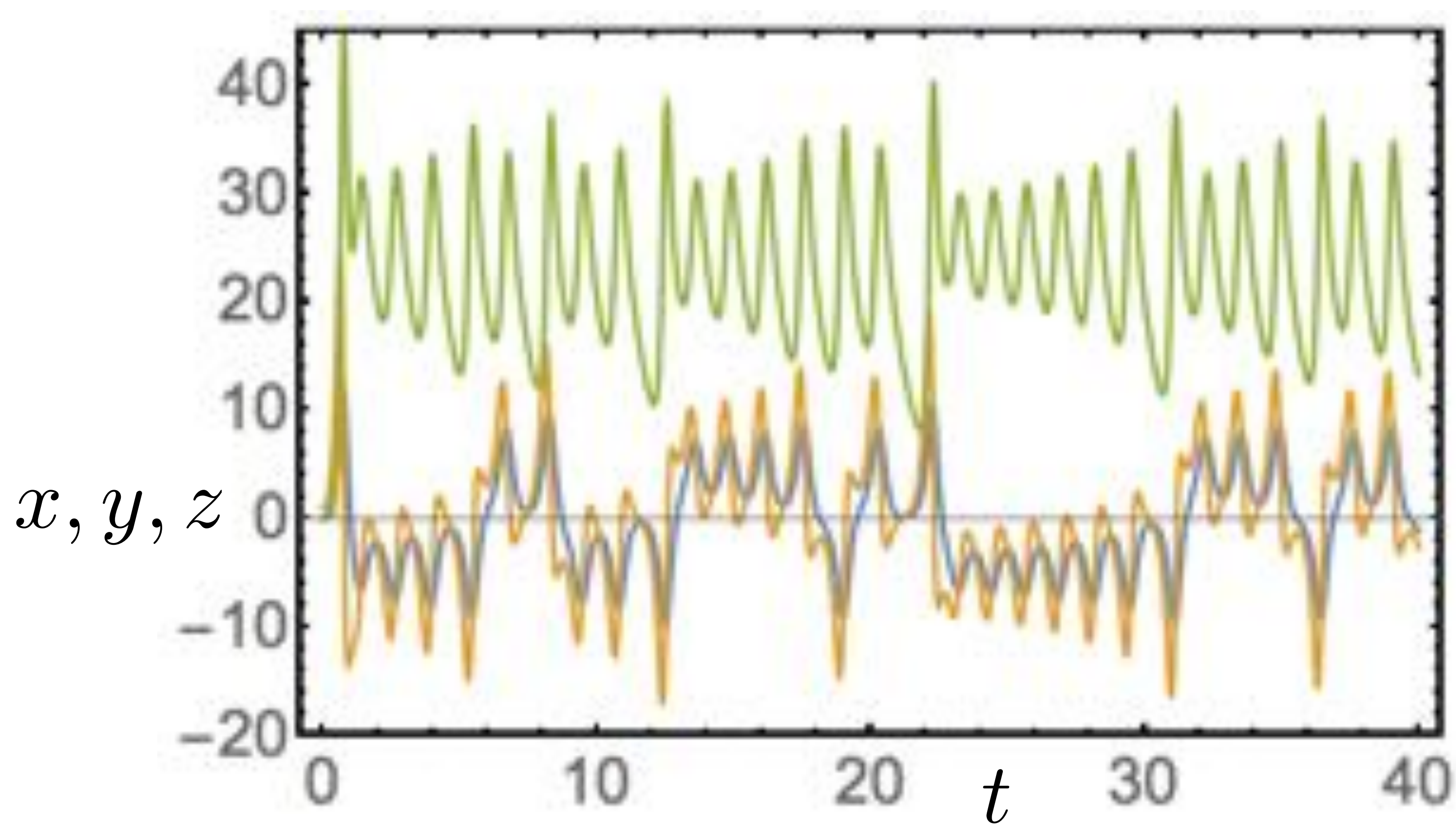
Convection under thermal and gravity gradients



$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$

$$y \propto T_{\uparrow} - T_{\downarrow}$$

$\{x, y, z\}$



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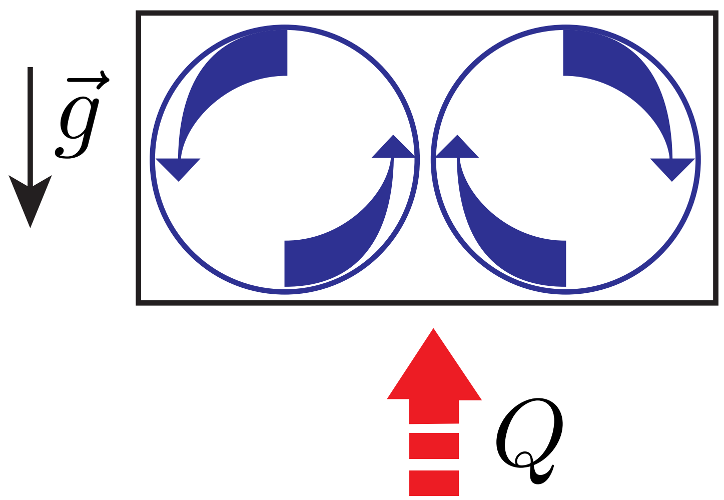

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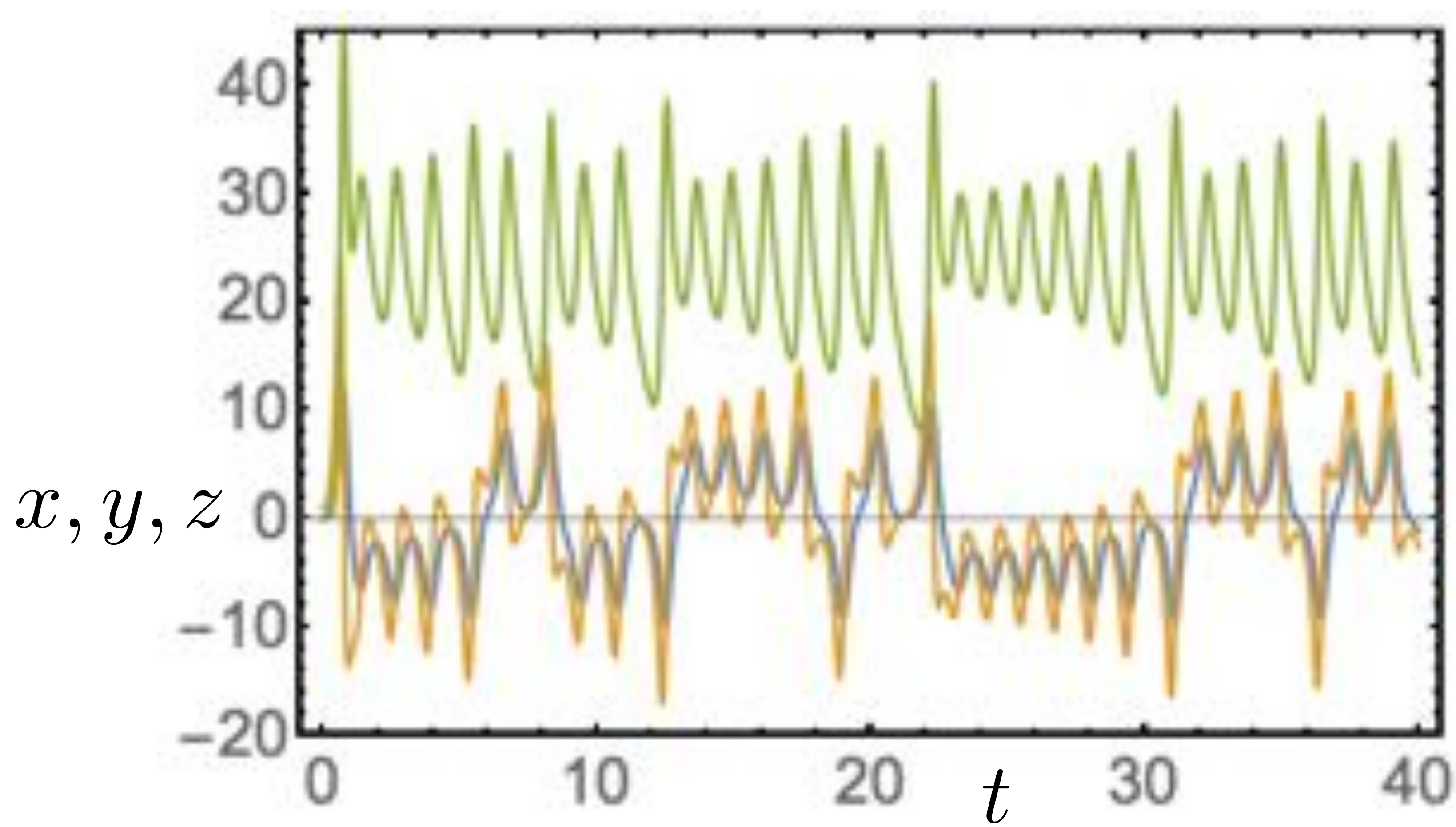

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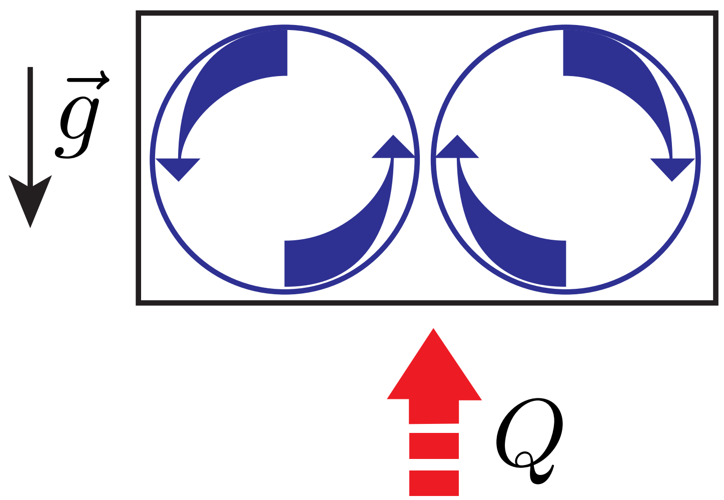

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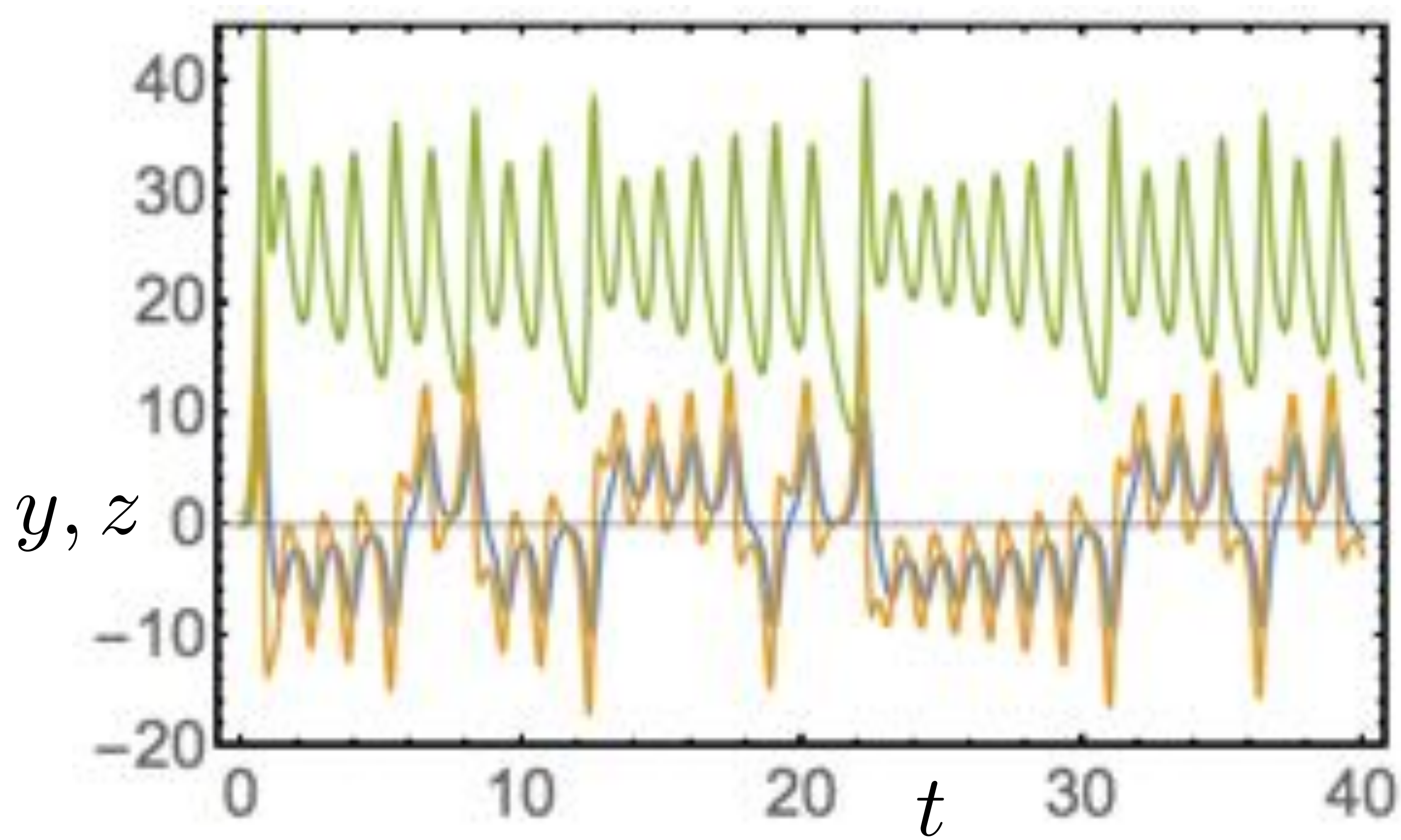
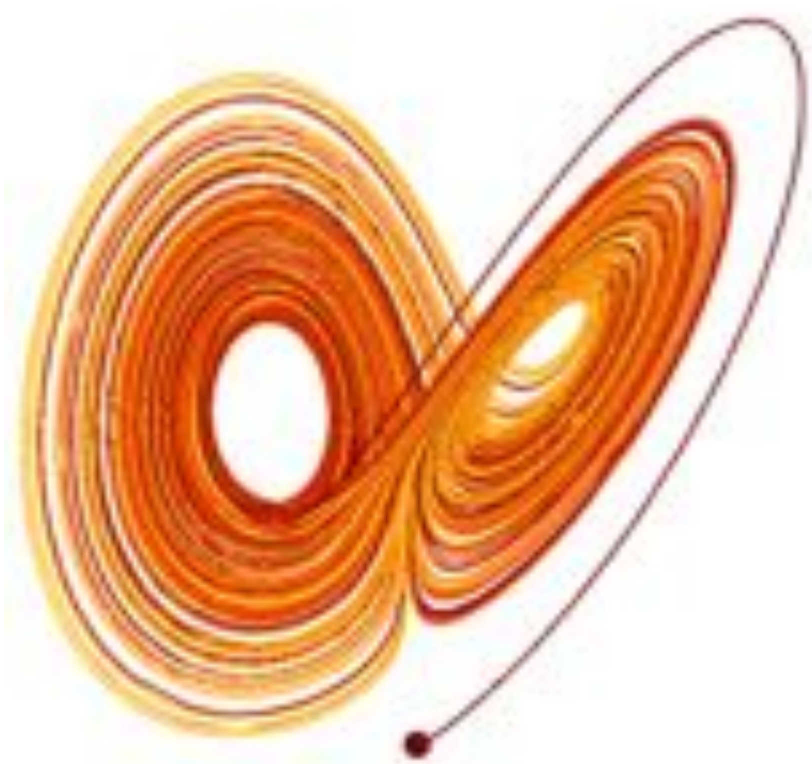
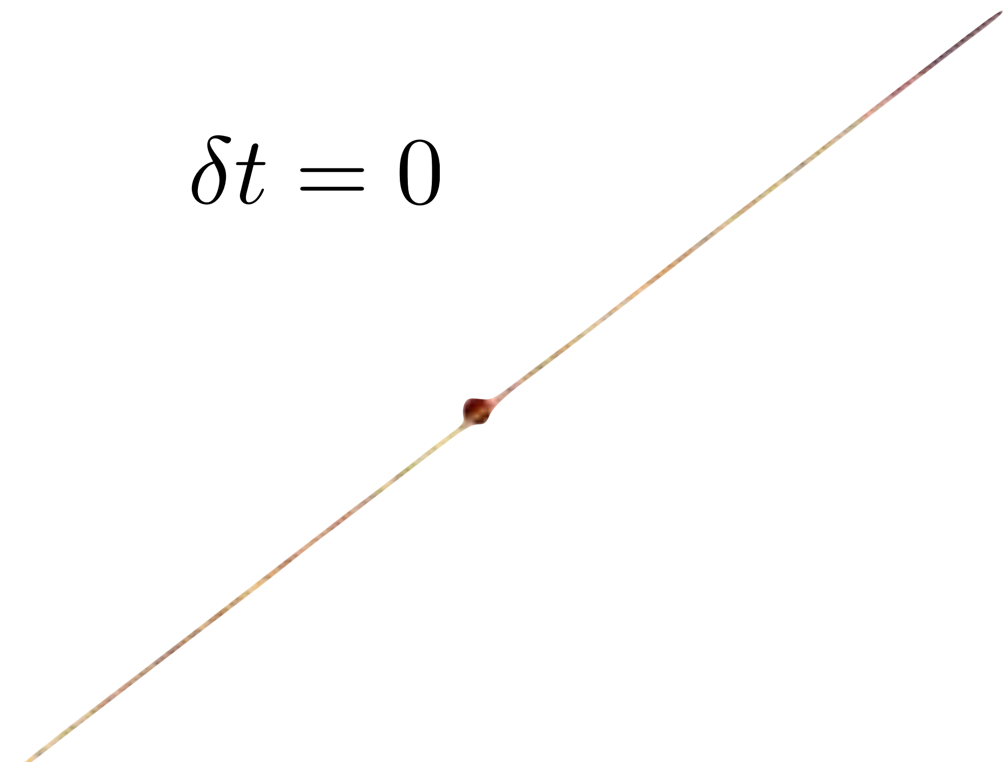


Takens Delay Embedding

Attractor reconstruction from time series



$\{x[t], y[t], z[t]\}$

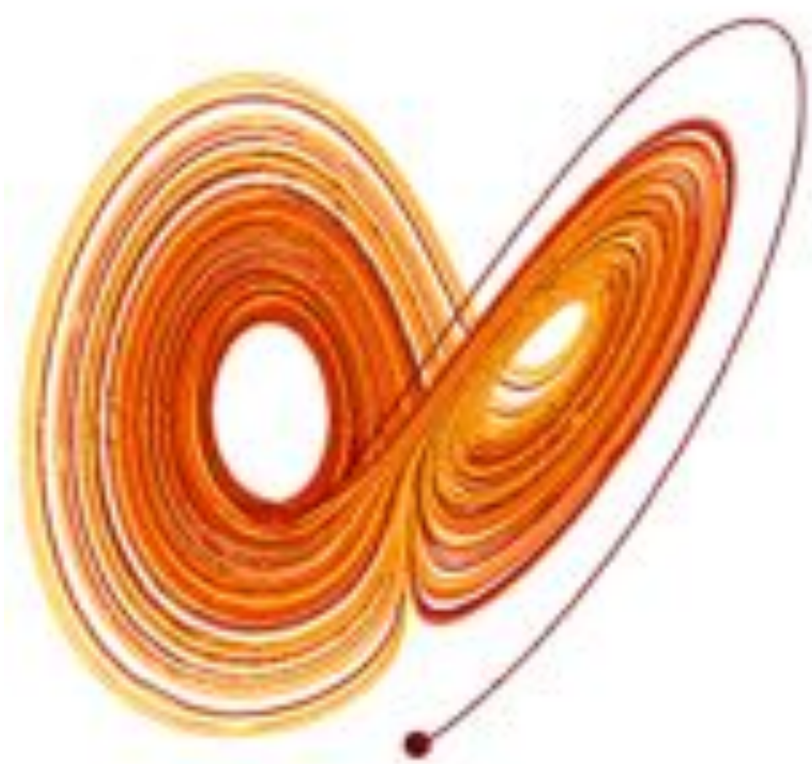


$\delta t = 0$

$\{x[t], x[t-\delta t], x[t-2\delta t]\}$

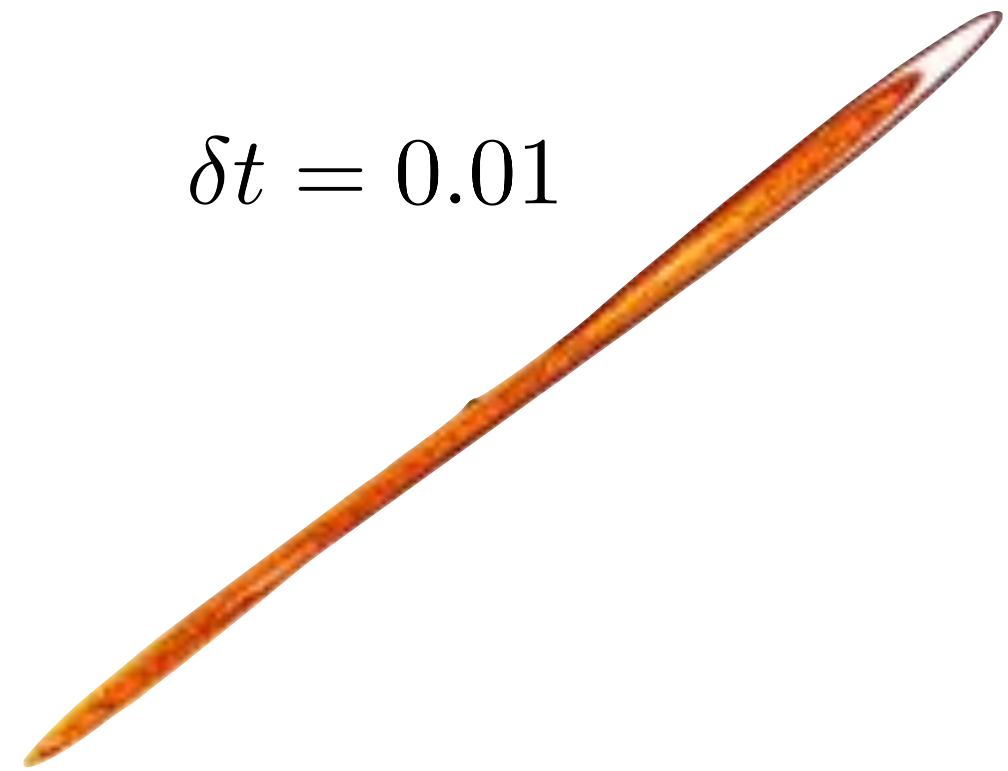
Takens Delay Embedding

Attractor reconstruction from time series



$\{x[t], y[t], z[t]\}$

$\delta t = 0.01$



$\{x[t], x[t-\delta t], x[t-2\delta t]\}$

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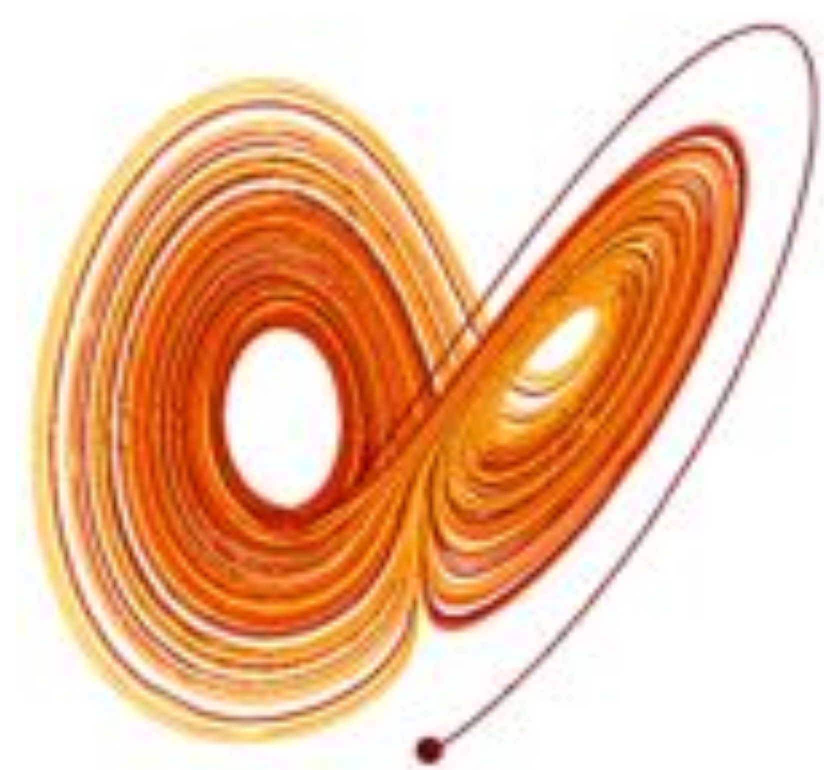
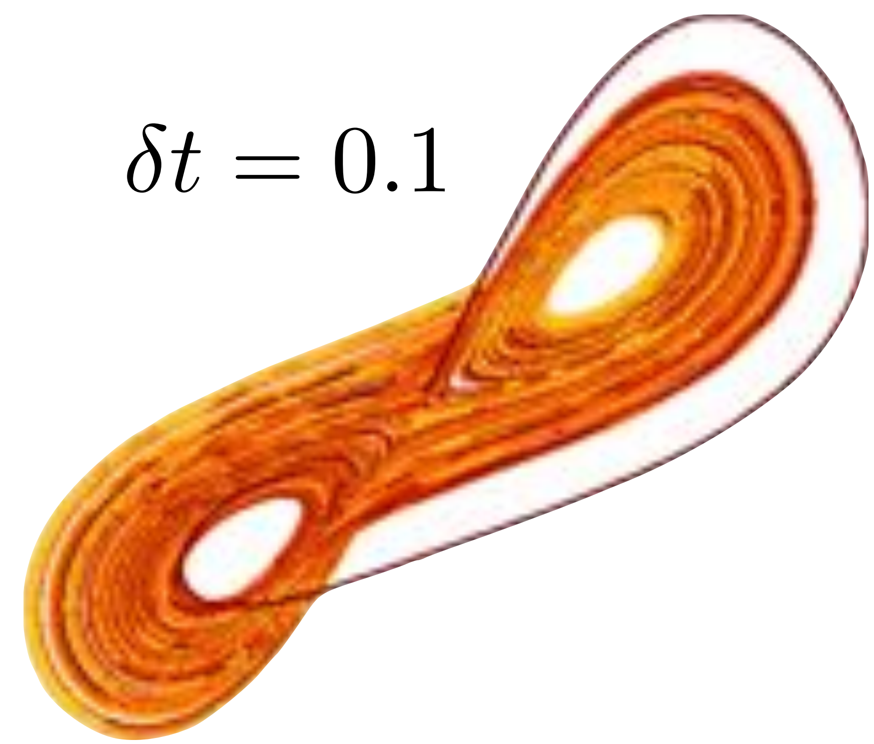


$$\{x[t], y[t], z[t]\}$$



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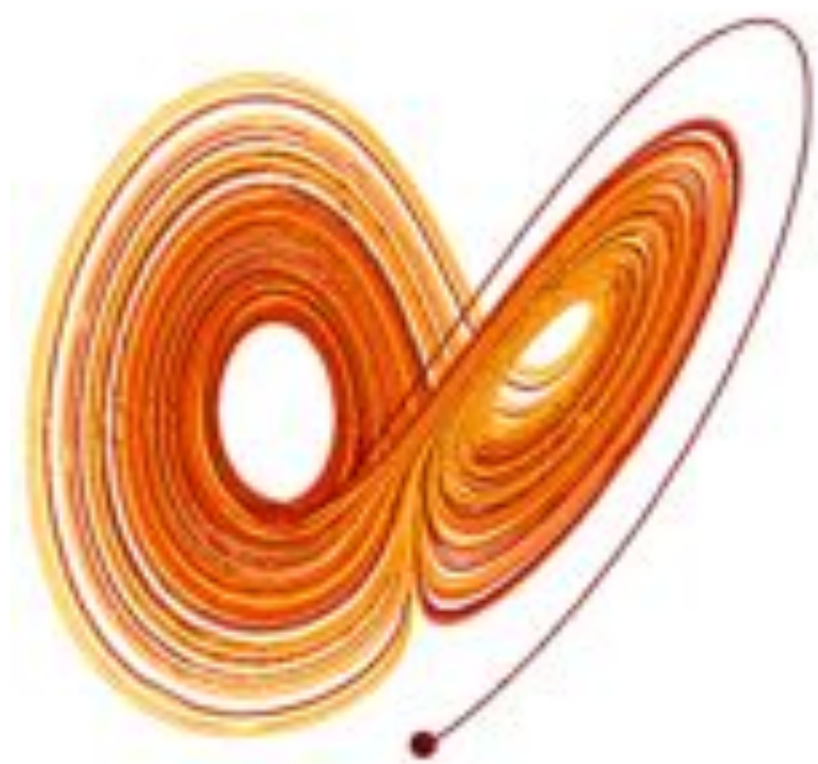

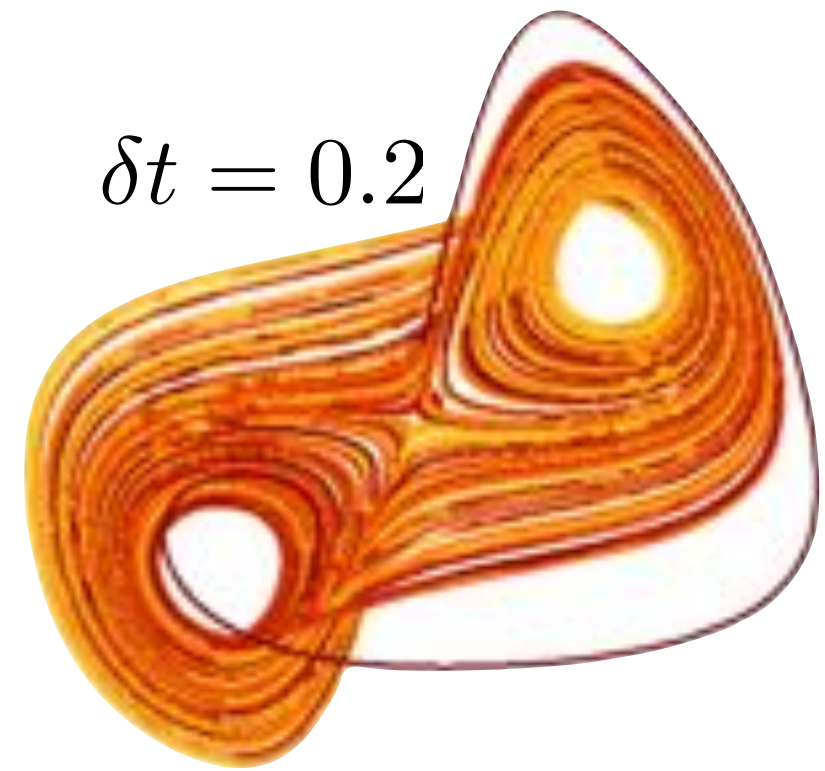

$\{x[t], y[t], z[t]\}$

$\{x[t], x[t-\delta t], x[t-2\delta t]\}$

Topologically equivalent

Same Lyapunov exponents & dimensions

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SNA Signatures

Despite Takens' theorem,
estimating the fractal dimension
and Lyapunov exponent
of a reconstructed attractor
from a **noisy** time serious
is difficult

Spectral scaling,
which is a measure of the roughness
of the time series Fourier transform,
is a better signature

Spectral Scaling Heuristics

Sample time series

$$x_n = x[n\Delta t]$$

Fourier series

$$x_m = \sum_{n=1}^{\infty} \hat{x}_n e^{i2\pi mn}$$

$$\partial_m^k x_m = \sum_{n=1}^{\infty} \hat{x}_n (i2\pi n)^k e^{i2\pi mn}$$

As exponential beats any power, expect

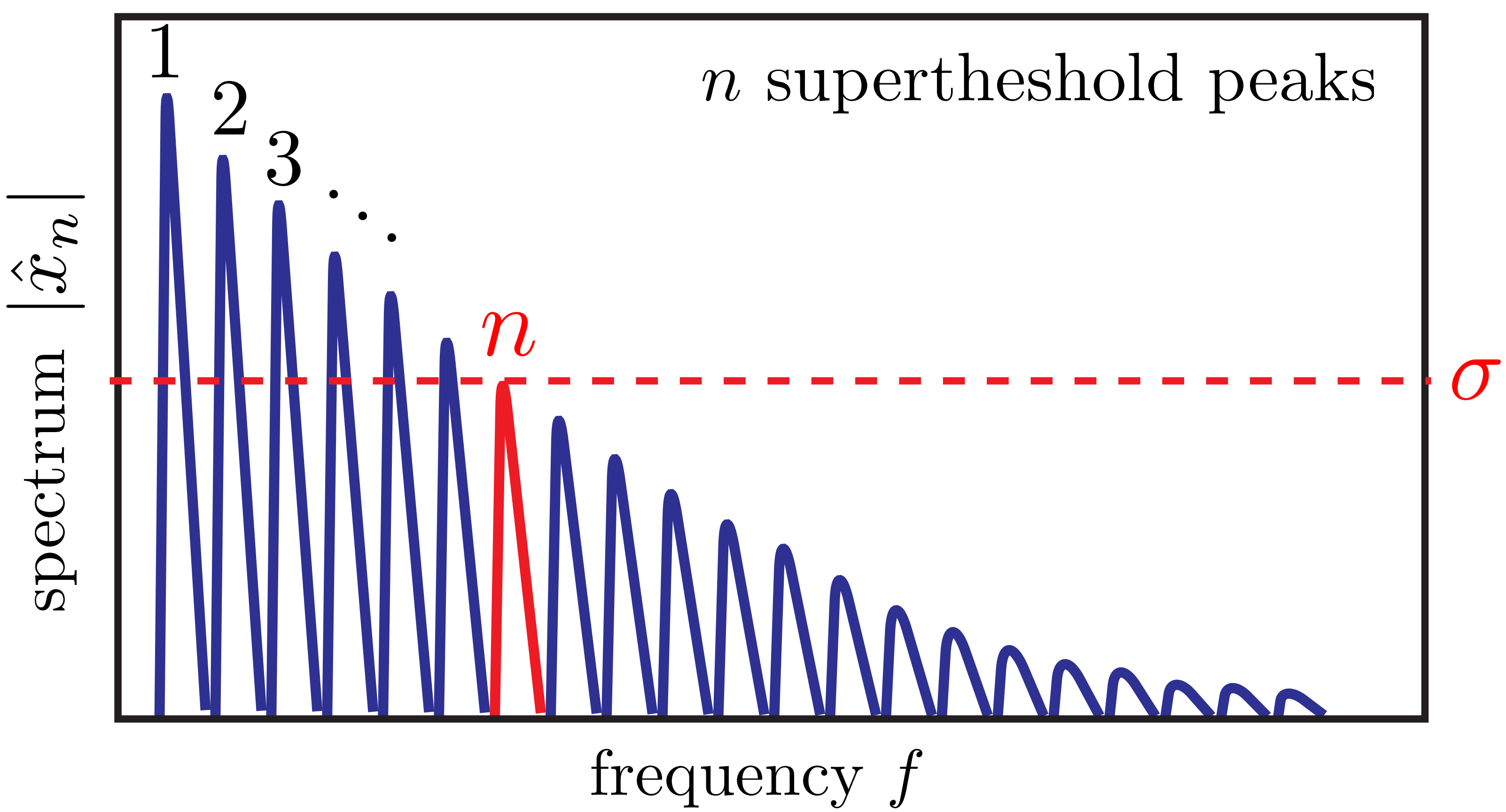
$$\sigma \equiv |\hat{x}_n| \sim \sigma_0 \begin{cases} e^{-\lambda n}, & \text{smooth derivatives} \\ n^{-\lambda}, & \text{non-smooth derivatives} \end{cases}$$

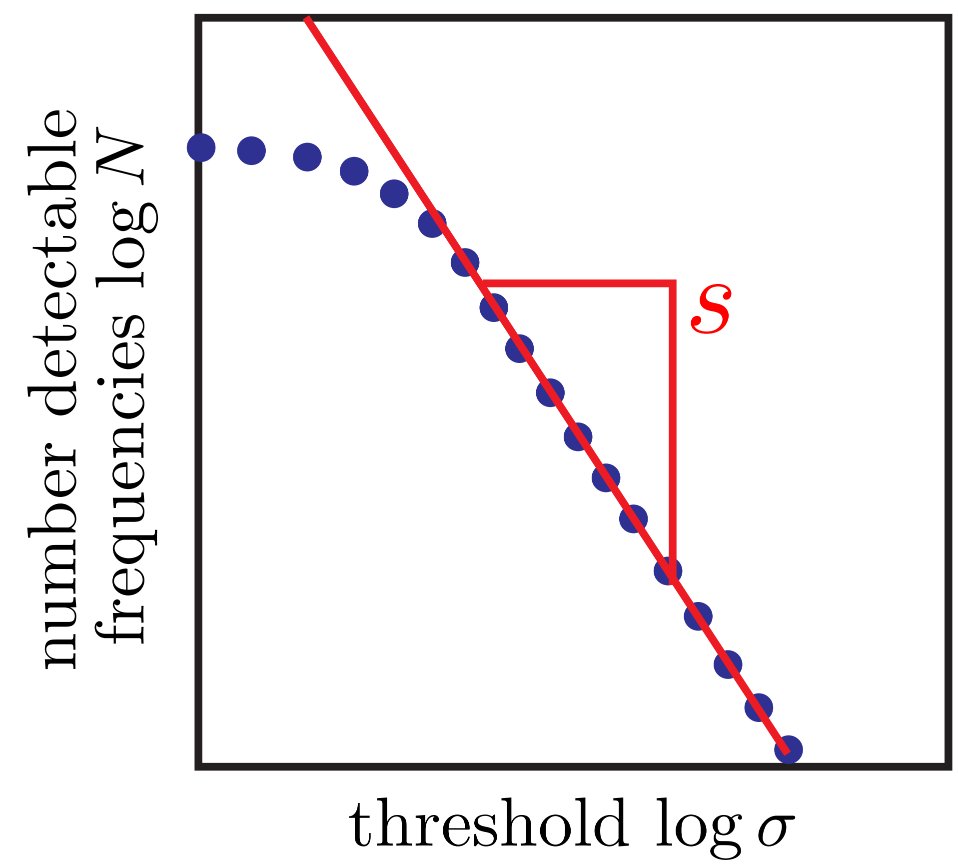
Spectral Scaling Heuristics

Invert to get number of peaks above threshold

$$N \equiv n \sim \begin{cases} s\log[\sigma/\sigma_0], & \text{smooth derivatives} \\ (\sigma/\sigma_0)^s, & \text{non-smooth derivatives} \end{cases}$$

$$-2 < s = -1/\lambda < -1 \quad \text{for SNA}$$

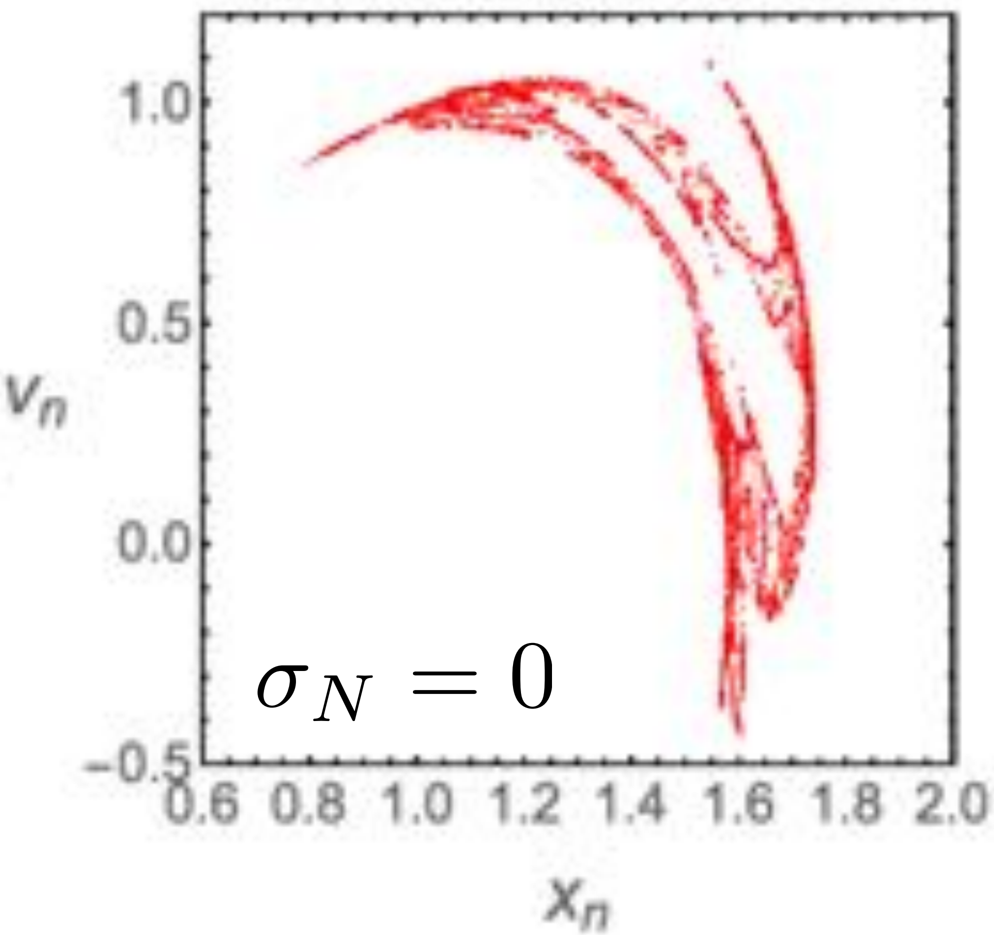


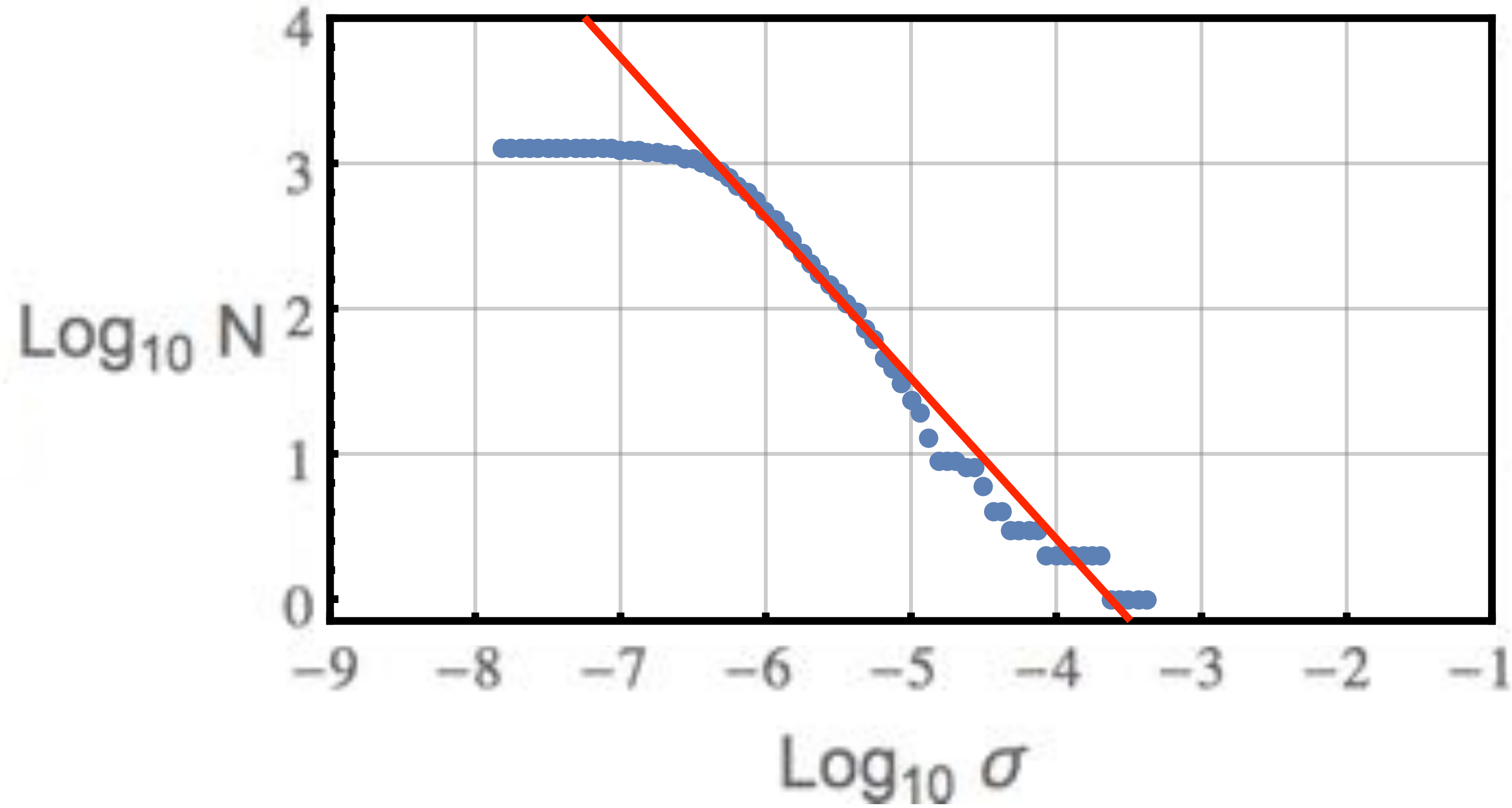


Noise Robustness

Quasiperiodically parametrically forced damped Duffing

$$f_2/f_1 = \varphi$$





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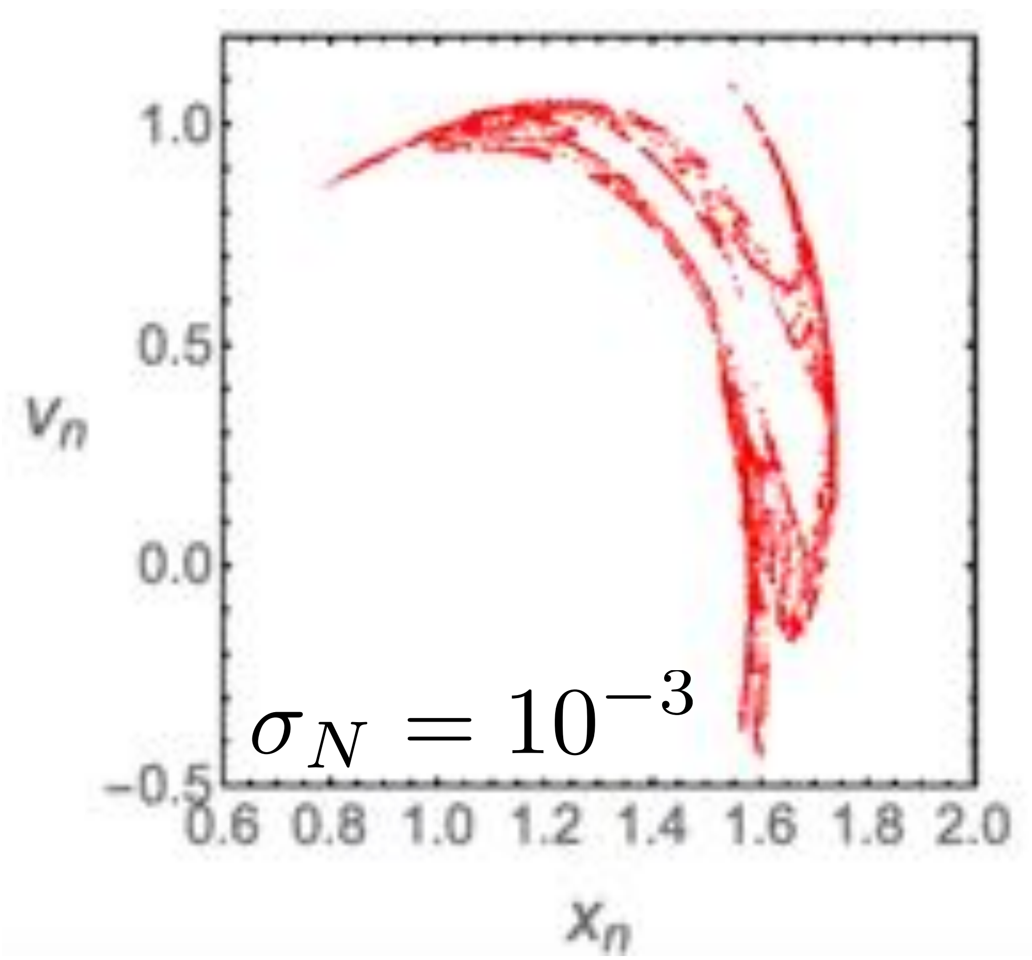

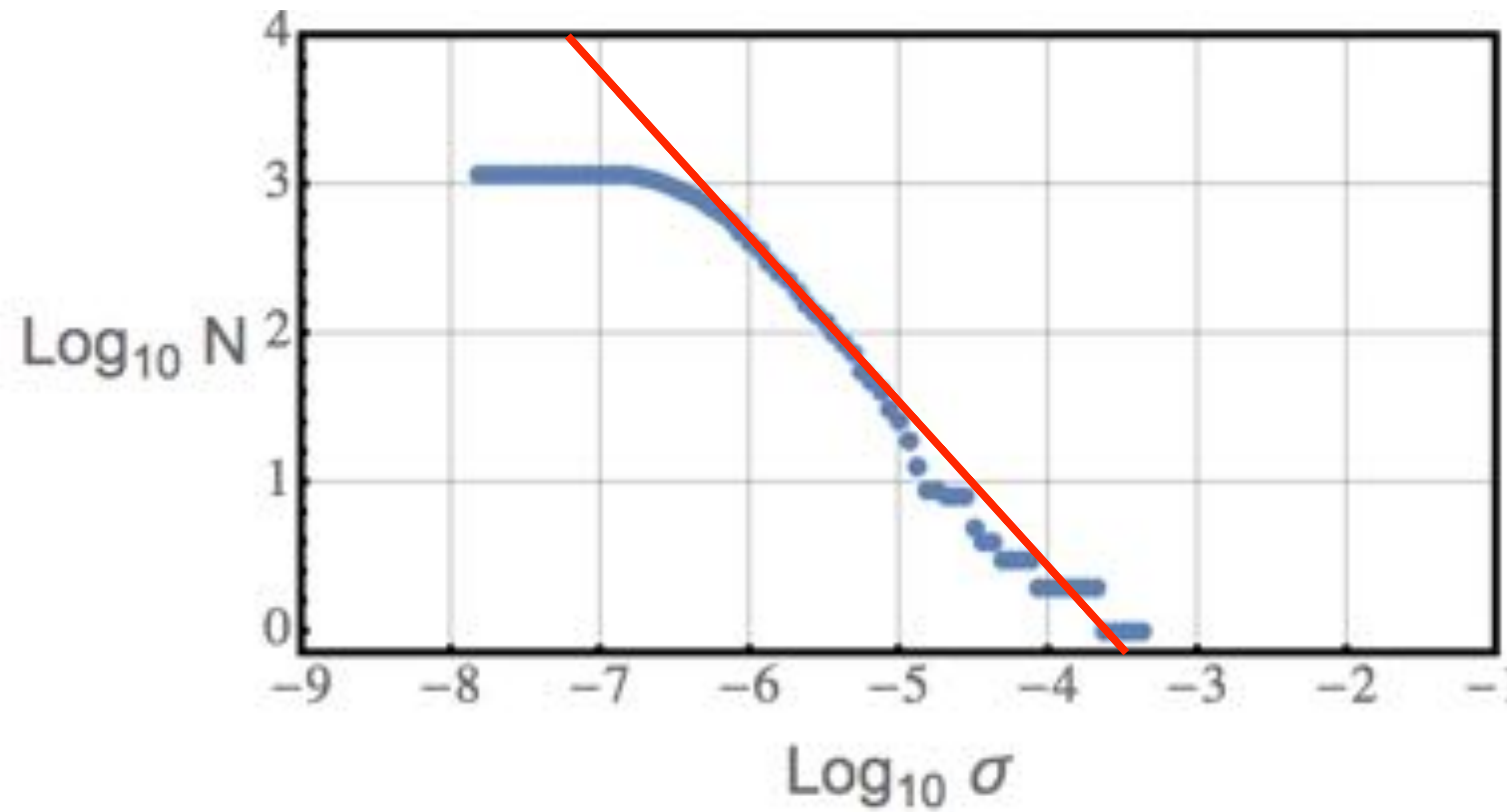

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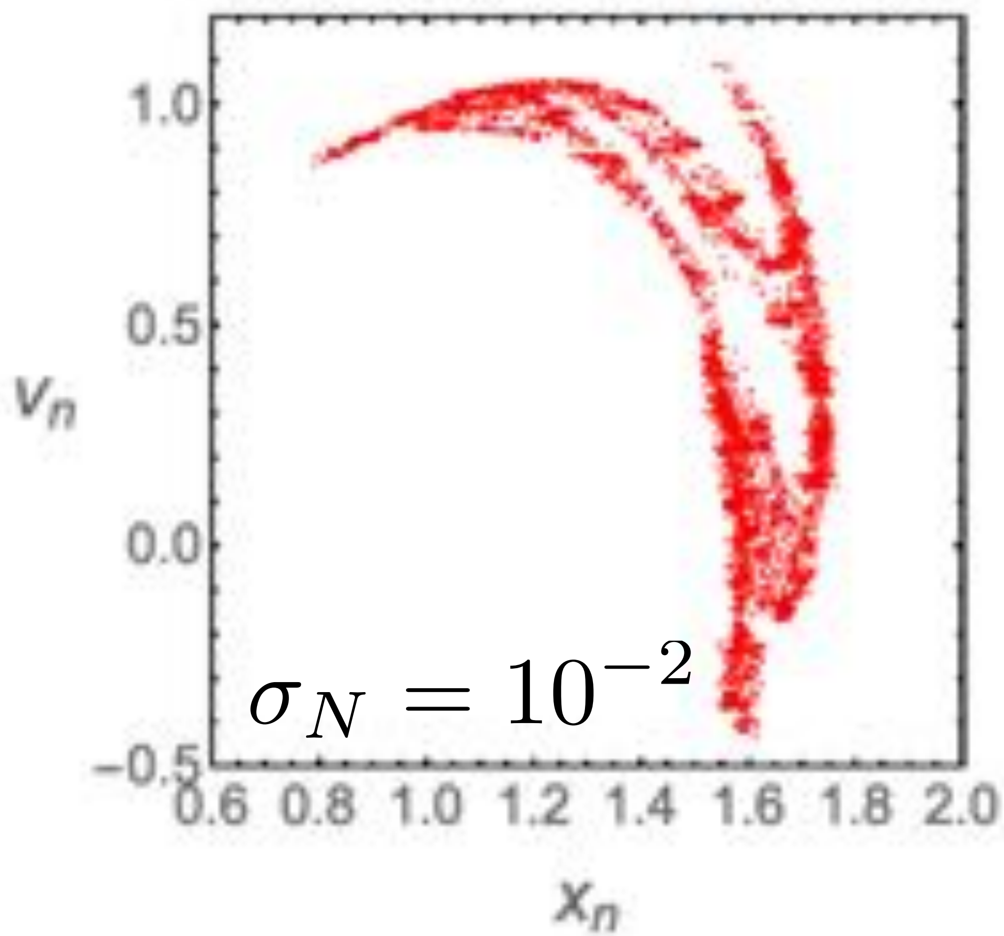

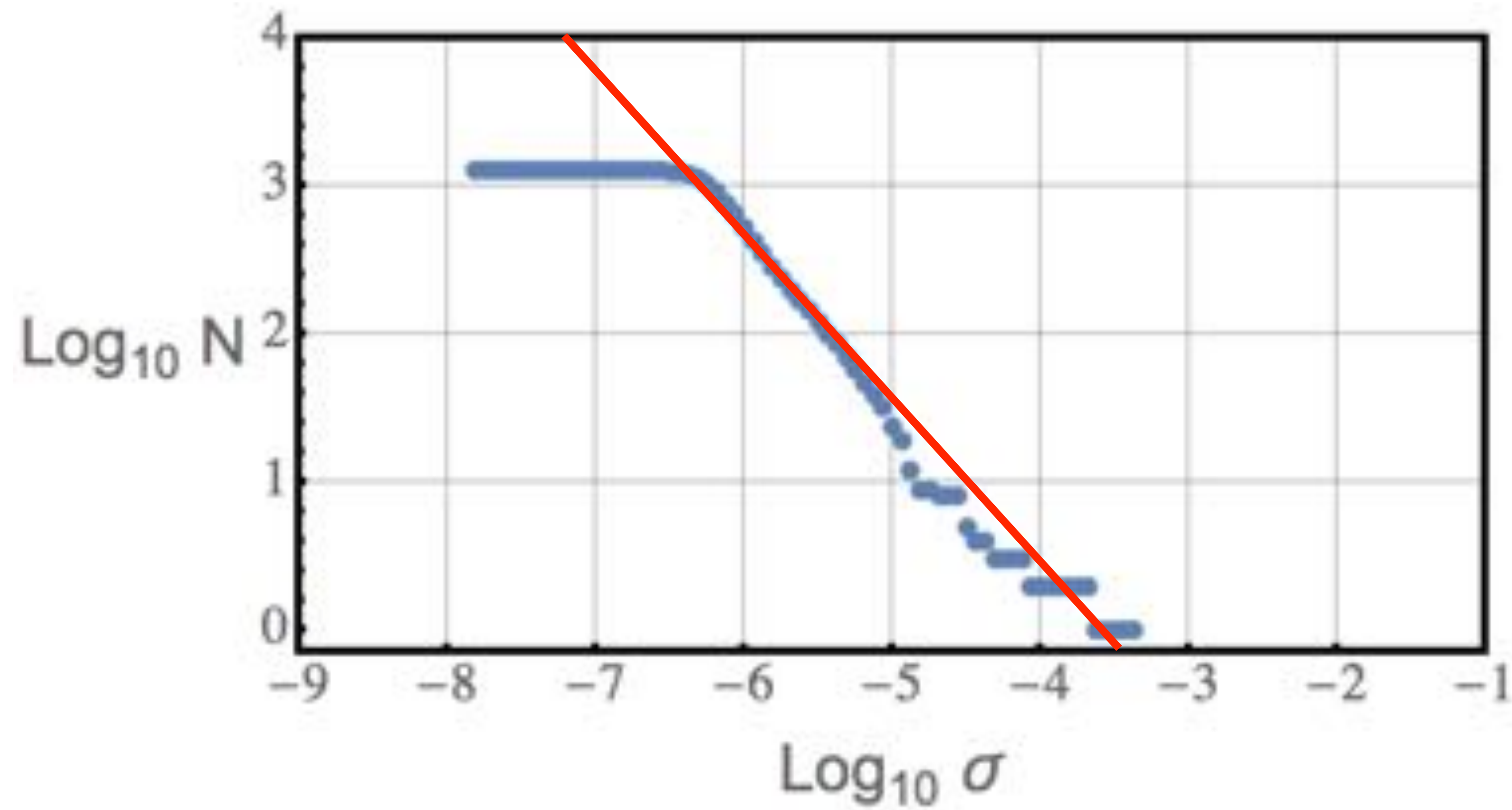

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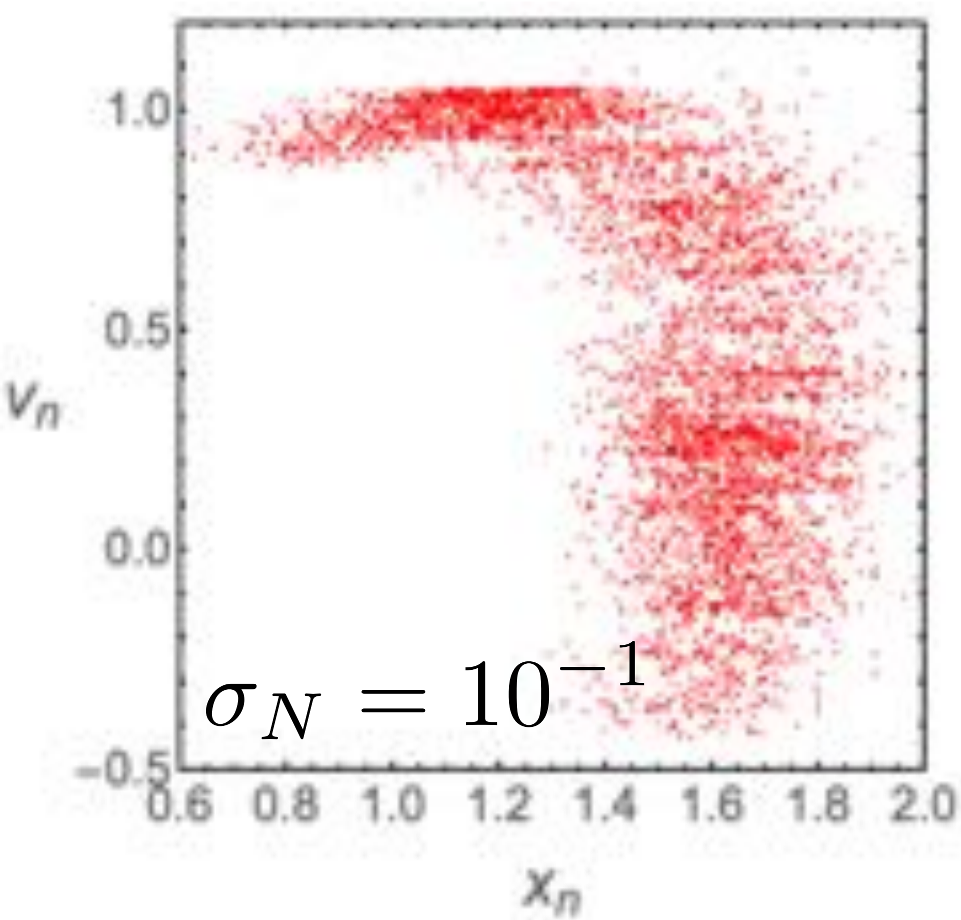

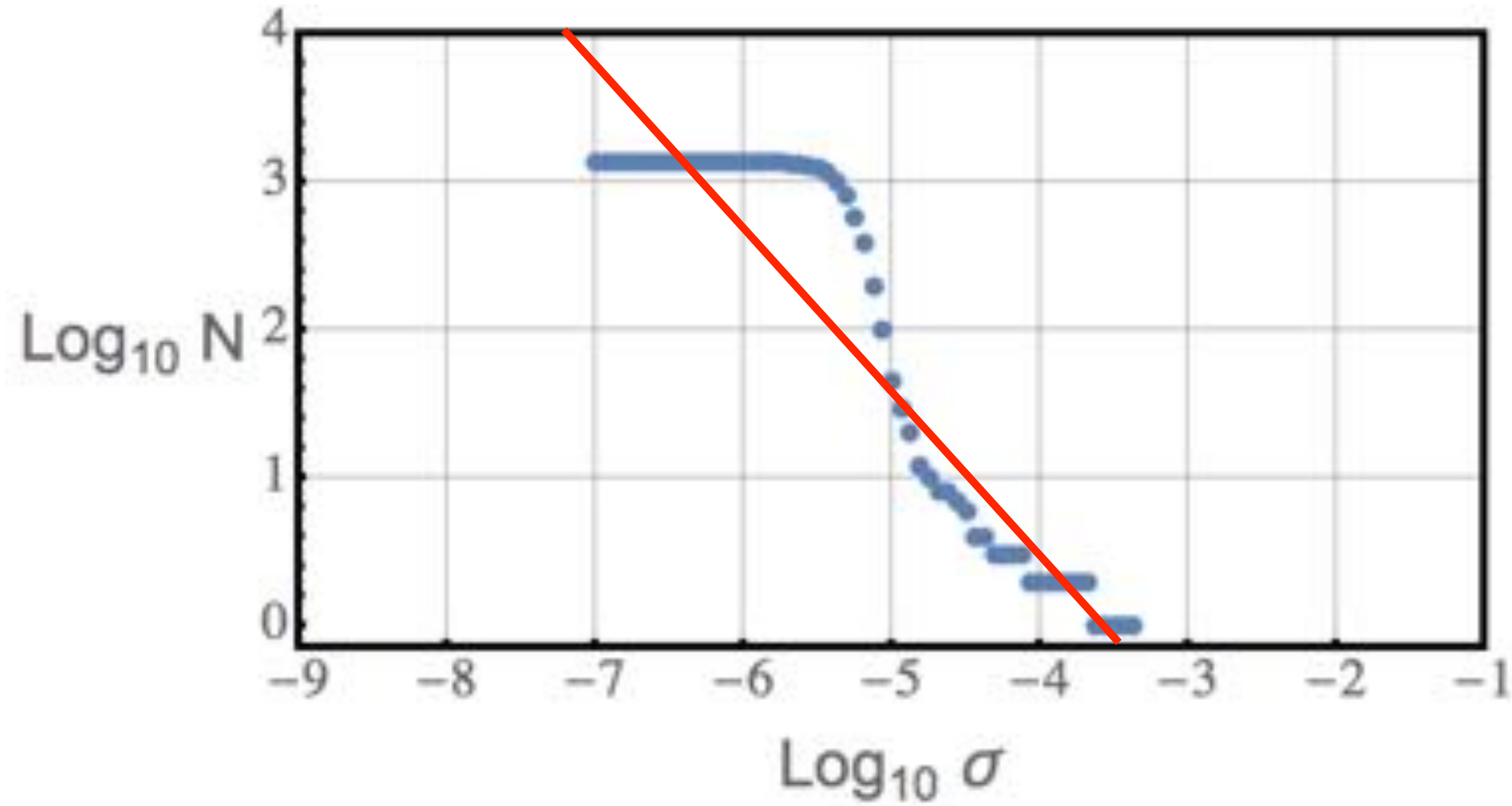

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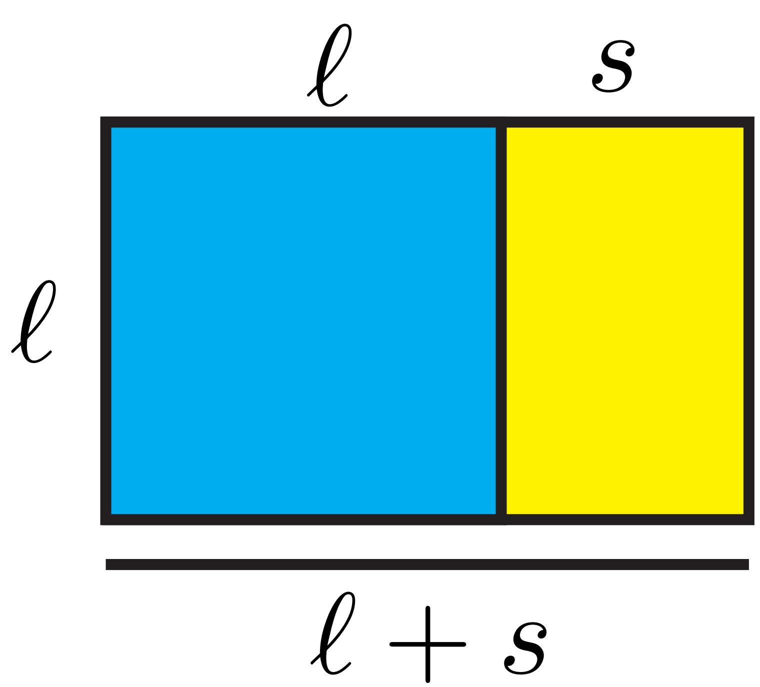
The Golden Rectangle

long : short :: whole : long

$$\varphi = \frac{\ell}{s} = \frac{\ell + s}{\ell} = 1 + \frac{1}{\varphi} = \frac{1+\sqrt{5}}{2} = 1.618\ldots$$

$$\frac{1}{\varphi} = \varphi - 1 = 0.618\ldots$$



Continued Fractions

$$\varphi = 1 + \frac{1}{\varphi} = 1 + \frac{1}{1 + \frac{1}{\varphi}} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}}$$

$\varphi = [1; 1, 1, 1, \cdots]$ **golden ratio**

$\nu = [0; a_1, a_2, \ldots, a_n, 1, 1, 1, \cdots]$ **noble numbers** have golden tails

All the 1s make for slow convergence

Degrees of Irrationality

By contrast,
Liouville's constant

$$L = \sum_{n=1}^{\infty} 10^{-n!} = 0.1100010000000000000000010000000000000000000000000\ldots$$

is very easily approximated
by rational numbers!

Most Irrational

For any irrational number,
there exists infinitely many relative prime integers such that

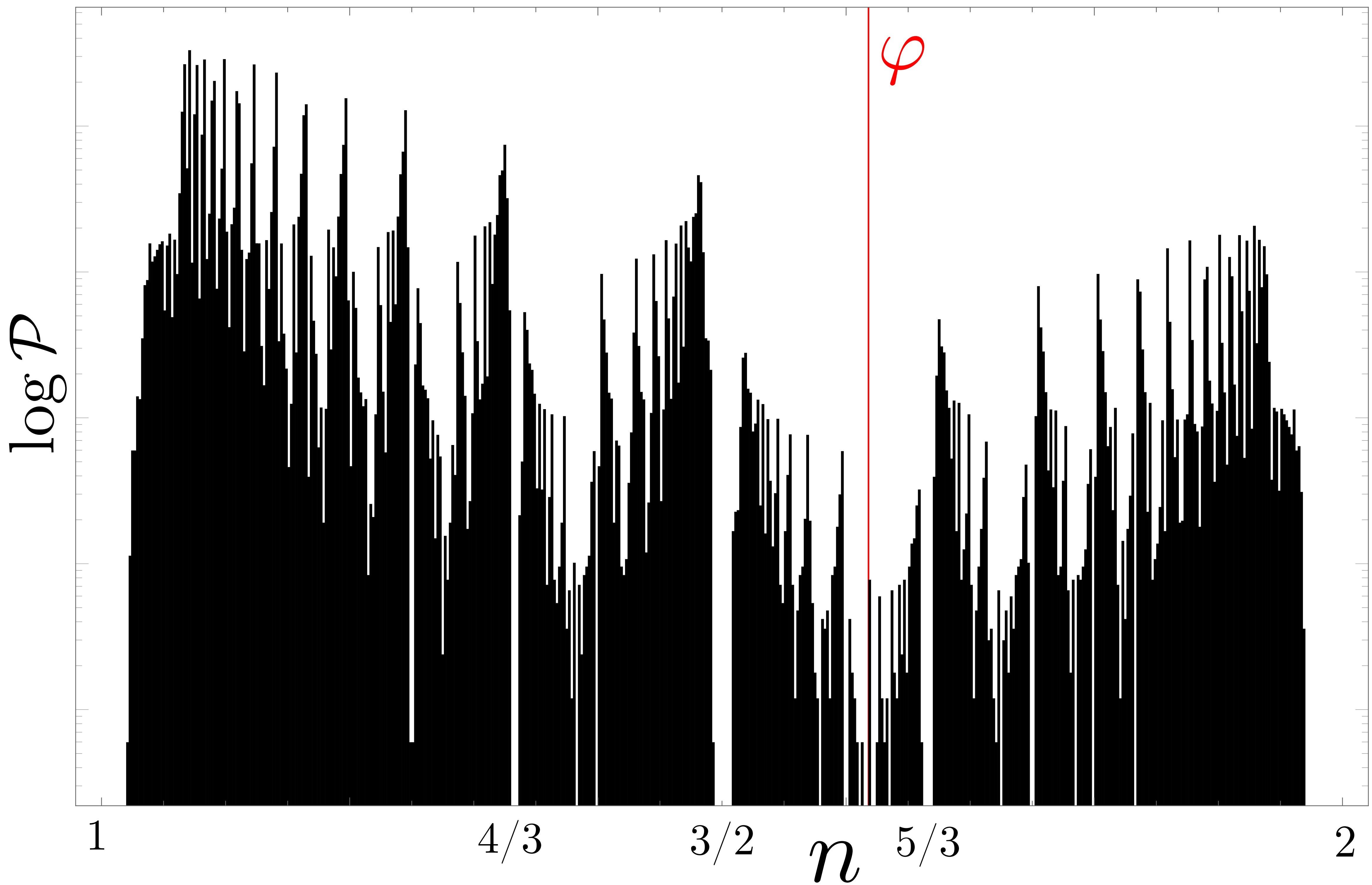
$$\left|\alpha - \frac{p}{q}\right| < \frac{1}{\sqrt{5}q^2}$$

Double denominator quarters error

Golden ratio saturates Hurwitz's inequality

Noble Number Distribution

of hard to rationally approximate irrationals

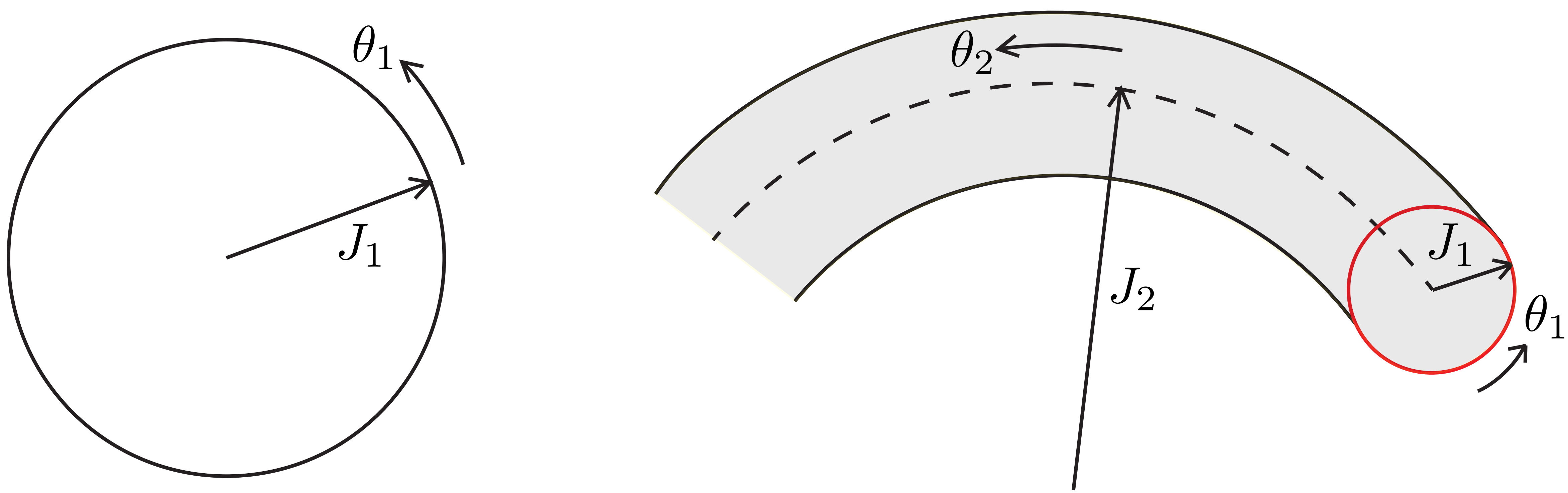


KAM Theorem

Action-angle $\{J, \theta\}$ variables for integrable systems

$$\frac{dQ_n}{dt} = +\frac{\partial H}{\partial P_n} \qquad \frac{d\theta_n}{dt} = +\frac{\partial H}{\partial J_n} = \omega_n \qquad \theta_n = \text{const} + \omega_n t$$

$$\frac{dP_n}{dt} = -\frac{\partial H}{\partial Q_n} \qquad \frac{dJ_n}{dt} = -\frac{\partial H}{\partial \theta_n} = 0 \qquad J_n = \text{const}$$

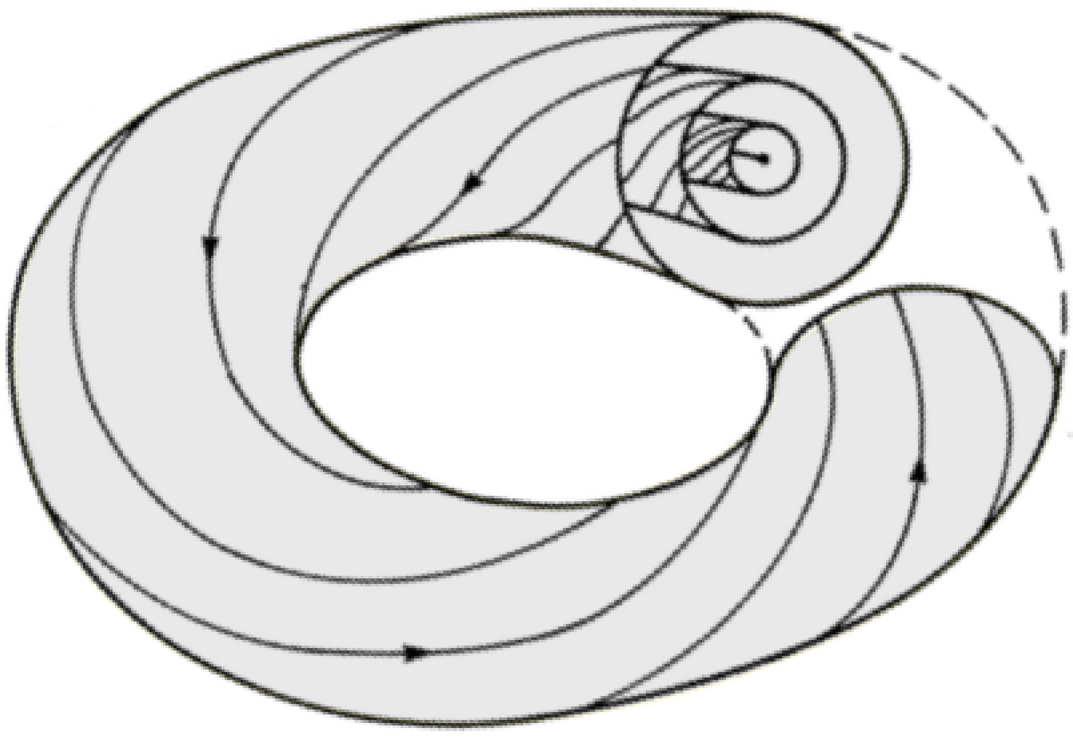


1-torus = circle

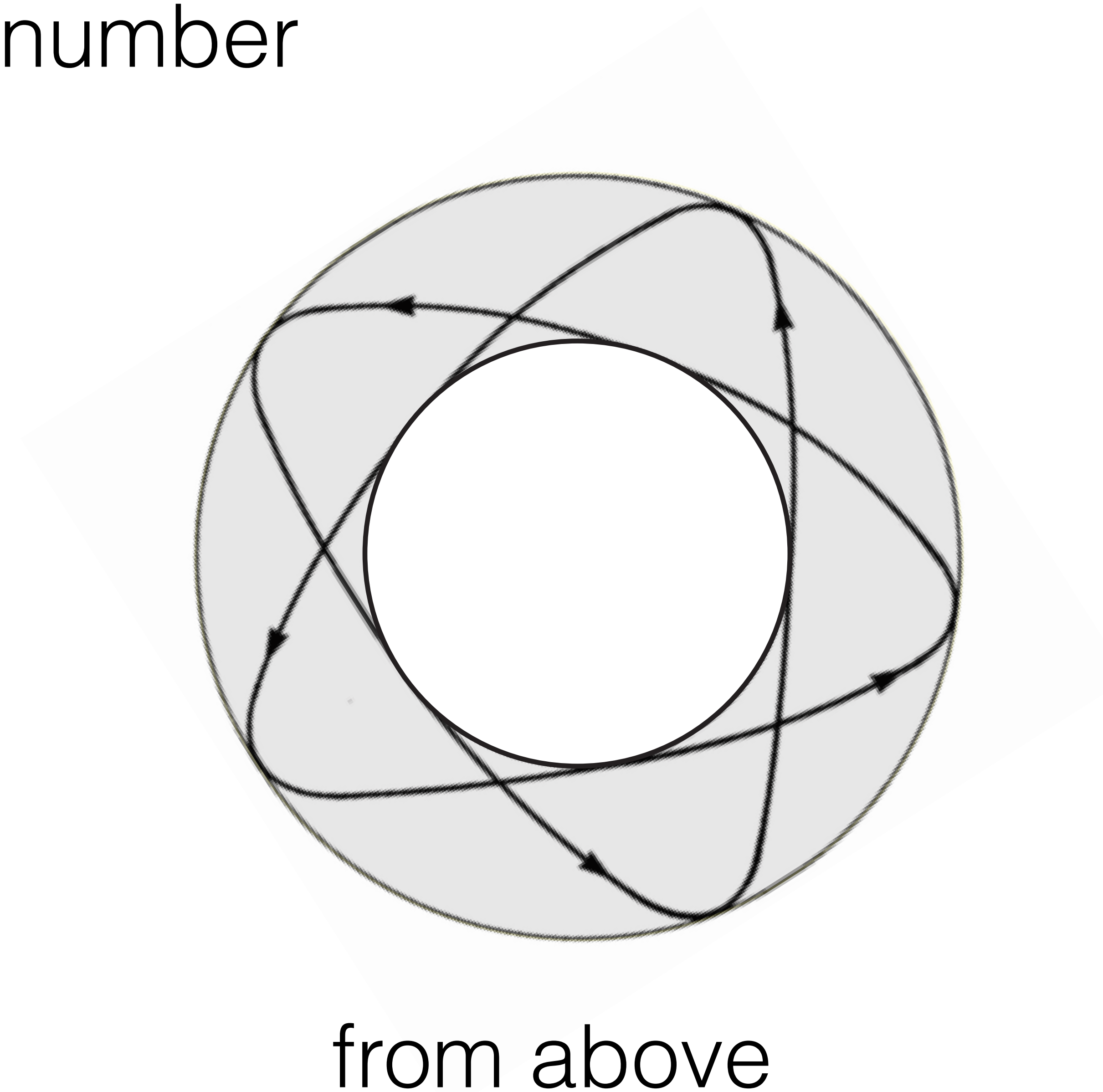
2-torus

KAM Theorem

Winding number



tori foliate phase space



from above

$$\frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = W$$

$$\frac{\omega_1}{\omega_2} = \frac{2}{5}$$

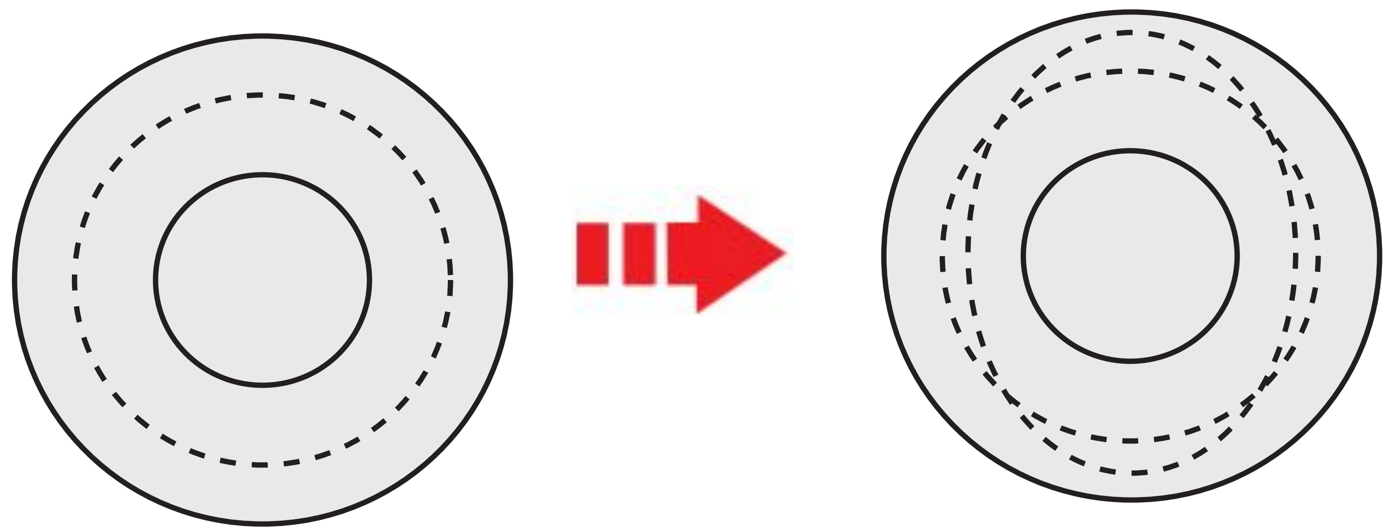
KAM Theorem

Perturbations destroy rational resonant tori

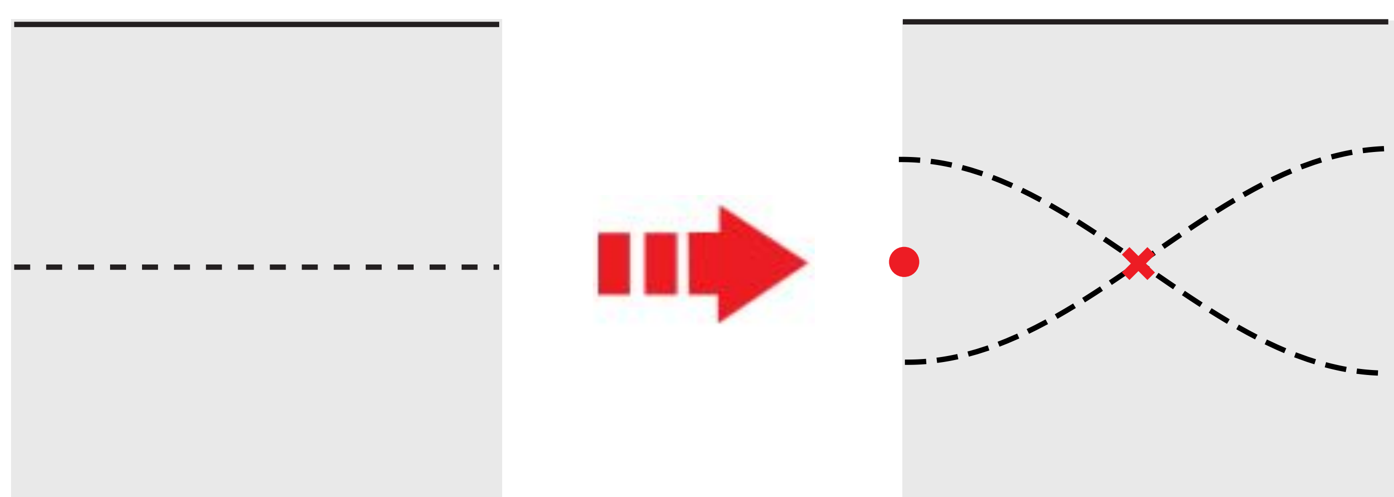
$$H' = H + \epsilon H_p$$

rational W

irrational W



Unwrapped



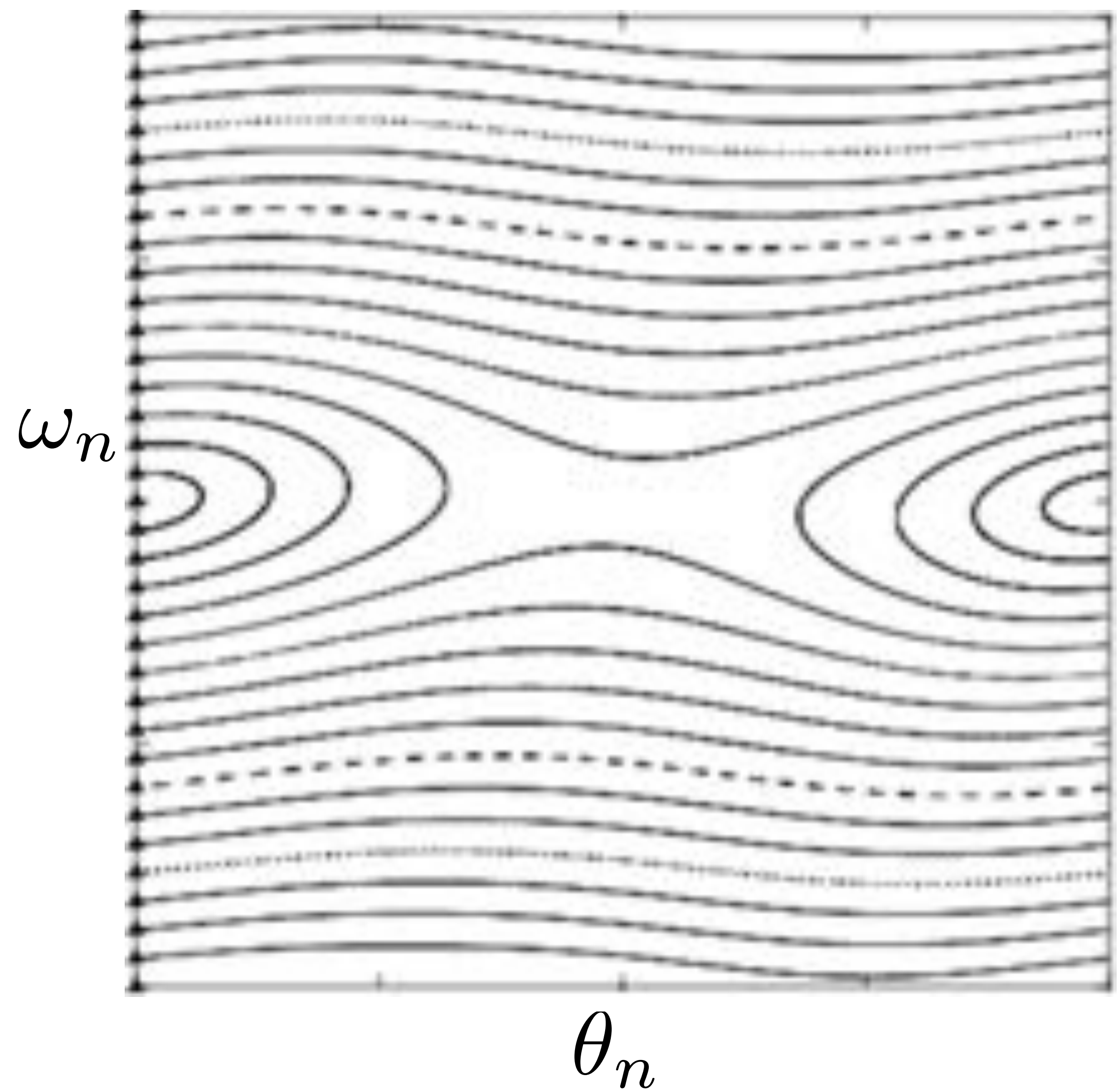
KAM Theorem

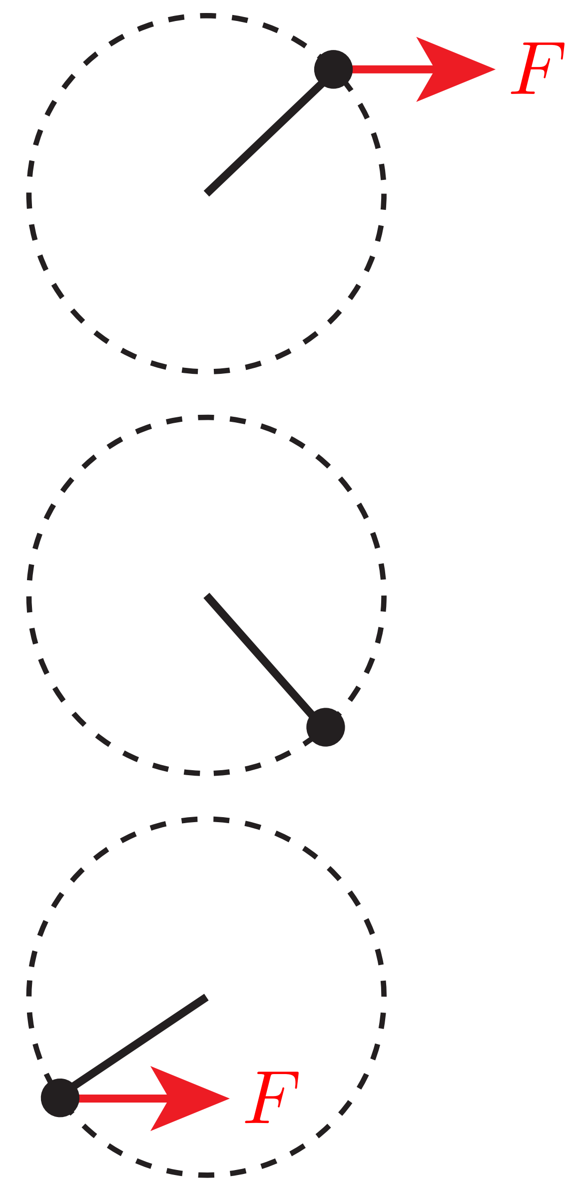
Perturbations destroy rational resonant tori

$$H' = H + \epsilon H_p$$

kicked rotor standard map

$\epsilon \propto F < F_c$





Torus with golden ratio winding number destroyed last

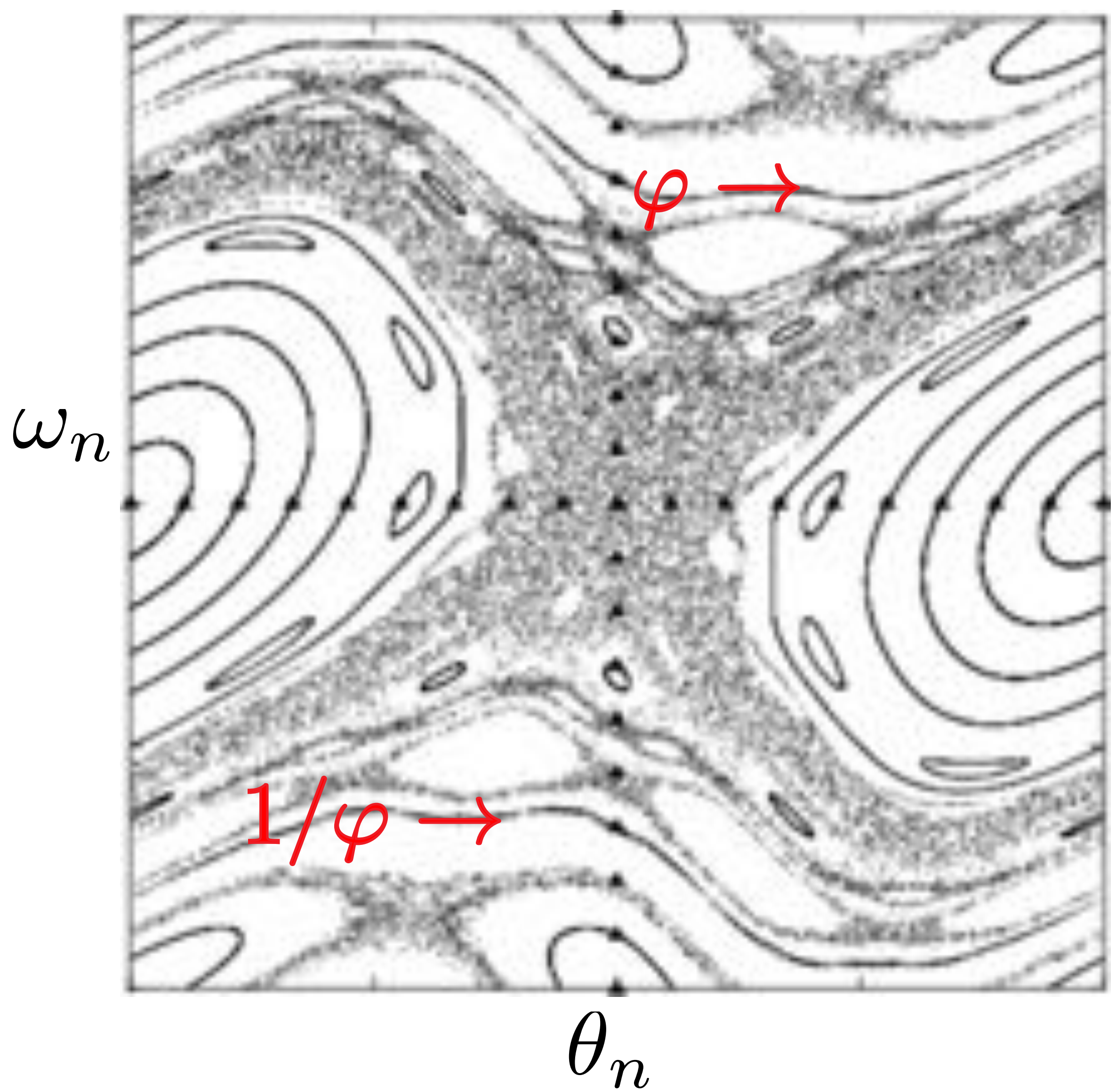
KAM Theorem

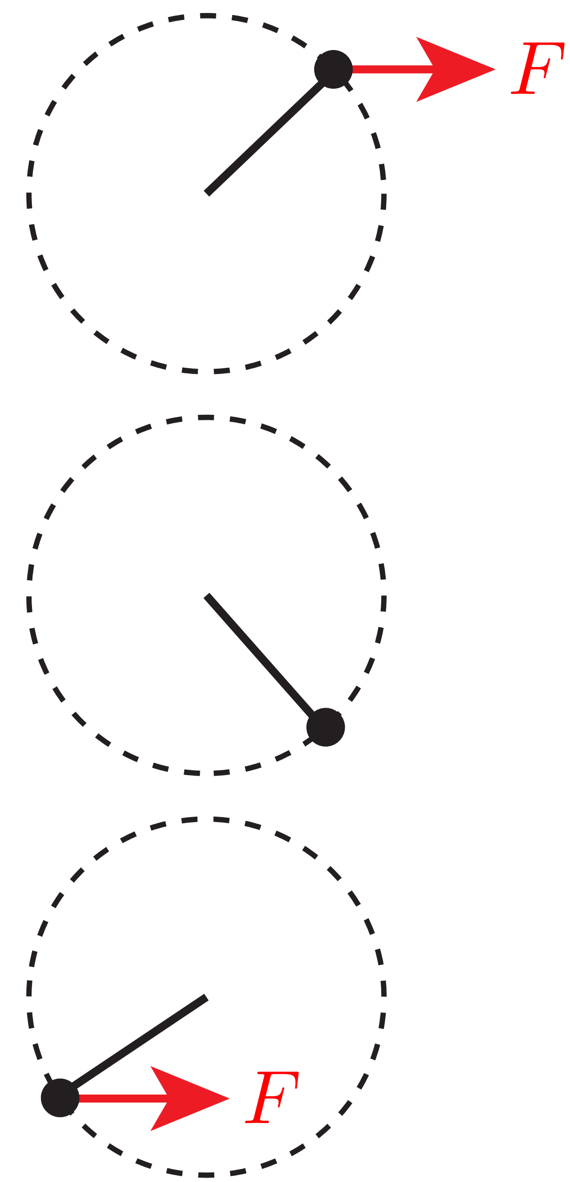
Perturbations destroy rational resonant tori

$$H' = H + \epsilon H_p$$

kicked rotor standard map

$\epsilon \propto \boldsymbol{F} = \boldsymbol{F_c}$





Torus with golden ratio winding number destroyed last

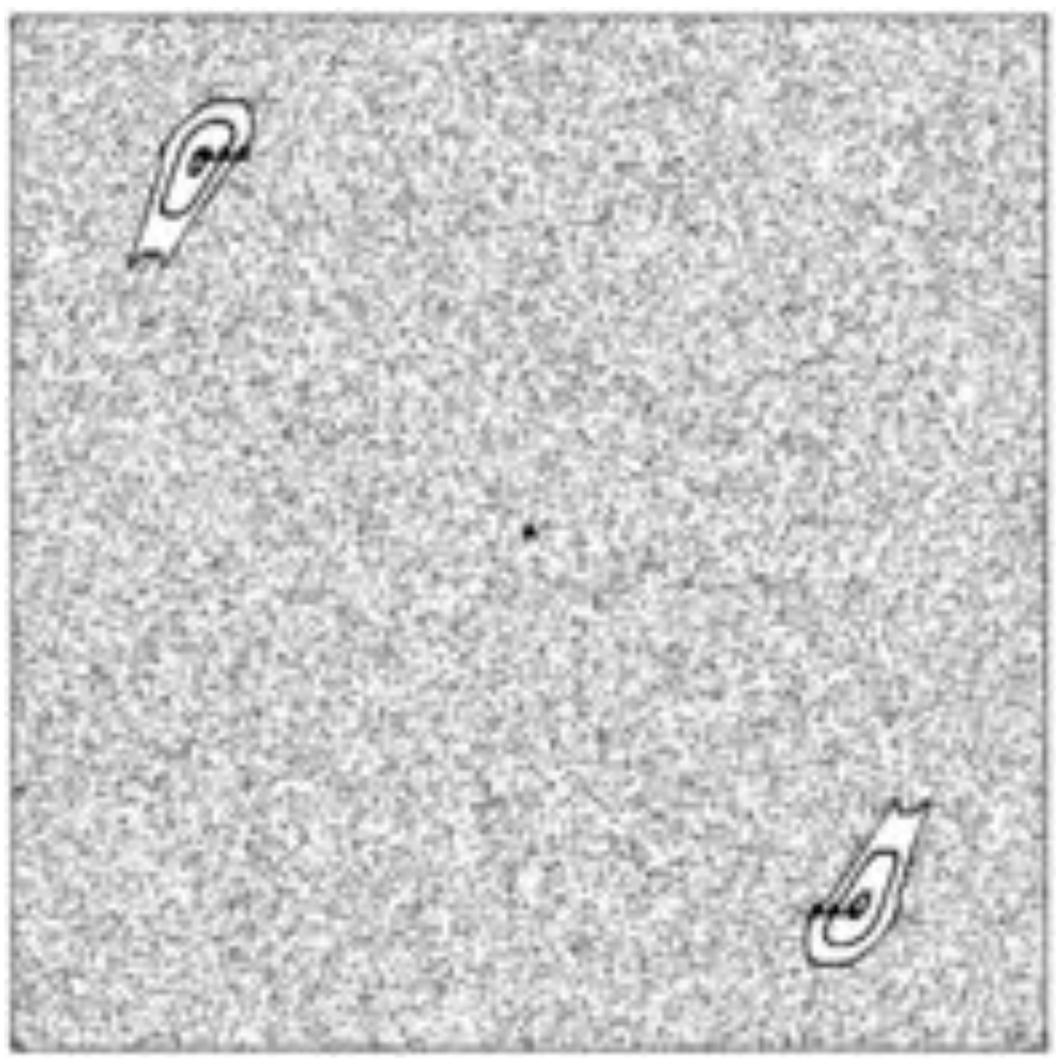
KAM Theorem

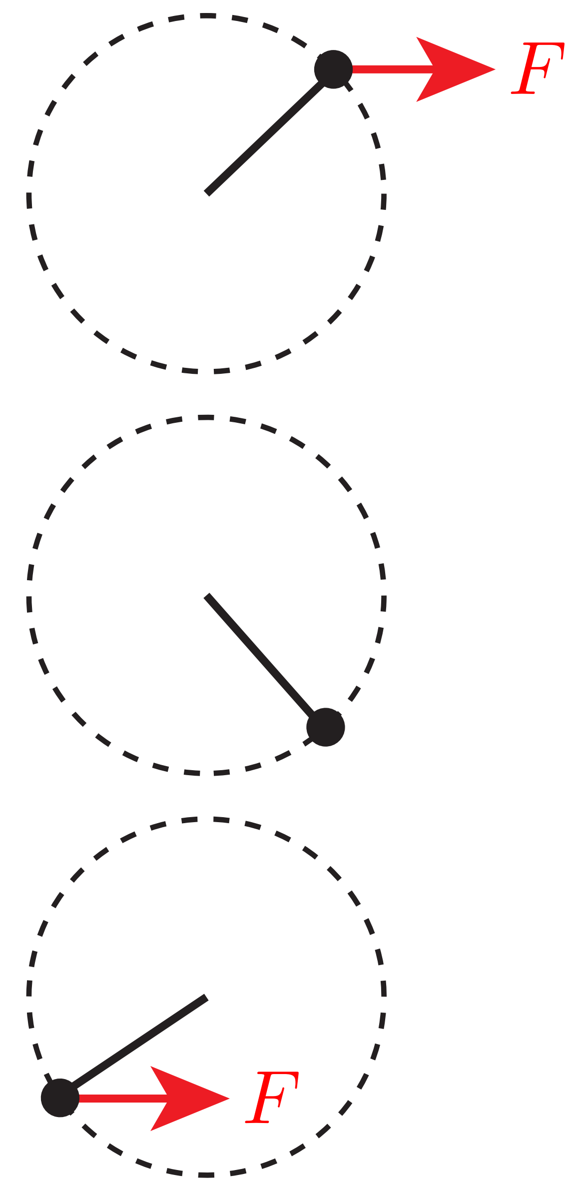
Perturbations destroy rational resonant tori

$$H' = H + \epsilon H_p$$

kicked rotor standard map

$\epsilon \propto \boldsymbol{F} > \boldsymbol{F_c}$





Torus with golden ratio winding number destroyed last

φ

painter’s most aesthetic ratio

number theory’s most irrational number

nonlinear dynamics most robust configuration

Outline

Dynamical Attractors

Spectral Scaling

Golden Ratio

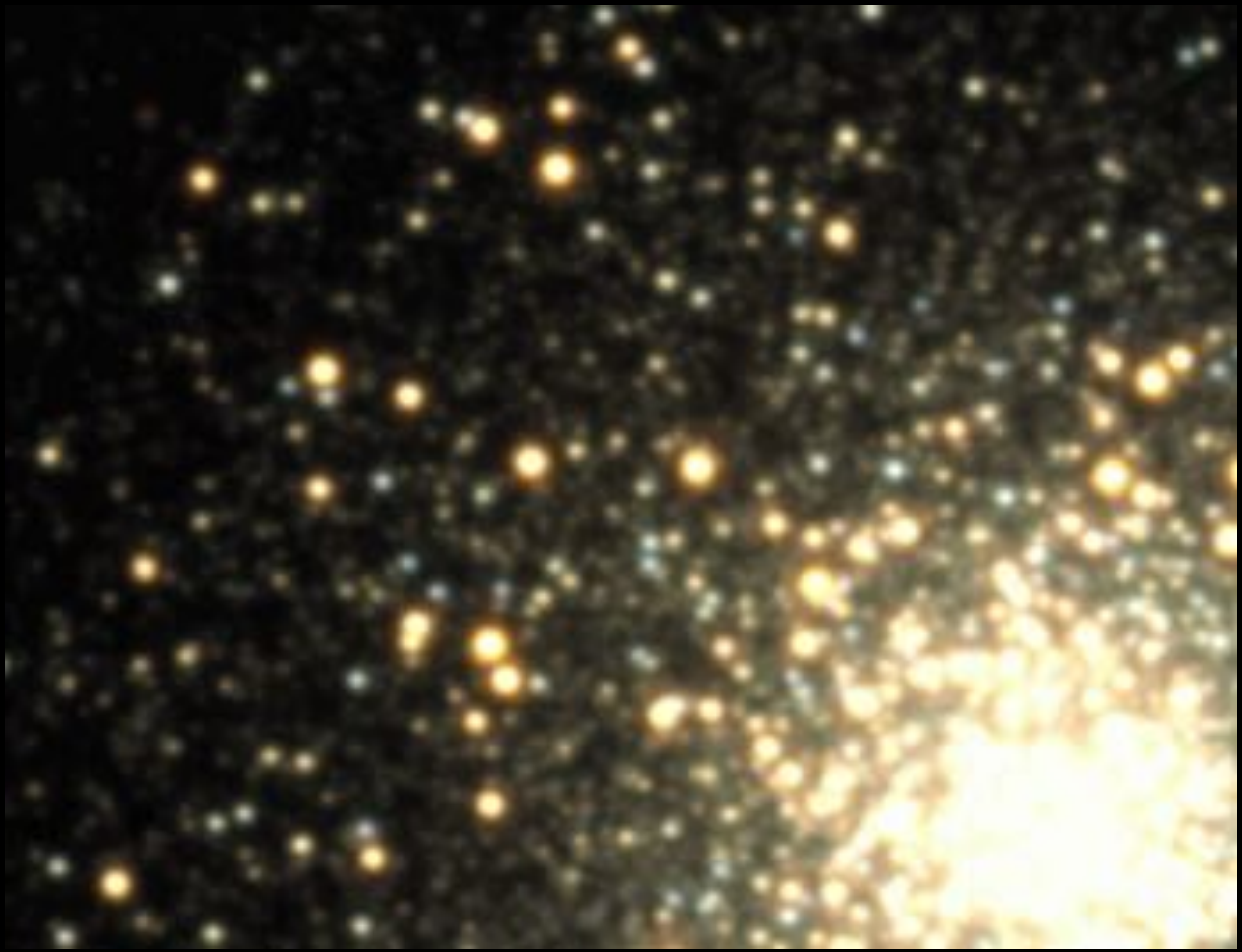
Variable Stars

Analysis

Toy Models

Conclusions

RR Lyrae in M3



http://www.astro.princeton.edu/~jhartman/M3_movies.html

RR Lyrae Summary

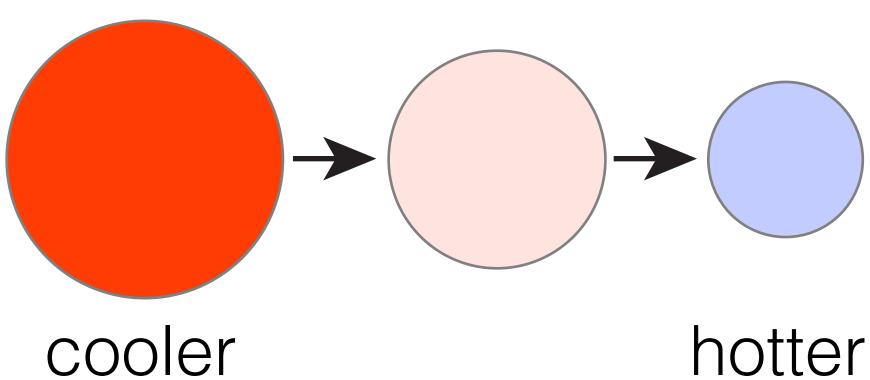
Old $T > 10^{10}$ years

MS → **RG** → PN → WD

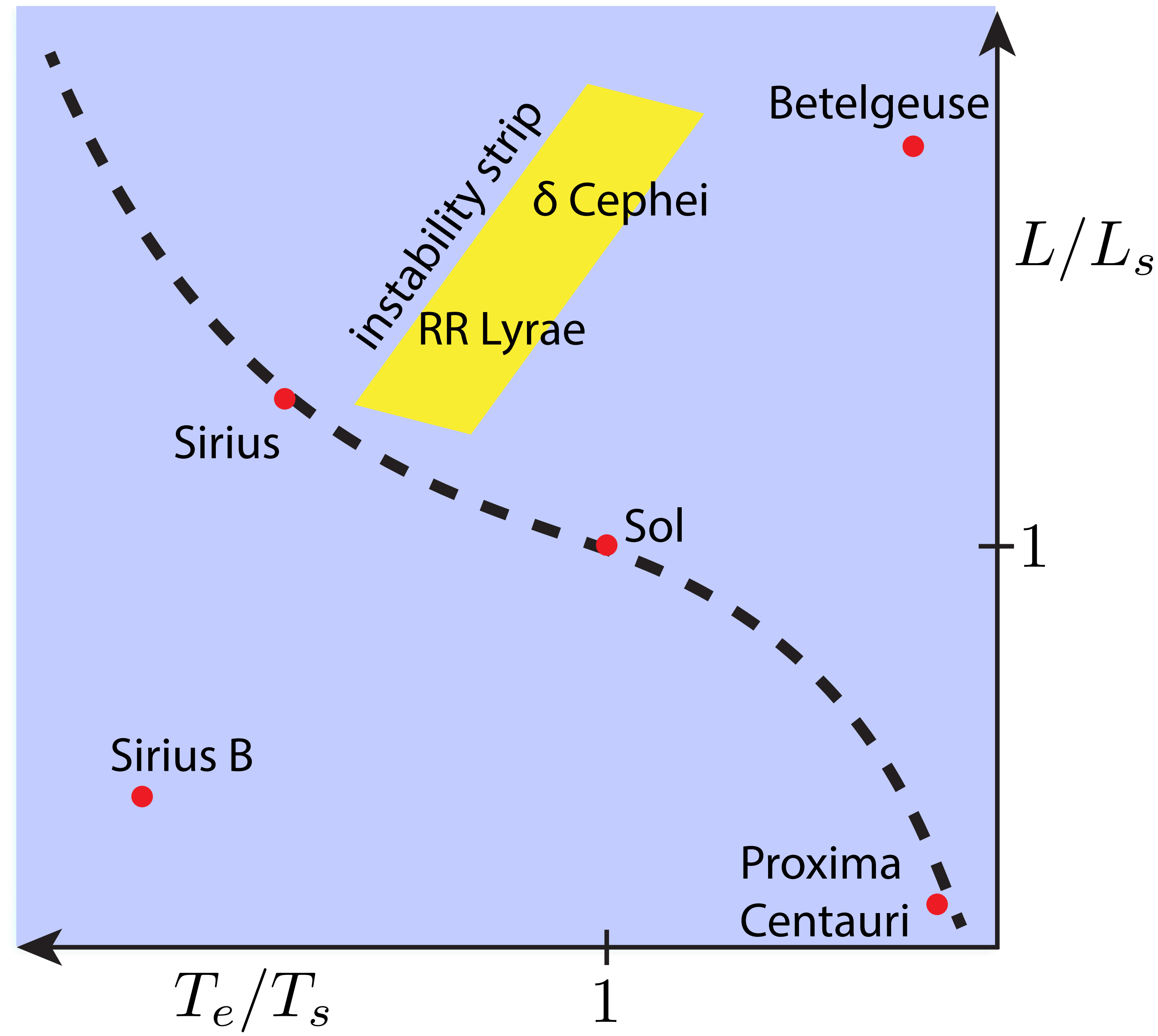
Helium fusing

Variable 0.2 d < δT < 1.1 d

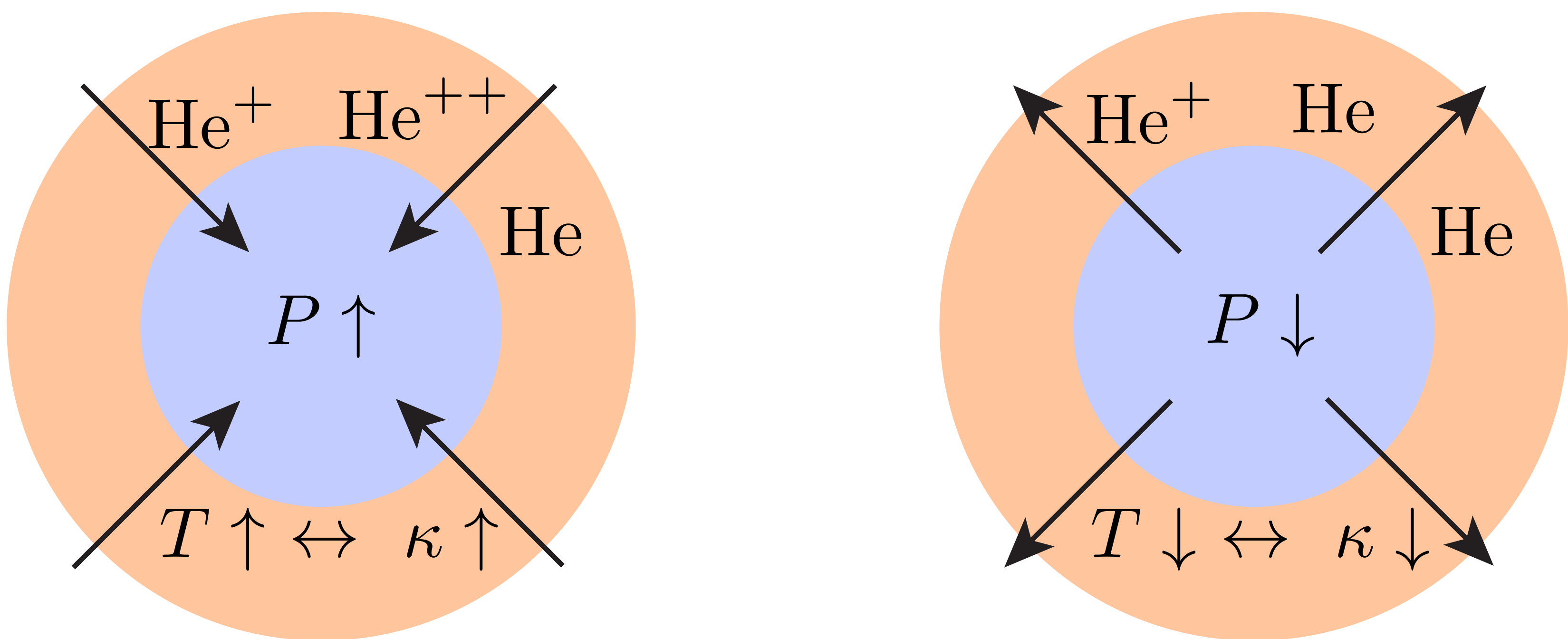
2X brightness



Hertzsprung-Russell Diagram



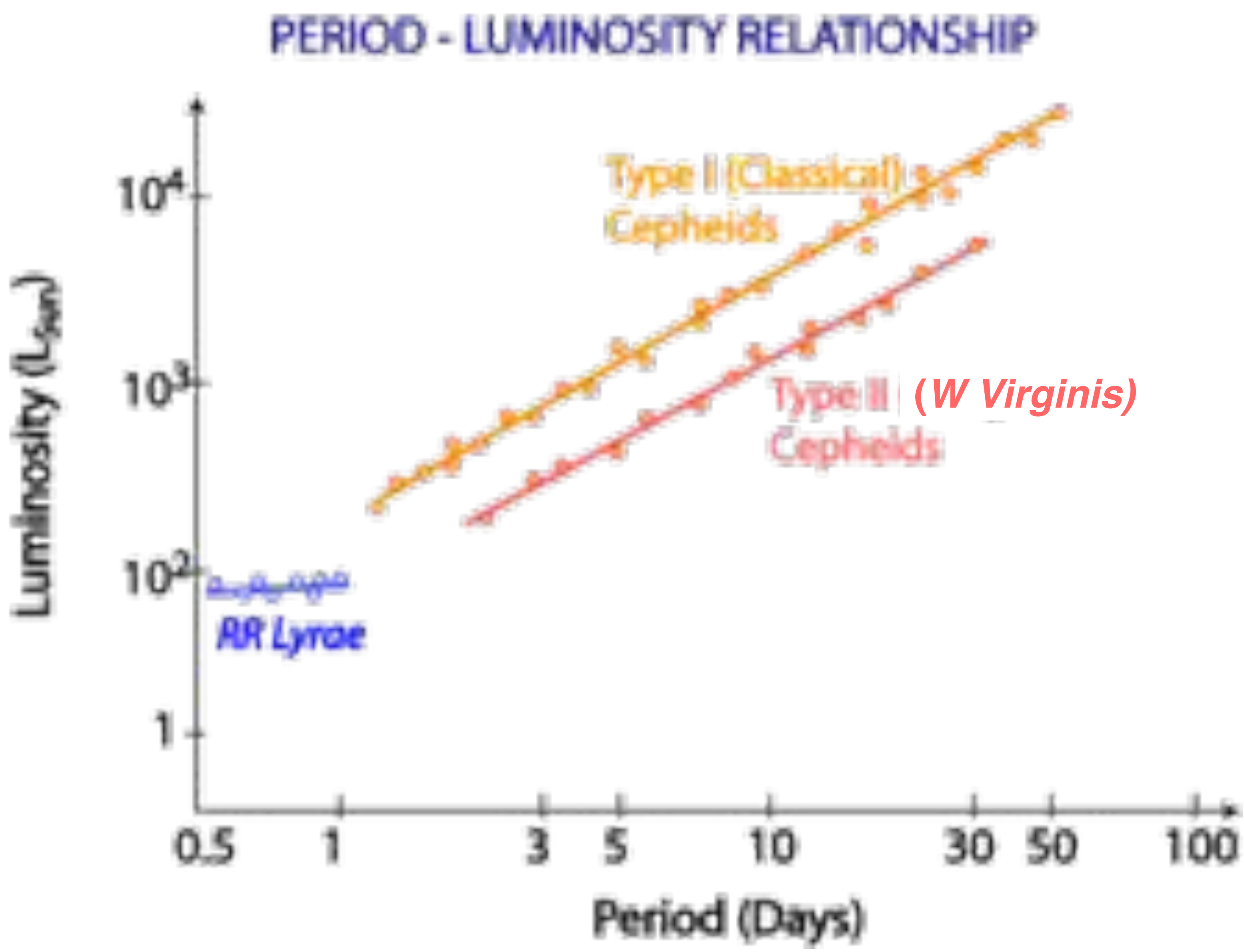
Edddington valve (opacity κ) mechanism



ionization increases opacity & chokes heat flow

recombination decreases opacity & releases heat

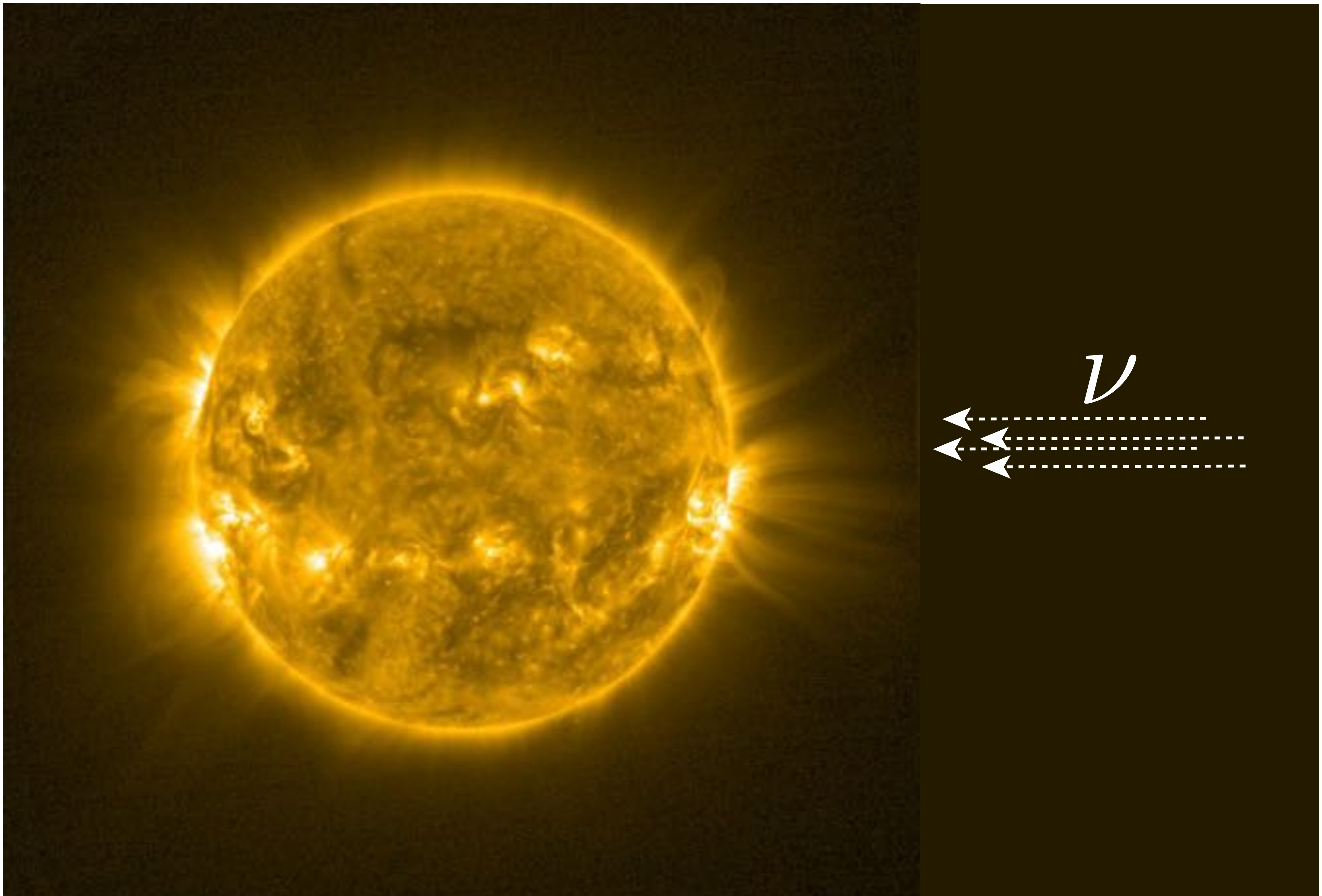
Standard Candles



http://scioly.org/wiki/index.php/Astronomy#Cepheids_and_RR_Lyrae

The Cepheid Galactic Internet

Communication via variable star neutrino beam modulation



Learned, Kudritzki, Pakvasa, & Zee, arXiv:0809.0339 (2008)

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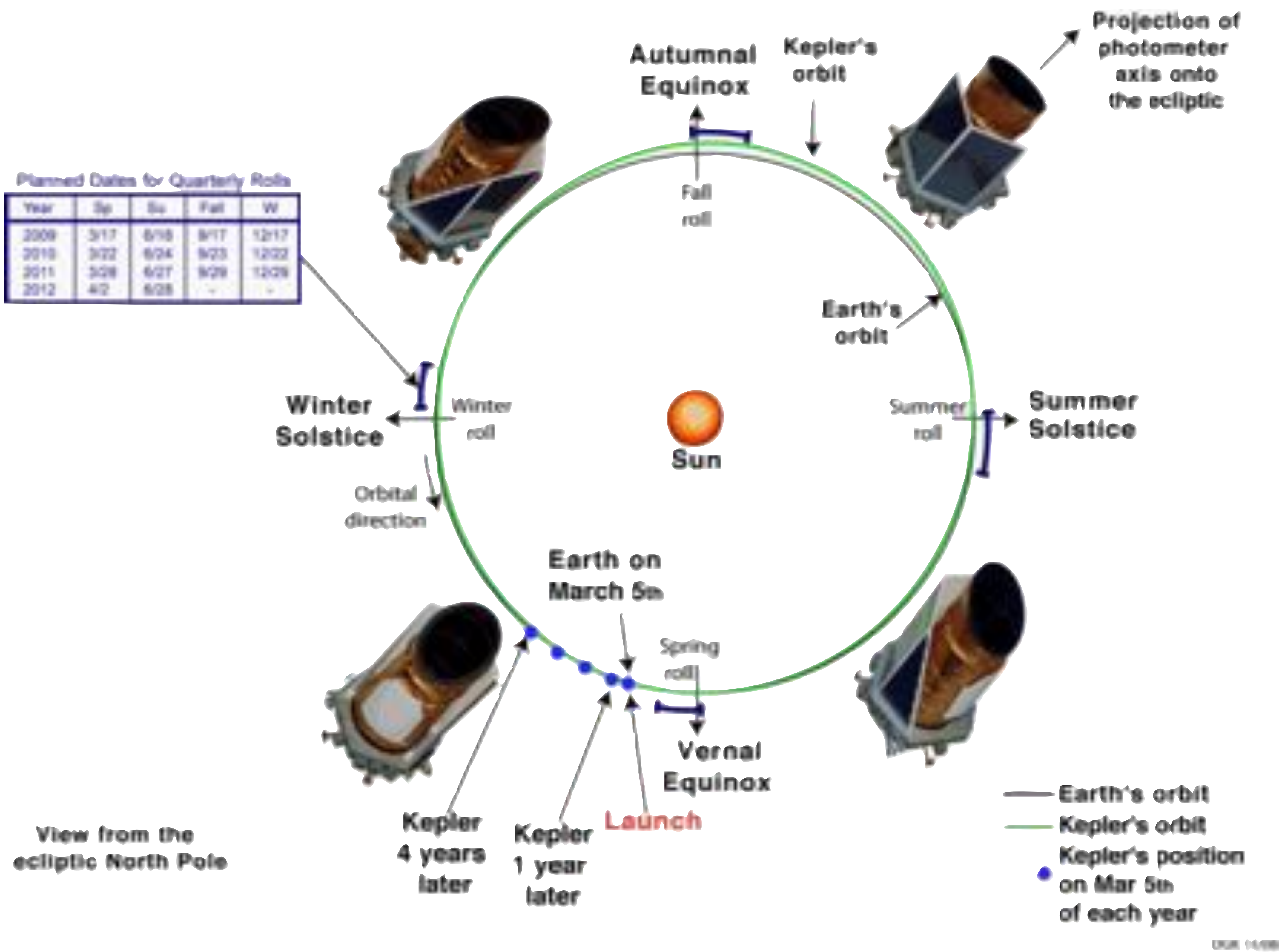
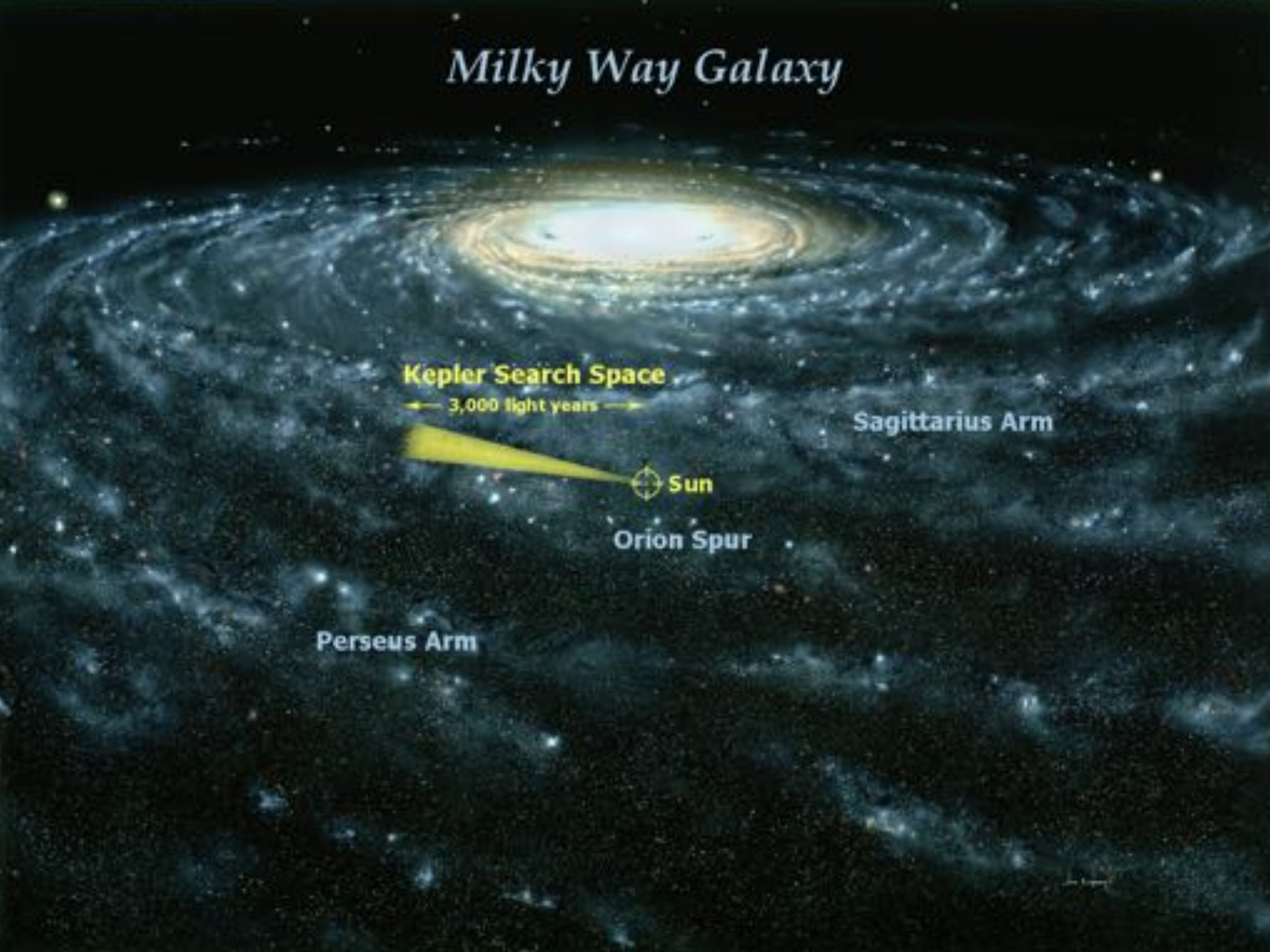
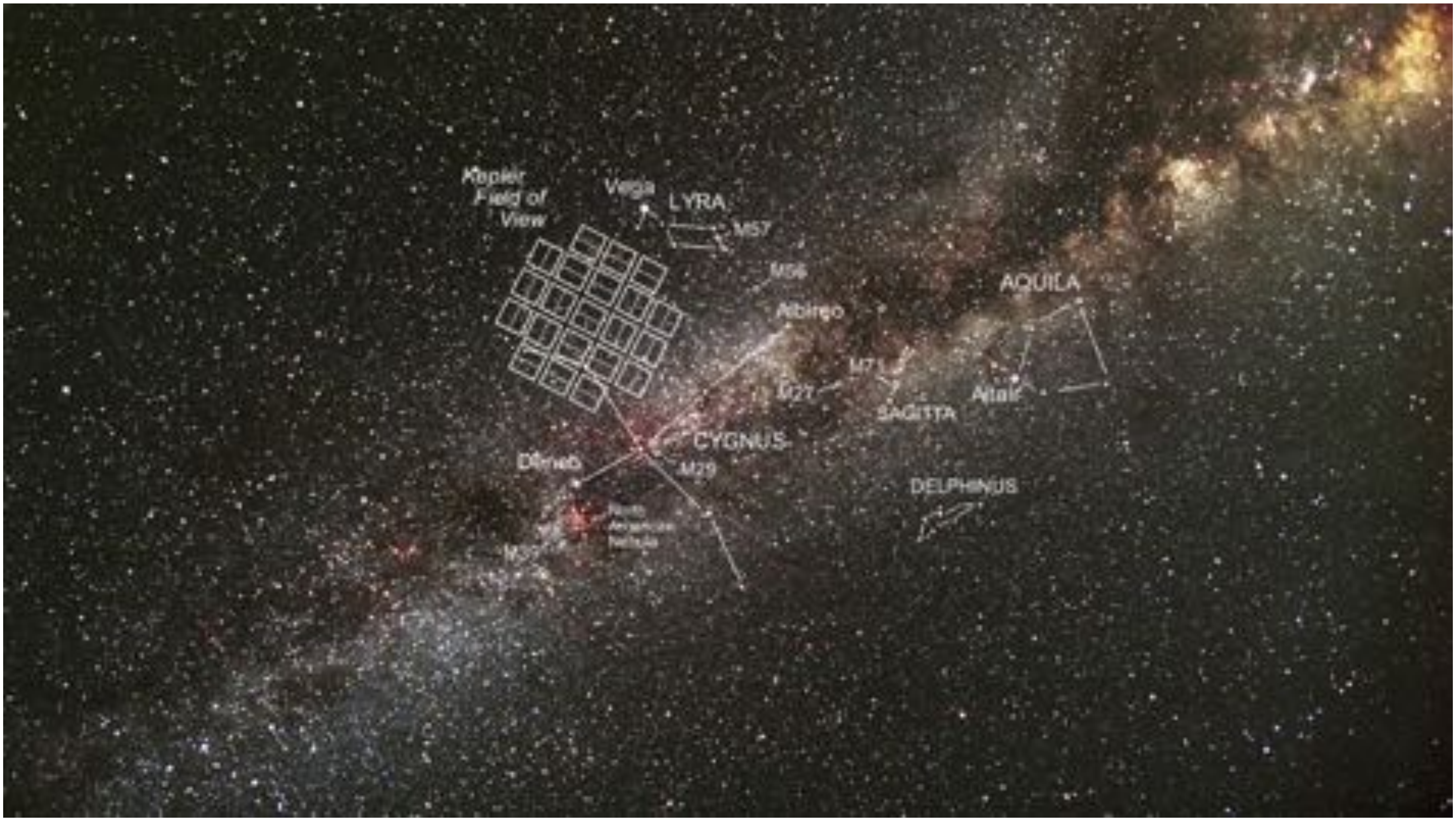
Kepler Space Telescope



Year	Sp	Su	Fall	W
2009	3/17	6/18	9/17	12/17
2010	3/22	6/24	9/23	12/22
2011	3/28	6/27	9/29	12/29
2012	4/2	6/28	-	-

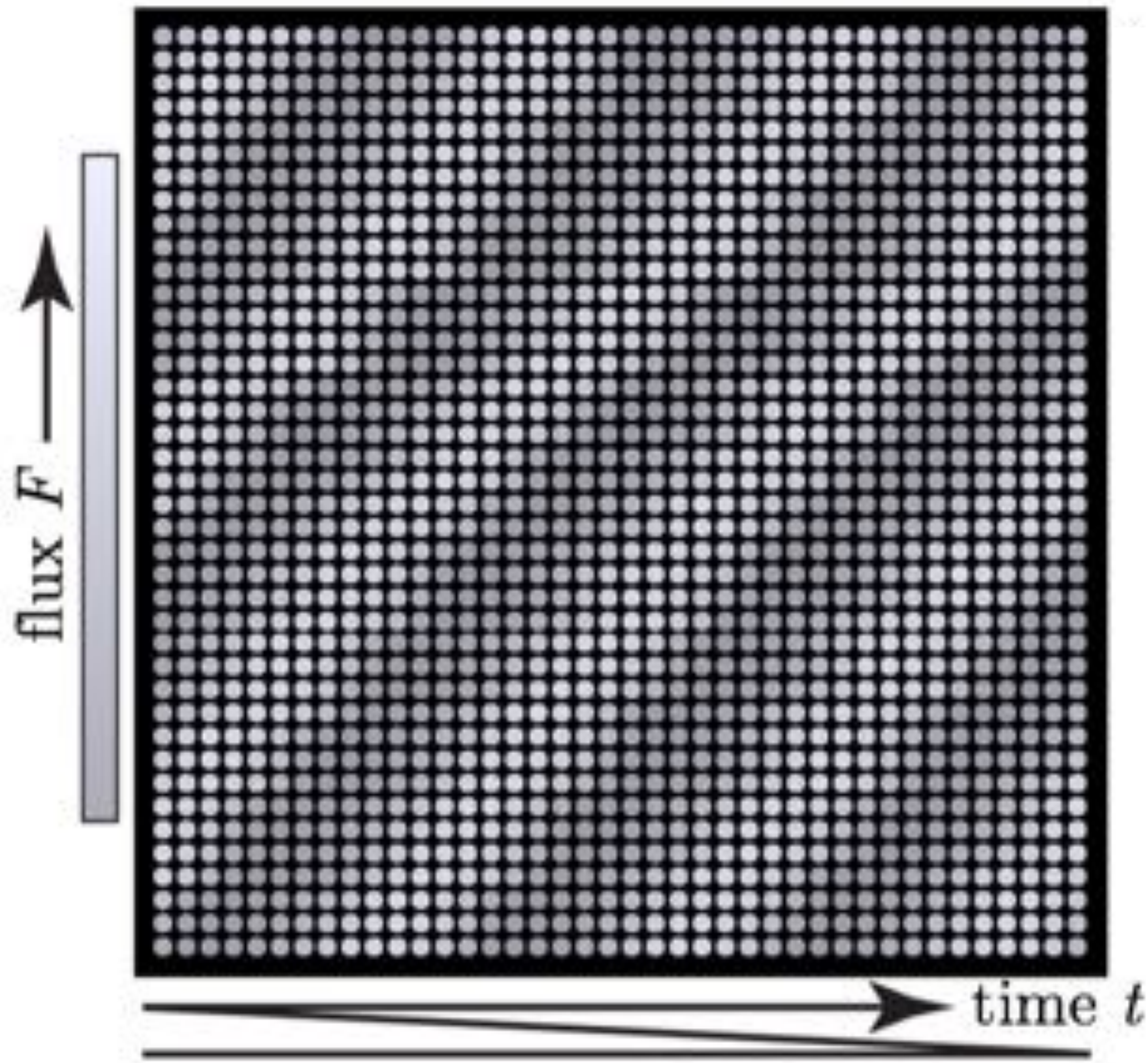

Milky Way Galaxy
Kepler Search Space
3,000 light years
Sagittarius Arm
Sun
Orion Spur
Perseus Arm

Kepler Field of View



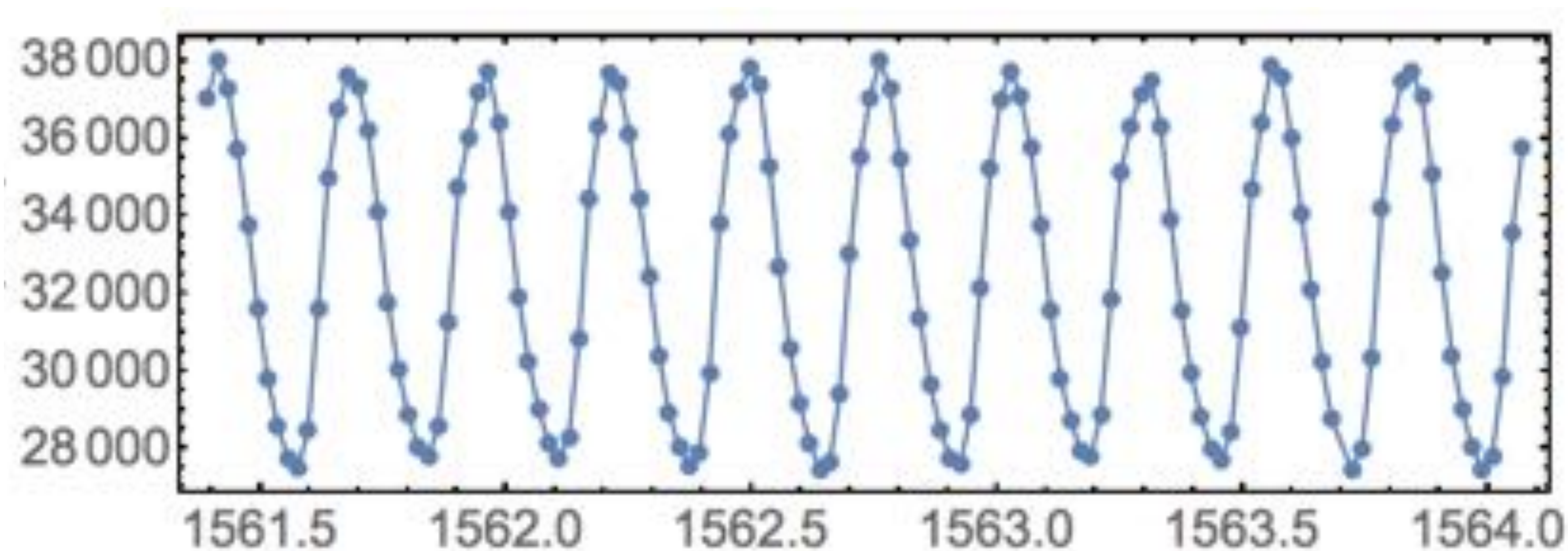
Hippke's Star: KIC 5520878

Brightness fluctuations



Flux Time Series

Long Cadence Data at 30-minute Intervals



Flux F (e/s)

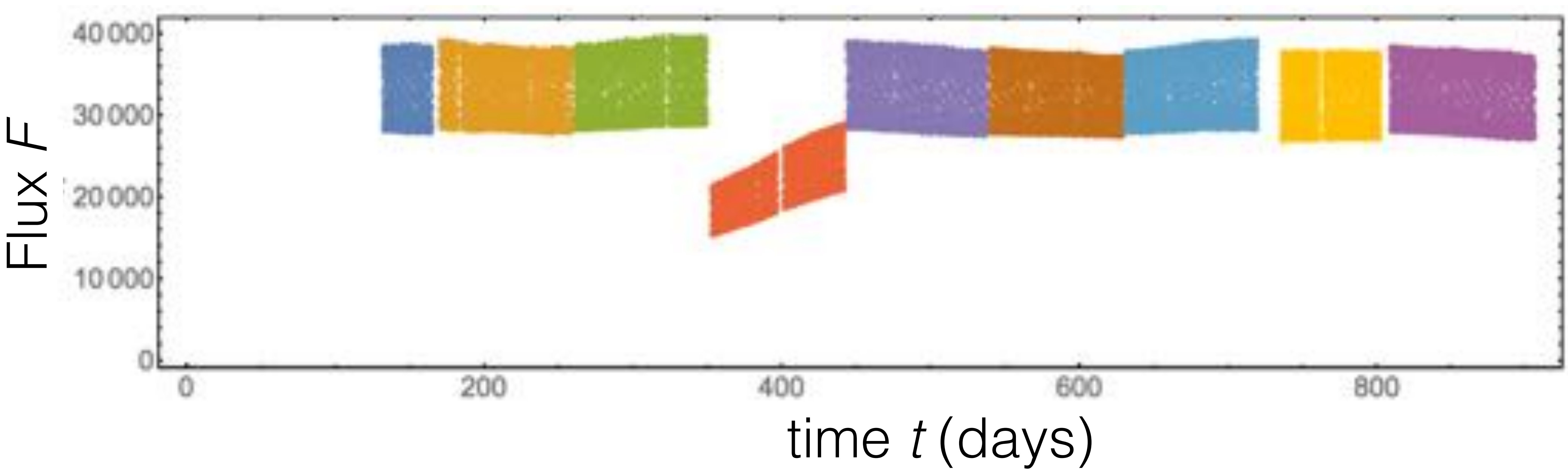
time t (days)

2 Data Analysis Pipelines

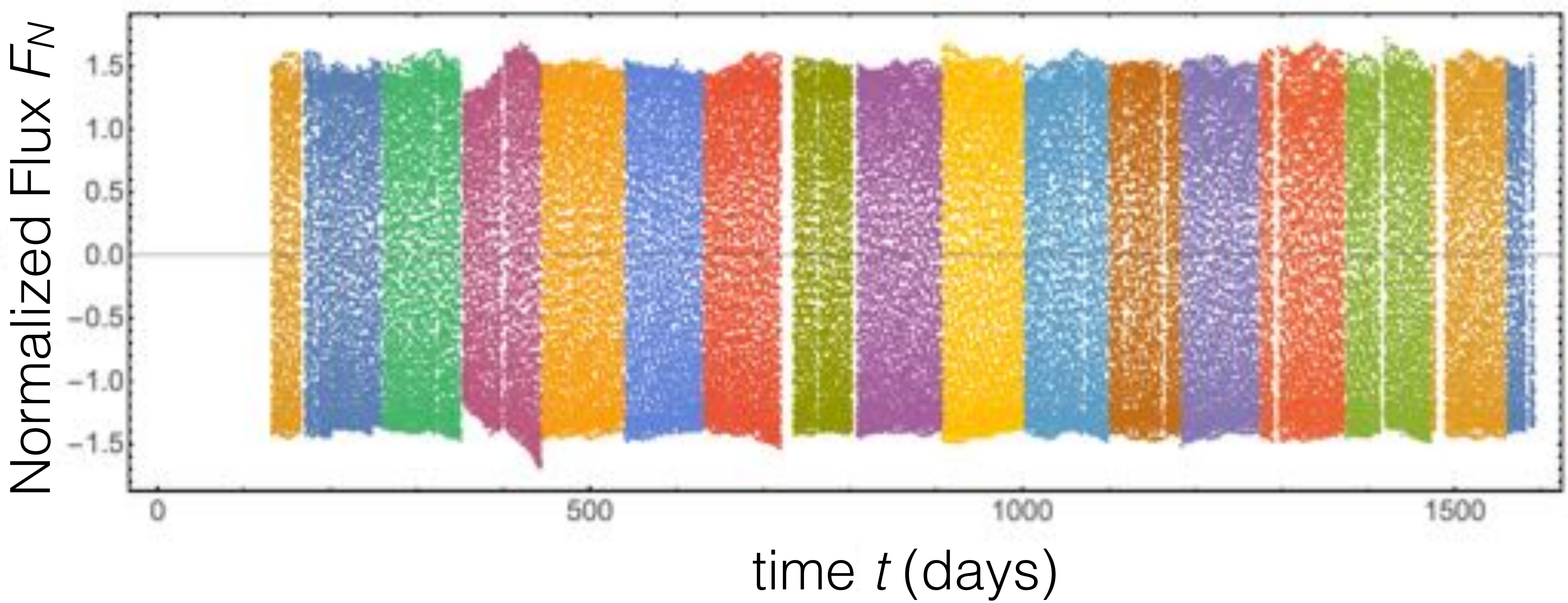
Mathematica 10

MatLab, Period04, C++

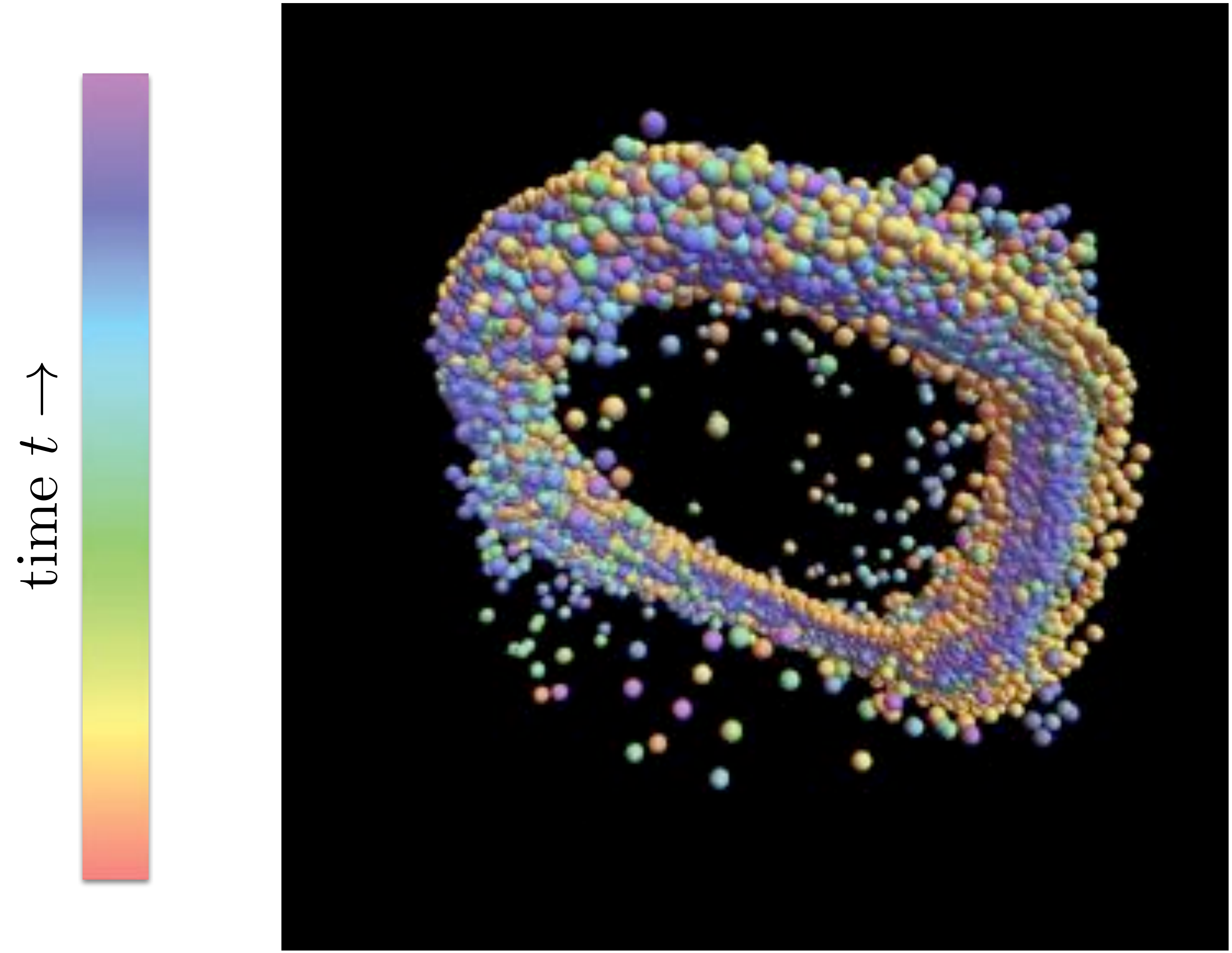
Raw Flux



Detrend & Rescale

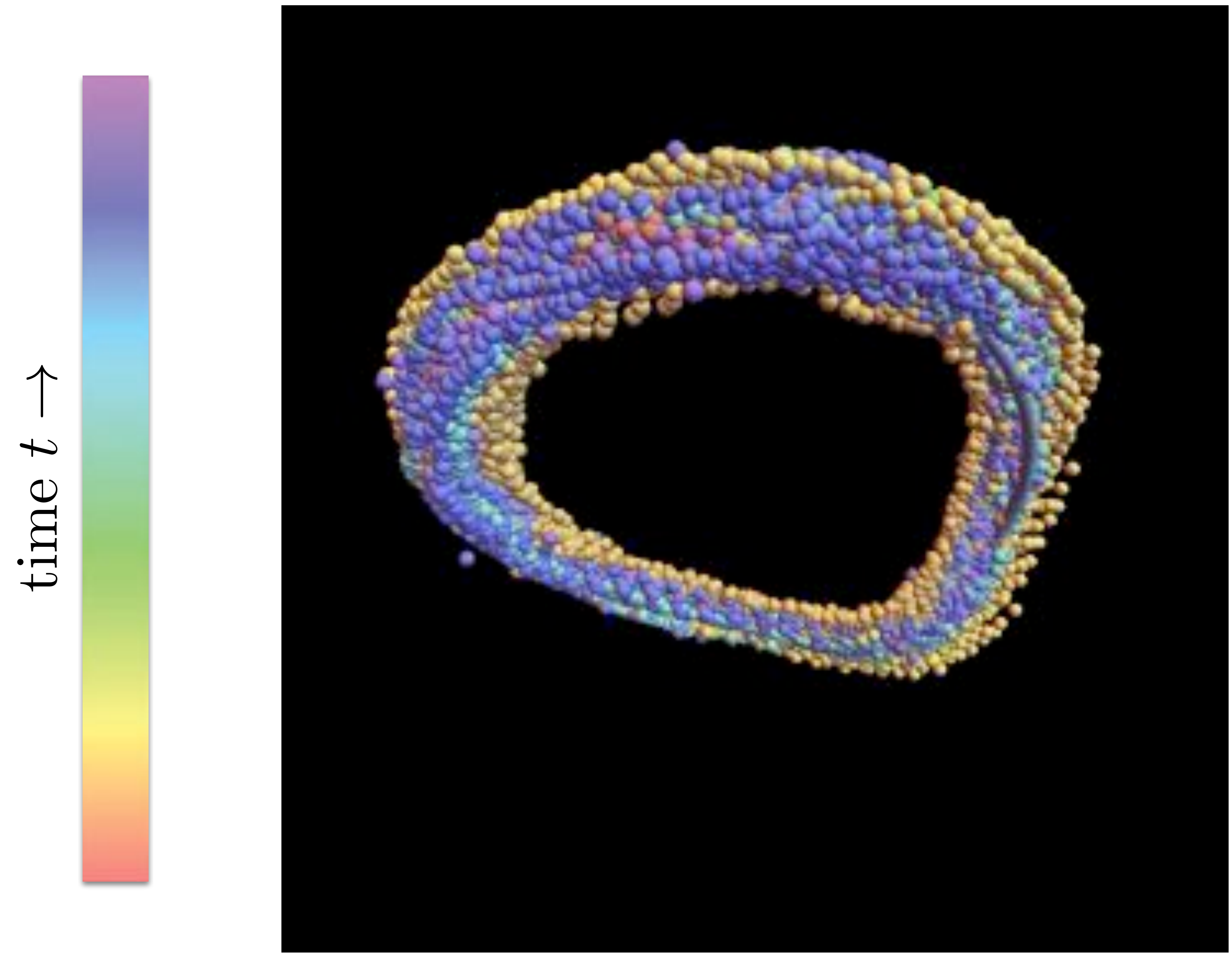


Attractor Reconstruction



$$\{F_N[t], F_N[t-\tau], F_N[t-2\tau]\}\,,\ \tau = 2\delta t$$

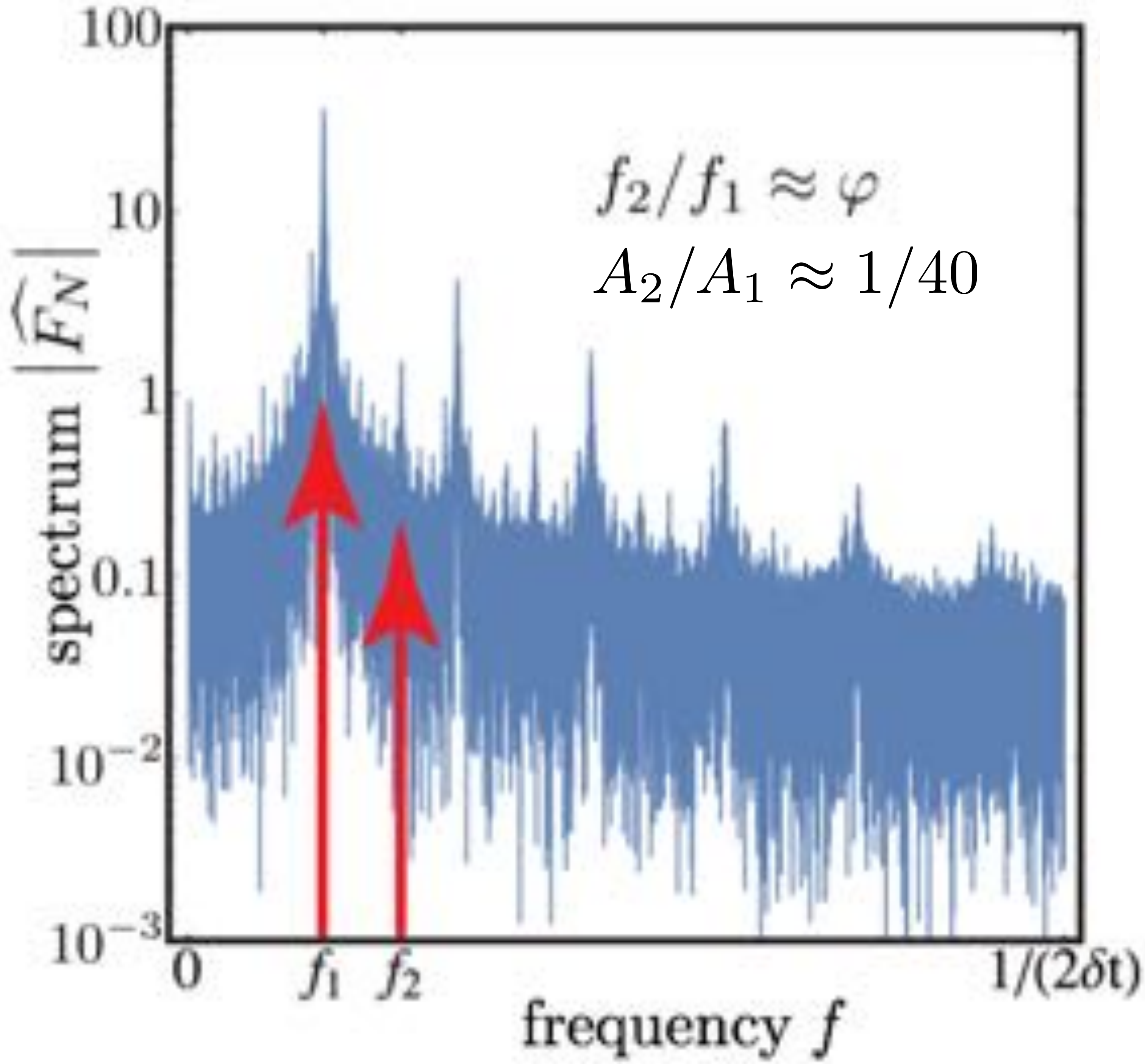
Attractor Gapless



$$\{F_N[t], F_N[t-\tau], F_N[t-2\tau]\}\,,\ \ \tau = 2\delta t$$

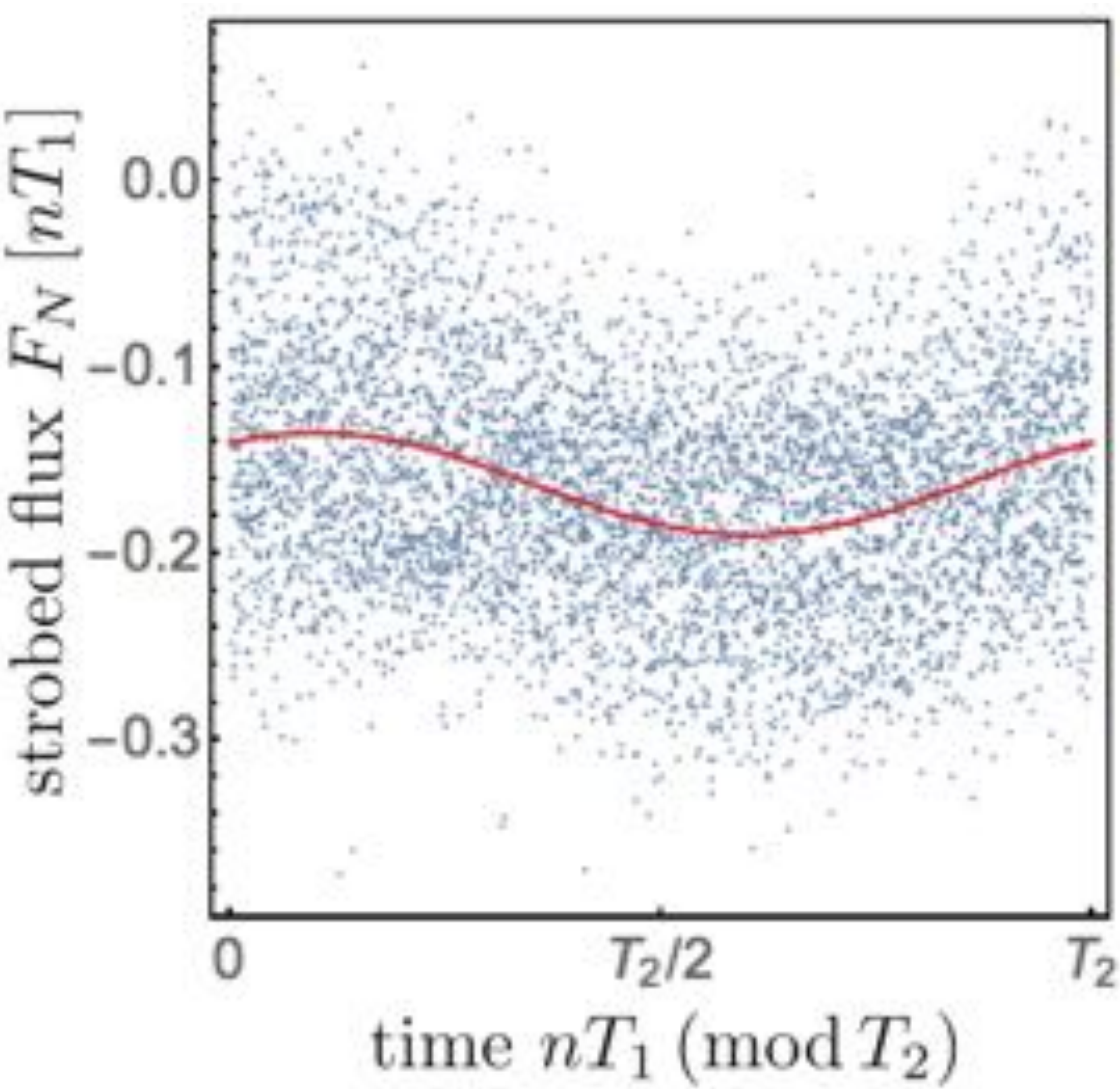
Spectrum

A golden star



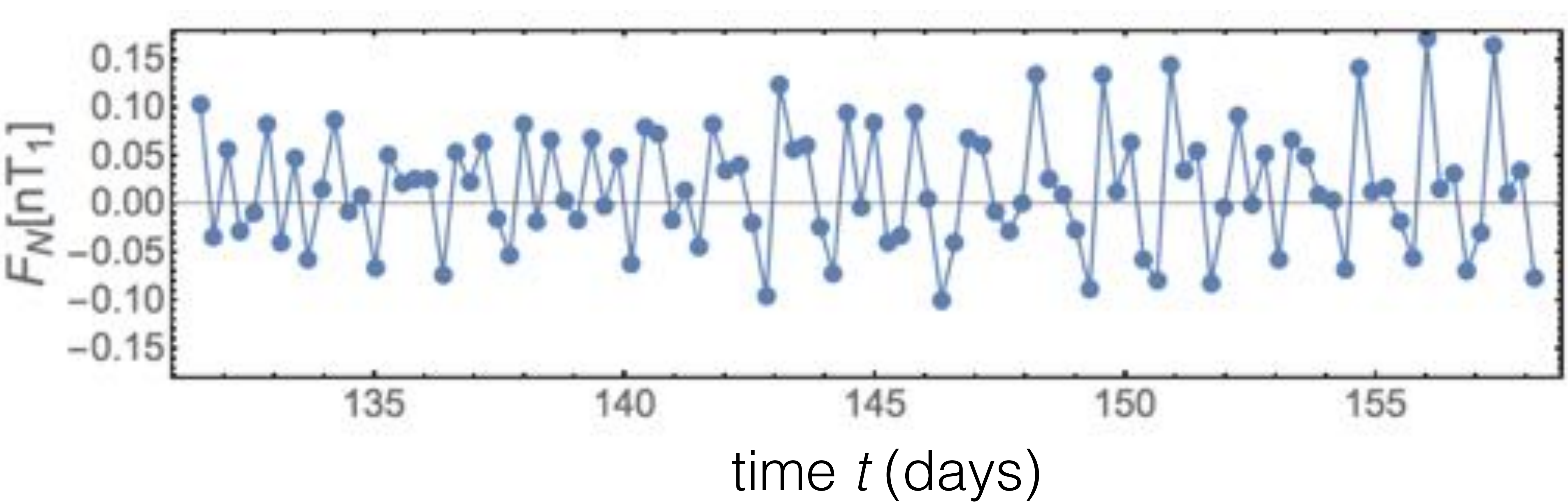
Sample & Wrap

Highlights secondary frequency



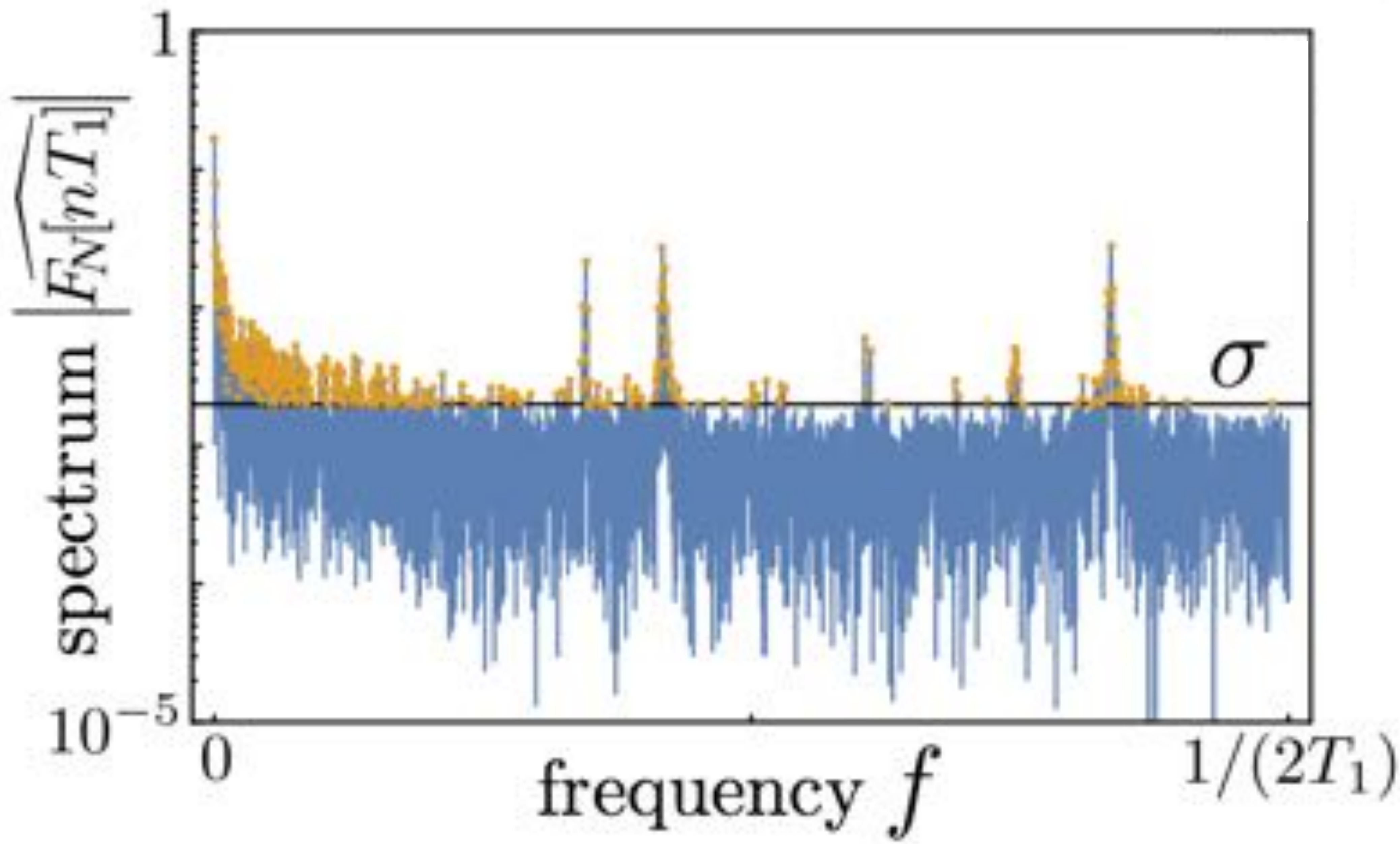
Strobed Time Series

Robust with respect to interpolation scheme

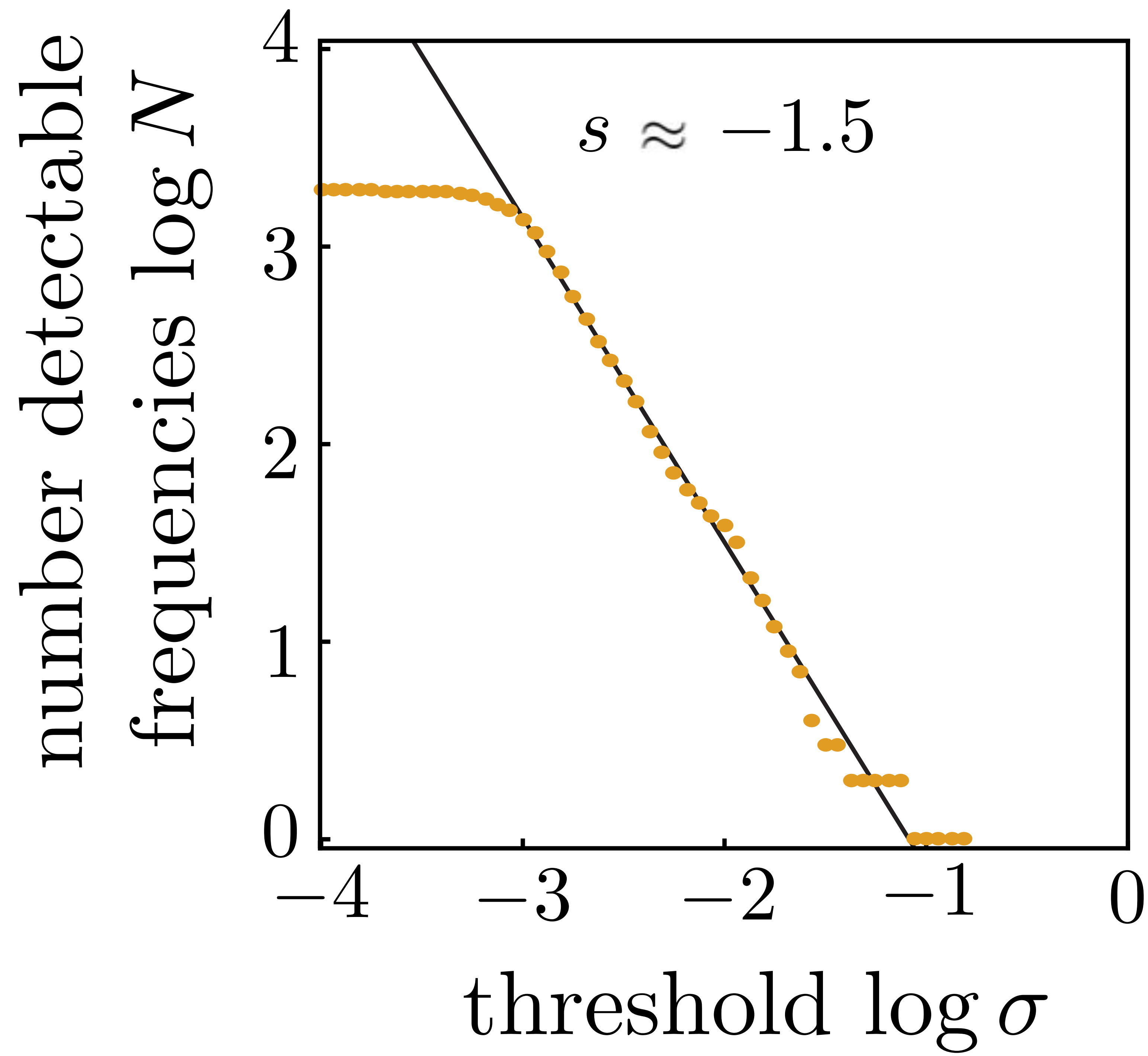


Strobed Spectrum

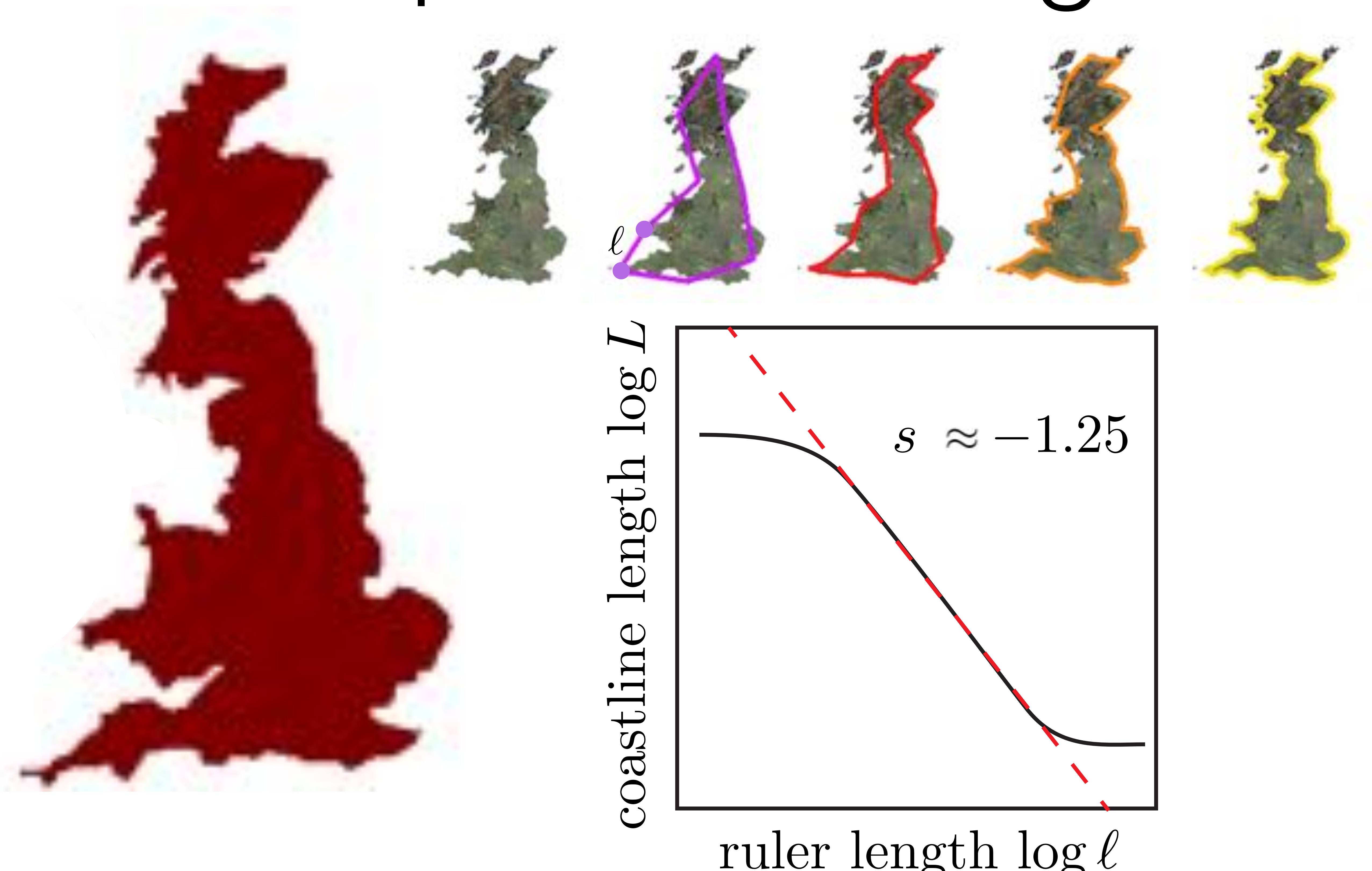
Gold peaks are super threshold



Spectral Scaling

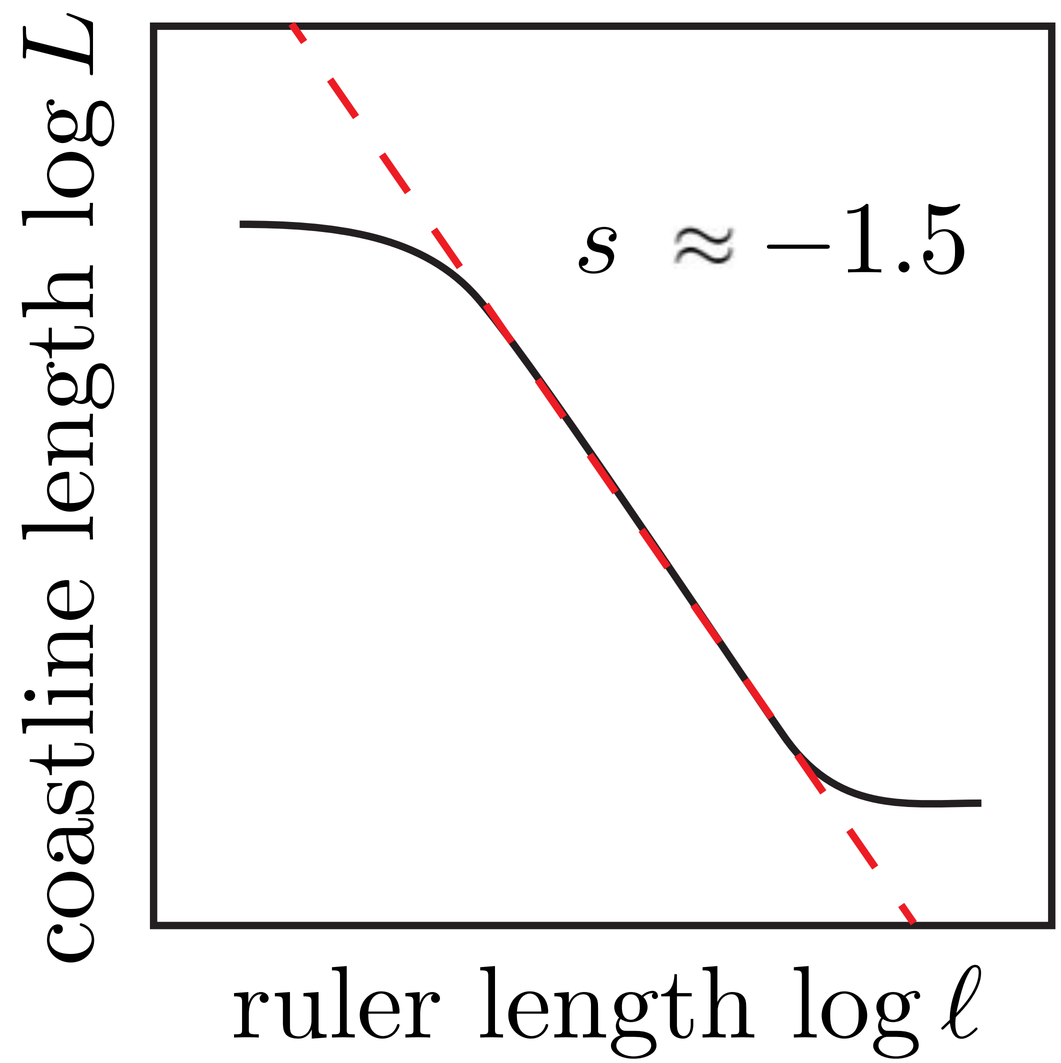


Spatial Scaling



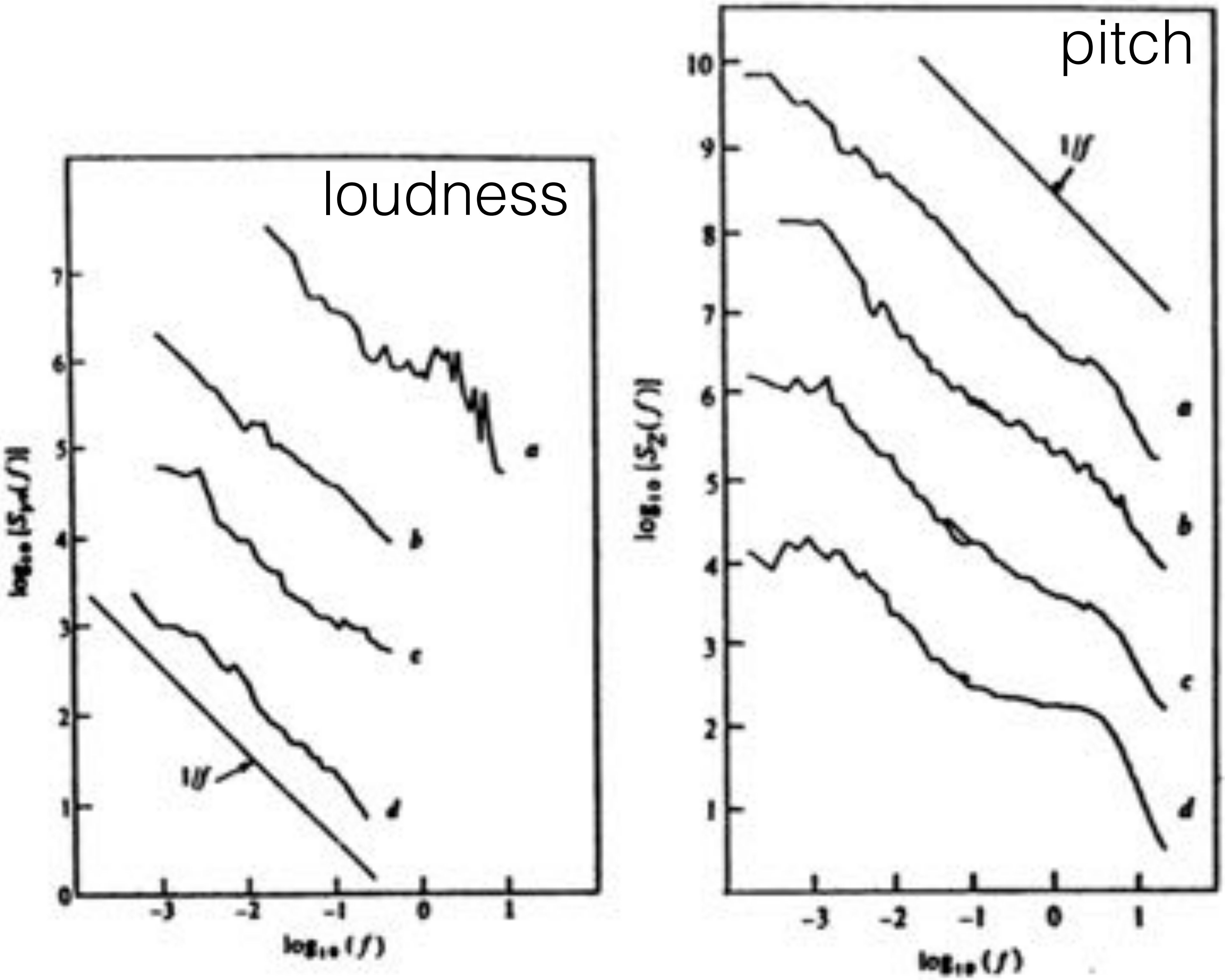
Mandelbrot, Science **156**, 636-638 (1967)

Spatial Scaling



spectral scaling
of secondary frequencies
is similar to
spatial scaling
of Norwegian coast

Music is Fractal Too



(a) ragtime, (b) classical, (c) rock, (d) talk

R. F. Voss and J. Clarke, Nature **258** (1975) 317

Fractal self-similarity can describe
processes in time as well as
patterns in space

and a music analogy elucidates our analysis

We removed the musical backbeat
to discover a subtle melody

Null Hypotheses

Tested a variety of **null hypotheses**
that generated **surrogate data** sets
of artificial light curves of flux versus time

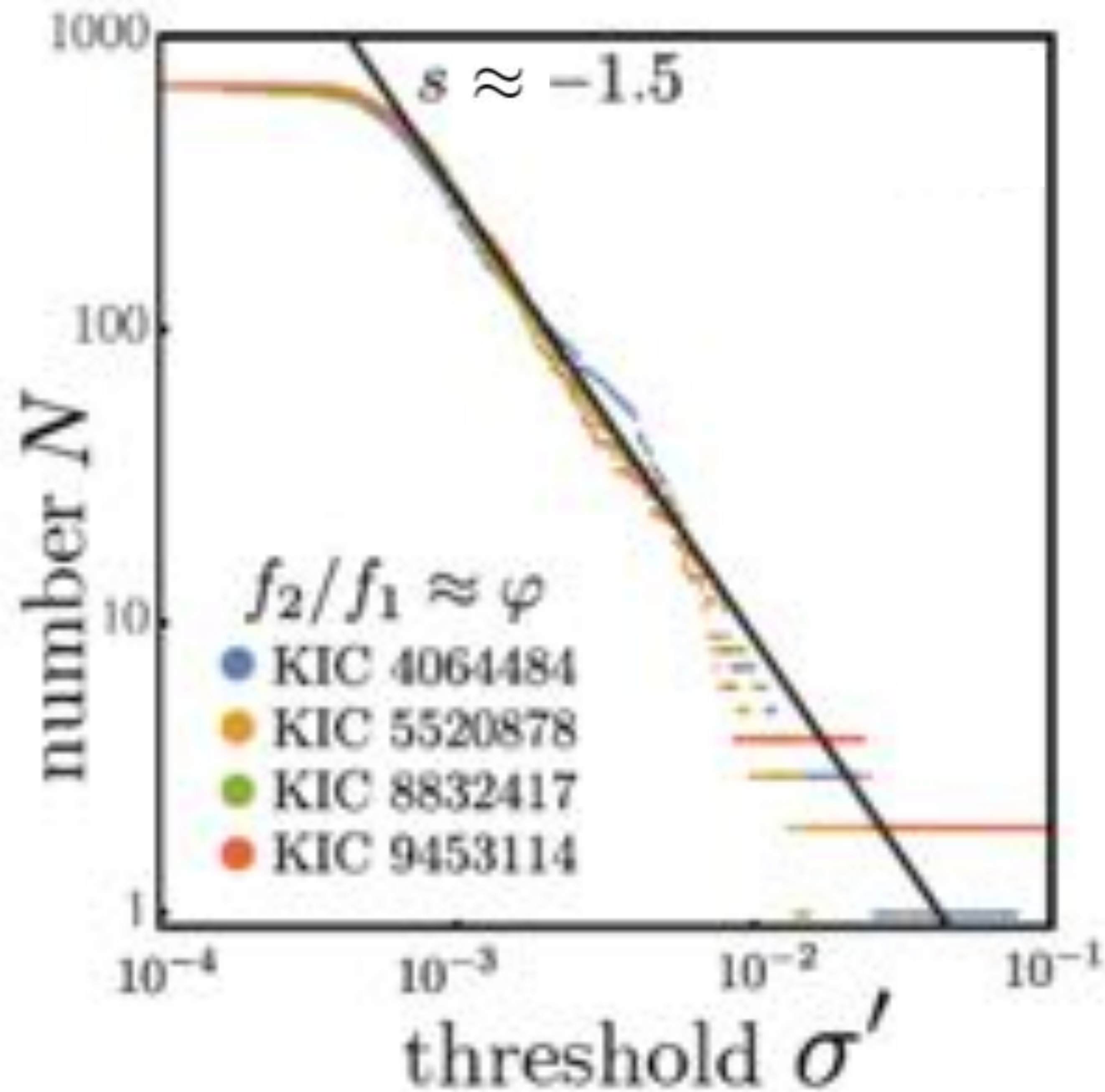
phase randomization of original time series

ideal quasiperiodic with noise

strobed section and spectral exponent
discriminated between
the surrogate and original data

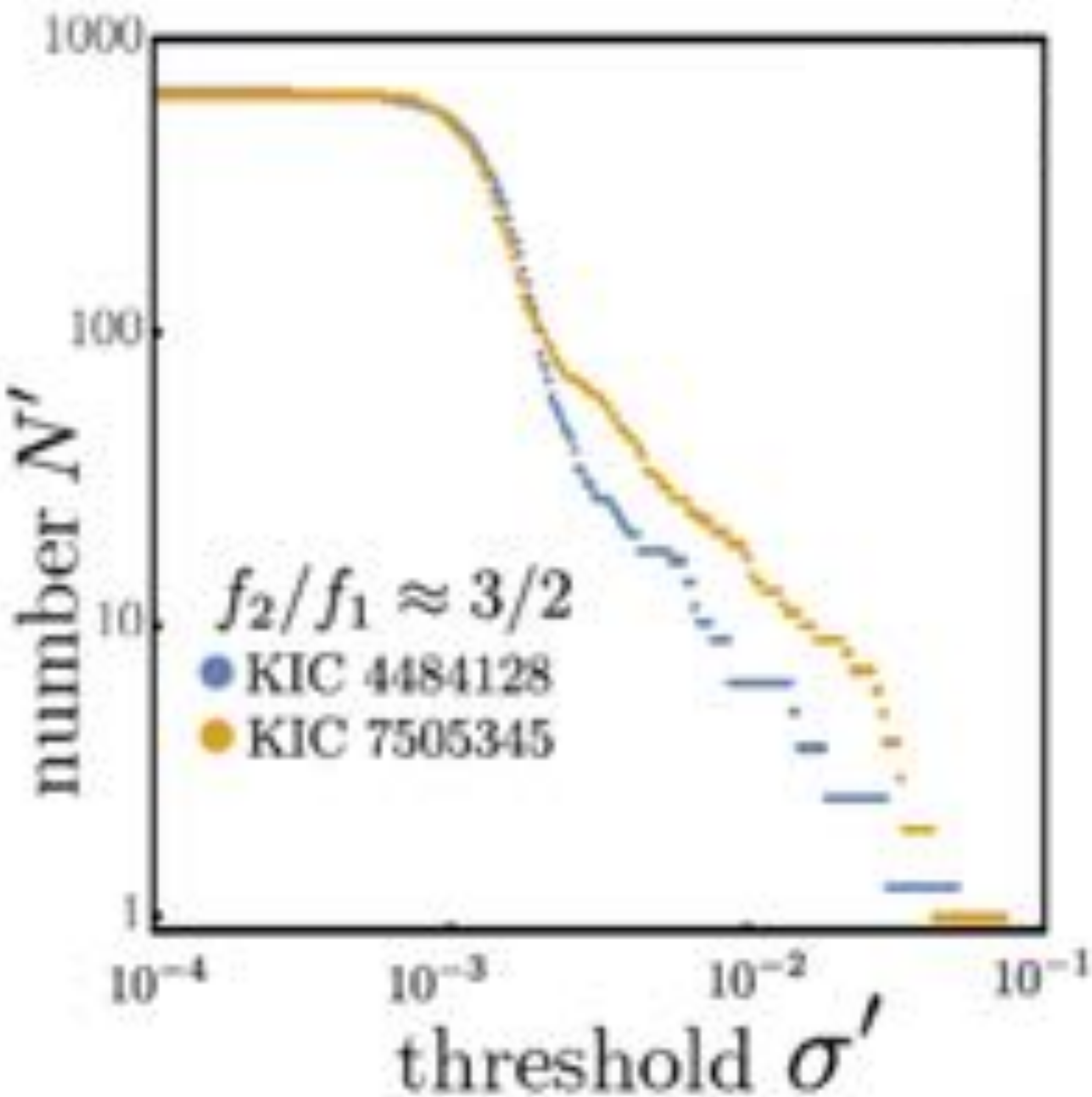
Other RRc Golden Stars

Norwegian scaling



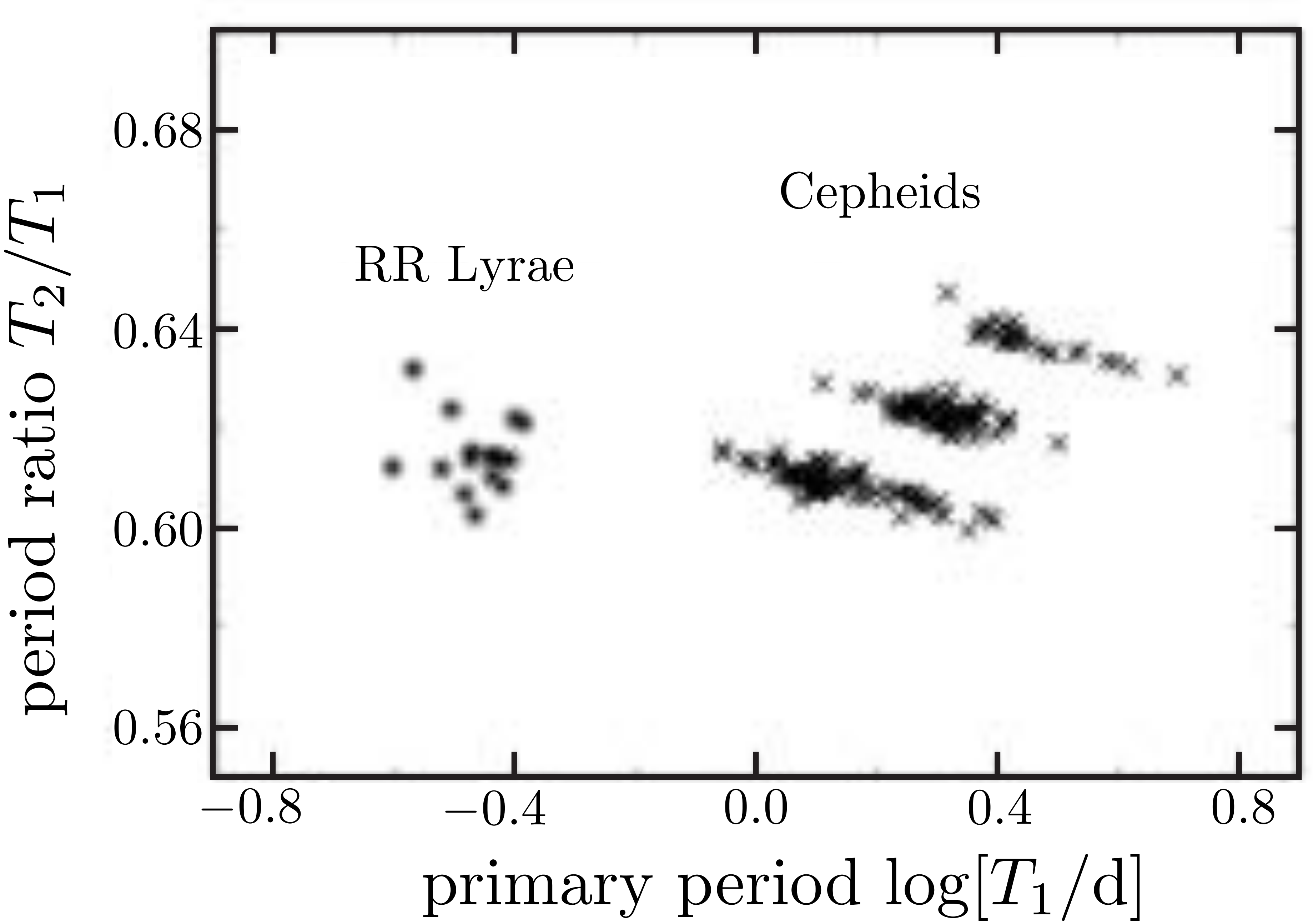
RRab Non-Golden Stars

No scaling or multiscaled?



Petersen Diagram

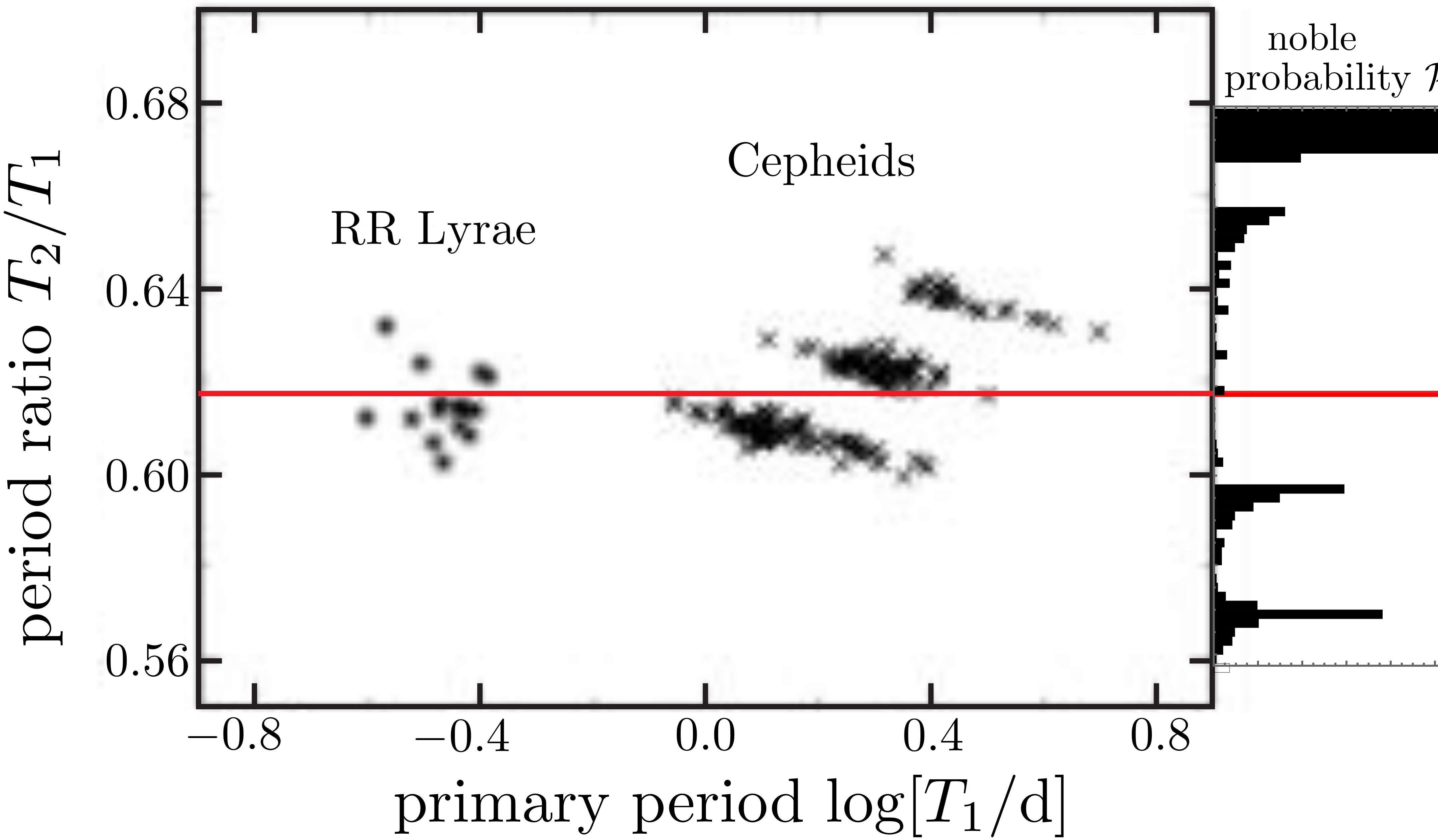
multi-frequency stars outside Kepler database



P. Moskalik, IAU Syposium **301** (2014)

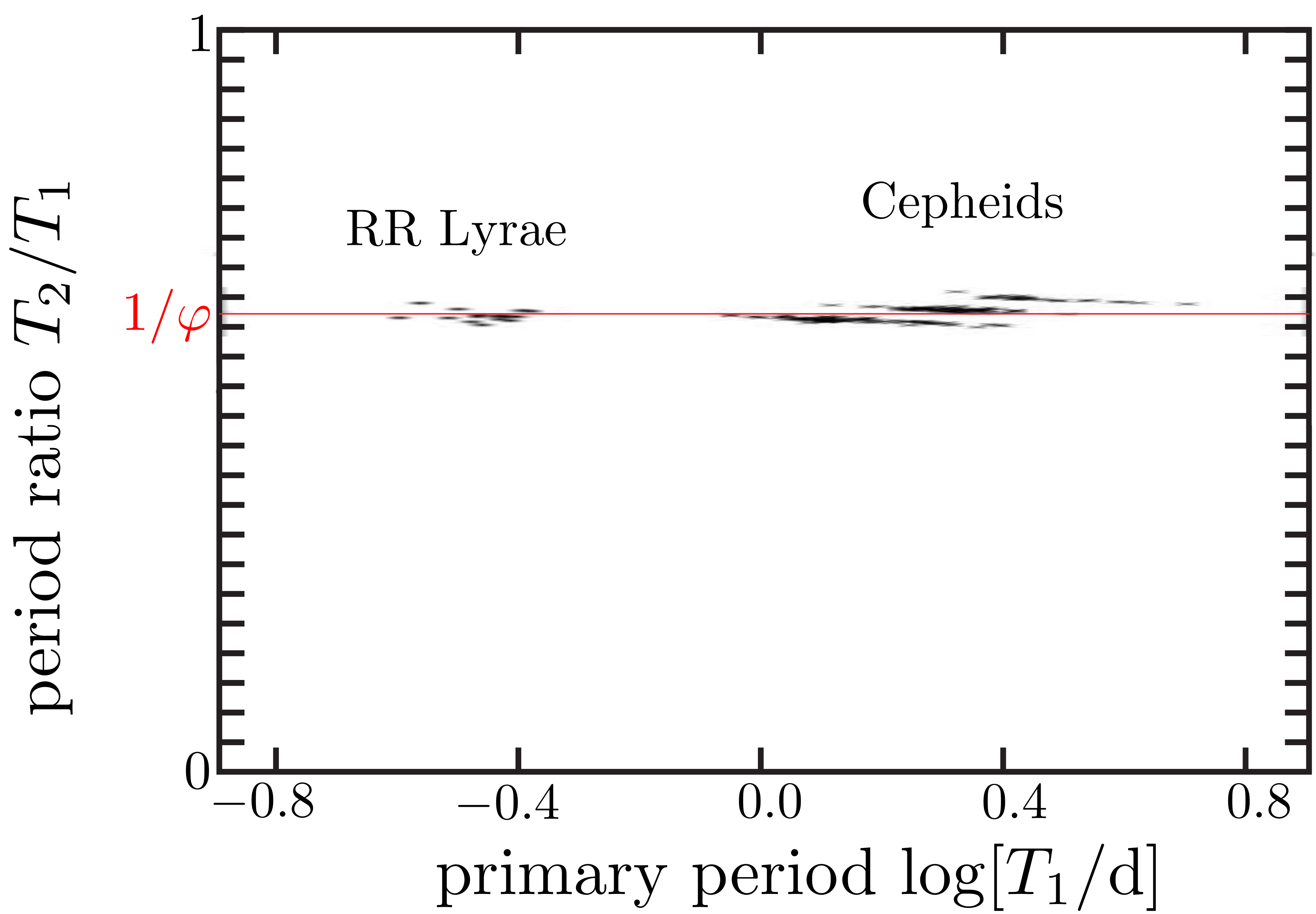
Petersen Diagram

multi-frequency stars outside Kepler database



Petersen Diagram

multi-frequency stars outside Kepler database



including period ratio zero

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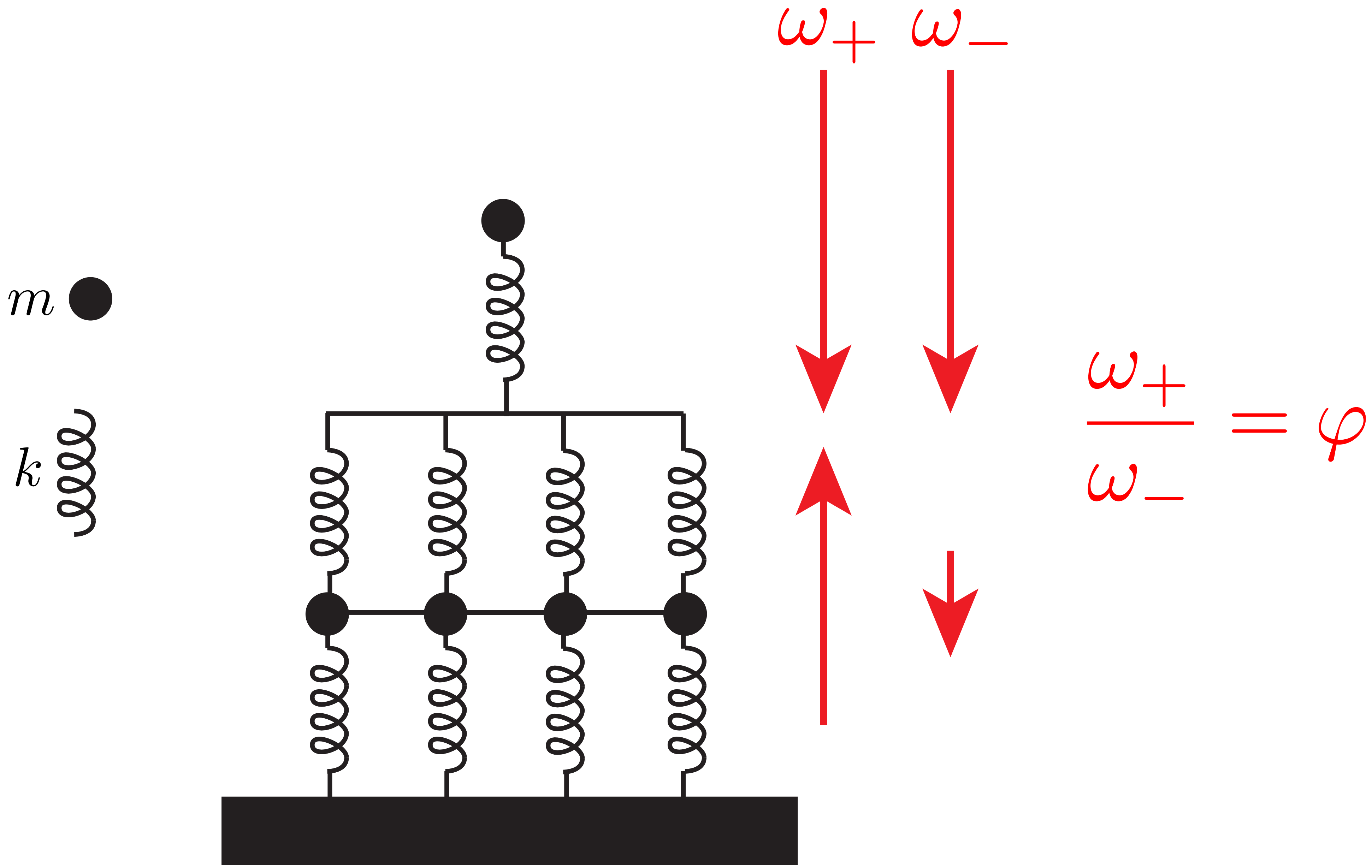
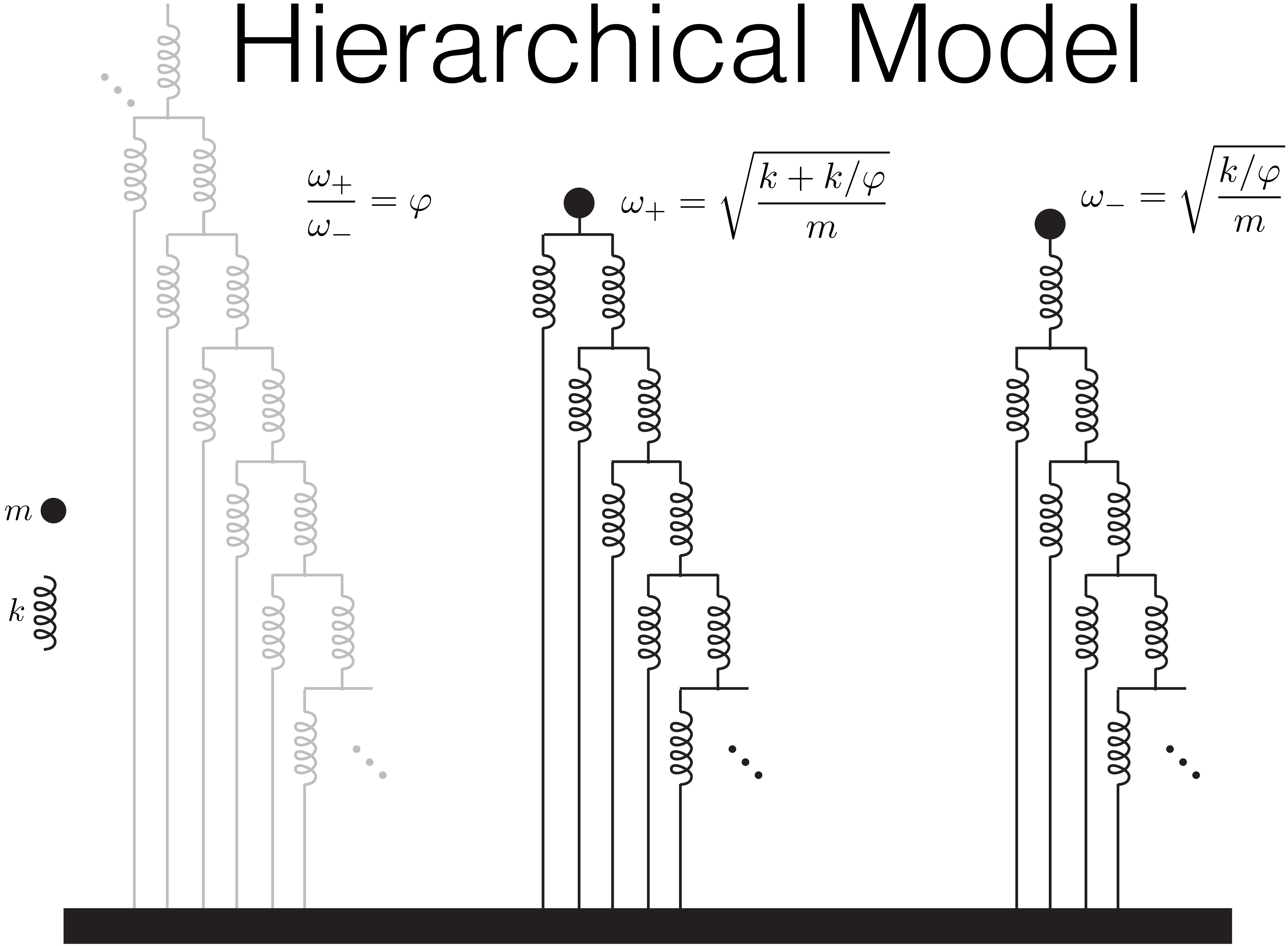
Why φ?

Lumped Model



Hierarchical Model



Truncated in practice so ratio only approximately golden

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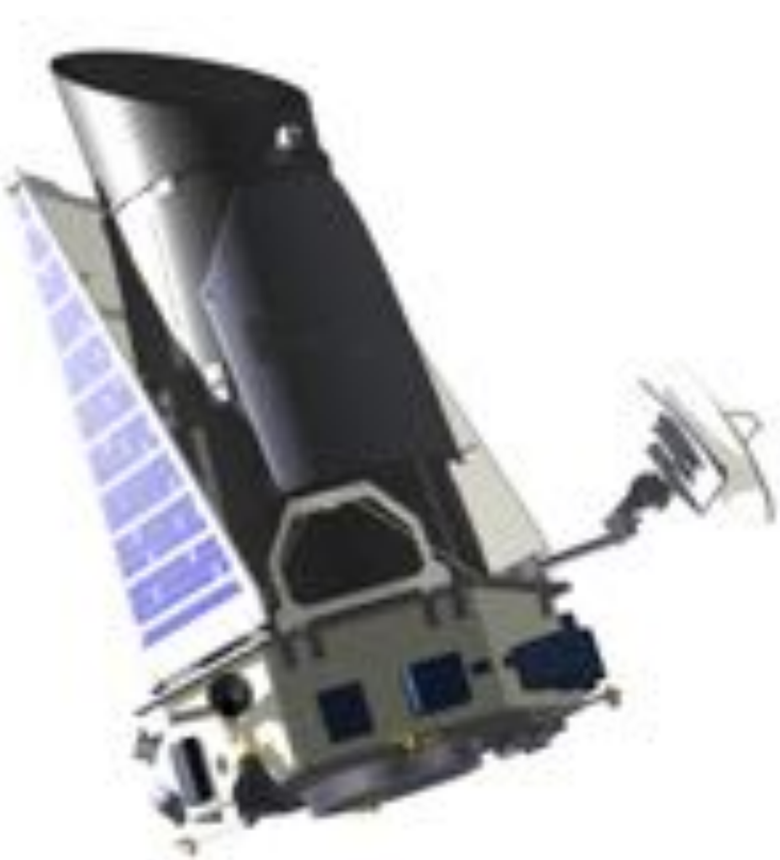

Fractals

A subtle dynamics between order and chaos may characterize RRc Lyrae variable stars

Secondary frequencies exhibit power law scaling, a temporal analog to fractal scaling of Norwegian coast

Do all golden stars exhibit Norwegian scaling?

Universality

Some natural dynamical patterns
result from universal features
common to even simple models

Other patterns are
peculiar to particular physical details

**Is the frequency distribution of variable stars
universal or particular?**

Sonification

Noise piano performance

KIC5520878 secondary frequencies

Sonification

Noise piano performance

KIC5520878 secondary frequencies

Sonification

Noise piano performance

KIC5520878 secondary frequencies

Sonification

Noise piano performance

KIC5520878 secondary frequencies

Thanks for Listening