Gravitational wave Oscillation in bi-metric gravity

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arXiv:1304.3920
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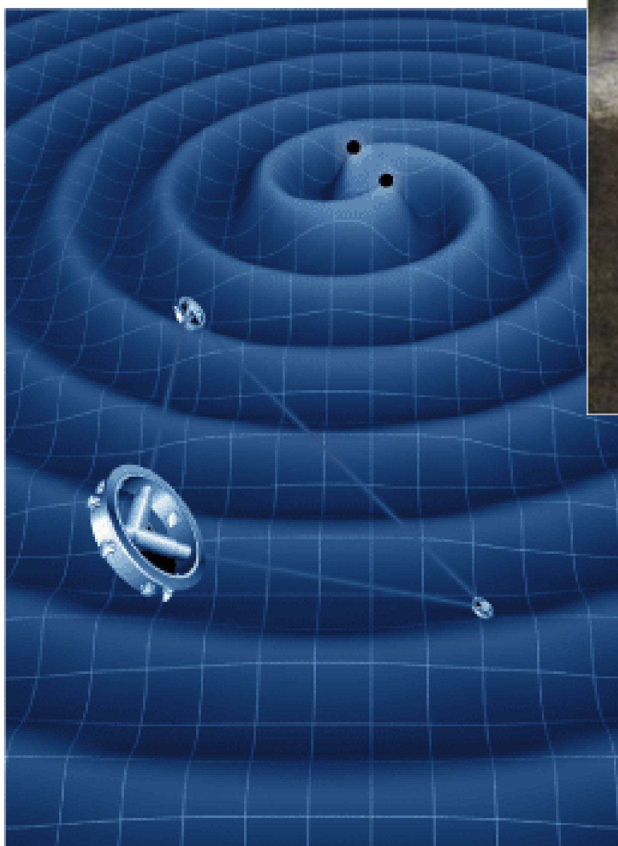
Gravitation waves will be detected soon!!!

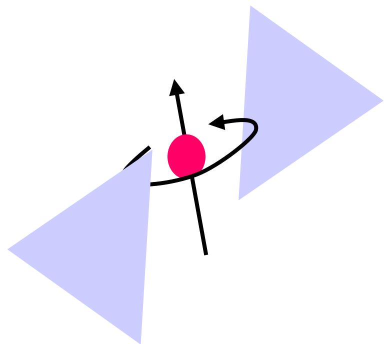


eLISA(NGO)
⇒DECIGO/BBO

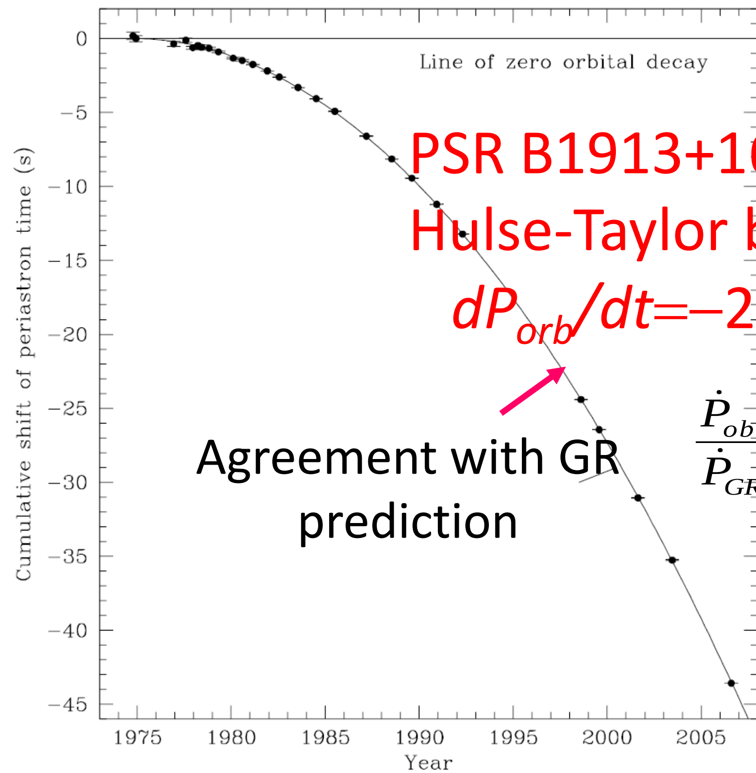
LIGO⇒adv LIGO

GW generation is almost confirmed

Pulsar : ideal clock



Periastron advance due to GW emission



PSR B1913+16

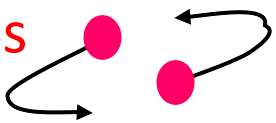
Hulse-Taylor binary

$dP_{orb}/dt = -2.423 \times 10^{-12}$

Agreement with GR prediction

$$\frac{\dot{P}_{obs}}{\dot{P}_{GR}} = 0.997 \pm 0.002$$

Test of GR by pulsar binaries



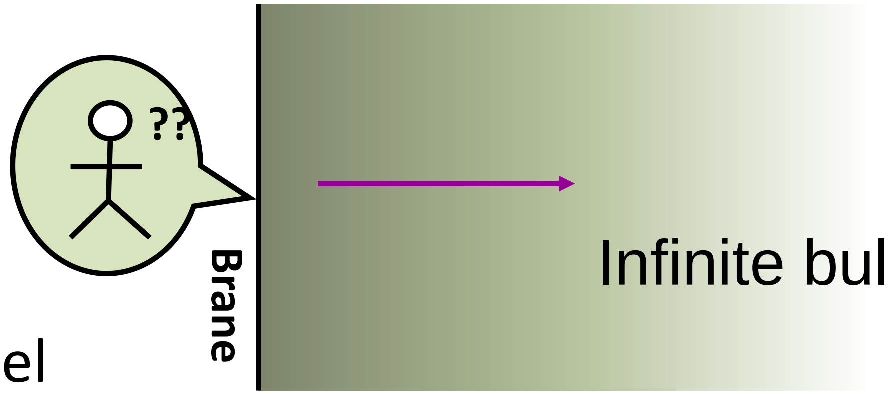
(J.M. Weisberg, Nice and J.H. Taylor, arXiv:1011.0718)

We know that GWs are emitted from binaries.

What is the possible big surprise when we directly detect GWs?

Is there possibility that graviton disappears during its propagation over cosmological distance?

Braneworld



Infinite extra-dimension

RS-II model, DGP model

Modification of GW propagation is small even if sources are placed at cosmological distances.

Chern-Simons Modified Gravity

$$S \supset \frac{\alpha}{4}\int d^4x\sqrt{-g}\,\theta\,\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta\rho\sigma}$$

Right-handed and left-handed gravitational waves are magnified differently during propagation, depending on frequencies.

However, the effect is large only in the strong coupling regime, outside the validity range of EFT.

Massive gravity

$$\Box h_{\mu\nu} = 0 \quad \Rightarrow \quad \left(\Box - m^2\right) h_{\mu\nu} = 0$$

Just adding mass to graviton seems theoretically inconsistent → ghost, instability, etc.

⇨ Bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{16\pi} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{16\pi\kappa} + \frac{L_{matter}(g,\phi)}{M_G^2} + \cdots$$

Both massive and massless gravitons exist.

→ ν oscillation-like phenomena?

First question is whether or not we can construct a viable cosmological model.

Ghost free bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{2} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{2\kappa} + \frac{\sqrt{-g}}{2}\sum_{n=0}^{4} c_n V_n + \frac{L_{matter}}{M_G^2}$$

$$V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2,\ \cdots$$

$$\tau_n \equiv Tr\left[\gamma^n\right] \quad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

When $\tilde{g}$ is fixed, de Rham-Gabadadze-Tolley massive gravity.

Even if $\tilde{g}$ is promoted to a dynamical field, the model remains to be free from ghost.

(Hassan, Rosen (2012))

FLRW background

(Comelli, Crisostomi, Nesti, Pilo (2012))

Generic homogeneous isotropic metrics

$$ds^2 = \underline{a^2(t)}\left(-dt^2 + dx^2\right)$$
$$d\tilde{s}^2 = \underline{b^2(t)}\left(-\underline{c^2(t)}dt^2 + dx^2\right)$$

$$\xi \equiv b/a$$

$$\Rightarrow \quad \underbrace{\left(6c_3\xi^2 + 4c_2\xi + c_1\right)}_{\text{branch 1}}\underbrace{\left(cba' - ab'\right)}_{\text{branch 2}} = 0$$

branch 1 : Pathological:

Strong coupling

Unstable for the homogeneous anisotropic mode.

branch 2 : Healthy

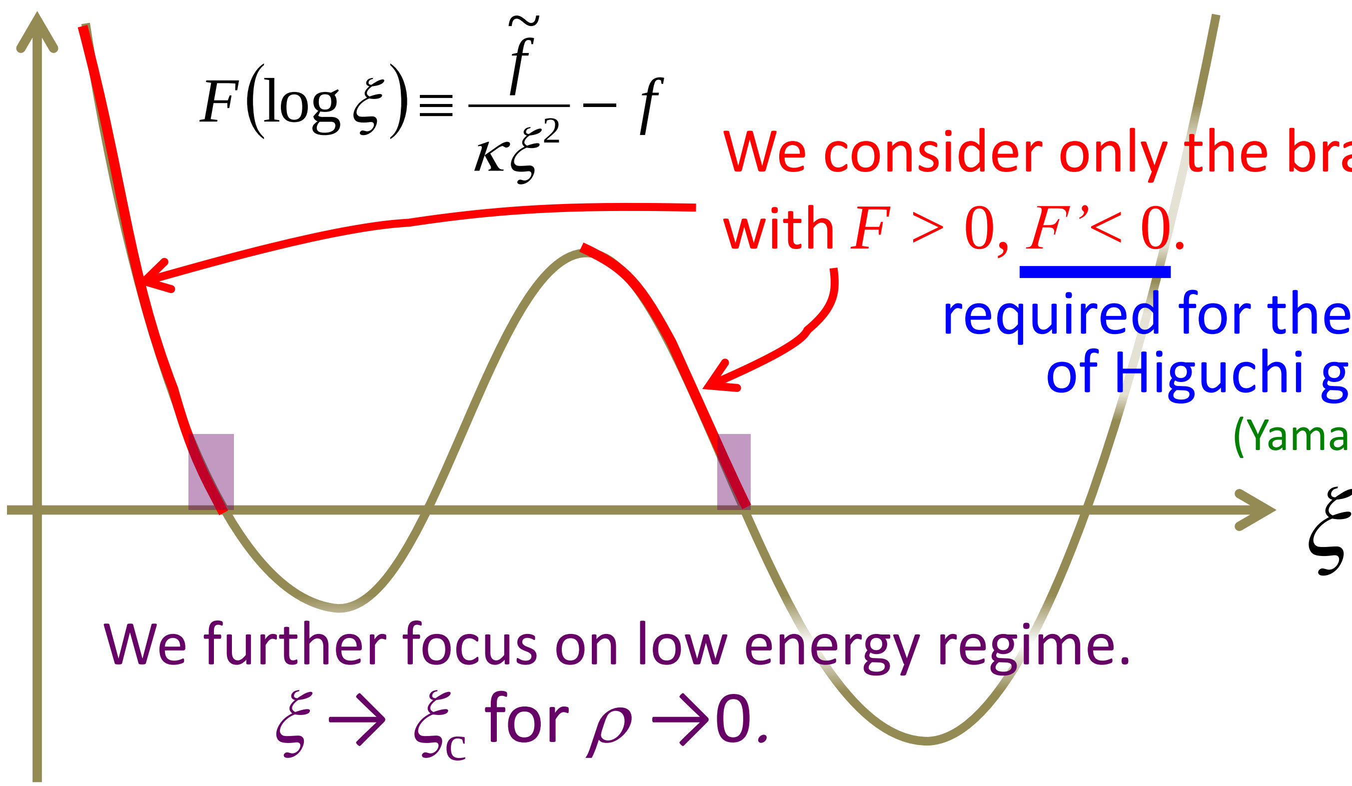
Branch 2 background

A very simple relation holds:

$$\frac{\rho}{M_G^2} + f - \tilde{f}/\kappa\xi^2 = 0 \qquad f(\log\xi) := c_0 + 3c_1\xi + 6c_2\xi^2 + 6c_3\xi^3$$

$$\tilde{f}(\log\xi) := c_1\xi + 6c_2\xi^2 + 18c_3\xi^3 + 24c_4\xi^4$$

$\xi \equiv b/a$ is algebraically determined as a function of ρ.



We further focus on low energy regime.

$\xi \to \xi_c$ for $\rho \to 0$.

Branch 2 background

We expand with respect to $\delta\xi = \xi - \xi_c$.

$$H^2 = \frac{\rho}{3M_G^2} + \frac{f}{3} \quad \Rightarrow \quad H^2 = \frac{\rho}{3\left(1+\kappa\xi_c^2\right)M_G^2}$$

effective energy density due to mass term

Effective gravitational coupling is weaker because of the dilution to the hidden sector.

$$\frac{1}{c-1}\frac{\xi'}{\xi} = \frac{a'}{a} \quad \Rightarrow \quad c-1 = \frac{3(\rho+P)}{\mu^2 M_G^2}$$

Effective graviton mass

$$\mu^2 = \left(1+\frac{1}{\kappa\xi_c^2}\right)f_c'$$

natural tuning to coincident light cones (c=1) at low energies ($\rho \to 0$)!

Solar system constraint: basics

◆ vDVZ discontinuity

In GR, this coefficient is 1/2

current bound <10^{-5}

$$\delta g_{\mu\nu} \propto \Box^{-1}\left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T\right)$$

To cure this discontinuity

we go beyond the linear perturbation (Vainshtein)

Schematically

Correction to the Newton potential Φ

$$\cancel{\Delta\delta\Phi} + \mu^{-2}(\partial\partial\delta\Phi)^2 = G_N \rho$$

$$\Rightarrow \frac{\delta\Phi}{\Phi} \approx \frac{\mu r^2 \sqrt{G_N \rho}}{r^2 G_N \rho} \approx \mu \sqrt{\frac{r^3}{r_g}}$$

$$10^{-10} \geq \mu \sqrt{\left(10^{13} cm\right)^3 / \left(10^5 cm\right)} \Rightarrow \mu^{-1} \geq 300 Mpc$$

Gravitational potential around a star in the limit $c \to 1$

Spherically symmetric static configuration:

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}\left(dr^2 + r^2 d\Omega^2\right)$$
$$d\tilde{s}^2 = \xi_c^2\left[-e^{\tilde{u}-\tilde{v}}dt^2 + e^{\tilde{u}+\tilde{v}}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)\right] \qquad \tilde{r} = e^R r$$

Erasing $\tilde{u}, \tilde{v}$ and R ,

$$\Rightarrow \quad \left(\Delta - \mu^2\right)u - \frac{C}{\mu^2}\left(\left(\Delta u\right)^2 - \left(\partial_i \partial_j u\right)^2\right) \approx \frac{\rho_m}{M_G^2}$$

$C \propto f_c''$, which can be tuned to be extremely large.

Then, the Vainshtein radius $r_V \approx \left(\frac{C r_g}{\mu^2}\right)^{1/3}$

can be made very large, even if $\mu^{-1} << 300\mathrm{Mpc}$.

Solar system constraint: $\sqrt{C}\mu^{-1} \geq 300\mathrm{Mpc}$

$\Delta v \approx \frac{\rho_m}{\tilde{M}_G^2}$ v is excited as in GR. $H^2 = \frac{\rho}{3\tilde{M}_G^2}$

Excitation of the metric perturbation on the hidden sector:

Erasing u, v and R

$$\Rightarrow \quad \left(\Delta-\mu^{2}\right)\tilde{u}-\frac{\tilde{C}}{\mu^{2}}\left(\left(\Delta\tilde{u}\right)^{2}-\left(\partial_{i}\partial_{j}\tilde{u}\right)^{2}\right)\approx\frac{\rho_{m}}{M_{G}^{2}}$$

$$\Delta\tilde{v}\approx\frac{\rho_{m}}{\tilde{M}_{G}^{2}}$$

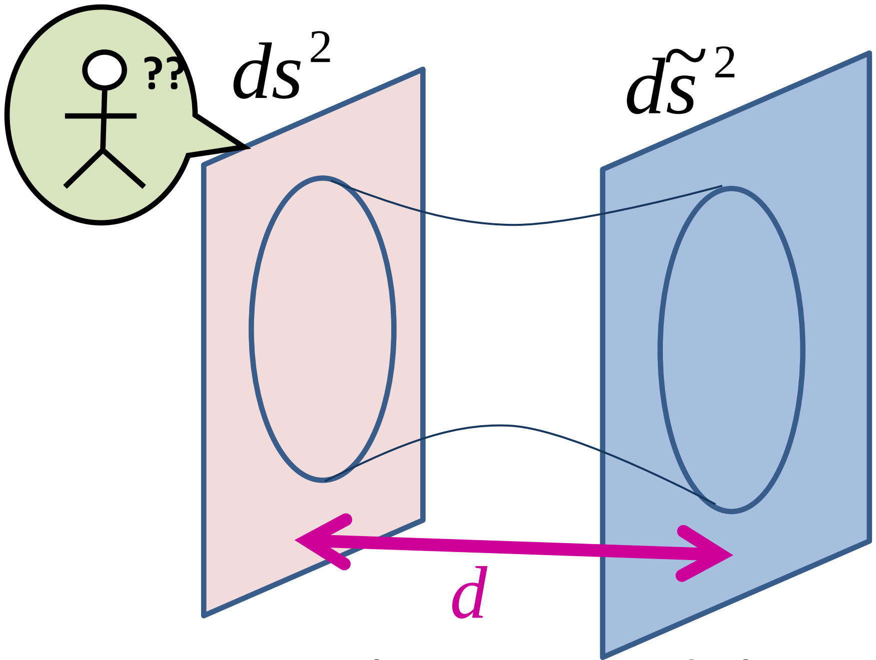
$\tilde{u}$ is also suppressed like u.

$\tilde{v}$ is also excited like v.

The metric perturbations are almost conformally related with each other: $d\tilde{s}^{2}\approx\xi_{c}^{2}ds^{2}$

Non-linear terms of $\tilde{u}$ (or equivalently u) play the role of the source of gravity.

Why do we have this attractor behavior, $c \to 1$ and $\xi \to \xi_c$, at low energies?

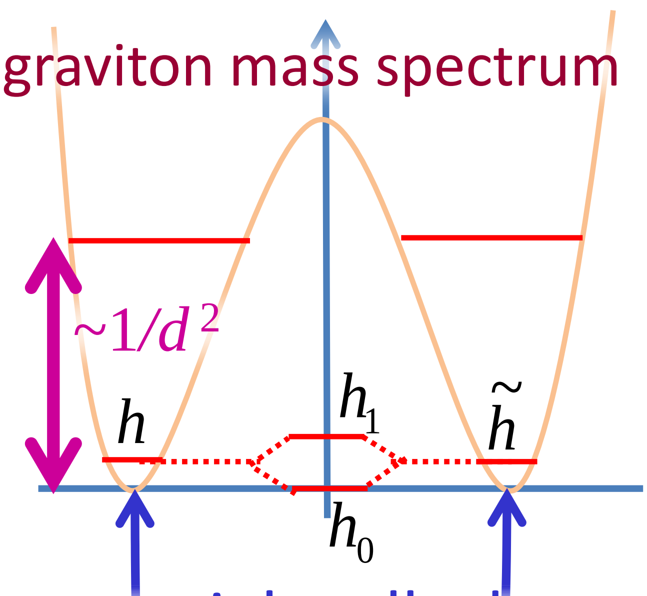


DGP 2-brane model?!

$$+\int d^4x\sqrt{-g}R \;\&\; \int d^4x\sqrt{-\tilde{g}}\tilde{R}$$

$\approx$

KK graviton mass spectrum



potential wells due to induced gravity terms

$d \to 0$ ⇒ Only first two modes remain at low energy

⇒ $d\tilde{s}^2 = \xi_c^2 ds^2$ ⇒ identical light cone $c = 1$

Gravitational wave propagation

Short wavelength approximation :

$$k >> m_g >> H$$

$$h'' - \Delta h + m_g^2 \left(h - \tilde{h}\right) = 0$$

$$\tilde{h}'' - c^2 \Delta \tilde{h} - \frac{c m_g^2}{\kappa \xi_c^2} \left(h - \tilde{h}\right) = 0$$

$$m_g^2 = \frac{f'}{3} + \frac{(c-1)}{6}\left(f'' - f'\right)$$

(Comelli, Crisostomi, Pilo (2012))

$$\mu^2 := m_g^2 \frac{1 + \kappa \xi^2}{\kappa \xi^2}$$

$$k_c := \frac{\mu}{\sqrt{2(c-1)}}$$

k

mass term is important.

Eigenmodes are

$$h + \tilde{h}, \quad \kappa \xi_c^2 h - \tilde{h}$$

modified dispersion relation
due to the effect of mass

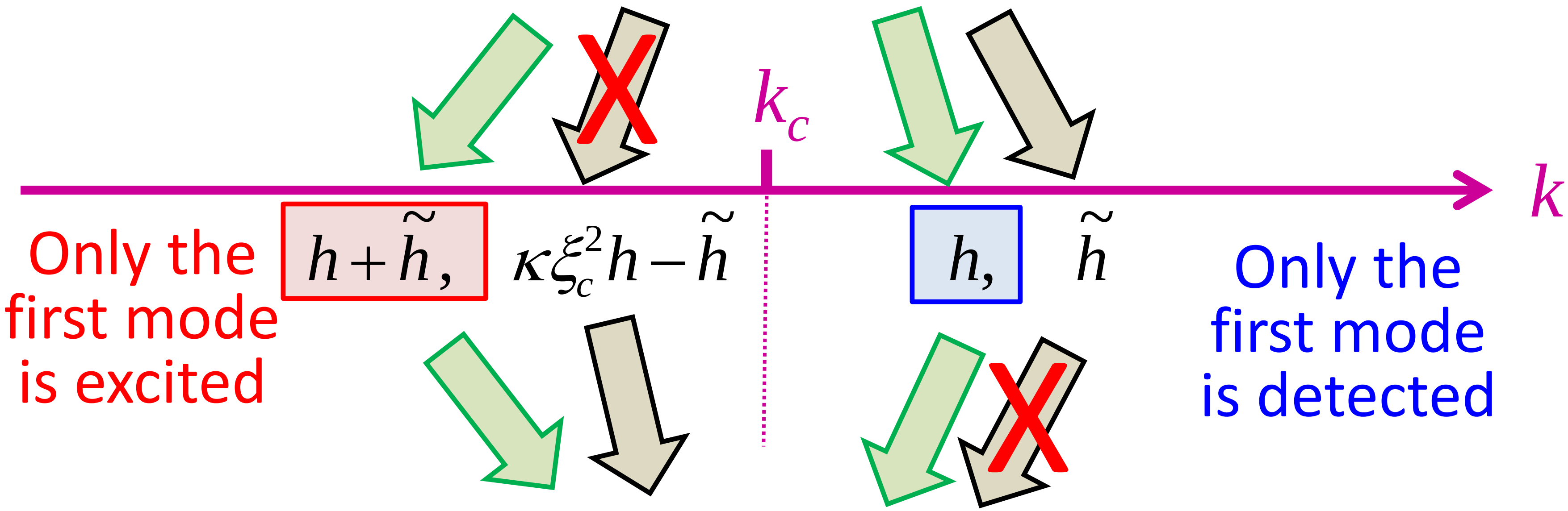
$C \neq 1$ is important.

Eigenmodes are

$$h, \quad \tilde{h}$$

modified dispersion relation
due to different light cone

At the GW generation, both h and $\tilde{h}$ are equally excited.



We can detect only h.

Only modes with $k \sim k_c$ picks up the non-trivial dispersion relation of the second mode.

Interference between two modes. $\Rightarrow$ Graviton oscillations

If the effect appears ubiquitously, such models would be already ruled out by other observations.

Summary

Gravitational wave observations open up a new window for modified gravity.

Even graviton oscillations are not immediately denied, and hence we may find something similar to the case of solar neutrino experiment in near future.

Although space GW antenna is advantageous for the gravity test in many respects, we should be able to find more that can be tested by KAGRA.

Solar system constraint

Ordinary Vainshtein mechanism is not good enough!

$$G_{\mu\nu} = M_G^{-2}\left(T_{\mu\nu} + T_{\mu\nu}^{(\mathrm{int})}\right)$$

Ordinary Vainshtain mechanism tells that $T_{\mu\nu}^{(\mathrm{int})}$ can be simply neglected on small length scales for $T_{\mu\nu}^{(\mathrm{int})} \to 0$.

Then, however,

"local effective gravitational coupling M_G^2"

≠ "cosmological one $\left(1+\kappa\xi_c^2\right)M_G^2$"

Here, we do not send $T_{\mu\nu}^{(\mathrm{int})} \to 0$,

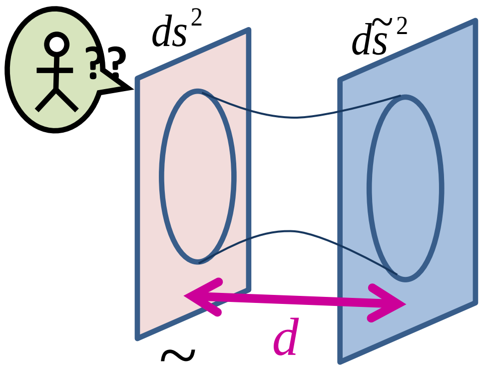
but we only tune the graviton mass to be small: $\mu^2 << c_i$

⇨ $h_{\mu\nu} \approx \tilde{h}_{\mu\nu}$

"local effective gravitational coupling"= $\left(1+\kappa\xi_c^2\right)M_G^2$

Vainshtein *a la* brane

In the DGP two-brane model stabilized at a small brane separation, this Vainshtein mechanism can be easily understood.



Junction conditions:

$$G_{\mu\nu} = M_G^{-2} T_{\mu\nu} + \left(K_{\mu\nu} - K g_{\mu\nu}\right)/r_c$$
$$\tilde{G}_{\mu\nu} = \left(\tilde{K}_{\mu\nu} - \tilde{K}\tilde{g}_{\mu\nu}\right)/\tilde{r}_c$$

small separation ⇨ $g_{\mu\nu} \approx \tilde{g}_{\mu\nu} \qquad K_{\mu\nu} \approx -\tilde{K}_{\mu\nu}$

⇨ $$G_{\mu\nu} = \left(1 + \tilde{r}_c / r_c\right)^{-1} M_G^{-2} T_{\mu\nu} = \left(1 + \kappa \xi_c^2\right)^{-1} M_G^{-2} T_{\mu\nu}$$

Furthermore,

$$r_c M_G^{-2} T_{\mu\nu} \approx K_{\mu\nu} \approx \left(\tilde{g}_{\mu\nu} - g_{\mu\nu}\right)/d$$

⇨ $$\tilde{g}_{\mu\nu} - g_{\mu\nu} \approx \frac{T_{\mu\nu}}{M_G^2 \mu^2}$$

d should shrink to maintain stabilization at large energies

Nearly identical metrics ⇔ tightly stabilized brane separation

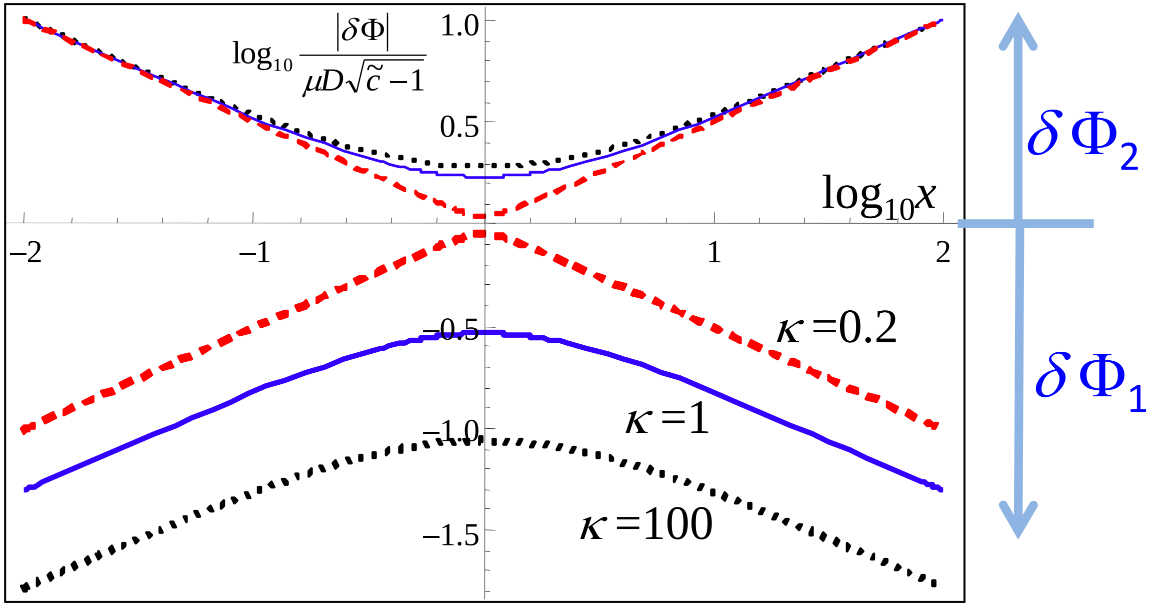
Gravitational wave propagation over a long distance D

Phase shift due to the modified dispersion relation:

$$\delta\Phi_{1,2} \equiv -\frac{\Delta k^2}{2\omega} D = -\frac{\mu D\sqrt{c-1}}{2\sqrt{2x}}\left(1+x \mp \sqrt{1+2x\frac{1-\kappa\xi^2}{1+\kappa\xi^2}+x^2}\right)$$

$$\mu D\sqrt{c-1} \approx HD\sqrt{3\left(1+\kappa\xi_c^2\right)\Omega_0}$$

becomes O(1) after propagation over the horizon distance



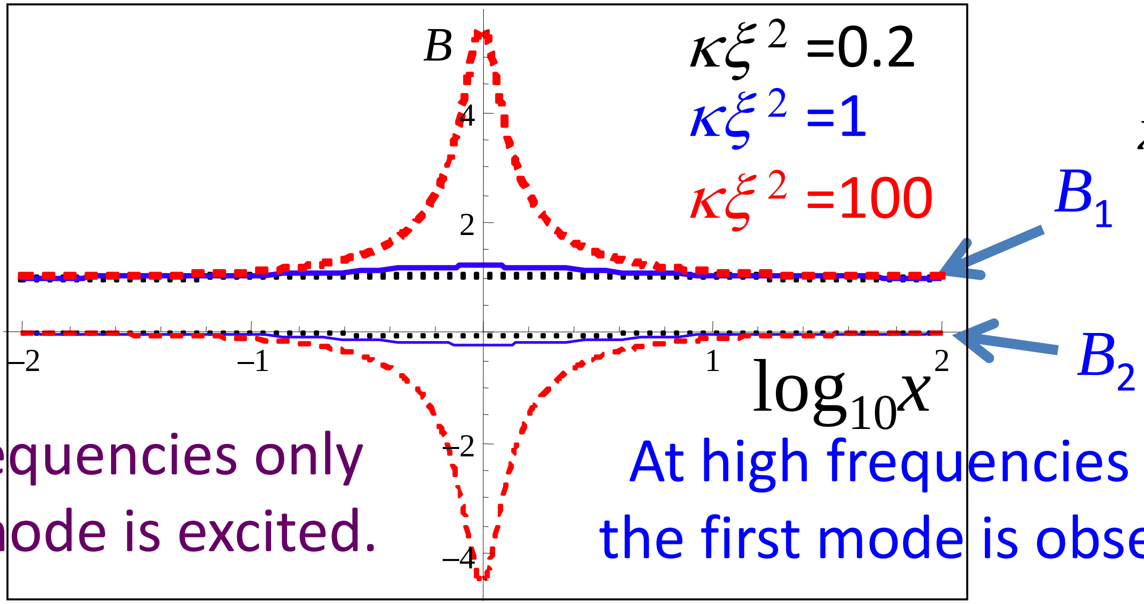
$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

Gravitational wave oscillations

1) At the time of generation of GWs from coalescing binaries, both h and $\tilde{h}$ are equally excited.

2) When we detect GWs, we sense h only.

$$\Rightarrow \quad h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$

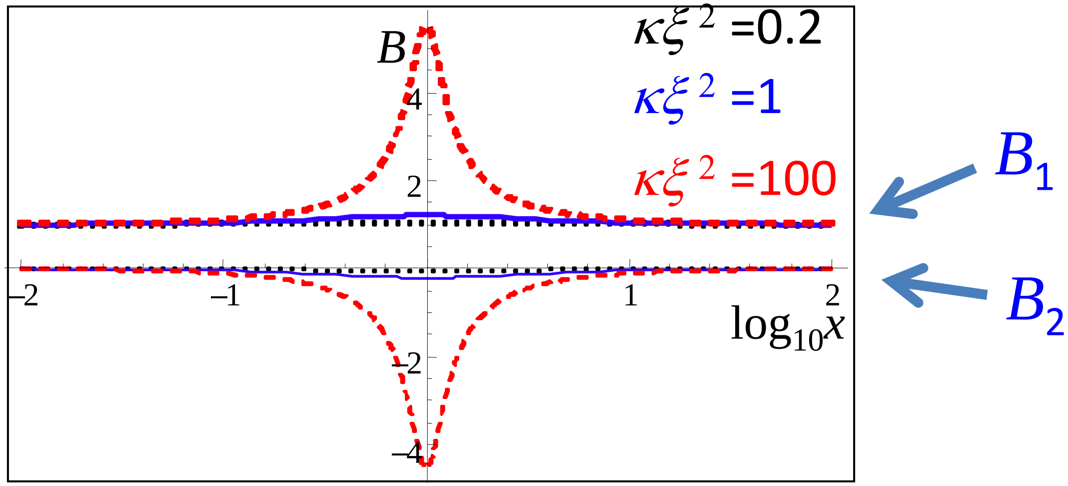


$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

At low frequencies only the first mode is excited.

At high frequencies only the first mode is observed.

$$h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$



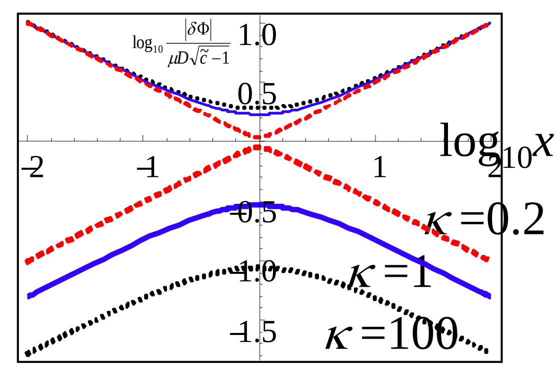
Graviton oscillations occur only around the frequency

$$\omega_{GW} \approx \frac{\mu^2}{\sqrt{6(1+\kappa\xi_c^2)\Omega_0}H} = 100\mathrm{Hz}\left(\frac{1+\kappa\xi_c^2}{100}\right)^{-1/2}\left(\frac{\mu}{(0.08pc)^{-1}}\right)^4$$

$x \approx 1$

Phase shift is as small as $HD\sqrt{3(1+\kappa\xi_c^2)\Omega_0}$?

No, $x << 1$ when the GWs are propagating the inter-galactic low density region.



Induced gravity on the brane

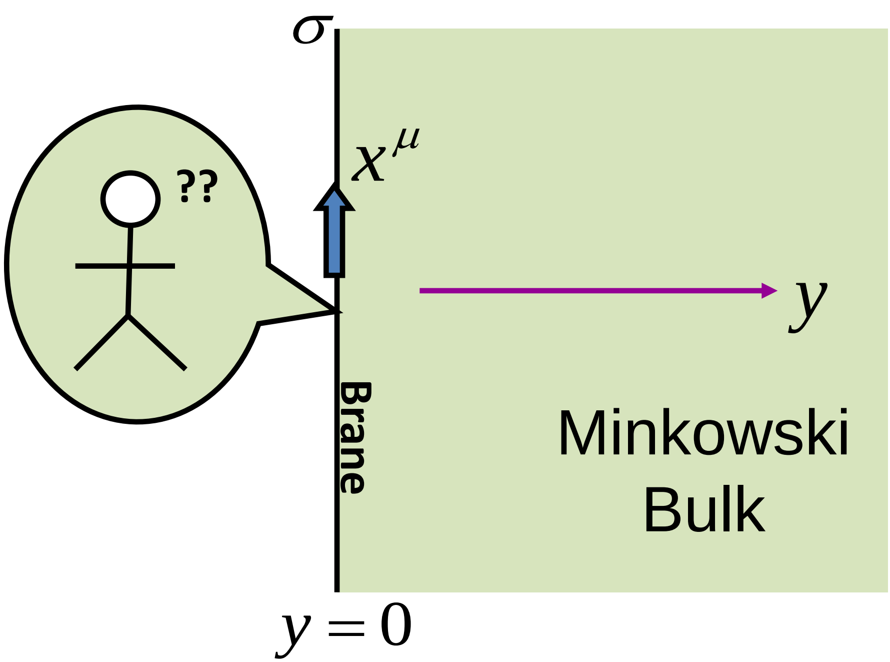
Dvali-Gabadadze-Porrati model (2000)

$$S = \underline{M_5^3} \int d^5x \sqrt{g} R + \int d^4x \sqrt{g^{(4)}} \left(M_4^2 R^{(4)} + L_{matt} \right)$$

$$M_5^3 = M_4^2 / 2r_c$$

Critical length scale

$$\frac{M_5^3}{r} : \frac{M_4^2}{r^2} = \frac{1}{r r_c} : \frac{1}{r^2}$$



- For $r < r_c$, 4-D induced gravity term dominates?
- Extension is infinite, but 4-D GR seems to be recovered for $r < r_c$.

very different from the other braneworld models

Gravitons are trapped to the brane but not completely.

5D scalar toy model:

$$\left[M_5^3\Box+\delta(y)M_4^2\Box^{(4)}\right]\phi=\delta(y)J$$

Source term

$$\phi=\tilde{\phi}(y)e^{ip_\mu x^\mu}$$

$$M_5^3=M_4^2/2r_c$$

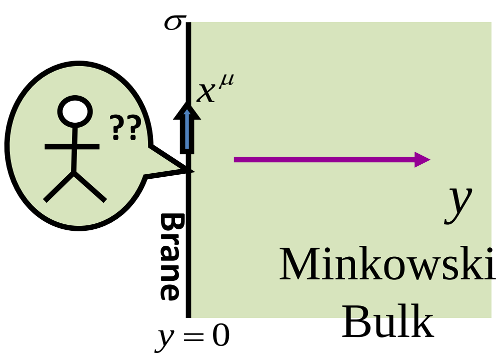
$$M_5^3\left(p^2-\partial_y^2\right)\tilde{\phi}+\delta(y)M_4^2p^2\tilde{\phi}=-\delta(y)\tilde{J}$$

$$\int_{-\varepsilon}^{\varepsilon}dy\,(\text{equation})$$

$$-2M_5^3\partial_y\tilde{\phi}+M_4^2p^2\tilde{\phi}=-\tilde{J}$$

Solution in the bulk is given by

$$\tilde{\phi}=\tilde{\phi}_0\,e^{-py}$$



$$\left[2M_5^3p+M_4^2p^2\right]\tilde{\phi}=-\tilde{J}$$

$$\tilde{\phi}\propto\frac{\tilde{J}}{p+r_cp^2}$$

$$\tilde{\phi} \propto \frac{\tilde{J}}{p + r_c p^2}$$

Static pointlike source on the brane

$$\boxed{\tilde{J} \propto \int dt\, e^{-i\omega t} \int d^3x\, e^{i\vec{k}\cdot\vec{x}} \delta(x) \propto \delta(\omega)}$$

large scale (small k) $\quad \phi \propto \int d^3k\, e^{ikr} \frac{1}{k} \propto \frac{1}{r^2}$

five dimensional behavior

small scale (small k) $\quad \phi \propto \int d^3k\, e^{ikr} \frac{1}{k^2} \propto \frac{1}{r}$

four dimensional behavior

After propagation over cosmological distance, GWs may escape into the bulk?

$$\int \frac{d^3k}{k^2-\omega^2+r_c^{-1}\sqrt{k^2-\omega^2}}\, e^{i\mathbf{kx}}$$

$$= \frac{2\pi}{r}\int_{-\infty}^{\infty}\frac{\sin(kr)\,k\,dk}{k^2-\omega^2+r_c^{-1}\sqrt{k^2-\omega^2}}$$

$$= \frac{2\pi}{ir}\int_{C}\frac{e^{ikr}\,k\,dk}{k^2-\omega^2+r_c^{-1}\sqrt{k^2-\omega^2}}$$

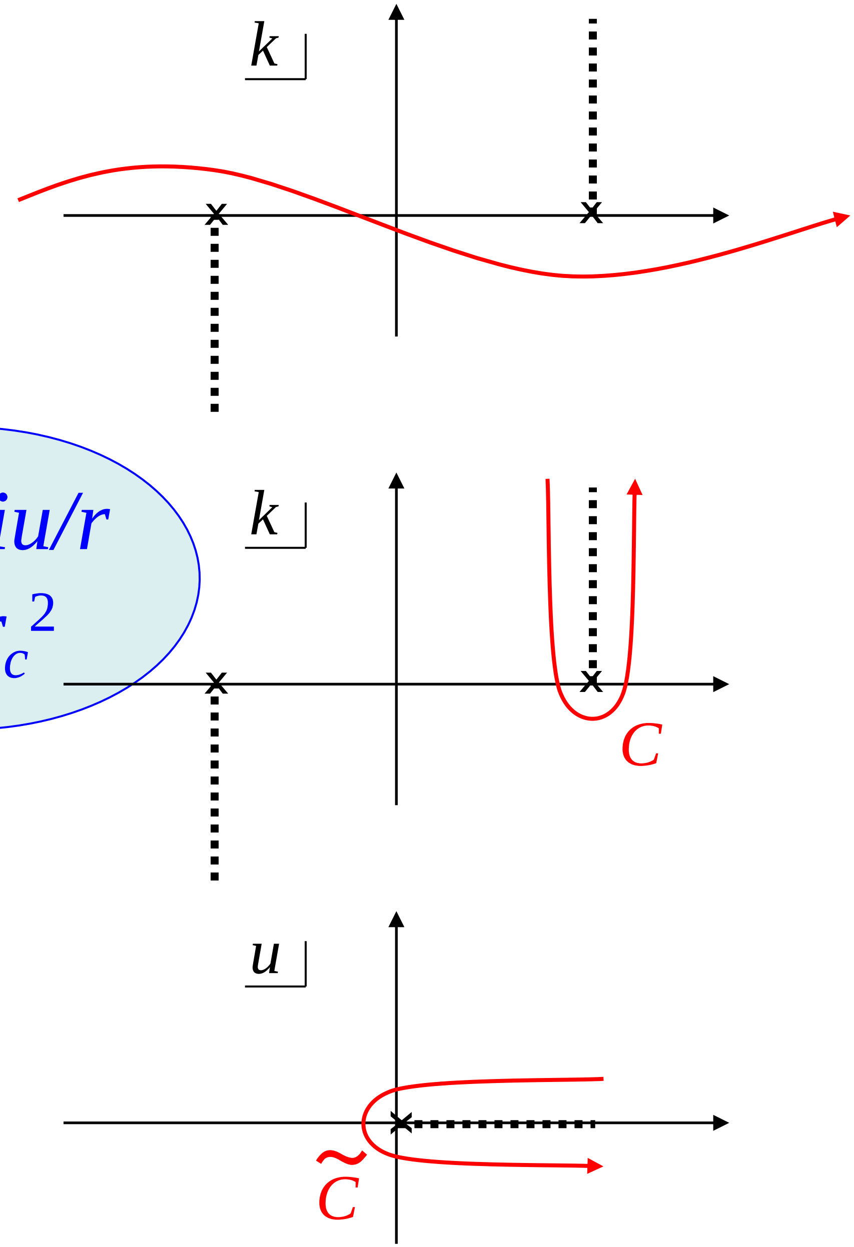
$$\approx \frac{2\pi}{r}\int_{\tilde{C}}\frac{e^{i\omega r}\,e^{-u}\,du}{2iu-\dfrac{e^{\pi i/4}}{\omega r_c}\sqrt{2\omega r u}}$$

$$\approx \frac{2\pi}{r}e^{i\omega r}\int_{\tilde{C}}\frac{e^{-u}\,du}{2iu}\left(1+\frac{e^{-\pi i/4}}{2\omega r_c}\sqrt{\frac{2\omega r}{u}}\right)$$

$$= \frac{2\pi^2}{r}e^{i\omega r}\left(1+\sqrt{\frac{2ir}{\pi\omega r_c^2}}\right)$$

$k=\omega+iu/r$

$r \ll \omega r_c^2$



Chern-Simons Modified Gravity

$$S \supset \frac{\alpha}{4}\int d^4x\sqrt{-g}\,\theta\,{}^*RR$$

Right-handed and left-handed gravitational waves are magnified differently during propagation, depending on the frequencies.

$$\boldsymbol{h}^{(L,R)} = \frac{1}{\sqrt{2}}\left(\boldsymbol{h}^{(+)} + i\boldsymbol{h}^{(\times)}\right)$$

$$\boldsymbol{h}^{(L,R)}_{\rm obs} \approx \boldsymbol{h}^{(L,R)}\sqrt{1\pm\omega\alpha\dot{\theta}}\Big|_{\rm emit}$$

Current constraint on the evolution of the background scalar field θ :

$$\left|\alpha\dot{\theta}\right| < \left(10^6\,{\rm Hz}\right)^{-1}$$: J0737-3039(double pulsar)

(Ali-Haimoud, (2011))

Moreover, $\left|\omega\alpha\dot{\theta}\right| \approx 1$ modes are in the strong coupling regime.

outside the validity range of EFT.

Ghost free bi-gravity

When $\tilde{g}$ is fixed, de Rham-Gabadadze-Tolley massive gravity.

$$L = \frac{\sqrt{-g}R}{16\pi G_N} + \sqrt{-g}\sum_{n=0}^{4} c_n V_n + L_{matter}$$

$$V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2,\ \cdots \quad \tau_n \equiv Tr\left[\gamma^n\right] \quad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

No gauge degrees of freedom, but by introducing a field

$$g_{\mu\nu} \to g_{\mu\nu} + 2\pi_{;\mu\nu} + \cdots$$

$$\begin{cases} \pi \to \pi - \Lambda \\ g_{\mu\nu} \to g_{\mu\nu} + 2\Lambda_{;\mu\nu} \end{cases}$$ becomes a gauge symmetry.

Fixing the gauge by π =0 ⇒ original theory

Imposing condition on $g_{\mu\nu}$ ⇒ π becomes dynamical

Setting $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, we consider flat metric + π perturbation:

$$g_{\mu\nu} \to \eta_{\mu\nu} + 2\pi_{;\mu\nu} + \pi_{;\mu\rho}\pi^{;\rho}{}_{;\nu}$$

$$\gamma^{\mu}{}_{\nu} = \sqrt{\eta^{\mu\nu} g_{\rho\nu}} = \delta^{\mu}{}_{\nu} + \pi^{;\mu}{}_{;\nu}$$

If $L \supset \pi^{;\mu}{}_{;\nu}\pi^{;\nu}{}_{;\mu}$, its variation gives higher derivative terms.

To avoid higher derivatives of π in the EOM,

$$\varepsilon_{\mu\nu\xi\zeta}\varepsilon^{\alpha\beta\xi\zeta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\beta}, \quad \varepsilon_{\mu\nu\rho\zeta}\varepsilon^{\alpha\beta\gamma\zeta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\beta}\gamma^{\rho}{}_{\gamma}, \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\beta}\gamma^{\rho}{}_{\gamma}\gamma^{\sigma}{}_{\delta},$$

$\Rightarrow$ $V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2, \ \cdots$

$$\tau_n \equiv Tr\left[\gamma^n\right] \quad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

In other words:

10 (metric components) – 4 (constraints) $= 6$

Since massive spin 2 field has 5 components, one scalar remains, which becomes a ghost (kinetic term with wrong sign).

If constraints do not completely fix the Lagrange multipliers, $g_{0\mu}$, their consistency relation gives an additional condition. As a result, the residual scalar degree of freedom disappears.

(Hassan, Rosen (2011))

Ghost free bi-gravity

$$\frac{L}{M_G^2}=\frac{\sqrt{-g}R}{2}+\frac{\sqrt{-\tilde{g}}\tilde{R}}{2\kappa}+\frac{\sqrt{-g}}{2}\sum_{n=0}^{4}c_nV_n+\frac{L_{matter}}{M_G^2}$$

$$V_0=1,\quad V_1=\tau_1,\quad V_2=\tau_1^2-\tau_2,\ \cdots$$

$$\tau_n\equiv Tr\left[\gamma^n\right]\qquad \gamma^i_j\equiv\sqrt{g^{ik}\tilde{g}_{kj}}$$

Now $\tilde{g}$ is promoted to a dynamical field.

Even in this case, it was shown that the model remains to be free from ghost.

(Hassan, Rosen (2012))

FLRW background

(Comelli, Crisostomi, Nesti, Pilo (2012))

$$ds^2 = a^2(t)\left(-dt^2 + dx^2\right)$$

$$d\tilde{s}^2 = b^2(t)\left(-c^2(t)dt^2 + dx^2\right)$$

$$T^{(mass)}_{\mu\nu} = 2\frac{\delta S^{(mass)}}{\delta g^{\mu\nu}}$$

$$\xi \equiv b/a$$

$$\nabla^\mu T^{(mass)}_{\mu\nu} = 0 \Rightarrow \underbrace{\left(6c_3\xi^2 + 4c_2\xi + c_1\right)}_{\text{branch 1}}\underbrace{\left(cba' - ab'\right)}_{\text{branch 2}} = 0$$

branch 1 :

Pathological: At the linear perturbation, expected scalar and vector perturbations are absent. Strong coupling? Unstable for the homogeneous anisotropic mode.

branch 2 :

Healthy: All perturbation modes are equipped.

Branch 2 background

$$\underbrace{\left(6c_3\xi^2+4c_2\xi+c_1\right)}_{\text{branch 1}}\underbrace{\left(cba'-ab'\right)}_{\text{branch 2}}=0 \qquad \xi\equiv b/a$$

branch 2 :

$$\rho-\frac{c_1}{\kappa\xi}+\left(c_0-\frac{6c_2}{\kappa}\right)+\left(3c_1-\frac{18c_3}{\kappa}\right)\xi+\left(6c_2-\frac{24c_4}{\kappa}\right)\xi^2+6c_3\xi^3=0$$

ξ becomes a function of ρ. $\xi \to \xi_c$ for $\rho \to 0$.

$$H^2=\frac{\rho+\rho_{mass}}{3M_G^2} \qquad \rho_{mass}:=c_0+3c_1\xi+6c_2\xi^2+6c_3\xi^3$$

effective energy density due to mass term

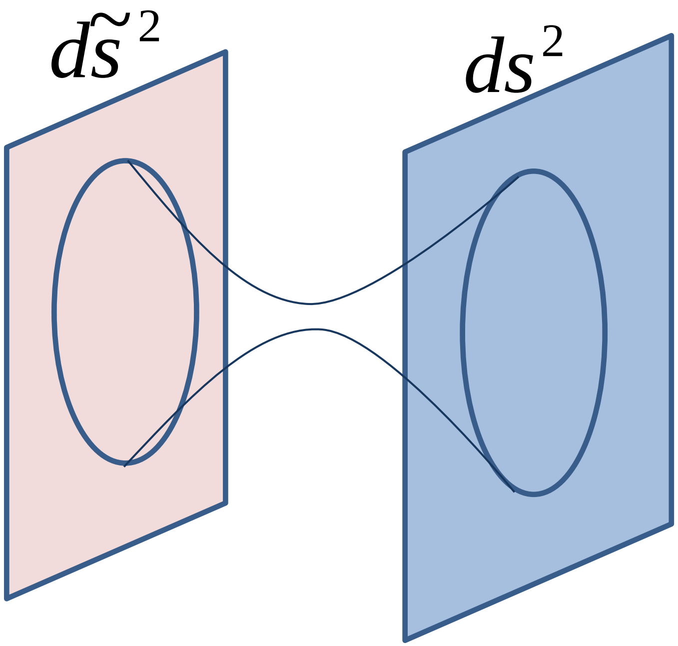
$$\frac{1}{c-1}\frac{\xi'}{\xi}=\frac{a'}{a} \Rightarrow c-1=\frac{3(\rho+P)\kappa\xi_c}{\Gamma_c\left(1+\kappa\xi_c^2\right)M_G^2} \qquad \Gamma=\frac{d\rho_{mass}}{d\xi}$$

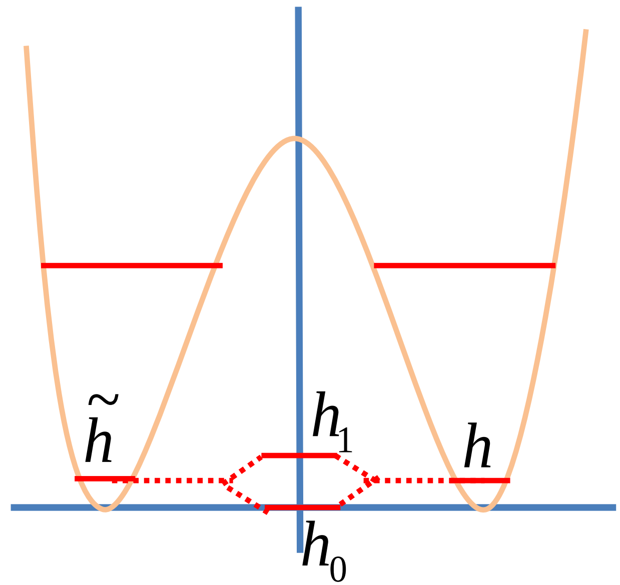
Natural Tuning to c=1 for $\rho \to 0$.

$$H^2=\frac{\rho}{3\left(1+\kappa\xi_c^2\right)M_G^2}$$

Effective gravitational coupling is weaker because of the dilution to the hidden sector.

Why do we have this attractor behavior, $c \to 1$ and $\xi \to \xi_c$, at low energies?





Higher dimensional model?!

Matter on right brane couples to h.

KK graviton spectrum

Only first two modes remain at low energy

If the internal space is stabilized

$$\Rightarrow \quad d\tilde{s}^2 = \xi_c^2 ds^2 \quad \Rightarrow \quad c = 1$$

Solar system constraint: basics

◈ vDVZ discontinuity

In GR, this coefficient is 1/2

$$\delta g_{\mu\nu} \propto \Box^{-1}\left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T\right)$$

current bound <10^{-5}

To cure this discontinuity

- Make the extra helicity 0 mode massive

$$\mu^{-1} \geq 0.1\mathrm{mm} \quad \text{(Chameleon)}$$

- Go beyond the linear perturbation (Vainshtein)

Schematically

$$\cancel{\Delta\delta\Phi} + \mu^{-2}(\partial\partial\delta\Phi)^2 = G_N \rho$$

$$\Rightarrow \quad \delta\Phi/\Phi \approx \mu\sqrt{G_N \rho}\, r^2 \big/ (r_g / r) \approx \mu\sqrt{r^3/r_g}$$

$$10^{-10} \geq \mu\sqrt{(10^{13} cm)^3 / (10^5 cm)} \quad \Rightarrow \quad \mu^{-1} \geq 300 Mpc$$

Gravitational potential around a star in the limit $c \to 1$

Spherically symmetric static configuration:

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}\left(dr^2 + r^2 d\Omega^2\right)$$

$$d\tilde{s}^2 = \xi_c^2\left[-e^{\tilde{u}-\tilde{v}}dt^2 + e^{\tilde{u}+\tilde{v}}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)\right] \qquad \tilde{r} = e^R r$$

Erasing $\tilde{u}, \tilde{v}$ and R ,

$$\Rightarrow \quad \left(\Delta - \mu^2\right)u - \frac{C}{\mu^2}\left((\Delta u)^2 - \left(\partial_i \partial_j u\right)^2\right) \approx \frac{\rho_m}{M_G^2}$$

$$C \propto \frac{d\left(\log \Gamma\right)}{d\left(\log \xi\right)}$$, which can be tuned to be extremely large.

Then, the Vainshtein radius $r_V \approx \left(\frac{C r_g}{\mu^2}\right)^{1/3}$

can be made very large, even if $\mu^{-1} << 300\mathrm{Mpc}$.

Solar system constraint: $\sqrt{C}\mu^{-1} \geq 300\mathrm{Mpc}$

$$\Delta v \approx \frac{\rho_m}{\tilde{M}_G^2}$$ v is excited as in GR. $$H^2 = \frac{\rho}{3\tilde{M}_G^2}$$

Excitation of the metric perturbation on the hidden sector:

Erasing u, v and R

$$\Rightarrow \quad \left(\Delta-\mu^2\right)\tilde{u}-\frac{\tilde{C}}{\mu^2}\left(\left(\Delta\tilde{u}\right)^2-\left(\partial_i\partial_j\tilde{u}\right)^2\right)\approx\frac{\rho_m}{M_G^2}$$

$$\Delta\tilde{v}\approx\frac{\rho_m}{\tilde{M}_G^2}$$

$\tilde{u}$ is also suppressed like u.

$\tilde{v}$ is also excited like v.

The metric perturbations are almost conformally related with each other: $d\tilde{s}^2\approx\xi_c^2 ds^2$

Non-linear terms of $\tilde{u}$ (or equivalently u) play the role of the source of gravity.

EOM of Gravitational waves

$$m_g^2 = \frac{f'}{3} + \frac{(c-1)}{6}\left(f'' - f'\right)$$

$$h'' + 2aHh'' - \Delta h + a^2 m_g^2\left(h - \tilde{h}\right) = 0$$

$$\tilde{h}'' + \left(2aH + 2\xi'/\xi - c'/c\right)\tilde{h}' - c^2\Delta\tilde{h} - a^2 m_g^2 \frac{c}{\kappa\xi^2}\left(h - \tilde{h}\right) = 0$$

(Comelli, Crisostomi, Pilo (2012))

Short wavelength approximation: $k >> m_g >> H$

$$\begin{pmatrix} -\omega^2 + k^2 + m_g^2 & -m_g^2 \\ -\frac{1}{\kappa\xi^2} m_g^2 & -\omega^2 + c^2k^2 + \frac{1}{\kappa\xi^2} m_g^2 \end{pmatrix}\begin{pmatrix} h \\ \tilde{h} \end{pmatrix} = 0$$

Two propagation speeds are not same for $c \neq 1$.
[$\neq \nu$−oscillation]

Eigen mode decomposition

$$\begin{cases} h_A = \cos\theta_g h + \sin\theta_g\left(\sqrt{\kappa}\xi\tilde{h}\right) \\ h_2 = -\sin\theta_g h + \cos\theta_g\left(\sqrt{\kappa}\xi\tilde{h}\right) \end{cases}$$

$$k_{1,2}^2 = \omega^2 - \frac{\mu^2}{2}\left(1 + x \mp \sqrt{1 + 2x\frac{1-\kappa\xi^2}{1+\kappa\xi^2} + x^2}\right) \qquad \mu^2 = m_g^2\frac{1+\kappa\xi^2}{\kappa\xi^2}$$

$$\cot 2\theta_g = \frac{\left(1+\kappa\xi_c^2\right)x + \left(1-\kappa\xi_c^2\right)}{2\sqrt{\kappa}\xi_c} \qquad x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

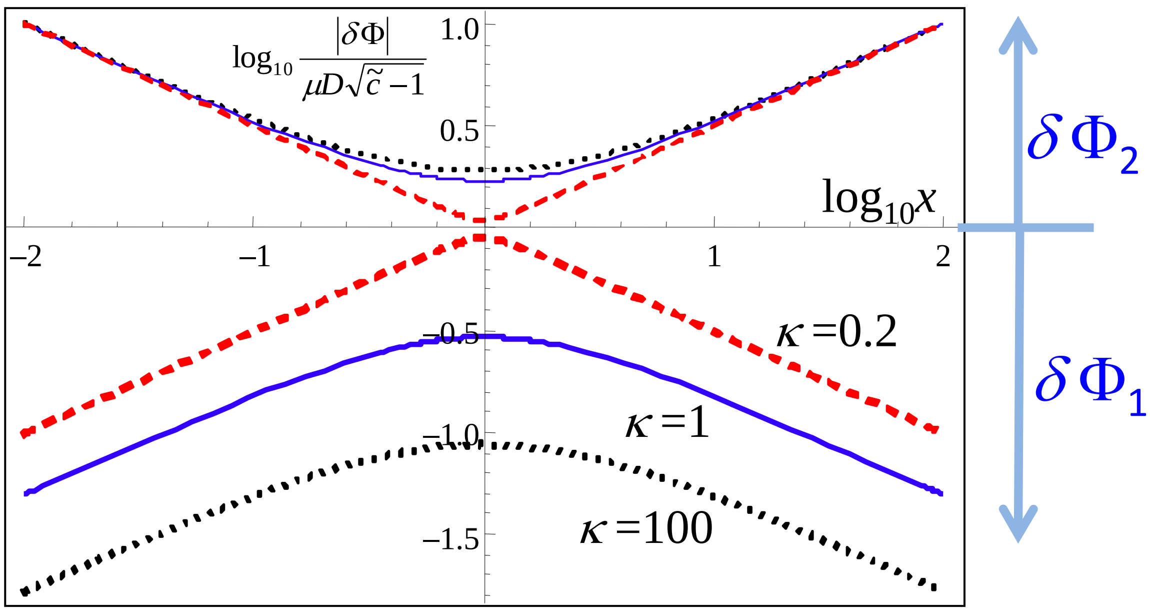
Gravitational wave propagation over a long distance D

Phase shift due to the modified dispersion relation:

$$\delta\Phi_{1,2} \equiv -\frac{\Delta k^2}{2\omega} D = -\frac{\mu D\sqrt{c-1}}{2\sqrt{2x}}\left(1 + x \mp \sqrt{1 + 2x\frac{1-\kappa\xi^2}{1+\kappa\xi^2} + x^2}\right)$$

$$\mu D\sqrt{c-1} \approx HD\sqrt{3\left(1+\kappa\xi_c^2\right)\Omega_0}$$

becomes O(1) after propagation over the horizon distance



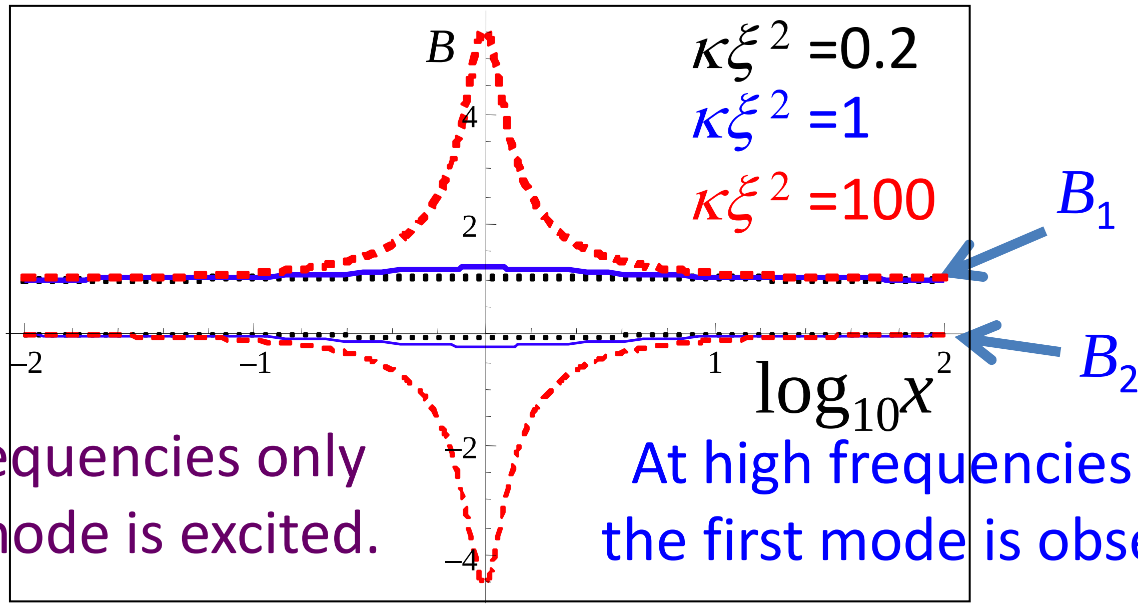
$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

Gravitational wave oscillations

1) At the time of generation of GWs from coalescing binaries, both h and $\tilde{h}$ are equally excited.

2) When we detect GWs, we sense h only.

$$\Rightarrow \quad h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$

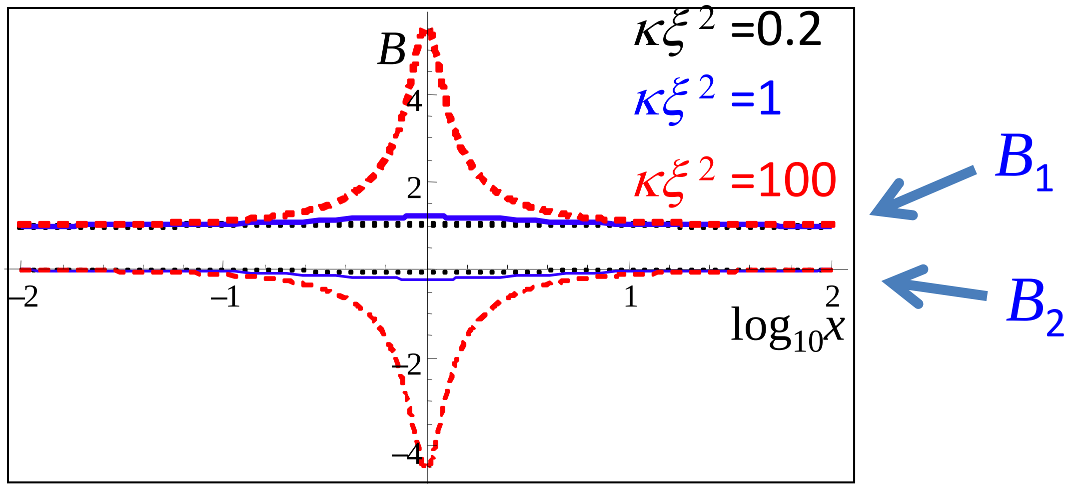


$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

At low frequencies only the first mode is excited.

At high frequencies only the first mode is observed.

$$h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$



Graviton oscillations occur only around the frequency

$$\omega_{GW} \approx \frac{\mu^2}{\sqrt{6(1+\kappa\xi_c^2)\Omega_0}H} = 100\mathrm{Hz}\left(\frac{1+\kappa\xi_c^2}{100}\right)^{-1/2}\left(\frac{\mu}{(0.08pc)^{-1}}\right)^4 \qquad x \approx 1$$

Phase shift is as small as $HD\sqrt{3(1+\kappa\xi_c^2)\Omega_0}$?

No, $x << 1$ when the GWs are propagating the inter-galactic low density region.