Investigating formation condition of primordial black holes for generalized initial perturbation profiles

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Introduction

- Let us consider an overdense region during the RD universe. If the density perturbation $\delta_{\mathrm{hc}} \gtrsim 1/3$ at the horizon crossing time, this region collapses to form a primordial black hole (PBH) (Carr, 1975)

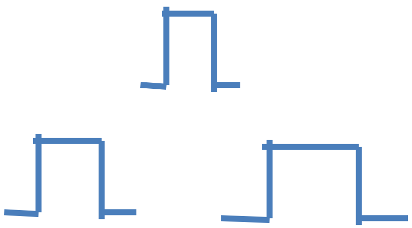
(cf. Harada, Yoo, Kohri 2013)

- **If we can predict the mass function of PBHs precisely,** we can falsify models of the early universe which predict too many PBHs and hence are inconsistent with observations.

Precise knowledge of PBH formation condition is necessary.

Perturbation profiles matter in PBH formation condition

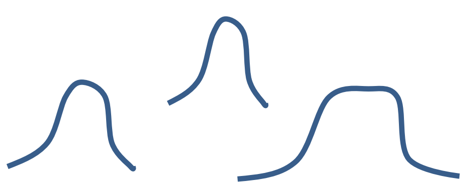
- The condition $\delta_{\mathrm{hc}} \gtrsim 1/3$ was obtained by approximating perturbation profiles by top-hat profiles, which amounts to assuming that ***the averaged density perturbation only*** determines PBH formation.



- This is strictly speaking incorrect and effects of profiles change the threshold value (that is, at least ***two*** quantities are necessary to describe PBH formation condition).

Polnarev, Musco, 2007 Shibata, Sasaki, 1999

We unify and extend these works.



Description of perturbation at the initial time

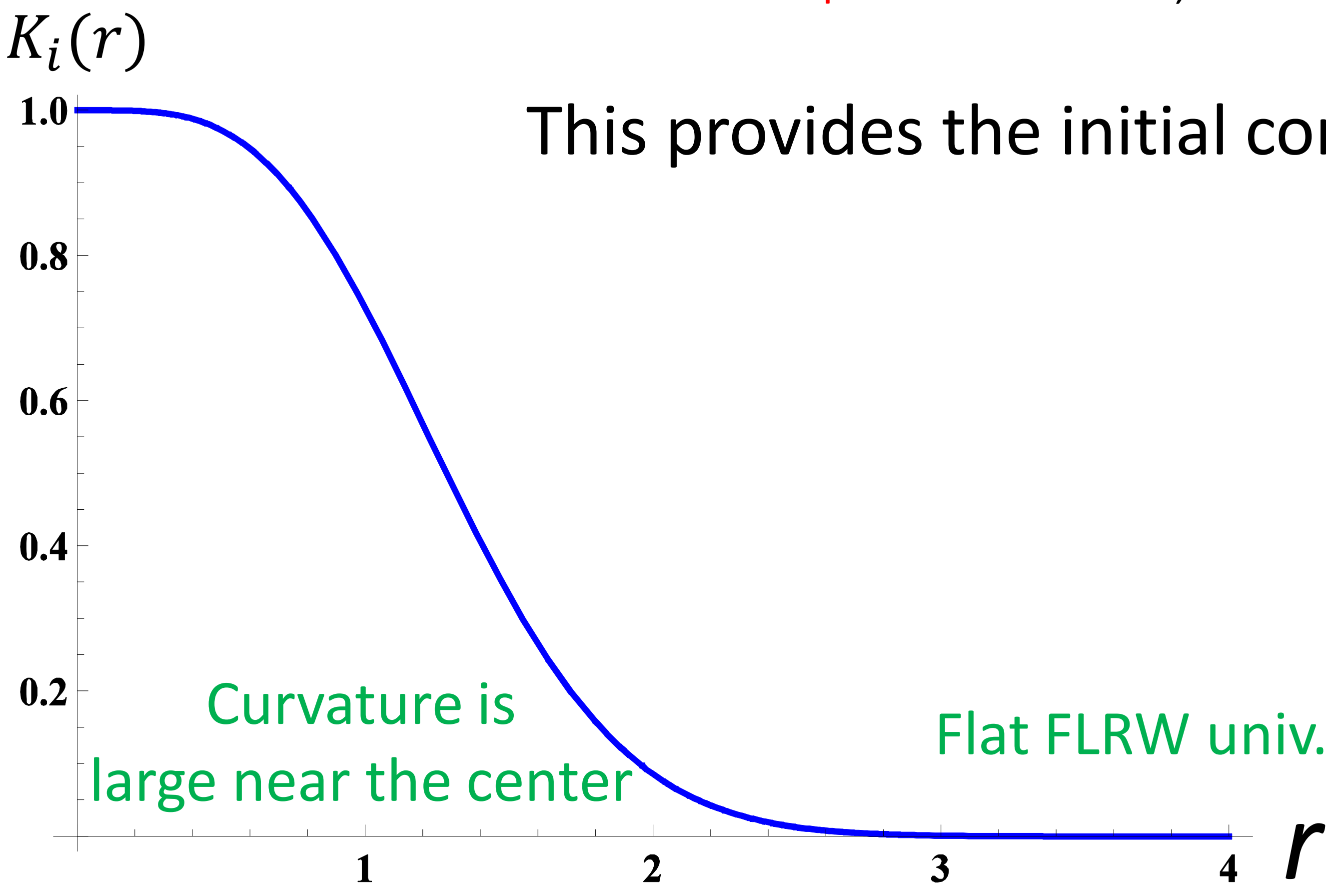
The solution when $perturbation\ scale \gg hubble\ radius$

$$ds^2 = -dt^2 + \frac{S^2 dr^2}{1 - K_i(r) r^2} + (Sr)^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2)$$

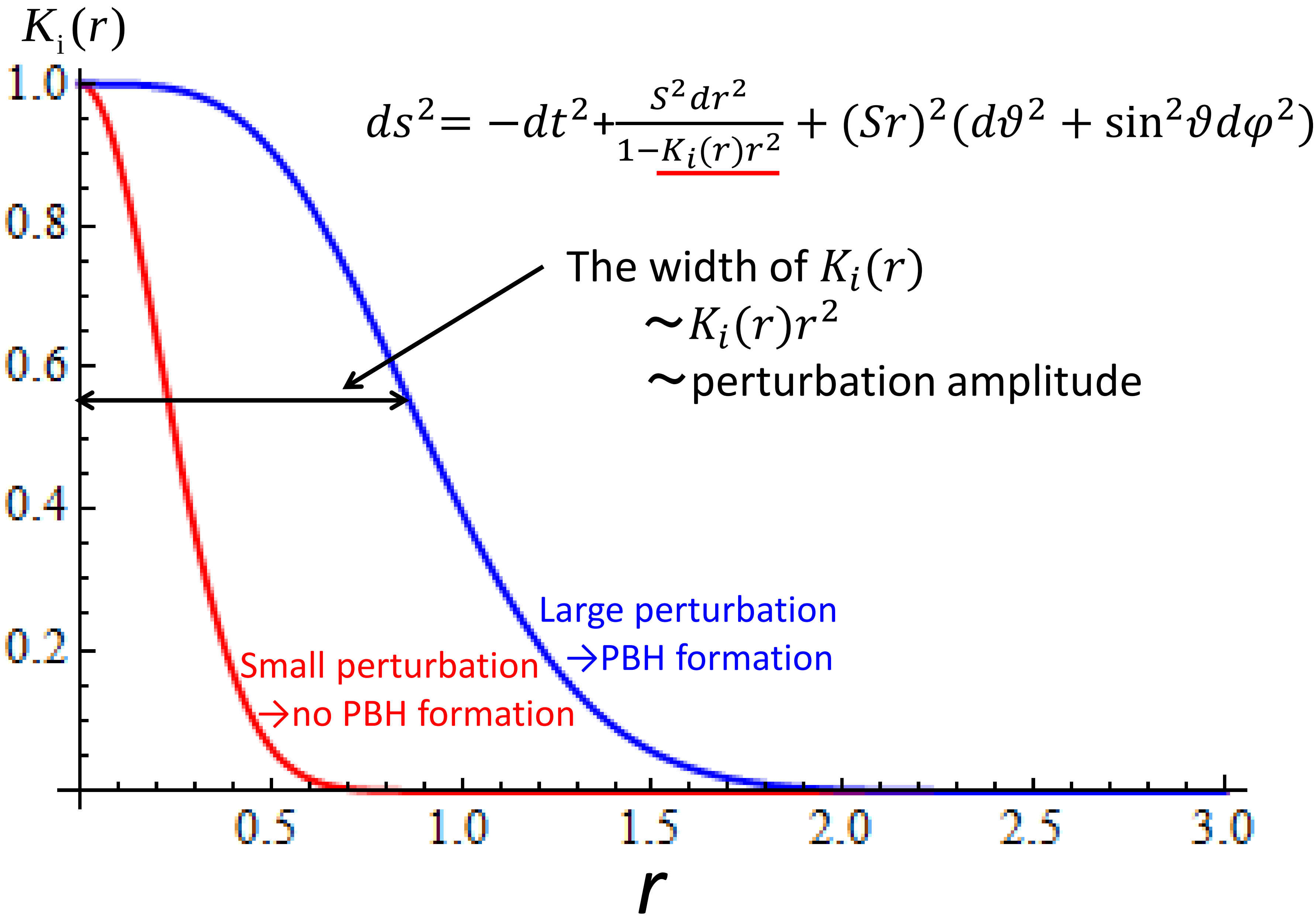
The initial curvature profile

Polnarev, Musco, 2007

This provides the initial condition.



The initial curvature profile $K_i(r)$ and PBH formation



The equations solved numerically

$$\boxed{\dot{R} = au}\quad \boxed{\Gamma = 4\pi\rho R^2 R'}\quad \boxed{w = E + P/\rho}$$

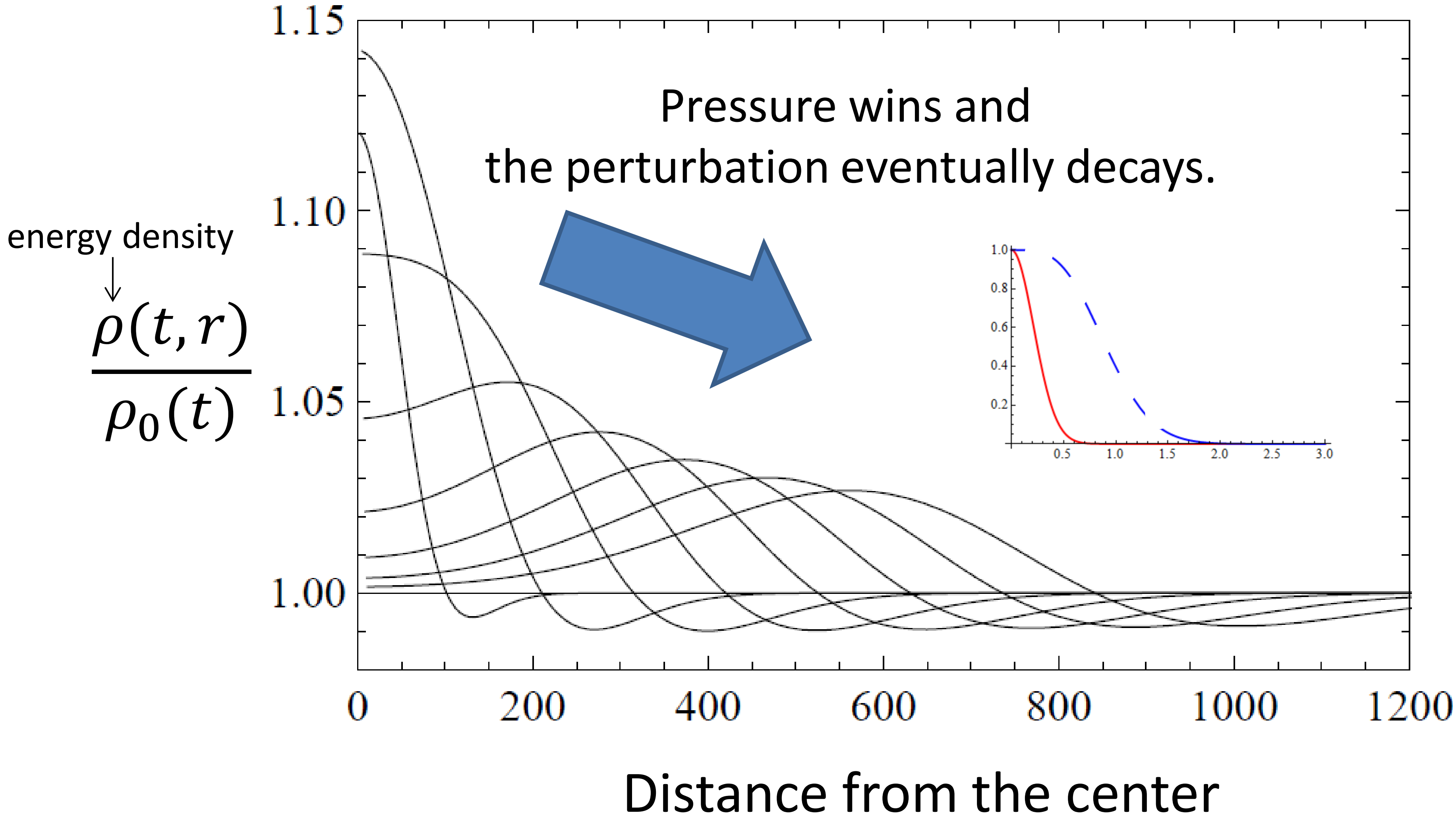
$$\boxed{\rho \equiv \frac{1}{4\pi b R^2}}\quad \boxed{\frac{(\rho R^2)^{\cdot}}{\rho R^2} = -a\frac{u'}{R'}}\quad \boxed{\dot{E} = -P\left(\frac{1}{\rho}\right)^{\cdot}}$$

$$\boxed{\Gamma^2 = 1 + U^2 - \frac{2M}{R}}\quad \boxed{m = 4\pi\int_0^r eR^2R'dr}$$

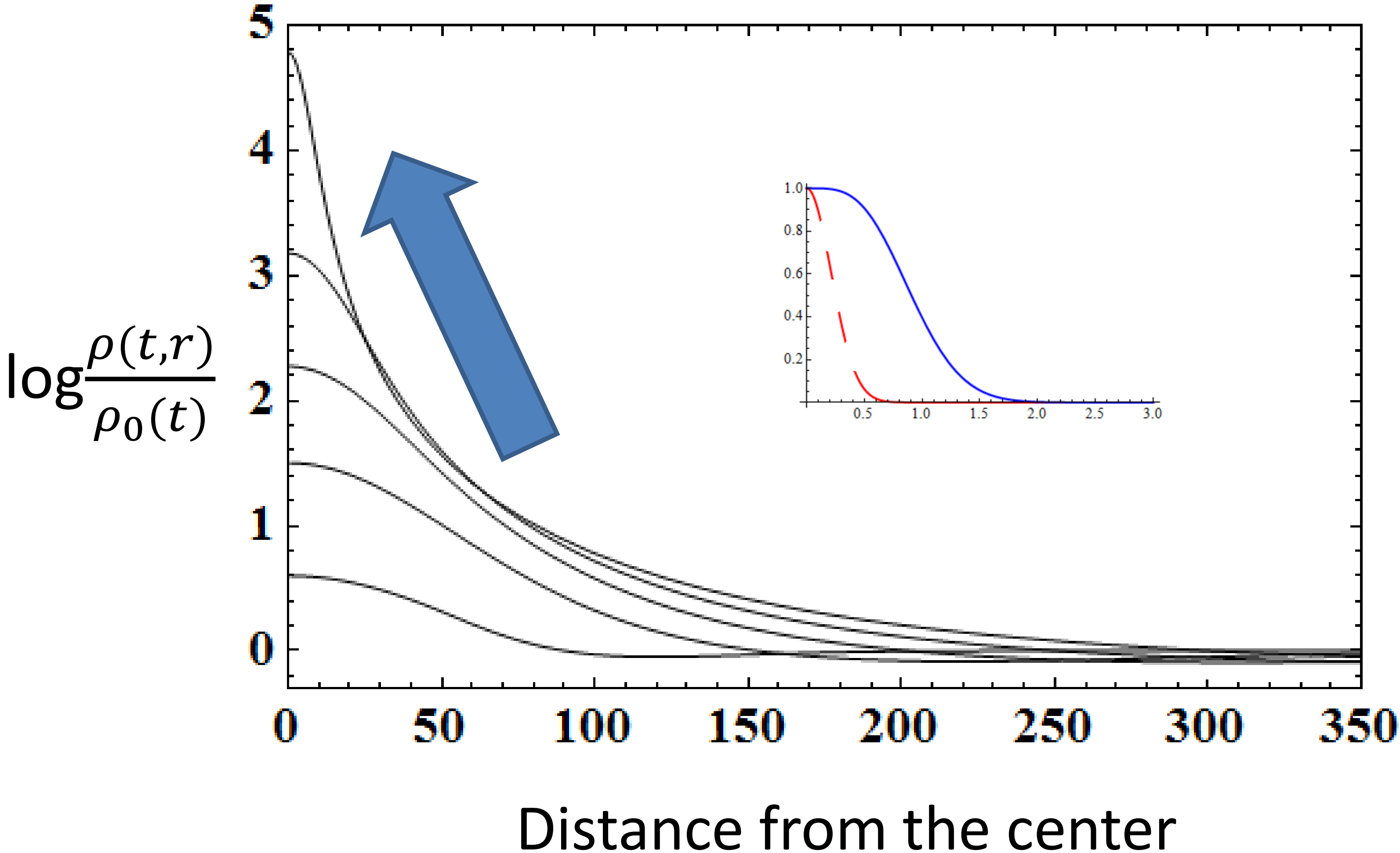
$$\boxed{\dot{u} = -a\left(4\pi R^2\frac{\Gamma}{w}P' + \frac{mG}{R^2} + 4\pi GPR\right)}\quad \boxed{P = \gamma e}$$

May, White(1967) Niemeyer, Jedamzik(1999)

A case where a PBH is *not* formed



A case where a PBH is formed



It is necessary to investigate various profiles

function1 : $$K(r) = \left(1+\alpha\frac{r^2}{2\Delta^2}\right)\exp\left(-\frac{r^2}{2\Delta^2}\right)$$

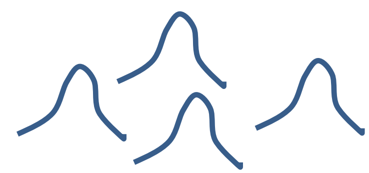
function2 : $$K(r) = \begin{cases} 1 & \text{if} \quad r \leqslant \Delta_* \\ \left(1+\frac{(r-\Delta_*)^2}{2\Delta^2}\right)\exp\left(-\frac{(r-\Delta_*)^2}{2\Delta^2}\right) & \text{if} \quad r > \Delta_* \end{cases}$$

Polnarev, Musco, 2007

function3 : $$\delta\psi^6 = C_\delta\left[\exp\left(-\frac{r^2}{r_0^2}\right) - \sigma^{-3}\exp\left(-\frac{r^2}{\sigma^2 r_0^2}\right)\right]\left(\frac{t}{r_0}\right)^{2-4/3\Gamma}$$

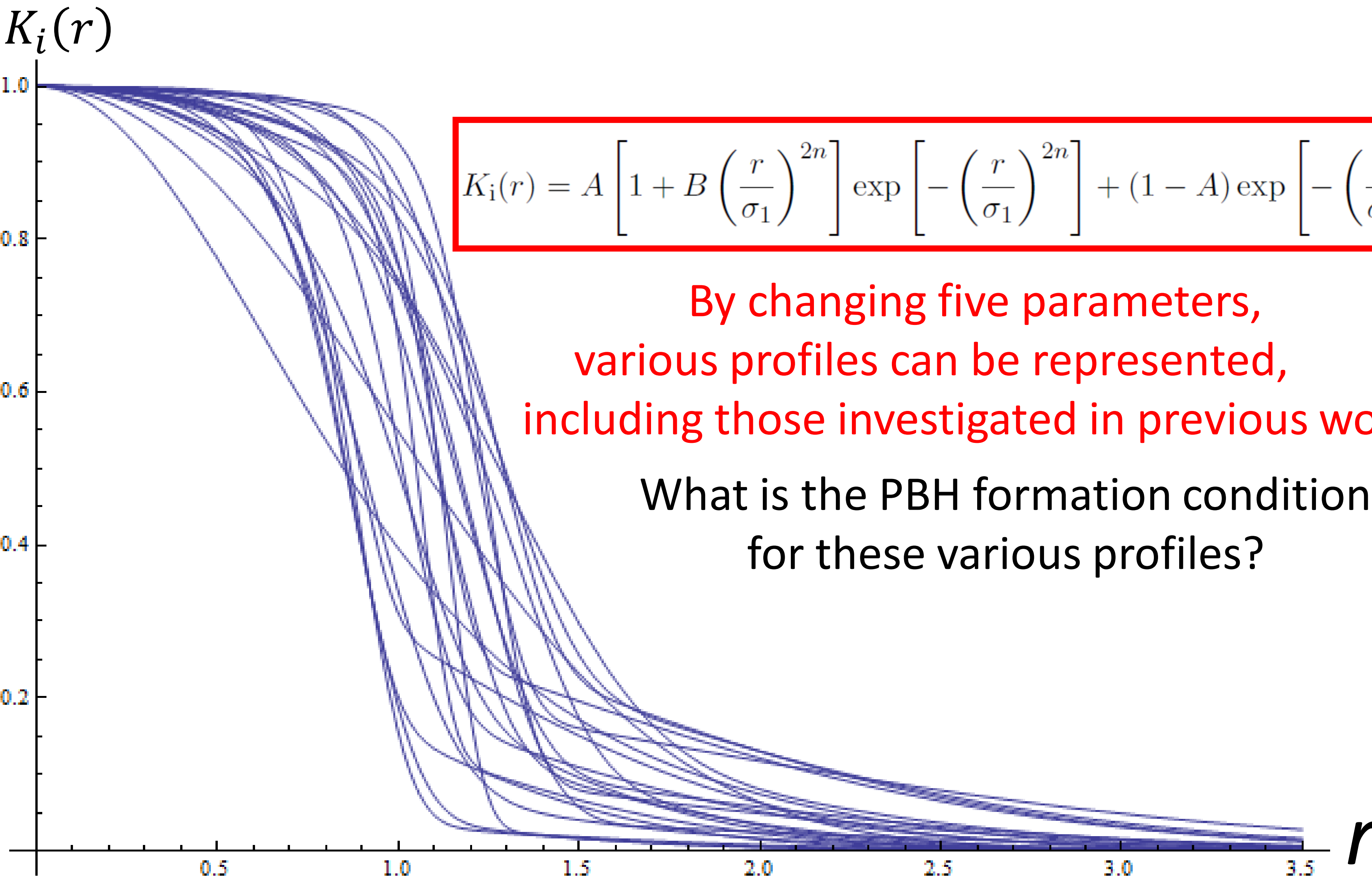
Shibata, Sasaki, 1999

- The variation of profiles represented by one of the above functions is limited.



- It is necessary to investigate various profiles since various profiles of perturbations should have been generated in the early universe.

Representation of various profiles

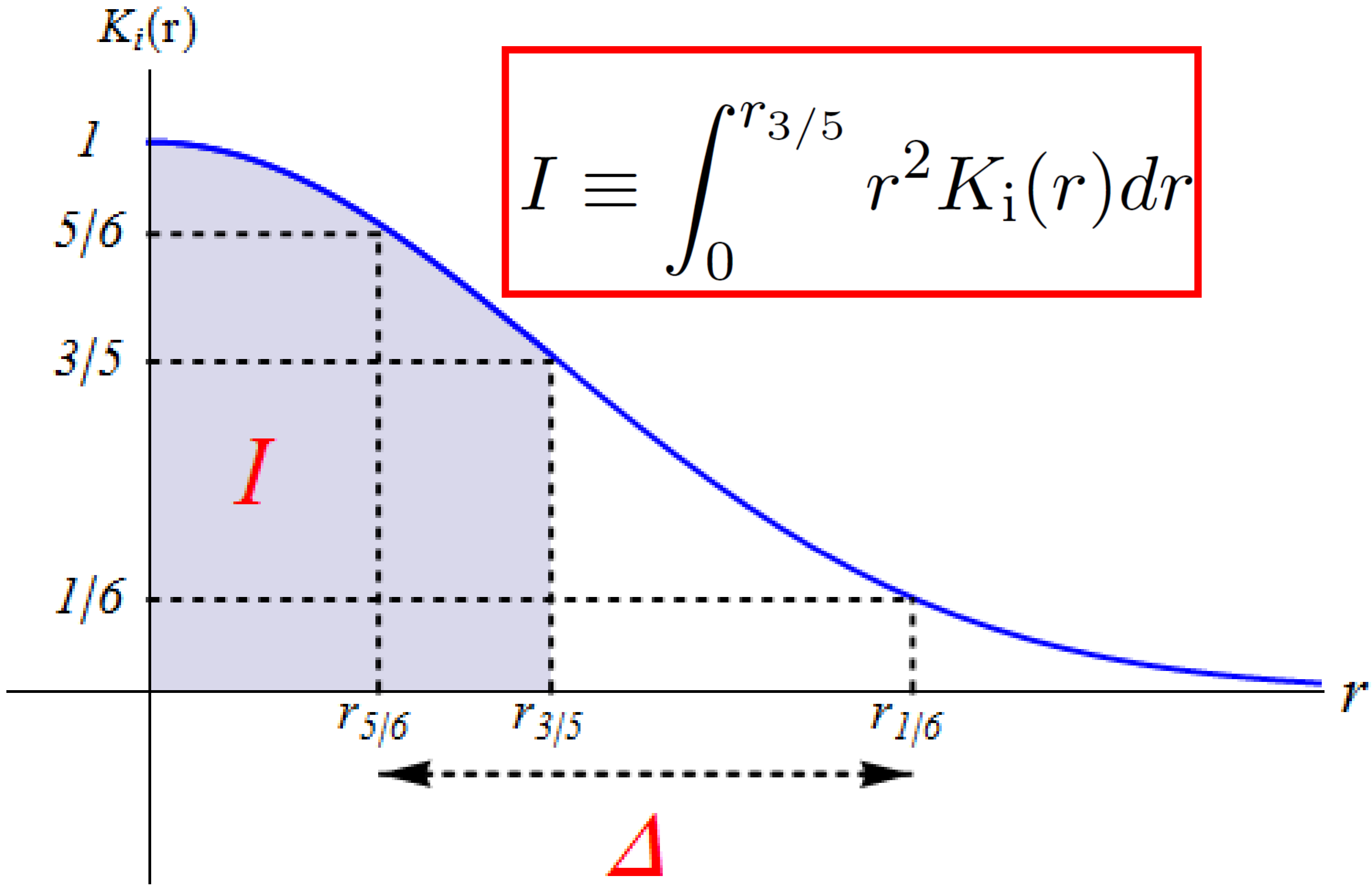


$$K_{\mathrm{i}}(r) = A\left[1 + B\left(\frac{r}{\sigma_1}\right)^{2n}\right]\exp\left[-\left(\frac{r}{\sigma_1}\right)^{2n}\right] + (1-A)\exp\left[-\left(\frac{r}{\sigma_2}\right)^{2}\right]$$

By changing five parameters,
various profiles can be represented,
including those investigated in previous works.

What is the PBH formation condition
for these various profiles?

Two quantities characterizing perturbation profiles

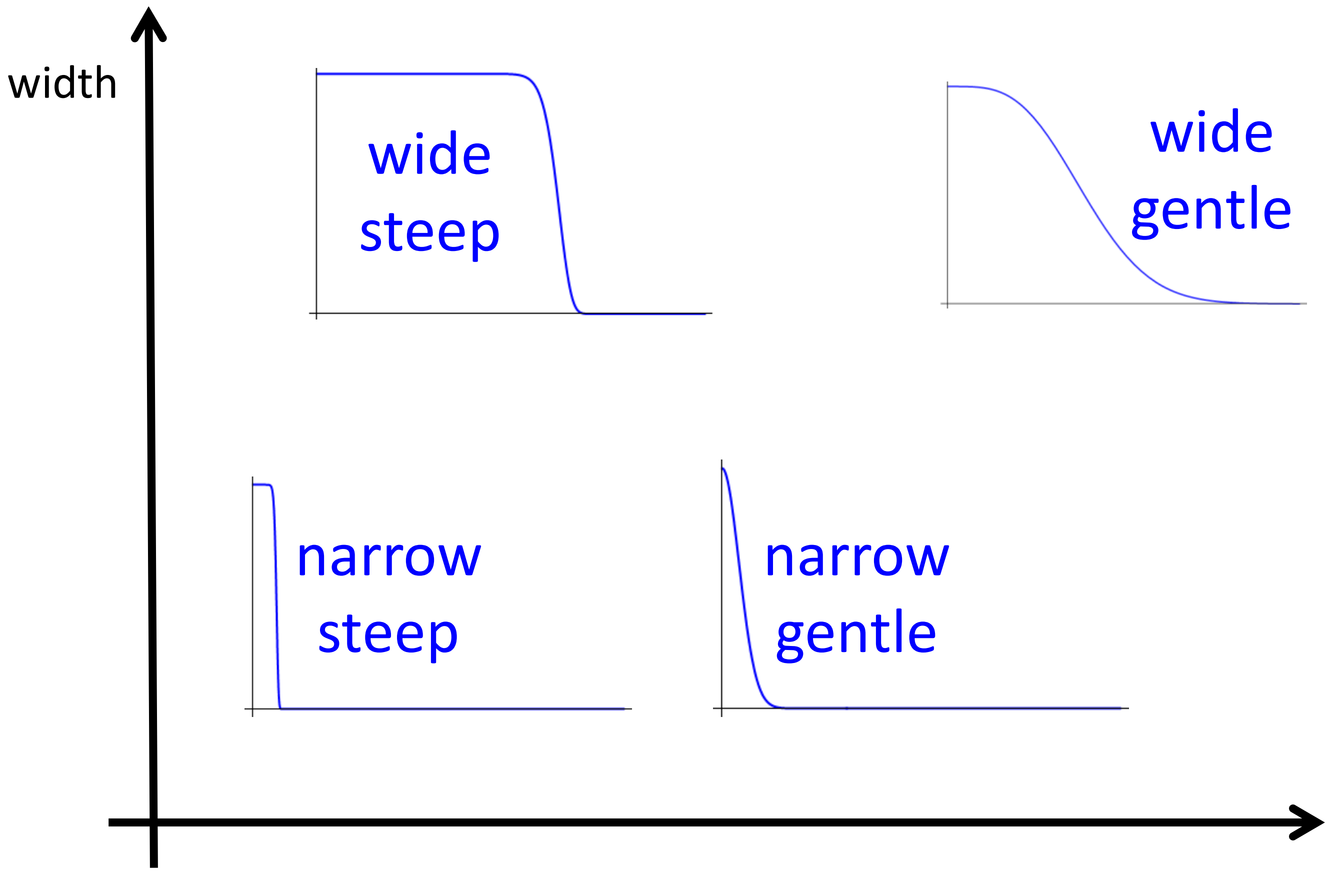


$$I \equiv \int_0^{r_{3/5}} r^2 K_{\mathrm{i}}(r)\,dr$$

$$\Delta \equiv r_{1/6} - r_{5/6}$$

Two quantities characterizing perturbation profiles

$$I \equiv \int_0^{r_{3/5}} r^2 K_{\mathrm{i}}(r)\,dr$$

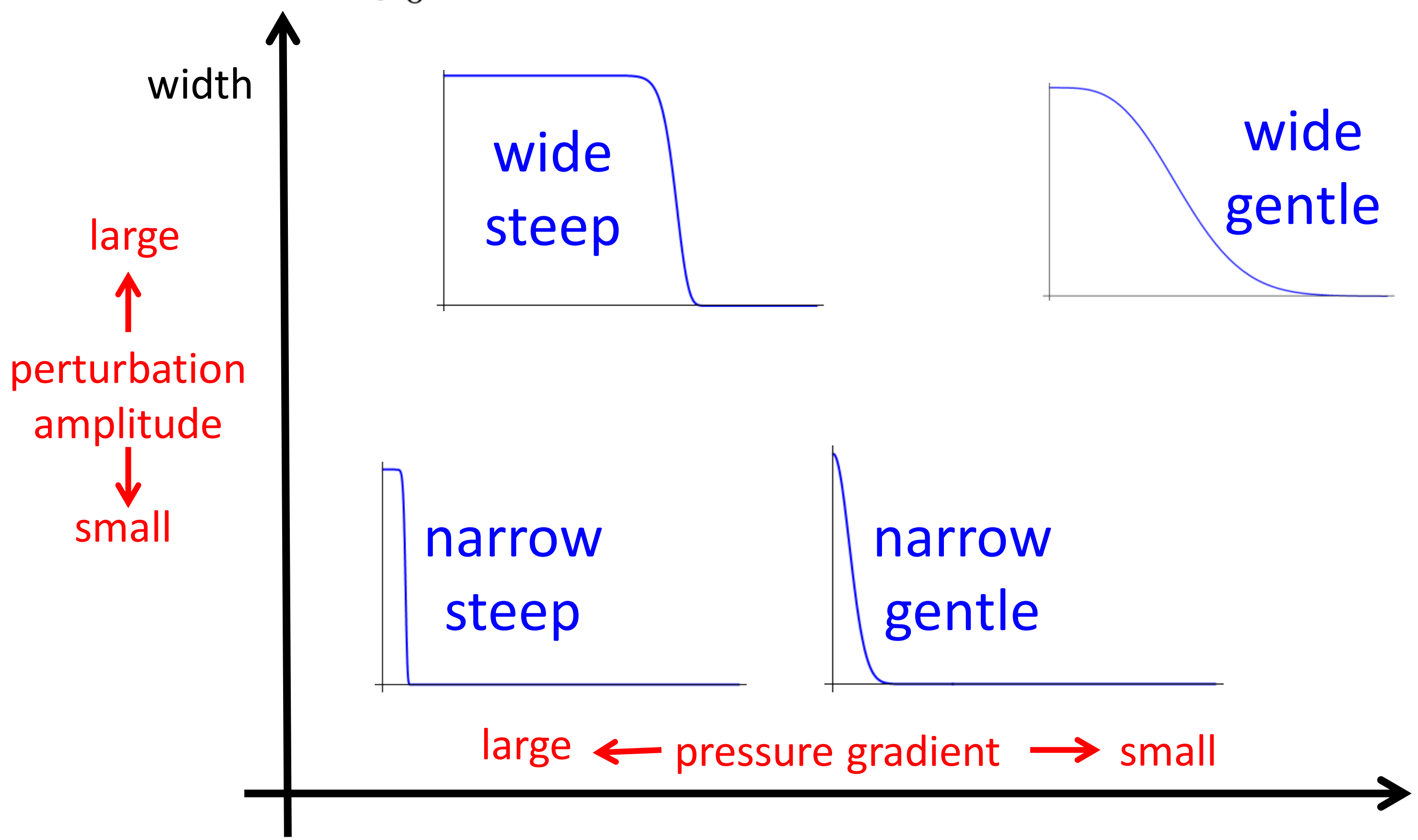


$$\Delta \equiv r_{1/6} - r_{5/6}$$

steepness

Two quantities characterizing perturbation profiles

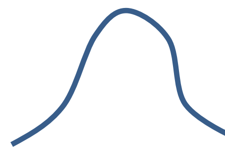
$$I \equiv \int_0^{r_{3/5}} r^2 K_{\mathrm{i}}(r)\,dr$$



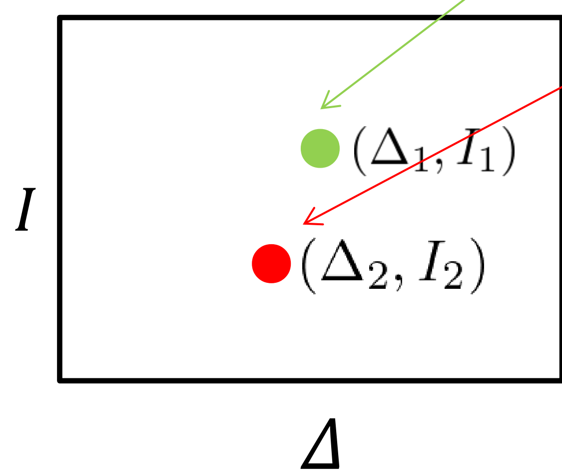
$$\Delta \equiv r_{1/6} - r_{5/6}$$ steepness

How to describe the PBH formation condition on *Δ-I* plane?

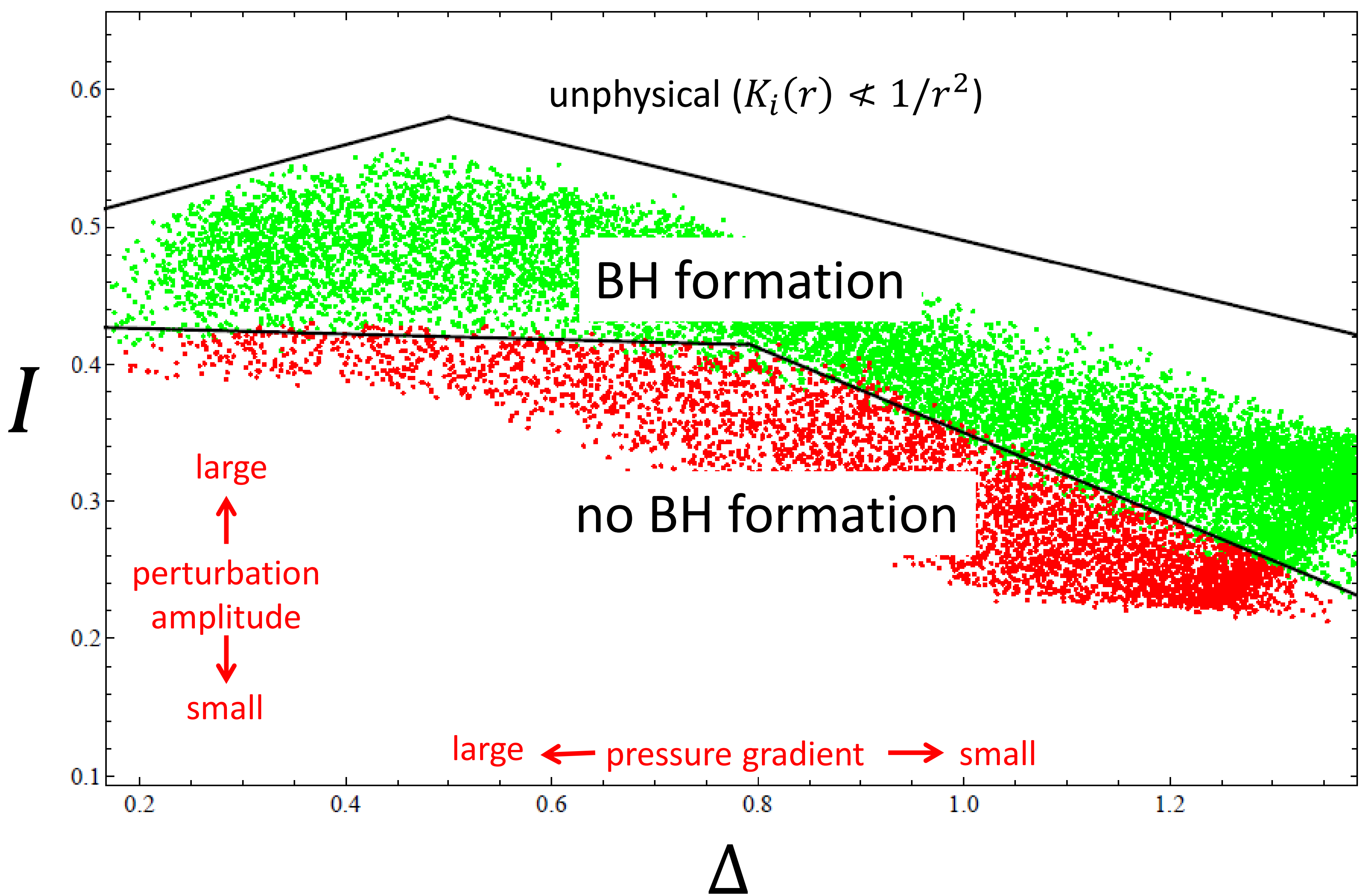
$$K_\mathrm{i}(r) = A\left[1 + B\left(\frac{r}{\sigma_1}\right)^{2n}\right]\exp\left[-\left(\frac{r}{\sigma_1}\right)^{2n}\right] + (1-A)\exp\left[-\left(\frac{r}{\sigma_2}\right)^{2}\right]$$

$(A_1, B_1, \sigma_{1,1}, \sigma_{2,1}, n_1) \longleftrightarrow$  $\longleftrightarrow (\Delta_1, I_1) \longrightarrow$ BH is formed

$(A_2, B_2, \sigma_{1,2}, \sigma_{2,2}, n_2) \longleftrightarrow$  $\longleftrightarrow (\Delta_2, I_2) \longrightarrow$ BH is *not* formed



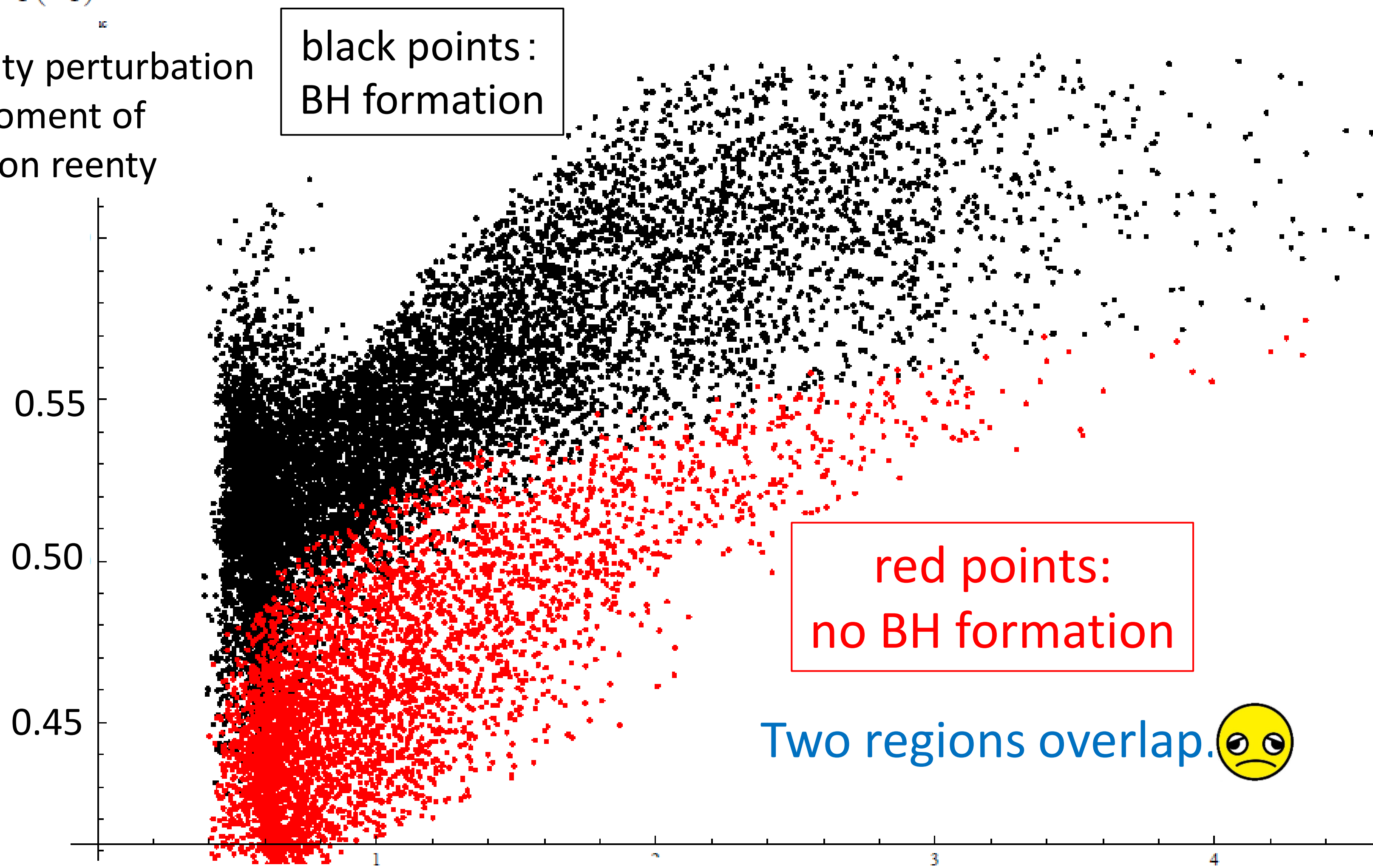
PBH formation condition for various profiles



Other quantities fail to seperate two regions clearly

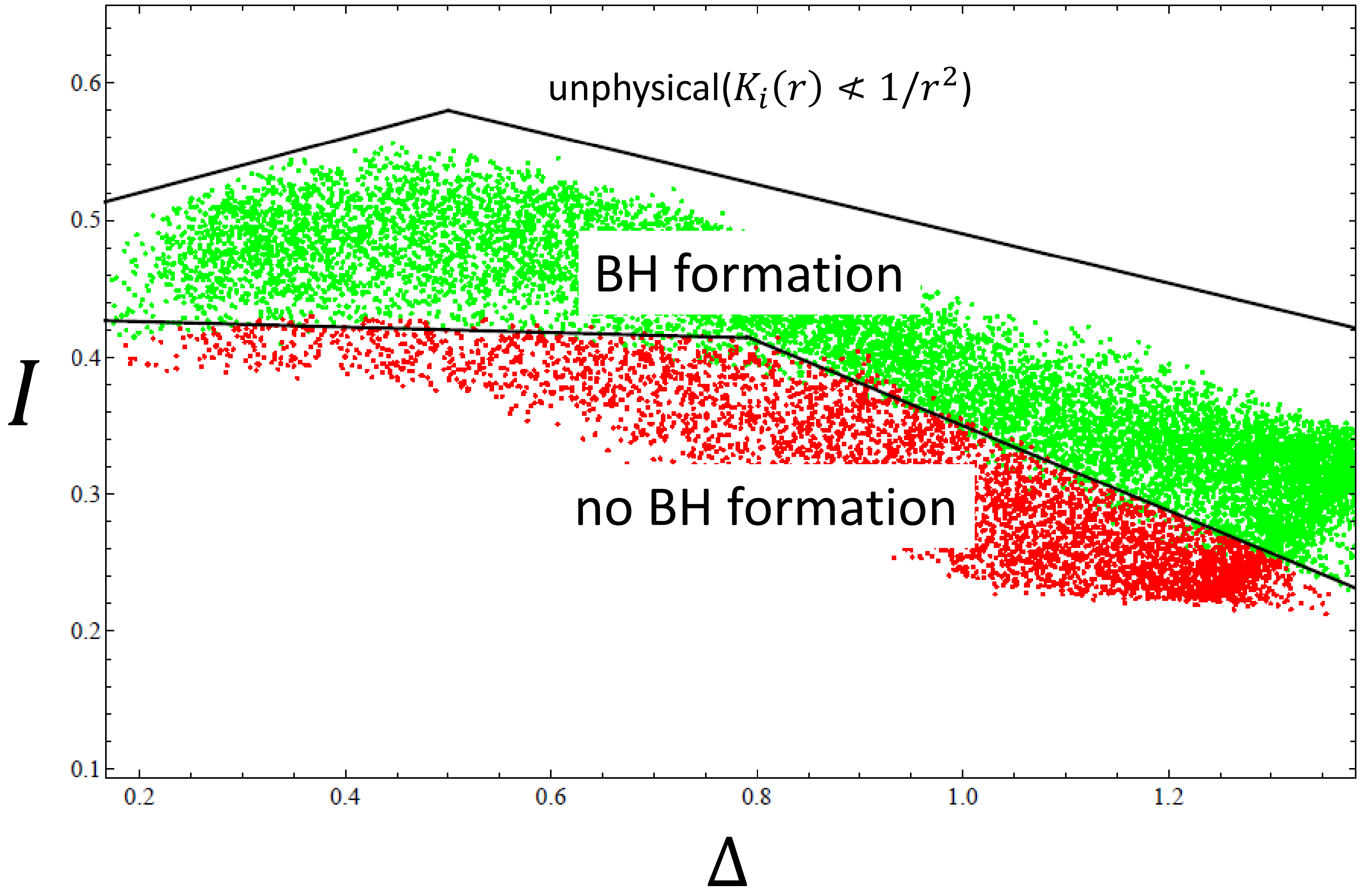
$$\bar{\delta}_{\rm hc} = \frac{2}{3} r_{\rm i}^2 K_{\rm i}(r_{\rm i})$$

averaged density perturbation
at the moment of
the horizon reenty

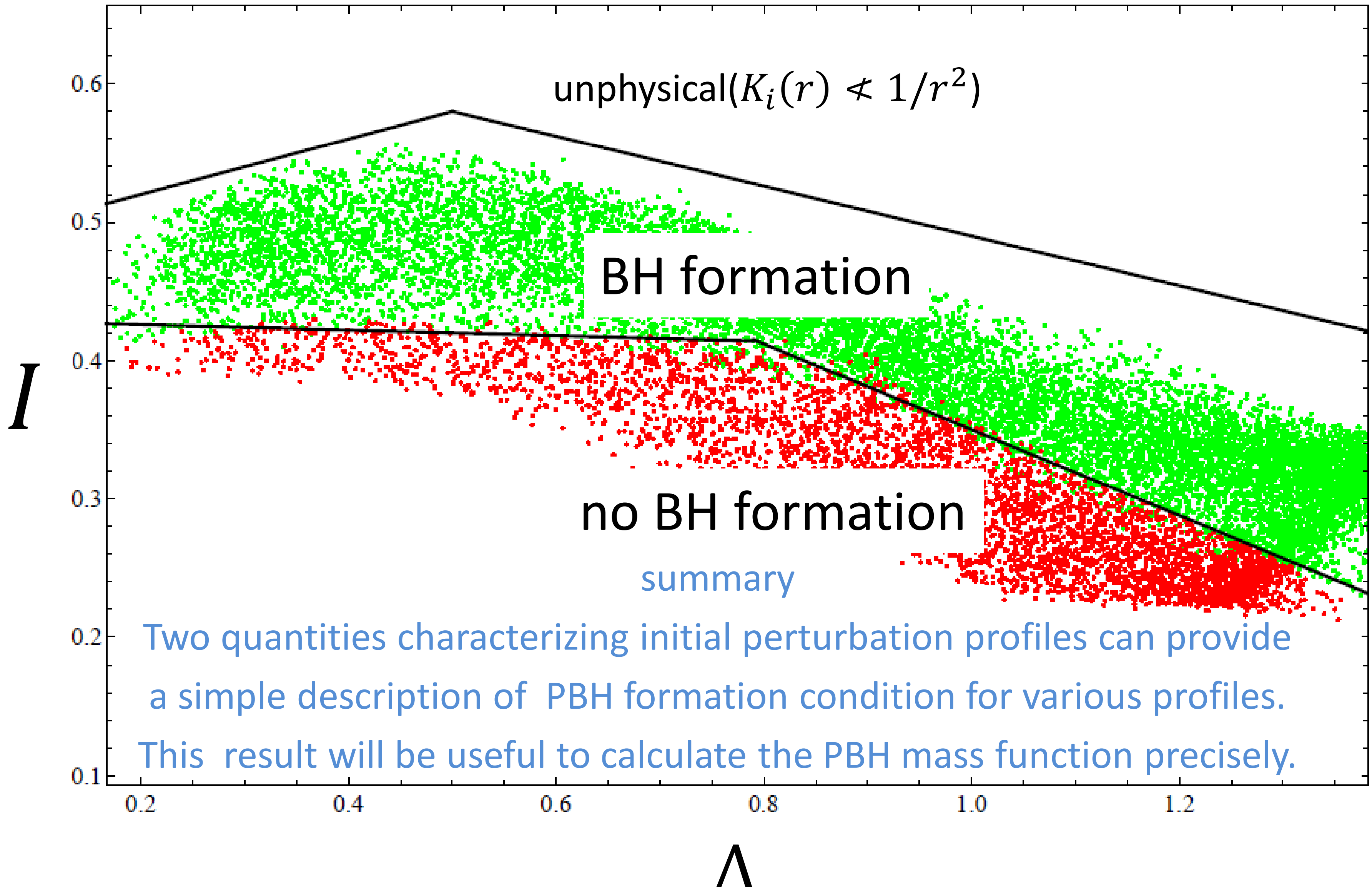


$$\Omega = \max_{\rm r} |K_{\rm i}'(r)|$$

I and Δ can separate two regions clearly



I and Δ can separate two regions clearly



summary

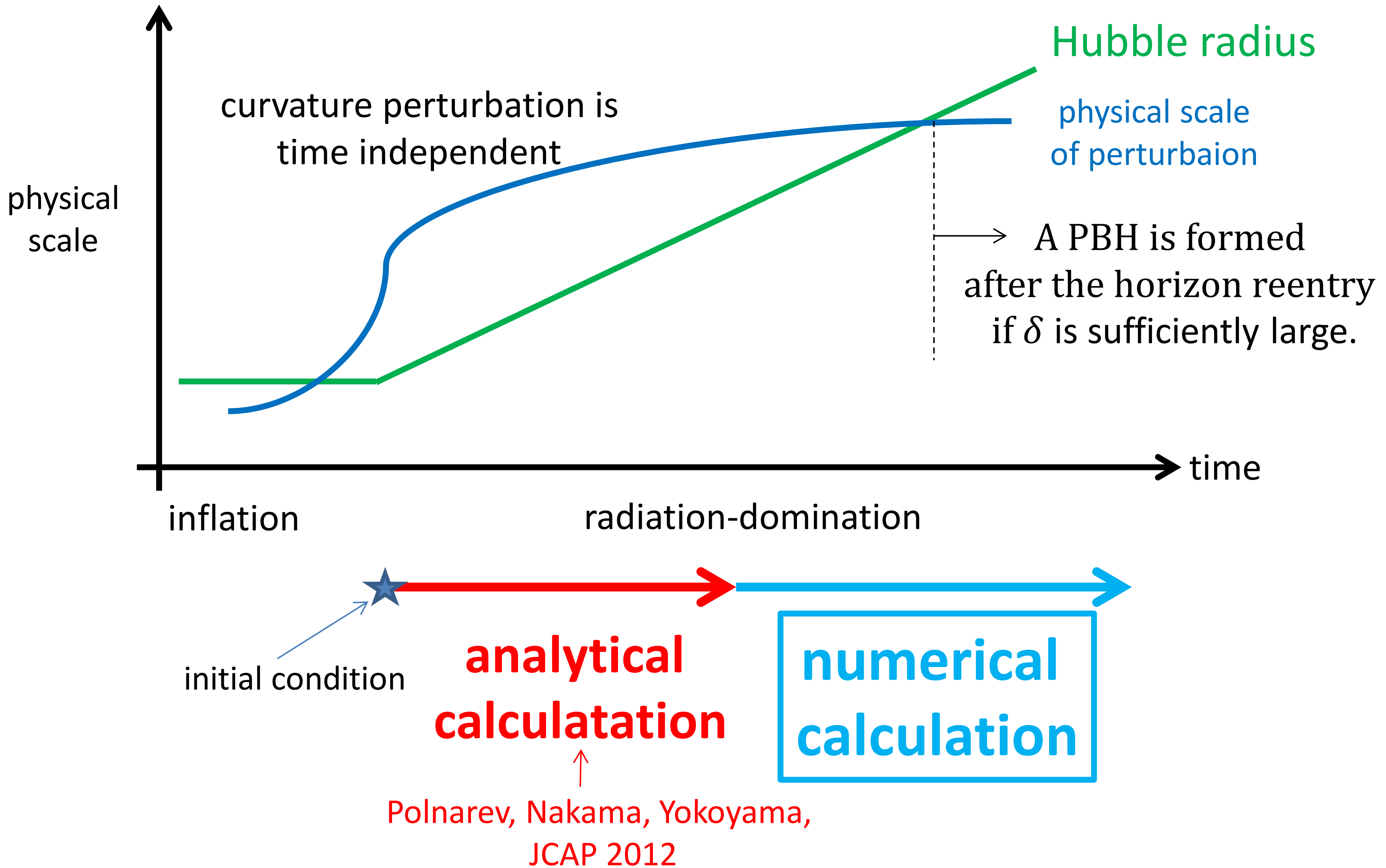
Two quantities characterizing initial perturbation profiles can provide a simple description of PBH formation condition for various profiles.

This result will be useful to calculate the PBH mass function precisely.

Nakama, Harada, Polnarev, Yokoyama, 1310.3007

Thank you

Two steps to analyze PBH formation process

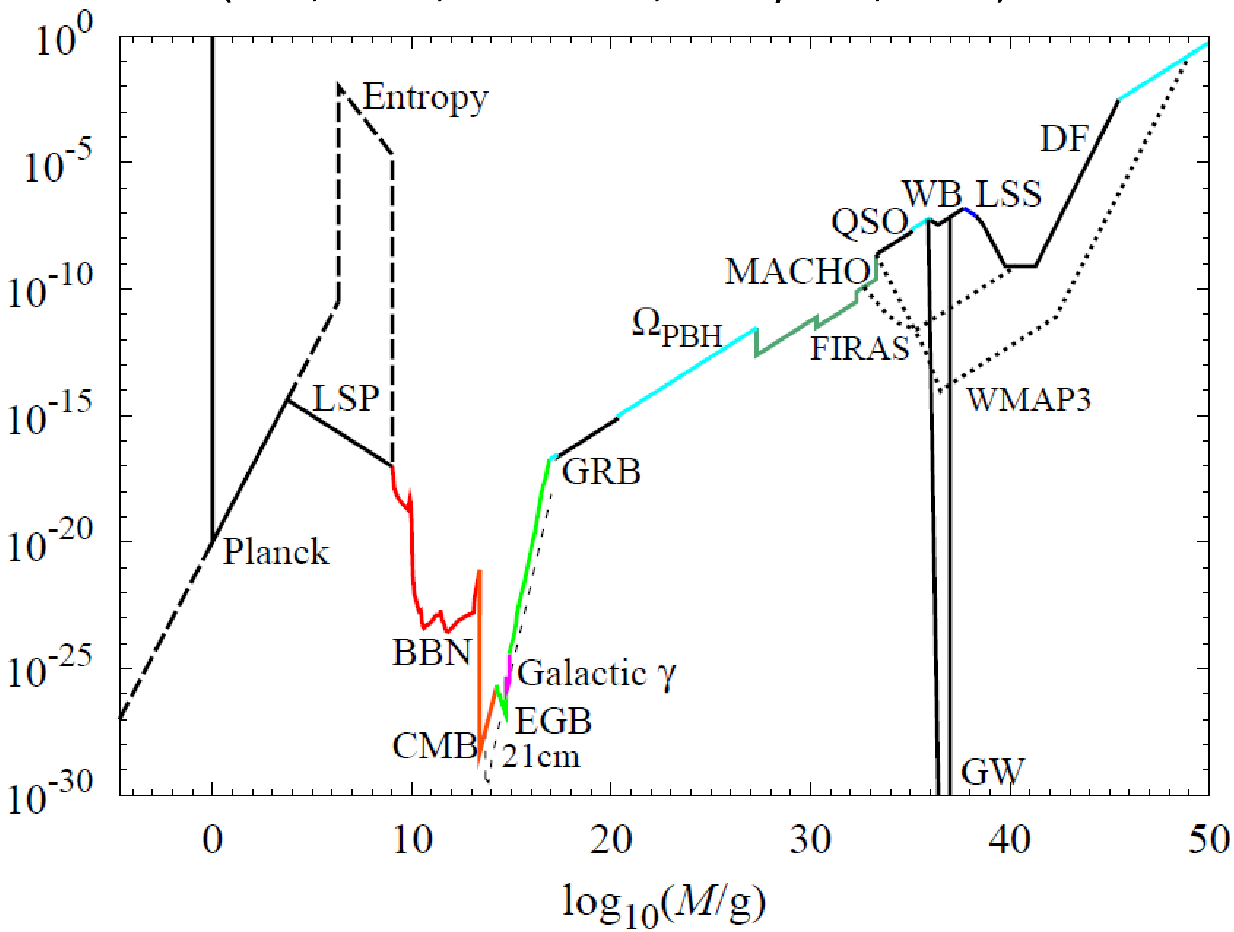


$$M_{\mathrm{PBH}} \sim \frac{4\pi}{3}\left(\frac{1}{H(t_{\mathrm{H}})}\right)^{3} \rho_{\mathrm{rad}}(t_{\mathrm{H}}) \sim 10^{5}\left(\frac{t}{1\sec}\right) M_{\odot}$$

Observational constraints on abundance of PBHs

(Carr, Kohri, Sendouda, Yokoyama, 2010)

PBH abundance at the moment of formations



PBHs have been evaporated due to Hawking radiation

PBHs have not completely evaporate

The solution at the super horizon limit

$$\epsilon \equiv \left(\frac{\text{Hubble radius}}{\text{pert. scale}}\right)^2 = \left(\frac{H_0^{-1}}{S(t)r_i}\right)^2 \propto t \ (\text{radiation-domination})$$

comoving radius of perturbed region

In the limit $\epsilon \to 0$, curvature perturbation is time independent and the solution of Einstein eqs. can be described by an arbitrary function $K_i(r)$:

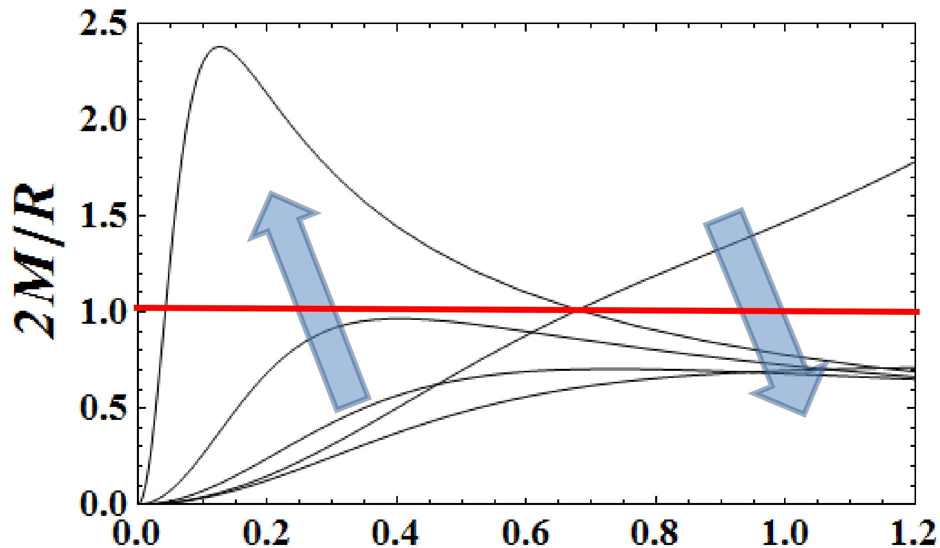
$$ds^2 = -dt^2 + \frac{S^2 dr^2}{1 - K_i(r) r^2} + (Sr)^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2)$$

initial curvature profile ($K_i(r) < \frac{1}{r^2}$, $K_i(\infty)=0$)

In the limit $\epsilon \to 0$, $\delta(t = 0, r) = 0$

Polnarev, Musco, 2007
Lifshits, Khalatnikov, 1964

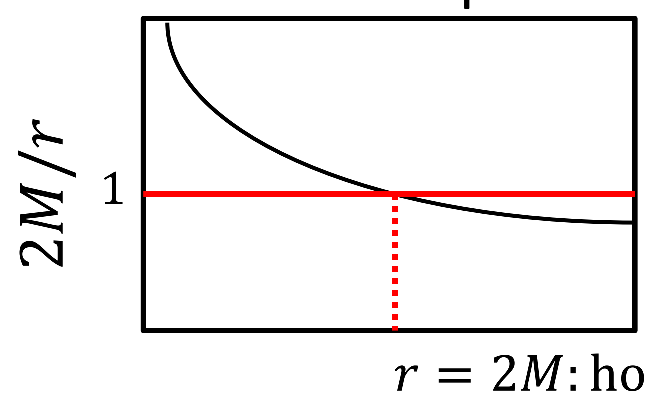
$\left.\frac{2M}{R}\right|_{\text{peak}} > 1$ marks formation of horizon

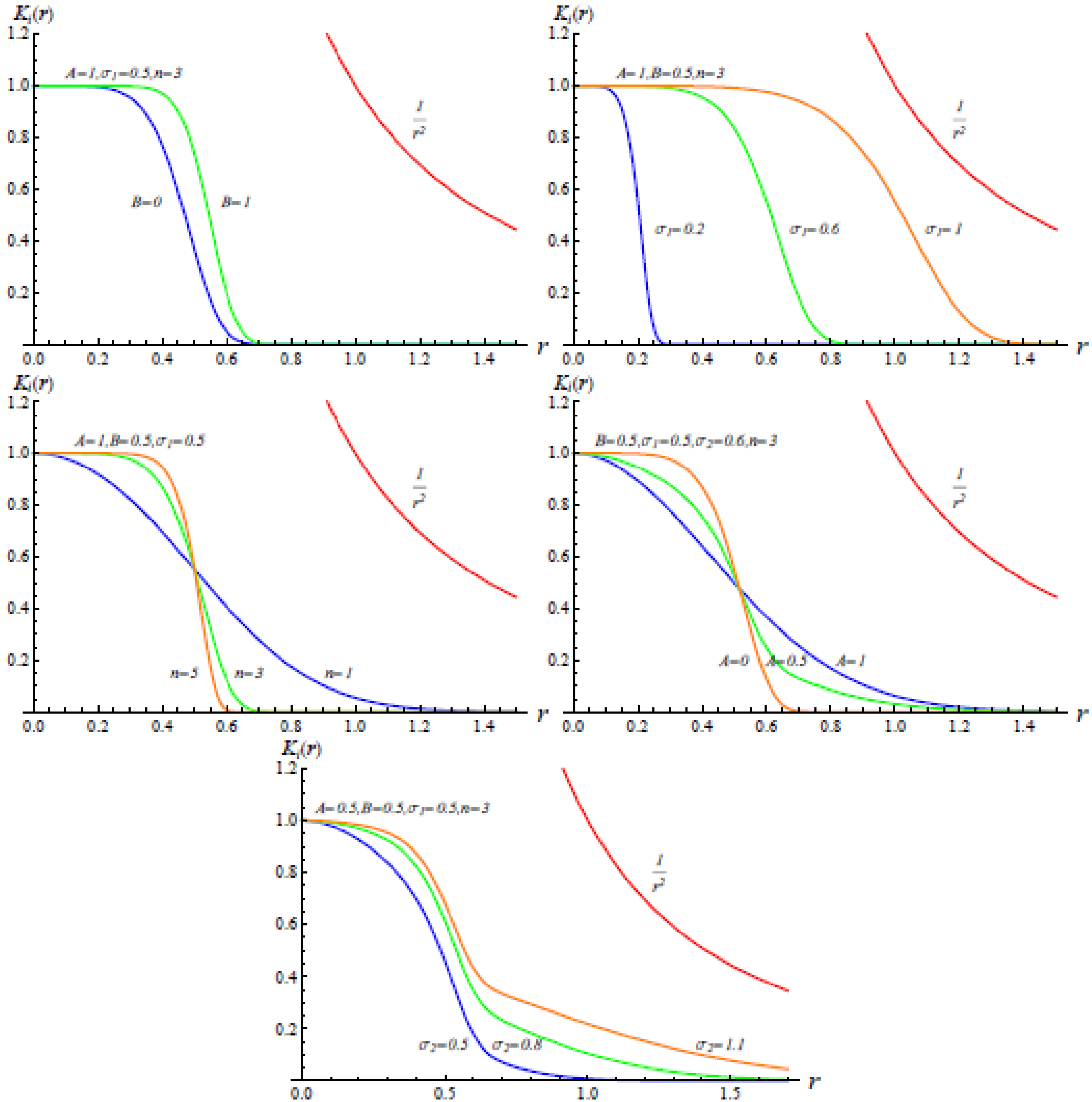


Distance from the center

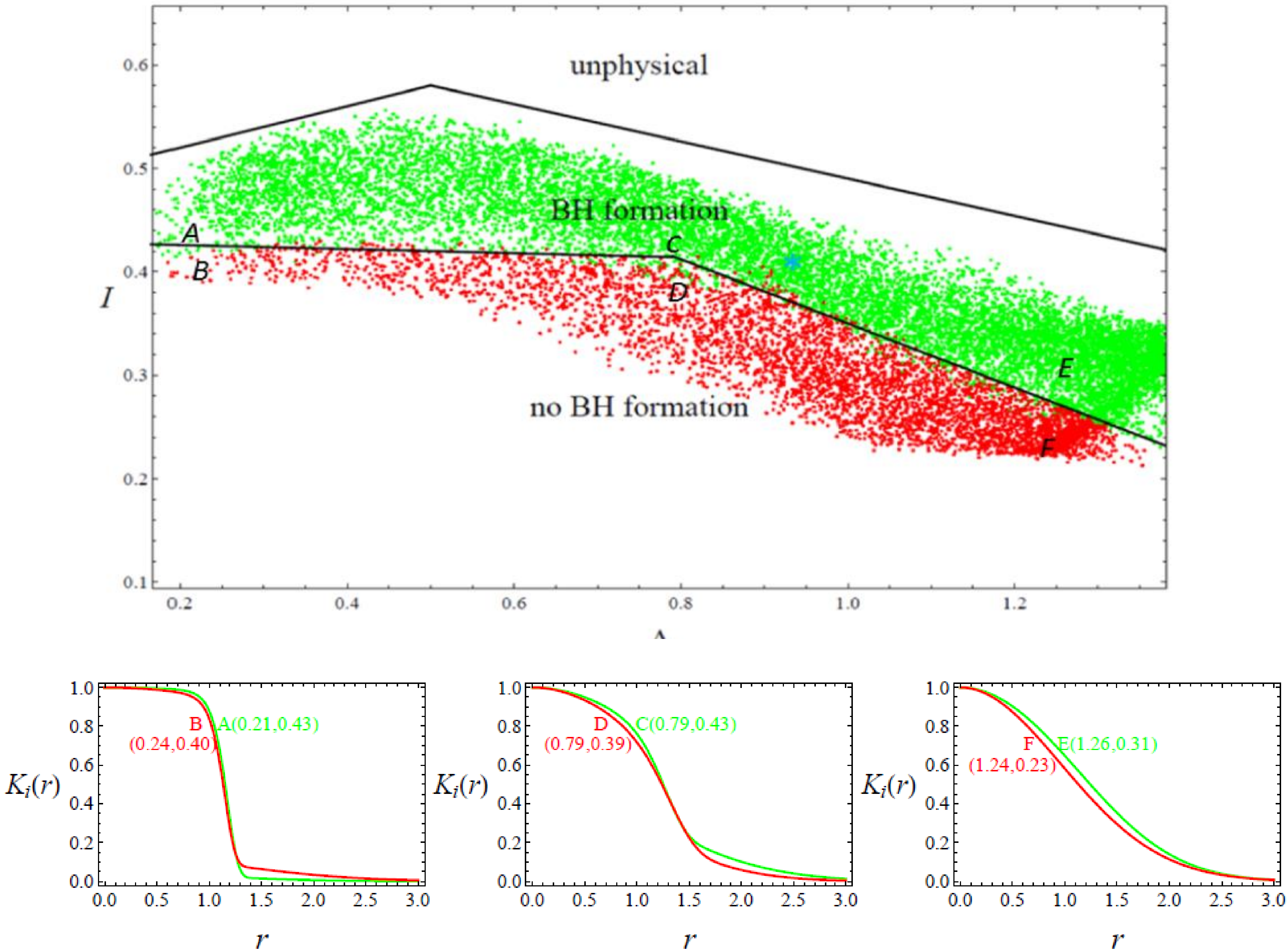
From some moment,
$(2M/R)_{\text{peak}} > 1$
→Horizon is formed

cf.Schwarzschild spacetime





$K_i(r)$
$A=1,\sigma_1=0.5,n=3$
$B=0$
$B=1$
$\frac{1}{r^2}$
r
$A=1,B=0.5,n=3$
$\sigma_1=0.2$
$\sigma_1=0.6$
$\sigma_1=1$
$A=1,B=0.5,\sigma_1=0.5$
$n=5$
$n=3$
$n=1$
$B=0.5,\sigma_1=0.5,\sigma_2=0.6,n=3$
$A=0$
$A=0.5$
$A=1$
$A=0.5,B=0.5,\sigma_1=0.5,n=3$
$\sigma_2=0.5$
$\sigma_2=0.8$
$\sigma_2=1.1$



unphysical
BH formation
no BH formation
A
B
C
D
E
F
I
0.1
0.2
0.3
0.4
0.5
0.6
0.2
0.4
0.6
0.8
1.0
1.2
B
A(0.21,0.43)
(0.24,0.40)
D
C(0.79,0.43)
(0.79,0.39)
F
E(1.26,0.31)
(1.24,0.23)
$K_i(r)$
r