Resolving the Inflationary Power Spectrum

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Outline

- Curvature Power Spectrum, Standard Expression
- Disagreement
- Resolution

Inhomogeneities in the Universe

Observed structure (matter inhomog.)

Observed anisotropy of the CMBR

Seeded by fluctuations in the energy density at early times (t<<1s)

Inflation in the very early universe
($t \sim 10^{-34}$ s or later)

Inflation and P(k)

$\delta\varphi \Rightarrow \delta\rho$

Primordial curvature fluctuation $\zeta(x)$, or ζ_k

Curvature power spectrum: $P(k) = |\zeta_k|^2$

Input in simulations of structure formation or cmb anisotropy

Inflation and P(k)

$$P(k) = |\zeta_k|^2 = \left[\frac{1}{3}\frac{\delta\rho(k)}{\rho + p}\right]^2$$

$$\varphi(\mathbf{x}, t) = \varphi_0(t) + \delta\varphi(\mathbf{x}, t)$$

Inflaton

$$P(k) = \left[\frac{1}{3}\frac{V'(\varphi_0)}{\dot{\varphi}_0^2}\right]^2 \frac{k^3}{2\pi^2}|\varphi_k|^2$$

φ_k is the Fourier transform of $\delta\varphi$(x)

Standard Result for $|\varphi_k|^2$

$$\varphi(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \, [a_k \, \varphi_k(t) e^{i\mathbf{k}.\mathbf{x}} + a_k^\dagger \, \varphi_k^*(t) e^{-i\mathbf{k}.\mathbf{x}}]$$

$$\ddot{\varphi}_k(t) + 3H\dot{\varphi}_k(t) + \frac{k^2}{a^2}\varphi_k(t) \;\; + \; V''(\varphi_0)\,\varphi_k(t) \quad = 0$$

H = Hubble parameter during inflation

Flat potential

Standard Result for $|\varphi_k|^2$

$$\varphi_k(t) = \frac{iH}{(2k^3)^{\frac{1}{2}}}[1 + ik\tau]\exp(-ik\tau) \qquad \tau = -1/[a(t)H]$$

$$|\varphi_k|^2 = \frac{1}{2ka^2} + \frac{H^2}{2k^3}$$

Ignore first term, k<<aH

$$P(k) = \left[\frac{1}{3}\frac{V'(\varphi_0)}{\dot{\varphi}_0^2}\right]^2 \frac{k^3}{2\pi^2}|\varphi_k|^2 = \left[\frac{1}{3}\frac{V'(\varphi_0)}{\dot{\varphi}_0^2}\right]^2 \left(\frac{H}{2\pi}\right)^2$$

TEXTBOOK RESULT FOR P(k)

Inflation and P(k)

$$P(k) = \left[\frac{1}{3}\frac{V'(\varphi_0)}{\dot{\varphi}_0^2}\right]^2 \frac{k^3}{2\pi^2} \ |\varphi_k|^2$$

Recent disagreement on $|\varphi_k|^2$

[Agullo, Navarro-Salas, Olmo, Parker;
Durrer, Marozzi, Rinaldi]

Amends curvature power spectrum substantially

Important to resolve

WHAT IS THE DISAGREEMENT ?

$|\varphi_k|^2$ and < $\varphi^2(x)$ >

$|\varphi_k|^2$ appears in expression for < $\varphi^2(x)$ >

$$\langle \varphi^2(x) \rangle = \frac{1}{(2\pi)^3} \int d^3k \, |\varphi_k|^2 = \frac{1}{(2\pi)^3} \int d^3k \left[\frac{1}{2ka^2} + \frac{H^2}{2k^3} \right]$$

MASSLESS, DE SITTER, BD VACUUM

Both terms lead to UV divergences

[First term in flat spacetime also]

Have to make < $\varphi^2(x)$ > finite -- Renormalise

$< \varphi^2(x) >_{REN}$ and $|\varphi_k|^2_{REN}$

$$\langle \varphi^2(x) \rangle_{REN} = \frac{1}{(2\pi)^3} \int d^3k \, |\varphi_k|^2_{REN}$$

- $< \varphi^2(x) >_{REN}$ is fundamental physical quantity
- So $|\varphi_k|^2$ in P(k) should be $|\varphi_k|^2_{ren}$

P(k) changes substantially

Parker, hep-th 2007; Agullo, Navarro-Salas, Olmo, Parker, PRL 2008, PRL 2009, PRD 2010, PRD 2011

Parker and Toms, “QFT in CST”, 2009

$< \varphi^2(x) >_{REN}$ and $|\varphi_k|^2_{REN}$

Adiabatic regularisation

$$\langle \varphi^2(x) \rangle_{REN} = \frac{1}{(2\pi)^3} \int d^3k \, |\varphi_k|^2_{REN}$$

$$|\varphi_k|^2_{REN} = |\varphi_k|^2 - |\varphi_K|^2$$

$\varphi_K(t) = \lim_{k\to\infty} \varphi_k(t)$ (upto 2nd adiabatic order)

Removes contribution of high k modes

Also alters the low k expression

Agullo, Navarro-Salas, Olmo, Parker , PRL 2009

Renormalised P(k)

$P(k) \sim |\zeta_k|^2 \sim \quad k^3\, |\varphi_k|^2{}_{REN}$

$= [0.9\ \varepsilon(t_k) + 0.45\ \eta(t_k)]\ k^3\, |\varphi_k|^2$ PRL 2009

t_k = horizon exit time for k mode

P(k) ↓ by ε and η. LESS FINE TUNING

Spectrum is still nearly scale free

$r = P_T/P$ changes

$\lambda\varphi^4$ still allowed

Objection, your Honour

$$P(k) \sim |\varphi_k(t)|^2_{REN} = |\varphi_k(t)|^2 - |\varphi_K(t)|^2$$ MS var Q

1. $\varphi_k(t)$ is constant after t_k, $\varphi_K(t)$ is not

   P(k) depends on t after horizon exit at which you evaluate

2. Different adiabatic subtraction schemes give different answers

   Most reasonable agrees with standard for $t \gg t_k$

Durrer, Marozzi, Rinaldi PRD 2009

Objection, your Honour

Conclusion:

Parker et al are wrong

$\varphi_K(t)$ not match eom solution $\varphi_k(t)$ for t> t_k,
i.e. for low k modes outside the horizon

Should only do adiabatic subtraction for high k modes

Durrer, Marozzi, Rinaldi PRD 2009

Rebuttal

1. Should do adiabatic subtraction for all k modes, including low momentum

2. Their adiabatic regularisation matches DeWitt-Schwinger renormalisation (in mom space, $m \to 0$)

Agullo, Navarro-Salas, Olmo, Parker , PRD 2010

(Includes metric pert in $\delta\varphi$ EOM, uses comov curv pert R not $\delta\varphi$)

Counter to the Rebuttal

DeWitt-Schwinger renormalisation unsuitable for renormalising the two point function during inflation and the power spectrum

Marozzi, Rinaldi, Durrer PRD 2011

Counter to the Counter

1. In adiabatic regularisation not trying to approximate solutions for all k.
   Durrer et al wrong.

2. Adiabatic subtraction has some ambiguities (high k: $l/(k^2+m^2)$ or $1/k^2$)
   Use some condition to fix.
   Durrer et al method is arbitrary, non-minimal

Agullo, Navarro-Salas, Olmo, Parker, PRD 2011

P(k)

Resolving the power spectrum is important

P(k) connects current observations (cmbr, structure) to early universe microphysics

HOW DO WE RESOLVE THIS?

Adiabatic Subtraction

Only subtract for high k modes - Durrer et al

EOM for φ_k implies $\dot{\rho}_k = -3H(\rho_k + p_k)$

$$\dot{\rho}_\varphi = -3H(\rho_\varphi + p_\varphi)$$ Energy cons eqn

$$\dot{\rho}_{REN} = \frac{d}{dt}\int \frac{d^3k}{(2\pi)^{3/2}}[\rho_k - \Theta(k - aH)\rho_K]e^{i\vec{k}.\vec{x}}$$

$$-3H(\rho_{REN} + p_{REN}) = -3H\int \frac{d^3k}{(2\pi)^{3/2}}[\rho_k - \Theta(k - aH)\rho_K$$

$$+p_k - \Theta(k - aH)p_K]e^{i\vec{k}.\vec{x}}$$

Adiabatic Subtraction

Only subtract for high k modes - Durrer et al

$$\dot{\rho}_{REN} = \frac{d}{dt}\int_0^{a(t)H} \frac{d^3k}{(2\pi)^{3/2}}\,\rho_k e^{i\vec{k}.\vec{x}}$$

$$-3H(\rho_{REN} + p_{REN}) = -3H\int_0^{a(t)H} \frac{d^3k}{(2\pi)^{3/2}}\,[\rho_k + p_k]e^{i\vec{k}.\vec{x}}$$

Adiabatic Subtraction

Only subtract for high k modes - Durrer et al

$$\dot{\rho}_{REN} = \frac{d}{dt}\int_0^{a(t)H} \frac{d^3k}{(2\pi)^{3/2}}\,\rho_k e^{i\vec{k}.\vec{x}}$$

$$\neq$$

$$-3H(\rho_{REN} + p_{REN}) = -3H\int_0^{a(t)H} \frac{d^3k}{(2\pi)^{3/2}}\,[\rho_k + p_k]e^{i\vec{k}.\vec{x}}$$

$|\varphi_k|^2_{REN}$ and $< \varphi^2(x) >_{REN}$

$$\langle \varphi^2(x) \rangle_{REN} = \frac{1}{(2\pi)^3} \int d^3k \, |\varphi_k|^2_{REN}$$

Parker et al: $|\varphi_k|^2_{REN}$

Is renormalisation of $|\varphi_k|^2$ needed ?

Curvature power spectrum $P(k) \sim |\varphi_k|^2$, not $< \varphi^2(x) >$

Look at expressions derived from physical observables. How does P(k) enter?

$|\varphi_k|^2_{REN}$ and $< \varphi^2(x) >_{REN}$

CMBR angular power spectrum

$$C_l = \frac{1}{4\pi}\int d^2\hat{n}\, d^2\hat{n}'\, P_l(\hat{n}.\hat{n}')\, \langle \Delta T(\hat{n}) \Delta T(\hat{n}') \rangle$$

$$\sim \quad \ldots \qquad \langle \delta\phi(\hat{n} r_L)\, \delta\phi(\hat{n}' r_L) \rangle$$

$$\sim \quad \ldots \qquad \int d^3q\, d^3q'\, e^{i\vec{q}.\hat{n} r_L}\, e^{i\vec{q}'.\hat{n}' r_L} \langle \delta\phi_{\vec{q}}\, \delta\phi_{\vec{q}'} \rangle$$

$$\sim \quad \ldots \qquad \int d^3q\, d^3q'\, e^{i\vec{q}.\hat{n} r_L}\, e^{i\vec{q}'.\hat{n}' r_L} P_\phi(q)\, \delta(\vec{q} + \vec{q}')$$

$$\sim \quad \ldots \qquad \int d^3q e^{i\vec{q}.\hat{n} r_L}\, e^{-i\vec{q}.\hat{n}' r_L} P_\phi(q)\,,$$

$$\langle \delta\phi_{\vec{q}} \delta\phi_{\vec{q}'} \rangle = P_\phi(q)\, \delta(\vec{q} + \vec{q}')$$

$P_\Phi(q)$ related to P(k) as $\delta\Phi$ related to $\delta\varphi$

$|\varphi_k|^2_{REN}$ and $< \varphi(x)\, \varphi(y)>_{REN}$

$$C_l \sim \quad \ldots \qquad \langle \delta\phi(\hat{n} r_L)\, \delta\phi(\hat{n}' r_L) \rangle$$

Coordinate space correlation function that is primary. Could be divergent

$\langle \varphi(\vec{x}, t) \varphi(\vec{y}, t) \rangle$ relevant rather than $\langle \varphi^2(x) \rangle$

Any renormalisation associated with $\langle \varphi(\vec{x}, t) \varphi(\vec{y}, t) \rangle$ would be relevant for P(k)

$|\varphi_k|^2_{REN}$ and $< \varphi(x)\, \varphi(y)>_{REN}$

$$\langle \varphi(\vec{x},t)\varphi(\vec{y},t)\rangle = \frac{1}{(2\pi)^3}\int d^3k\, \left[\frac{1}{2ka^2} + \frac{H^2}{2k^3}\right] e^{i\vec{k}.(\vec{x}-\vec{y})}$$

$$= \frac{1}{2\pi^2}\int dk\, k^2 \left[\frac{1}{2ka^2} + \frac{H^2}{2k^3}\right] \frac{\sin[k|\vec{x}-\vec{y}|]}{k|\vec{x}-\vec{y}|}$$

$|\varphi_k|^2_{REN}$ and $<\varphi(x)\,\varphi(y)>_{REN}$

$$\langle \varphi(\vec{x},t)\varphi(\vec{y},t)\rangle = \frac{1}{(2\pi)^3}\int d^3k\,\left[\frac{1}{2ka^2}+\frac{H^2}{2k^3}\right]e^{i\vec{k}.(\vec{x}-\vec{y})}$$

$$= \frac{1}{2\pi^2}\int dk\,k^2\left[\frac{1}{2ka^2}+\frac{H^2}{2k^3}\right]\frac{\sin[k|\vec{x}-\vec{y}|]}{k|\vec{x}-\vec{y}|}$$

1. Integral is UV finite

No need of renormalisation

$|\varphi_k|^2_{REN}$ and $<\varphi(x)\,\varphi(y)>_{REN}$

$$\langle \varphi(\vec{x},t)\varphi(\vec{y},t)\rangle = \frac{1}{(2\pi)^3}\int d^3k \left[\frac{1}{2ka^2}+\frac{H^2}{2k^3}\right] e^{i\vec{k}.(\vec{x}-\vec{y})}$$

$$= \frac{1}{2\pi^2}\int dk\, k^2 \left[\frac{1}{2ka^2}+\frac{H^2}{2k^3}\right] \frac{\sin[k|\vec{x}-\vec{y}|]}{k|\vec{x}-\vec{y}|}$$

2. P(k) is standard expression

$$P(k) = \left[\frac{1}{3}\frac{V'(\varphi_0)}{\dot{\varphi}_0^2}\right]^2 \frac{k^3}{2\pi^2}|\varphi_k|^2 = \left[\frac{1}{3}\frac{V'(\varphi_0)}{\dot{\varphi}_0^2}\right]^2 \left(\frac{H}{2\pi}\right)^2$$

Conclusion

Standard expression for the curvature power spectrum is correct

Modifications to the power spectrum as suggested are not valid

The arguments by others against these modifications are also not valid

Final Comments

Temporarily set aside the need to renormalise $\langle \varphi^2 \rangle$

$$\langle \varphi(x)\varphi(y)\rangle_{\rm int} = \langle \varphi(x)\varphi(y)\rangle + i\lambda \int d^4z \, \langle \varphi(x)\varphi(y)\varphi^4(z)\rangle$$

Also need renormalised $\langle \varphi^2 \rangle$ for renormalised $T_{\mu\nu}$

Conclusion

Standard expression for the curvature power spectrum is correct

Modifications to the power spectrum as suggested are not valid

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Thank you