Gravitational waves from curvaton

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OUTLINE

1. Introduction
2. Curvaton model with blue spectrum
3. Scalar-induced gravitational waves
4. Results

I. Introduction

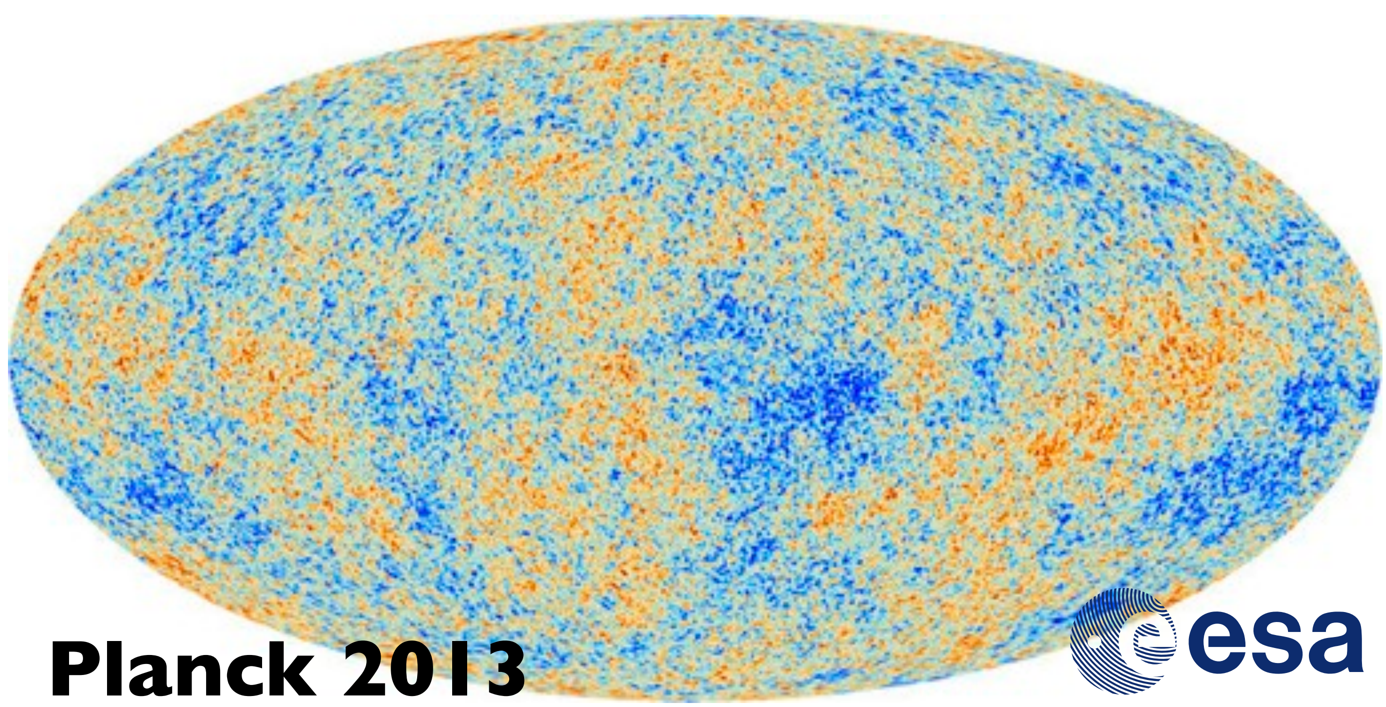
CMB temperature anisotropy probe (Planck, WMAP)

has revealed the large scale primordial curvature perturbations

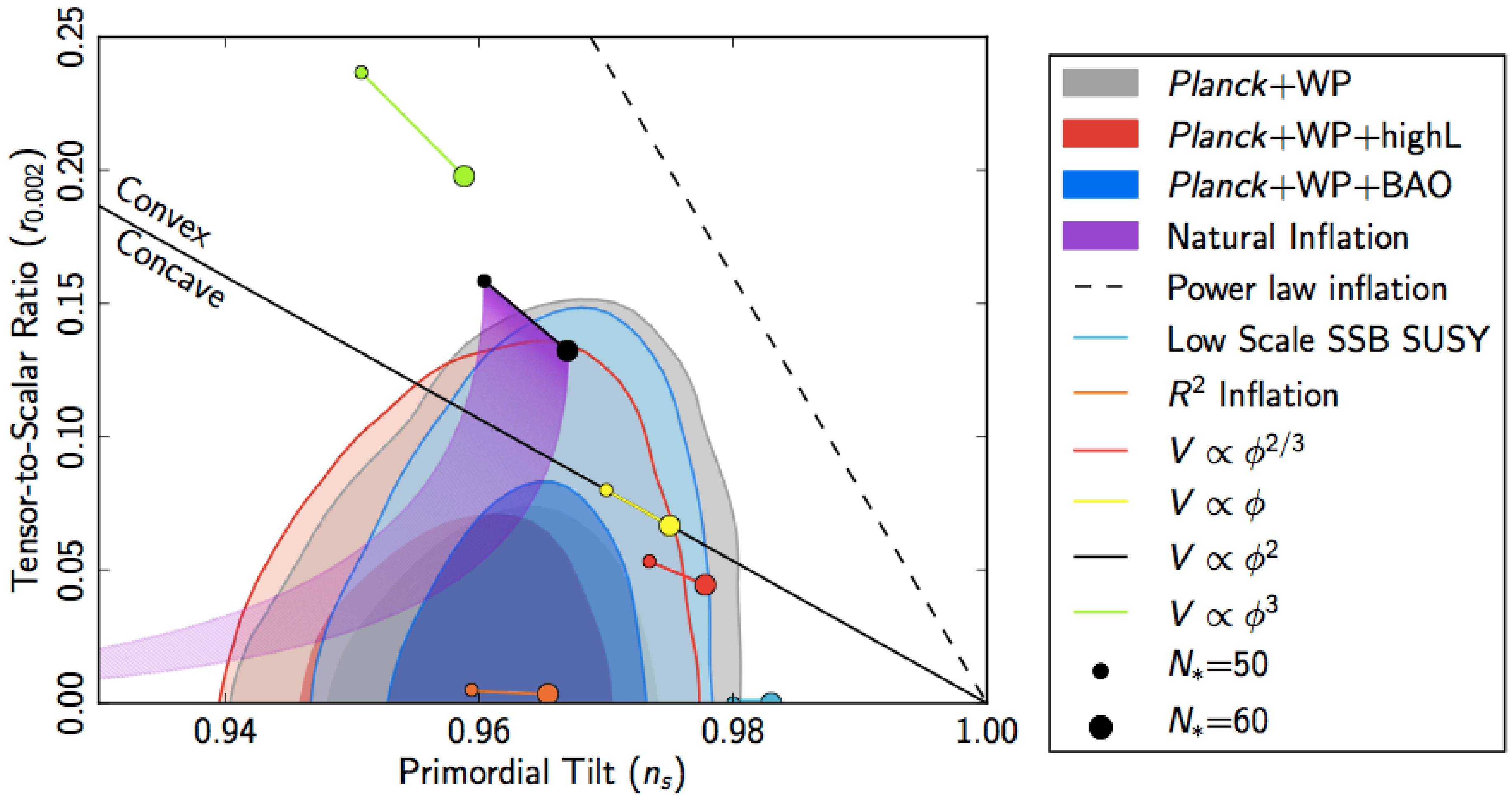
☞ nearly scale-invariant & obeying Gaussian statistics

amplitude, spectral index, tensor-to-scalar ratio

We can constrain inflation models!



Constraint on the inflation models



Planck 2013 results. XXII. Constraints on inflation [arXiv:1303.5082]

☺☺ CMB is a powerful tool to see the very early universe! ☺☺

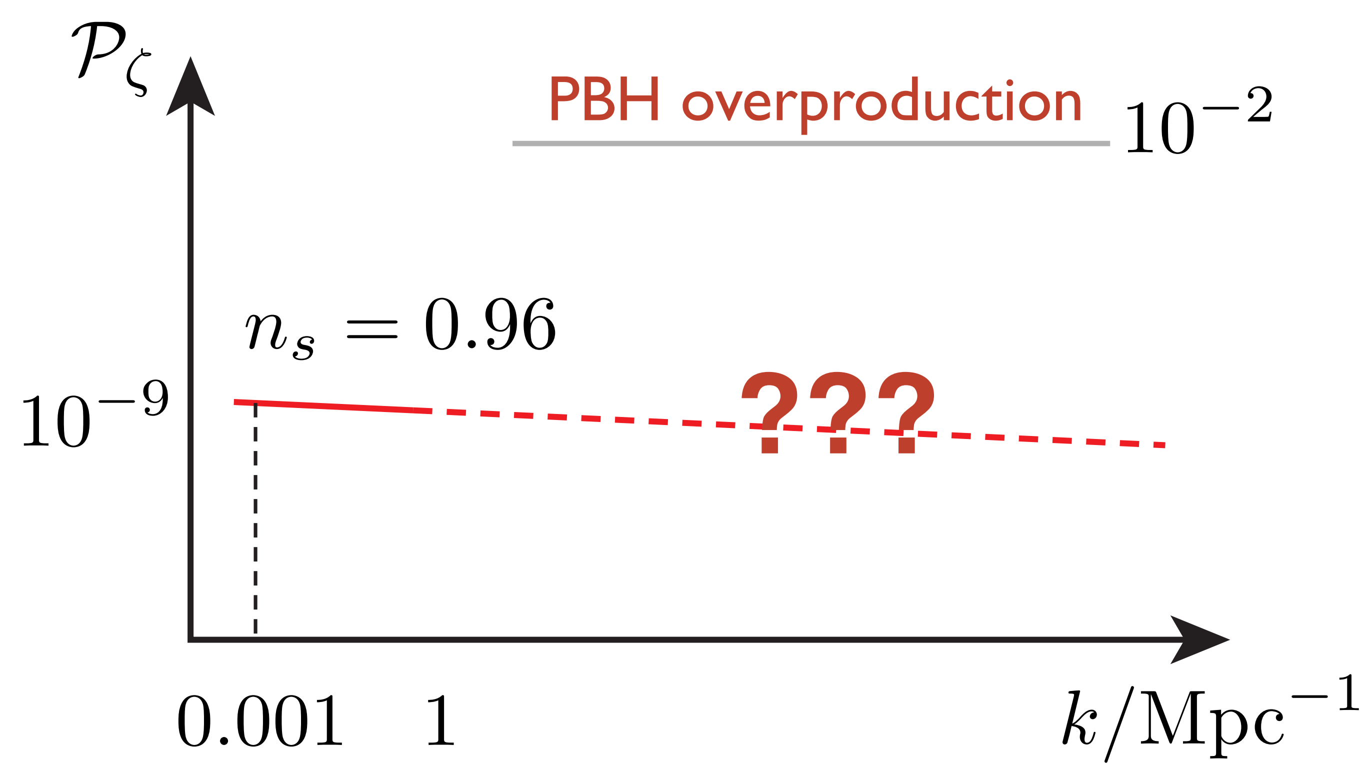
But it is not enough!

Are there any other scalar fields during inflation?

☹ CMB can't tell us the small scale perturbations.
(diffusion damping)

We know only the curvature perturbation on scales $k \lesssim 1\ \mathrm{Mpc}^{-1}$

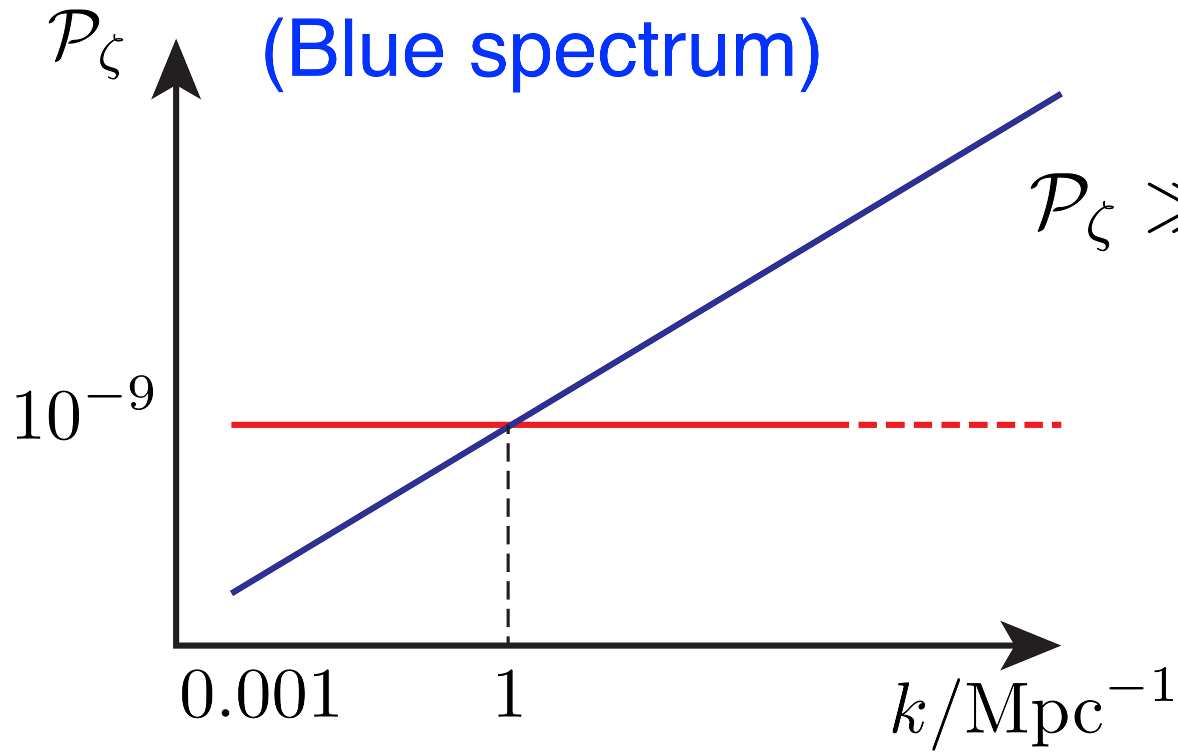
$\mathcal{P}_\zeta = 2 \times 10^{-9}$ & $n_s = 0.96$ for $k < 1\ \mathrm{Mpc}^{-1}$



☆ Scales: $k > 1\ \mathrm{Mpc}^{-1}$ are free from the observation

(exceptions: PBH/UCMH formation, CMB distortion)

It may be ...



$\mathcal{P}_\zeta \gg 10^{-9}$ & $n_s > 1$

for $k > 1\ \mathrm{Mpc}^{-1}$

Why blue spectrum?

High energy physics (SUSY, SUGRA)
predicts many scalar fields in the very early universe

◆ Inflaton ▶▶ drive inflation, primordial adiabatic perturbation

◆ Other scalar fields ▶▶ primordial isocurvature perturbation

Heavy scalar field (ex. Hubble-induced mass in SUGRA)

$H \sim m_\sigma$

✦ Heavy scalar fields predict
large curvature perturbation on small scales

If we can see the small-scale perturbations,
we will get some hints about the high energy physics.

Q. How can we see the small scale perturbations?

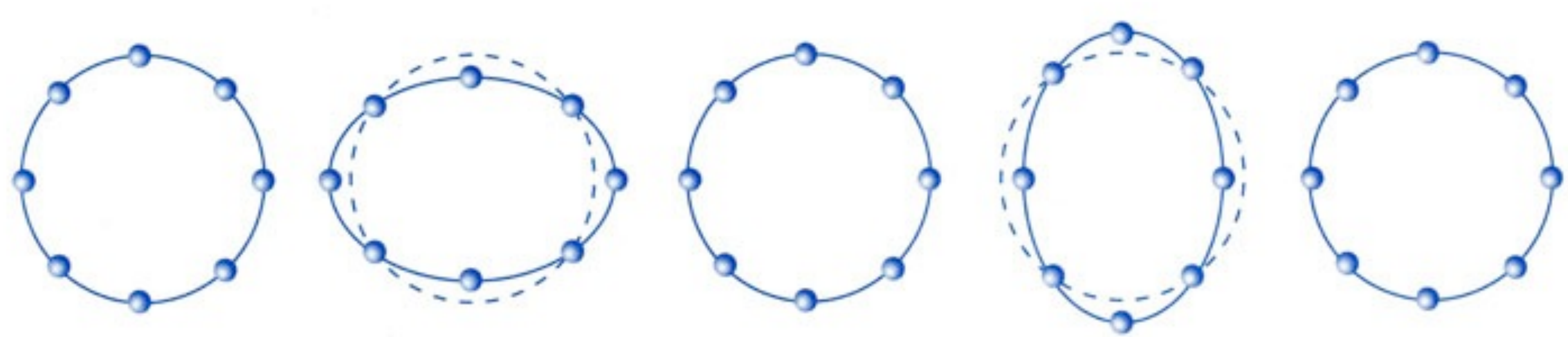
Stochastic Gravitational Wave Background (SGWB)

Metric perturbation from FRW metric

$$ds^2 = a^2(\eta)\Big[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\Big] \quad \text{with} \quad h^i_i = 0 \ \ \& \ \ \partial_i h^i_j = 0$$

(trace-free & transverse)

Space is distorted by GW



GW detection

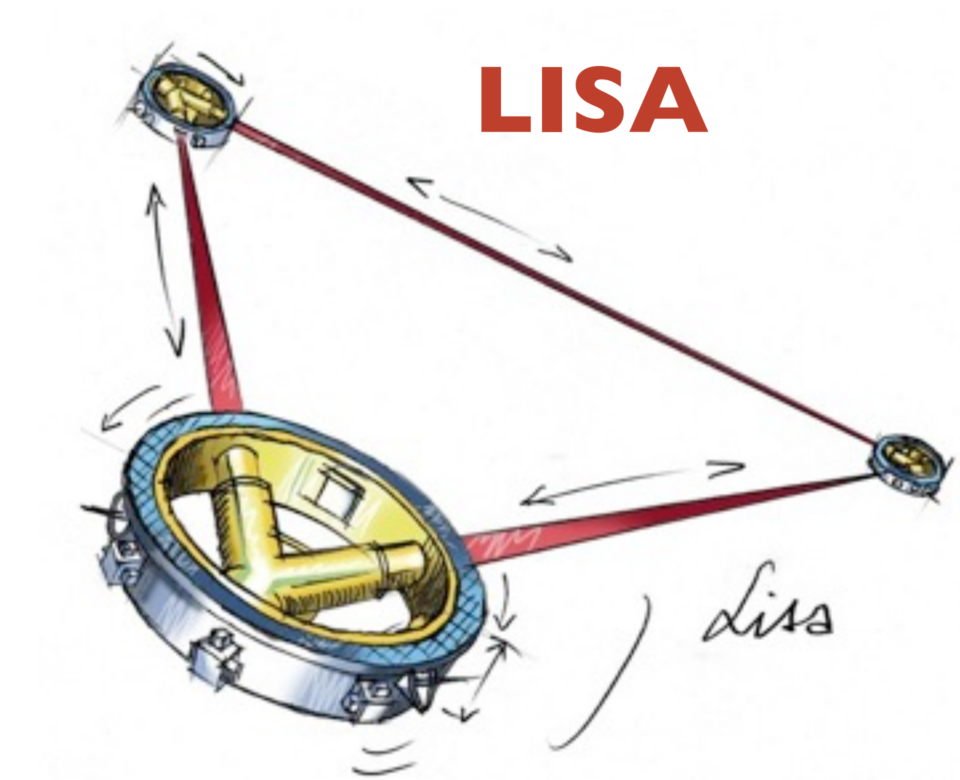
Because the amplitude of SGWB is so small, we must rely on the space-based detection or pulsar timing experiment.

$$10^{11}\ \mathrm{Mpc}^{-1} \lesssim k \lesssim 10^{15}\ \mathrm{Mpc}^{-1} \quad \text{(Direct detection)}$$

$$k \sim 10^{7}\ \mathrm{Mpc}^{-1} \quad \text{(Pulsar timing)}$$

Future space-based GW observations

http://tamago.mtk.nao.ac.jp/decigo/index.html



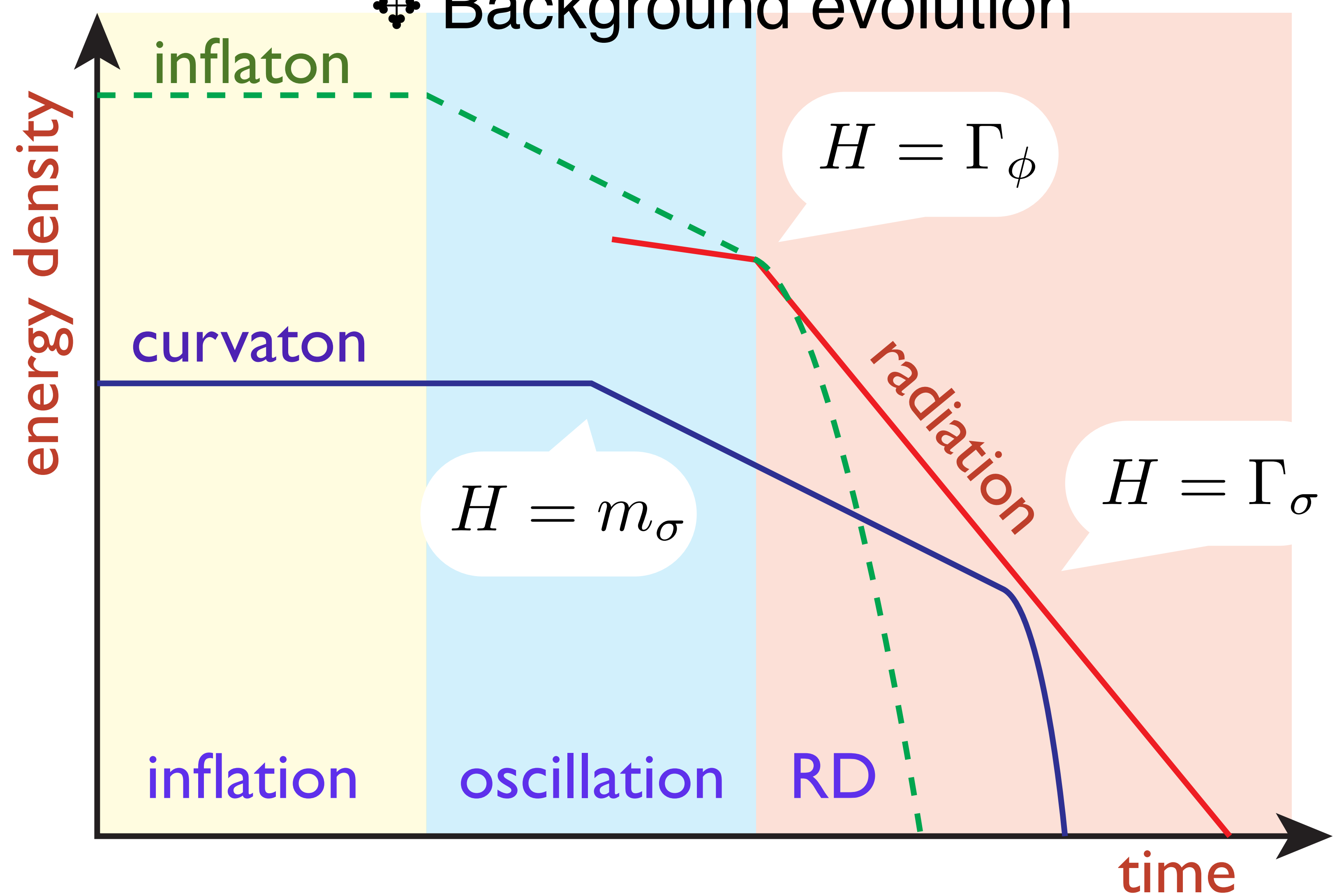
http://www.esa.int/Our_Activities/Space_Science/LISA

II. Curvaton model with blue spectrum

❖ Curvaton scenario

1. inflation
2. inflaton coherent oscillation
   - curvaton starts to oscillate at $H = m_\sigma$
3. radiation domination (after inflaton decay)
   - curvaton decays at $H = \Gamma_\sigma$

✣ Background evolution



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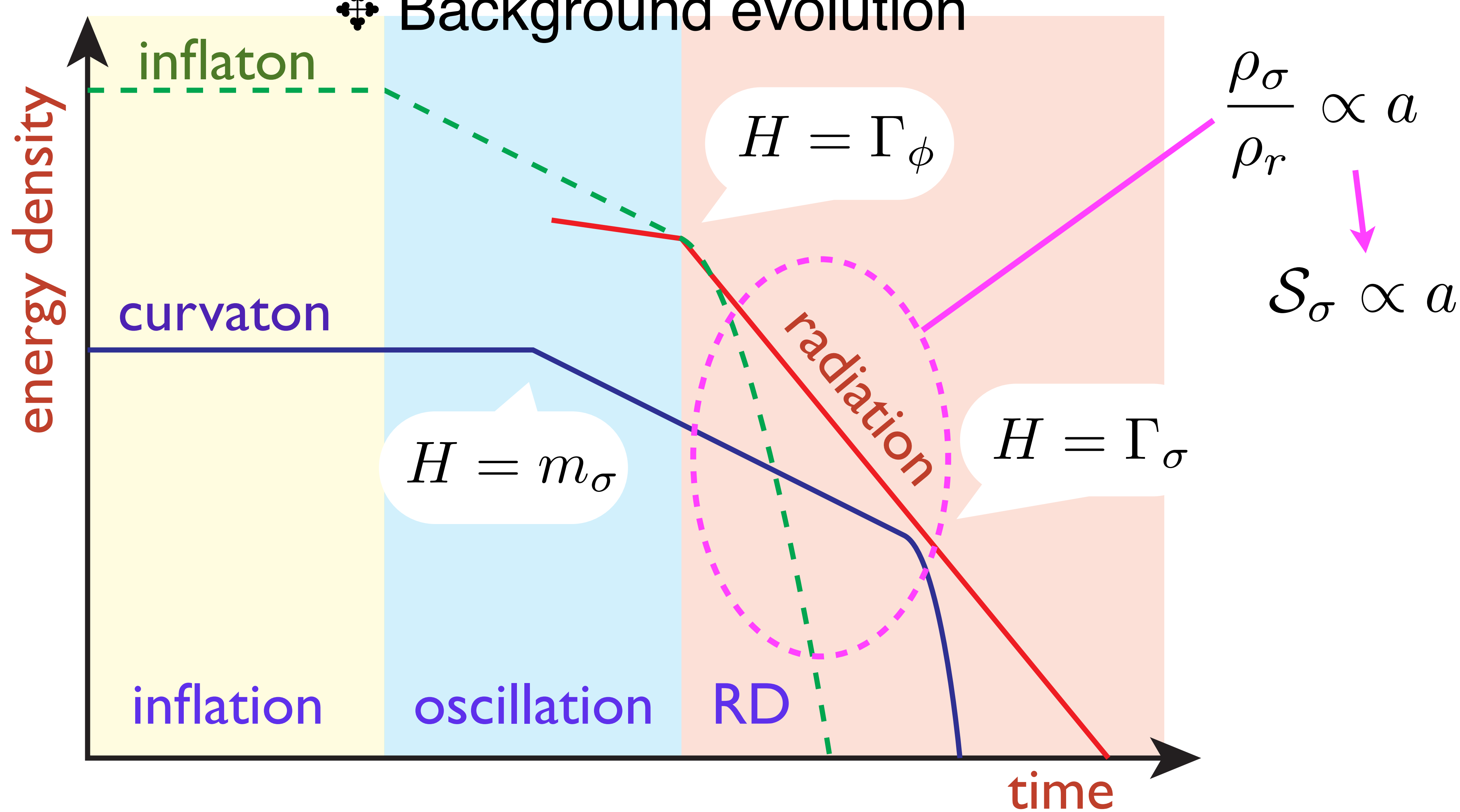


Curvature perturbation from multi scalar field

【inflaton ⊕ curvaton】

δN formalism: $\zeta = \delta N = N_\phi \delta\phi_* + N_\sigma \delta\sigma_* + \ldots$

There are 2 contributions ☛ inflaton curvaton

Power spectrum of curvature perturbation is defined via

$$\langle \zeta(\mathbf{k},\eta)\zeta(\mathbf{k}',\eta)\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k,\eta)\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$\zeta \ni \zeta_{\rm inf},\ \mathcal{S}_\sigma$

$$\mathcal{P}_\zeta(k,\eta) = \mathcal{P}_{\zeta,\rm inf}(k) + \mathcal{P}_{\zeta,\rm curv}(k)$$

We assume

Inflaton part reproduces CMB results: $\mathcal{P}_\zeta = 2\times 10^{-9}$ & $n_s = 0.96$

$\oplus$

Curvaton part is required to be $\mathcal{P}_{\zeta,\rm curv} < \mathcal{P}_{\zeta,\rm inf}$ for $k < 1\ \mathrm{Mpc}^{-1}$

Curvaton part of the power spectrum

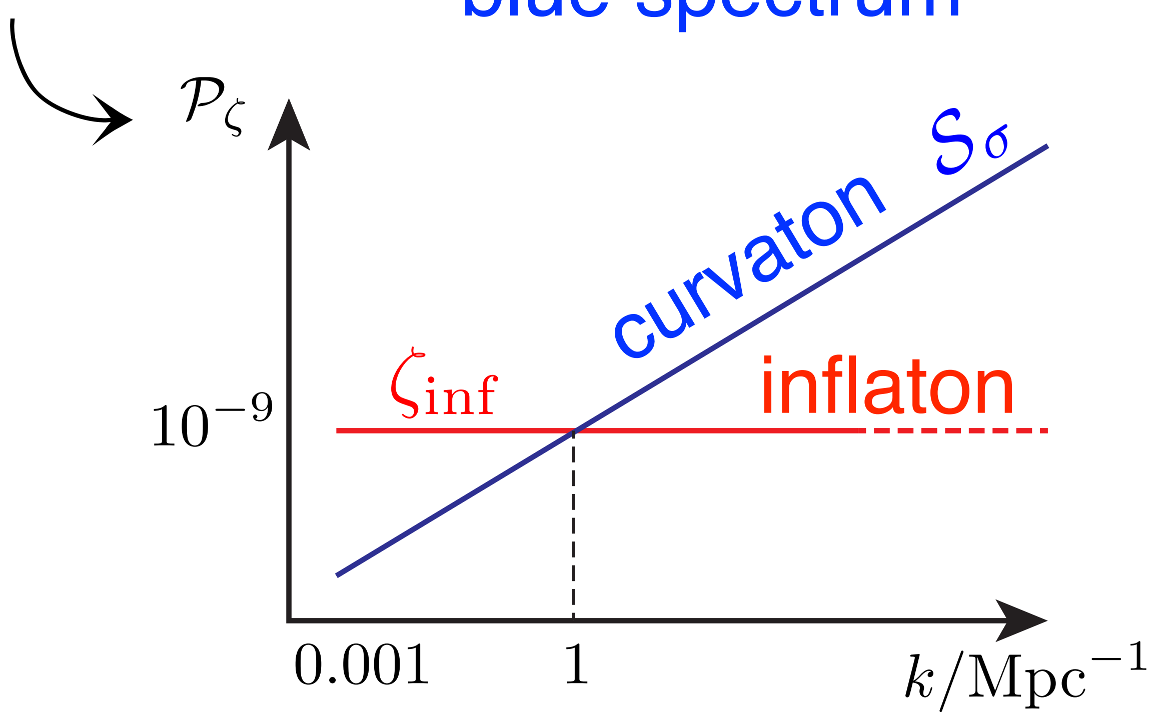
$$\mathcal{P}_{\zeta,\mathrm{curv}}(k) = \mathcal{P}_{\zeta,\mathrm{curv}}(k_c)\left(\frac{k}{k_c}\right)^{n_\sigma - 1}$$

with

$$\mathcal{P}_{\zeta,\mathrm{curv}}(k_c) = \mathcal{P}_{\zeta,\mathrm{inf}} \sim 2\times 10^{-9} \quad \& \quad k_c = 1\ \mathrm{Mpc}^{-1}$$

$$\& \quad n_\sigma > 1$$

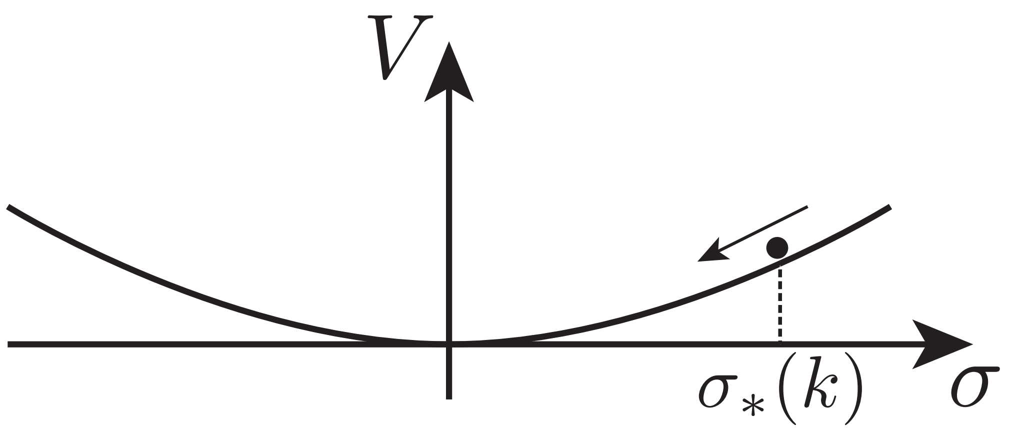
blue spectrum

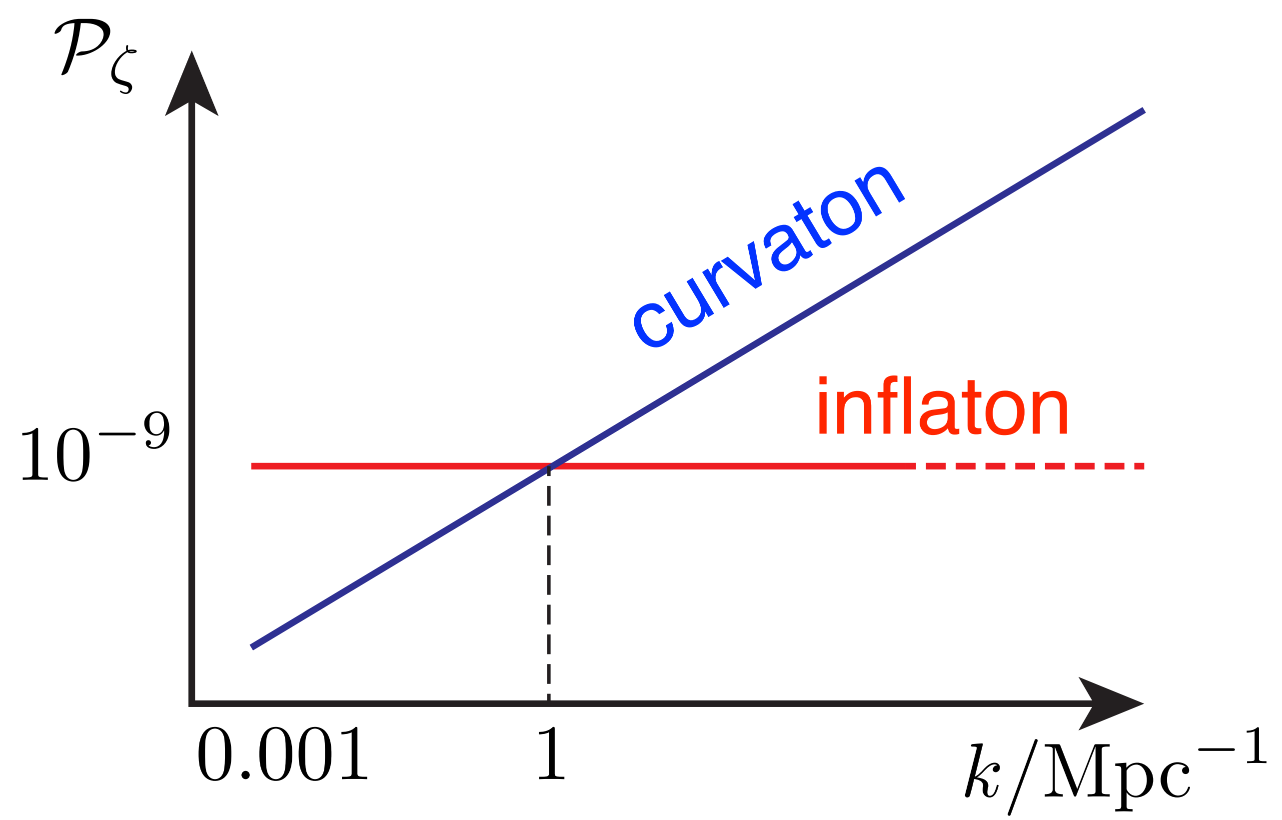


✣ Models

1. Quadratic curvaton model

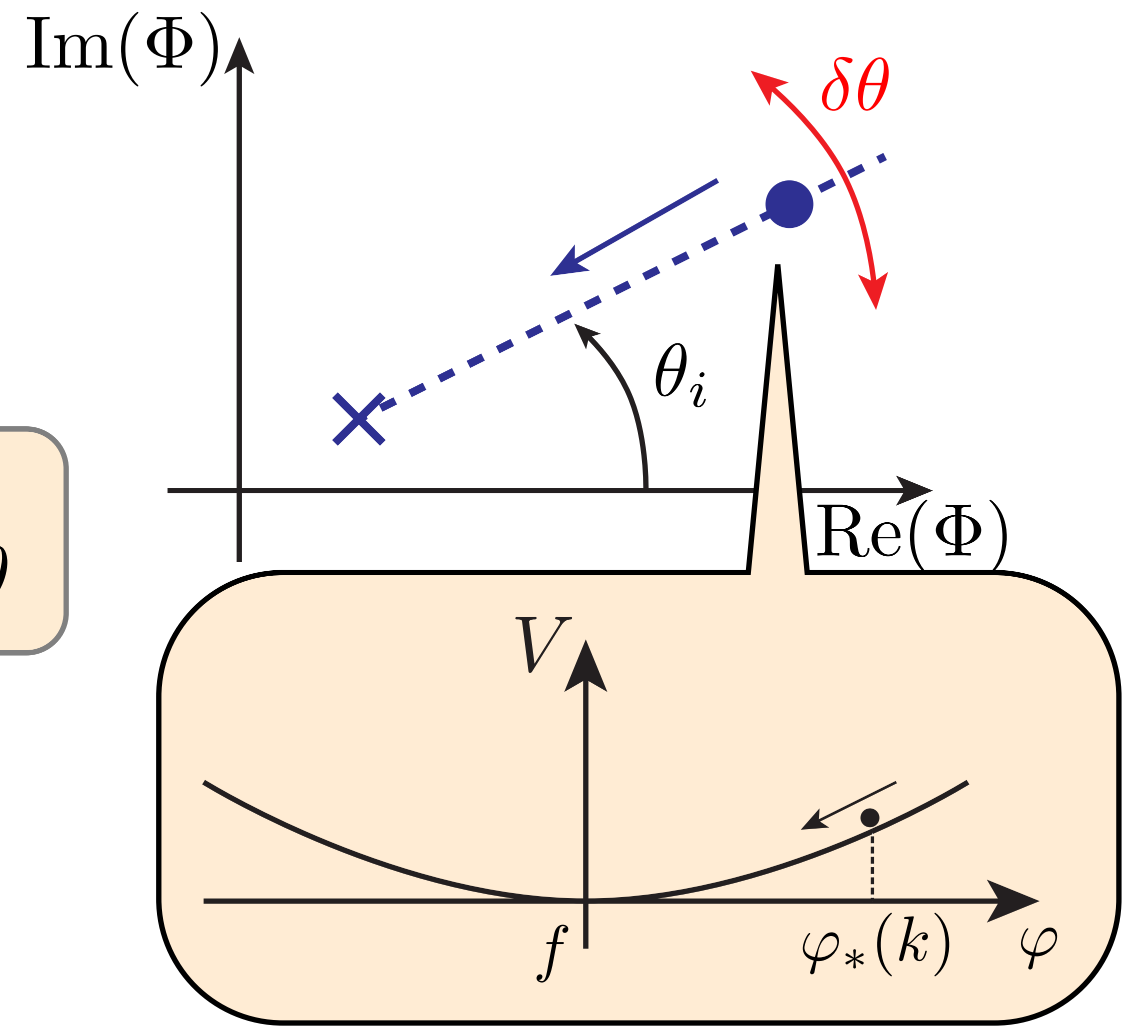
$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 \quad \Rightarrow \quad n_\sigma \simeq 1 + \frac{2m_\sigma^2}{3H_{\rm inf}^2}$$





2. Axion-like curvaton model

[Kasuya & Kawasaki, 0904.3800]



Complex scalar field: $\Phi = \varphi e^{i\theta}/\sqrt{2}$

Curvaton lives in the phase component of the scalar field: $\sigma = f\theta$

Potential of the curvaton:

$$V(\sigma) = \Lambda^4 \left[1 - \cos\left(\frac{\sigma}{f}\right)\right] \simeq \frac{1}{2} m_\sigma^2 \sigma^2$$

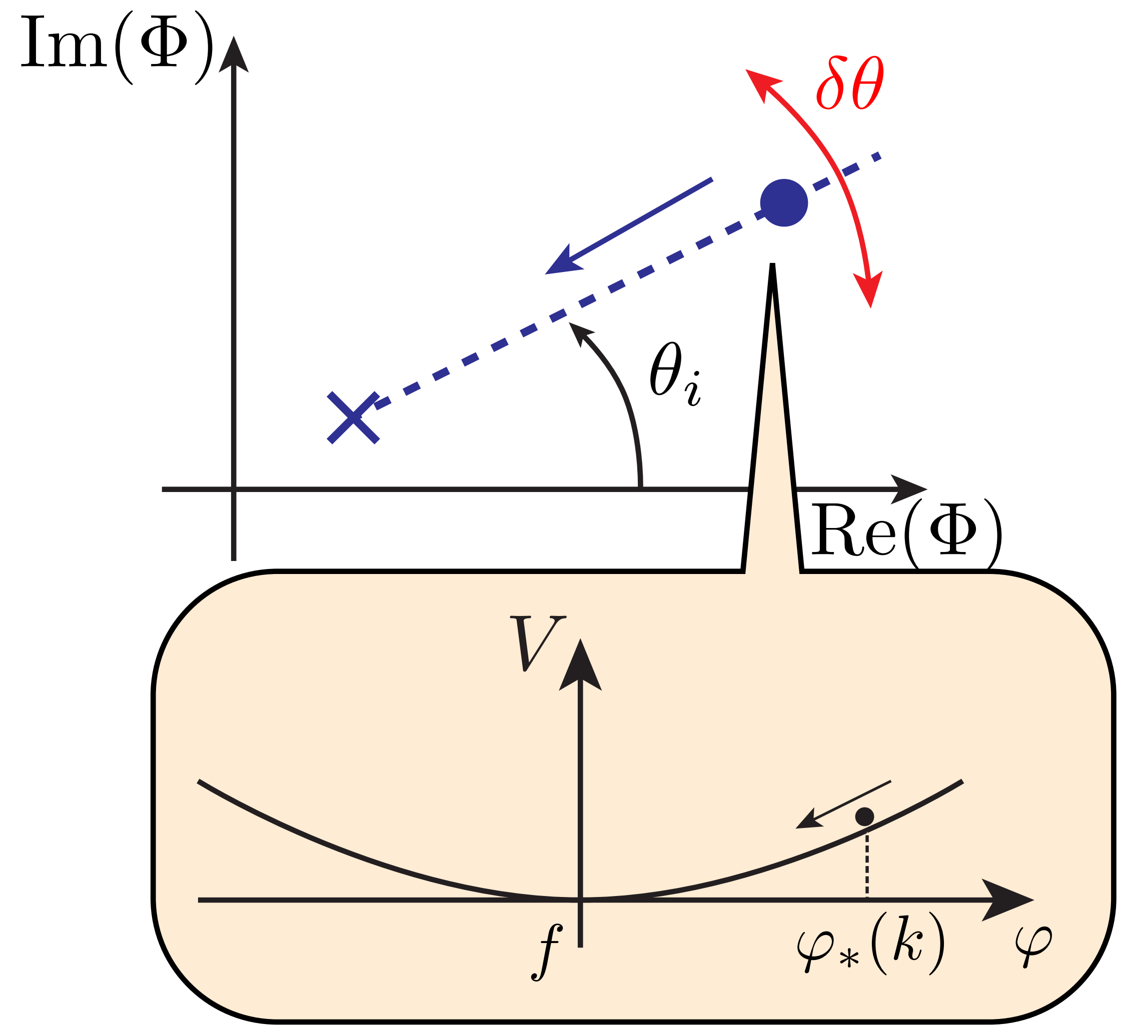
↳ $m_\sigma \ll H_{\rm inf}$ ▶▶ θ is unchanged during inflation

Potential of radial component: $V(\varphi) = \dfrac{1}{2} c H_{\rm inf}^2 (\varphi - f)^2$

$m_\varphi \sim H_{\rm inf}$ ▶▶ φ rolls down the potential somewhat rapidly.

Fluctuations

$\delta\theta/\theta$ is fixed at the value of
horizon exit during inflation

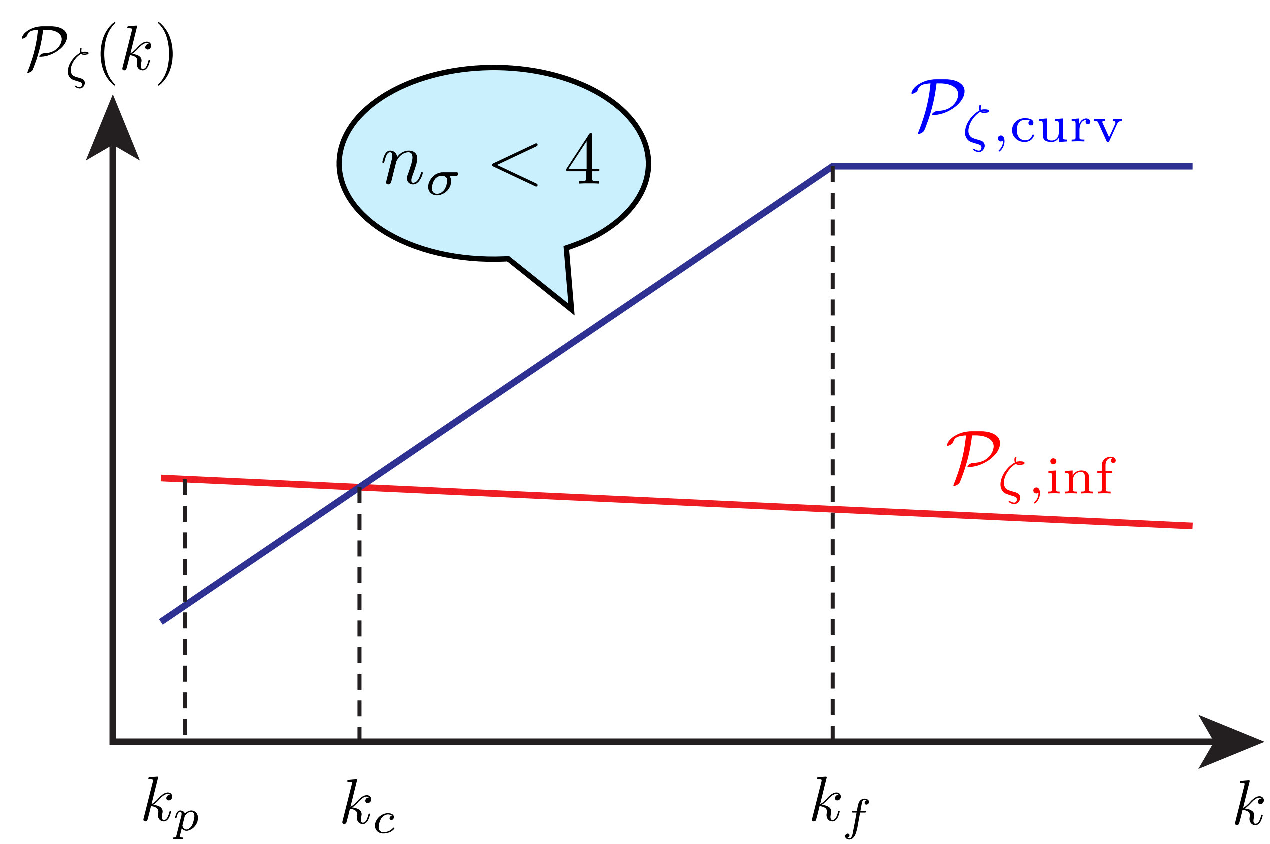


$$\frac{\delta\sigma_*}{\sigma_*} = \frac{\delta\theta_*}{\theta_i} = \frac{H_{\rm inf}}{2\pi\varphi_*\theta_i}$$

fluctuation of the curvaton
on superhorizon scale

$\varphi_* = \varphi_*(k)$ is the value
when the scale k exit the horizon

$$\mathcal{P}_{\zeta,\rm curv} \propto \varphi_*^{-2}$$



III. Scalar-induced gravitational waves

Our setup

❊ Perturbed metric (scalar & tensor modes)

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + \left[(1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right]dx^i dx^j\right]$$

h_{ij} : tensor mode ➤ stochastic background of GW

❊ Energy-momentum tensor (curvaton part)

$$T_{\mu\nu} = \partial_\mu\sigma\partial_\nu\sigma - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma + V(\sigma)\right)$$

Einstein equation: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

source term

Evolution for SGWB: $h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}^{lm}\mathcal{S}_{lm}$

$\mathcal{H} = a'/a, \quad \hat{\mathcal{T}}_{ij}^{lm}$: projection tensor into TT tensor

★ source term is zero at 1st order (No late-time GW production!)

✶ Source term for GW ▶ 2nd order perturbations

(scalar)×(scalar) can be a source term for GW

2 kind of source terms

♦ GWs from scalar metric perturbations (lhs of Einstein eq.)

$$\mathcal{S}_{ij}^{\Phi} = -2\partial_i \Phi \partial_j \Phi - \mathcal{H}^{-2} \partial_i (\Phi' + \mathcal{H}\Phi) \partial_j (\Phi' + \mathcal{H}\Phi)$$

♦ GWs from energy-momentum tensor (rhs)

$$\mathcal{S}_{ij}^{\mathrm{kin}} = M_P^{-2} \partial_i \delta\sigma \partial_j \delta\sigma$$

“anisotropic stress”

We have calculated the GW in blue-tilted curvaton model!

What can we observe?

Power spectrum of h ☞ $\langle h_{\mathbf{k}}(\eta) h_{\mathbf{p}}(\eta)\rangle \equiv \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{p})\mathcal{P}_h(k,\eta)$

$$\rho_{\rm GW}(\eta) = \frac{1}{32\pi G a^2}\langle h'_{ij}h'_{ij}\rangle = \frac{k^2}{16\pi G a^2}\int d\ln k\,\mathcal{P}_h(k,\eta)$$

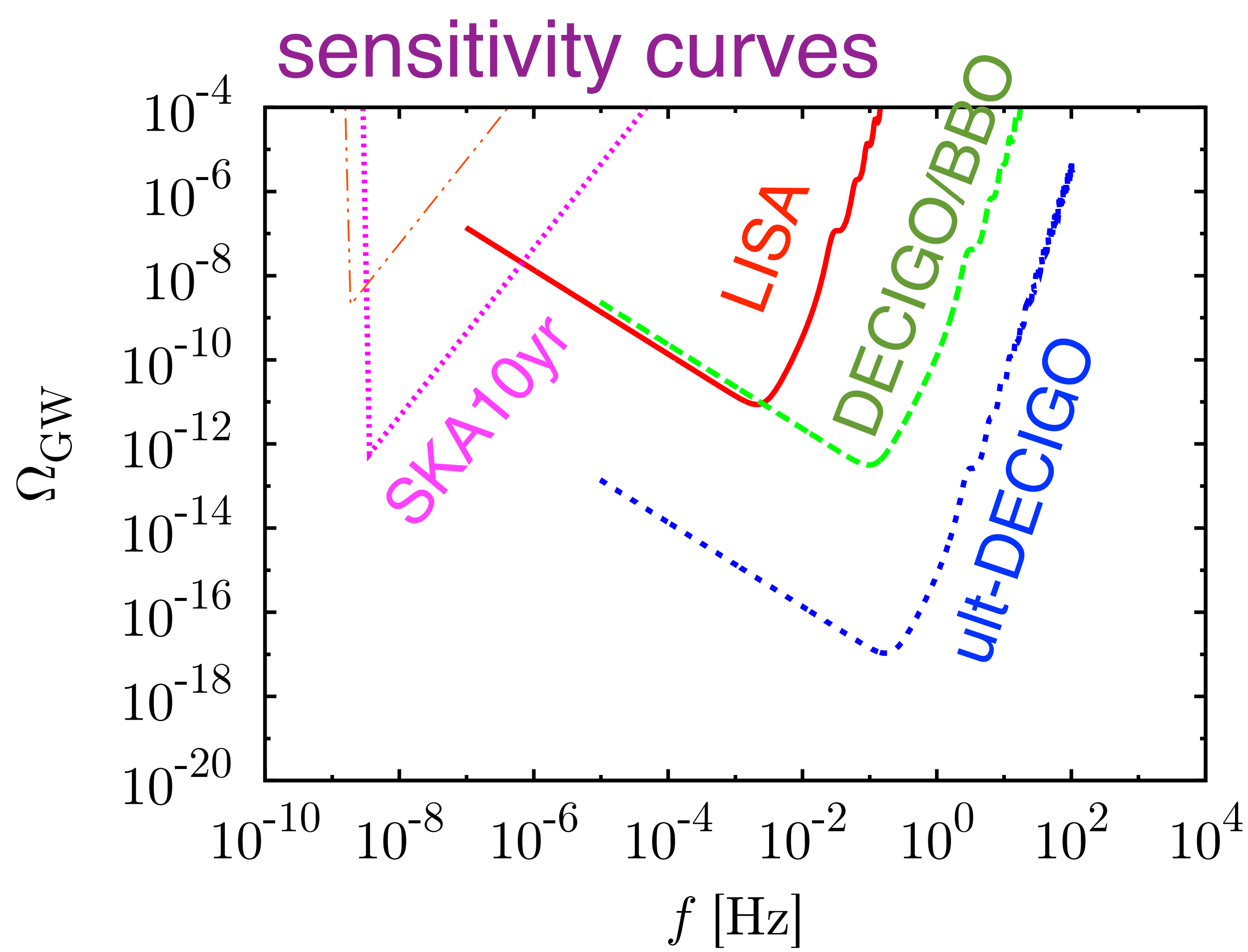
▶ $$\Omega_{\rm GW}(k,\eta) = \frac{1}{\rho_{\rm cr}(\eta)}\frac{d\rho_{\rm GW}(\eta)}{d\ln k} = \frac{k^2}{6\mathcal{H}^2(\eta)}\mathcal{P}_h(k,\eta)$$

Energy spectrum today :

$$\Omega_{\rm GW}(k) = \frac{k^2\Omega_\gamma}{6\mathcal{H}^2(\eta_\star)}\mathcal{P}_h(k,\eta_\star)$$

$\Omega_\gamma \simeq 4.8\times 10^{-5}$

(density parameter of radiation)



IV. Result

✻ GW from curvaton : contribution from EM tensor

Approximated formula : $\Omega_{\rm GW} \sim 10^{-19} \frac{\Gamma}{r_D^2 m_\sigma} \left(\frac{\mathcal{P}_\zeta}{2 \times 10^{-9}} \right)^2$

$\left(r_D = \frac{\rho_\sigma}{\rho_r} \text{ at decay} \right)$ Bartolo, Matarrese, Riotto & Vaihkonen (2007)

$$\Omega_{\rm GW} \sim 10^{-25} \left(\frac{4}{4+3r_D} \right)^4 \left(\frac{\sigma_{\rm osc}}{\sigma(k)} \right)^4 \left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}} \right)^4$$

➢ negligible contribution !

$r \lesssim 0.1$ or $H_{\rm inf} \lesssim 10^{14}\ {\rm GeV}$

cf. $\mathcal{S}_{ij}^{\rm kin} = M_P^{-2} \partial_i \delta\sigma \partial_j \delta\sigma \sim k^2 \left(\frac{H_{\rm inf}}{M_P} \right)^2$

✻ GW from curvaton : contribution from curvature

(i) Approximated formula : $\Omega_{\rm GW} \sim 10^{-19} \left(\dfrac{\mathcal{P}_{\zeta,{\rm curv}}(k)}{\mathcal{P}_\zeta(k_c)} \right)^2$

Ananda, Clarkson & Wands (2007),
Baumann, Steinheardt, Takahashi & Ichiki (2007)

(ii) GW is emitted at the horizon reentering

because Φ decays after horizon reentering (RD)

(In MD universe, Φ doesn't decay even after horizon reentering
& GWs are continuously emitted)

(iii) Peaked spectrum at k_dec

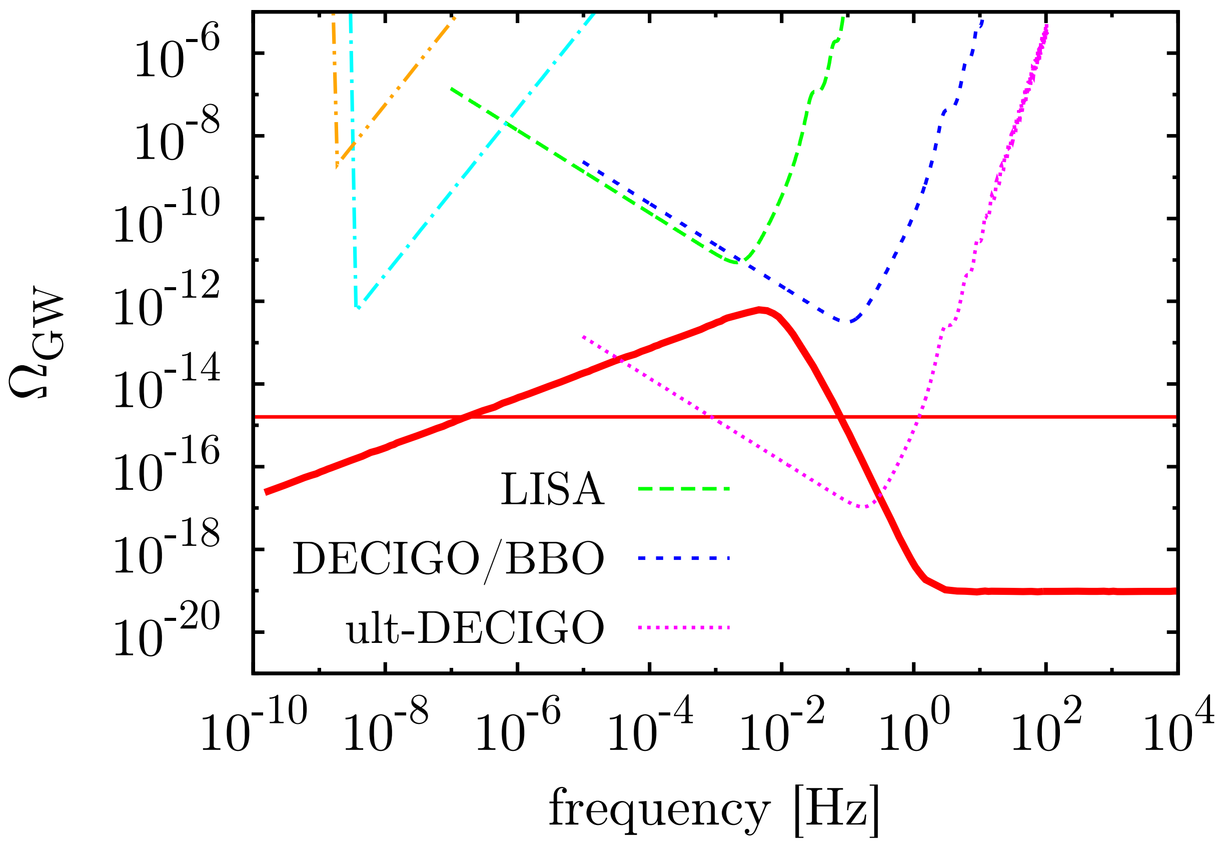
the mode reentering the horizon at the curvaton decay

because $\mathcal{S}_\sigma \propto a$ before curvaton decay

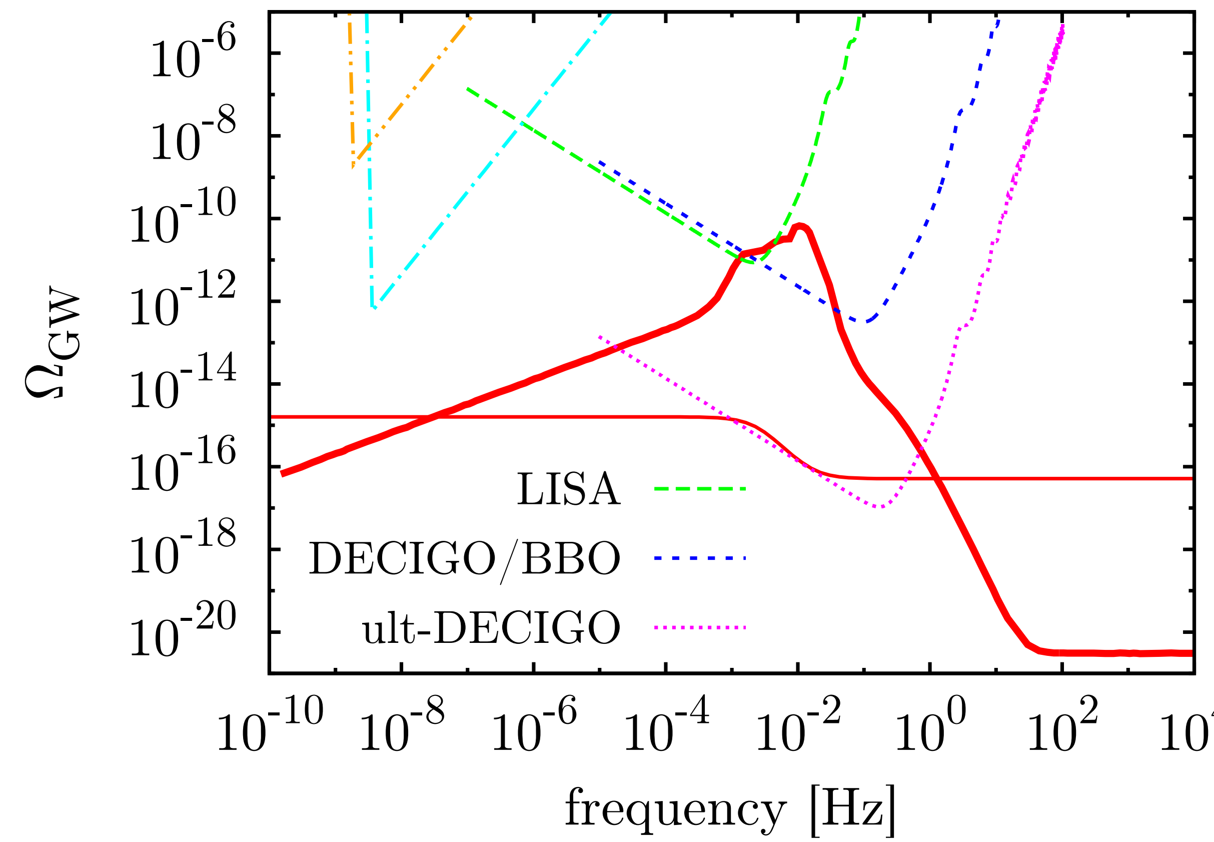
(1) Quadratic curvaton model

$H_{\rm inf} = 3 \times 10^{-5} M_P, n_\sigma = 1.3$ $\quad r_D = \rho_\sigma/\rho_r$ at decay

$r_D = 1$



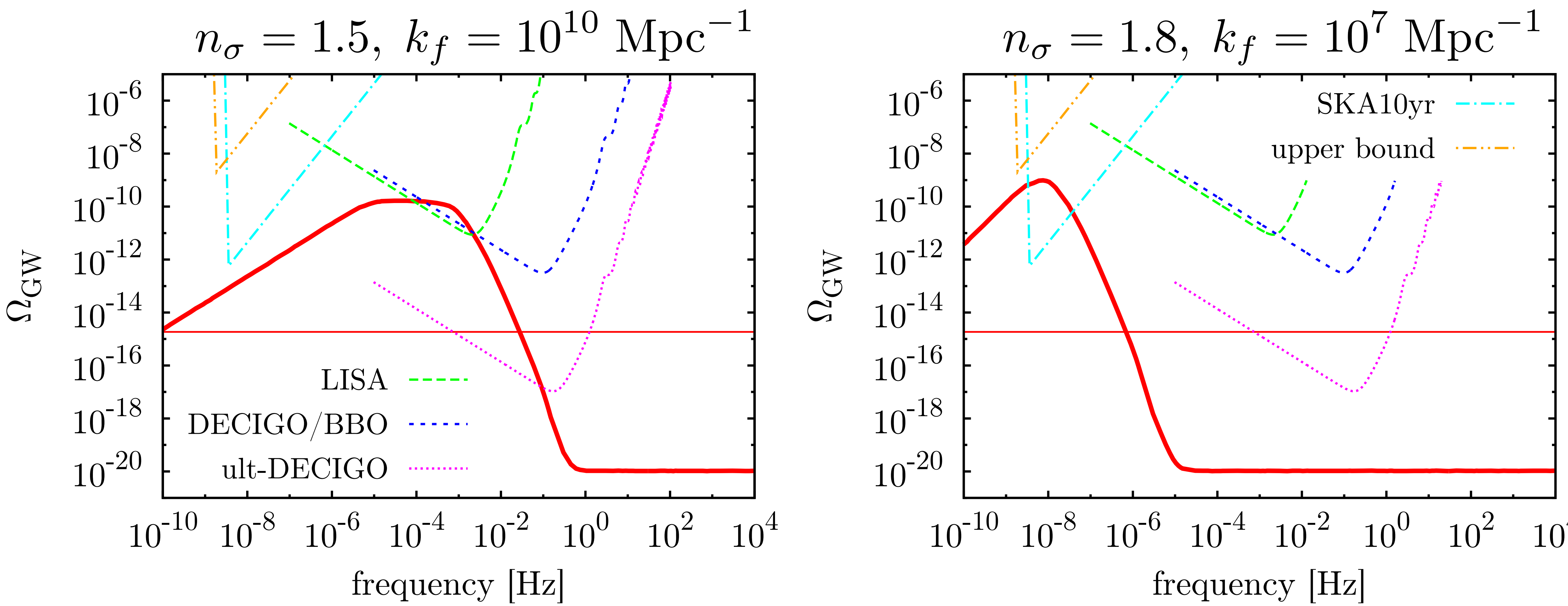
$r_D = 10$



- ☞ Spectrum has a peak corresponding to the curvaton decay!
- ☞ Signal is detectable by future observation!
- ☞ We can distinguish whether r_D < 1 or not.

(2) Axion-like curvaton model

$$H_{\rm inf} = 3 \times 10^{-5} M_P,\ r_D = 1$$



$f \simeq 2 \times 10^{14}$ GeV, $m_\sigma \simeq 3 \times 10^{10}$ GeV

$f \simeq 4 \times 10^{13}$ GeV, $m_\sigma \simeq 9 \times 10^{3}$ GeV

☛ Characteristic shape (there is a plateau)

☛ Signal is detectable by pulsar timing obs.

Summary

Motivation

Heavy scalar fields existing at the inflationary epoch (curvaton) can generate the large curvature perturbation on small scales

▶ Can we detect their imprints by observation?

What we did

We have calculated the amount of GW sourced by the scalar perturbations

(quadratic curvaton model & axion-like curvaton model)

Results

Detectable GW is predicted

- curvaton decay epoch
- curvaton domination

Conclusions:

We can see the imprints of curvaton scenario or constrain the heavy scalar fields during inflation by GW obs.

✻ GW from curvaton : contribution from curvature

(i) Approximated formula : $\Omega_{\rm GW} \sim 10^{19}\left(\frac{\mathcal{P}_{\zeta,{\rm curv}}(k)}{\mathcal{P}_{\zeta}(k_c)}\right)^2$

Ananda, Clarkson & Wands (2007),
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(ii) GW is emitted at the horizon reentering

because Φ decays after horizon reentering (RD)

(iii) Peaked spectrum at k_dec

the mode reentering the horizon at the curvaton decay

$k_{\rm dom} \sim 1/\eta_{\rm dom}$ or $k_{\rm NL} \sim \mathcal{P}_{\zeta,{\rm curv}}^{-1/4}(k_{\rm dec})/\eta_{\rm dec}$ for $r_D > 1$

➤ the mode reentering the horizon at curvaton domination
or the mode becoming nonlinear at decay