Tidal Forces and the Roche Limit of Planetary Bodies

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PHYS 305

Introduction

- The Roche Limit is the distance at which an orbiting satellite will disintegrate due to the tidal forces exerted on the satellite from the mass it's orbiting

- Tidal force: differential force from the effects of gravity

Equations

$$F = -\frac{GMm}{r^2} \qquad r = \sqrt{x^2 + y^2} \qquad G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

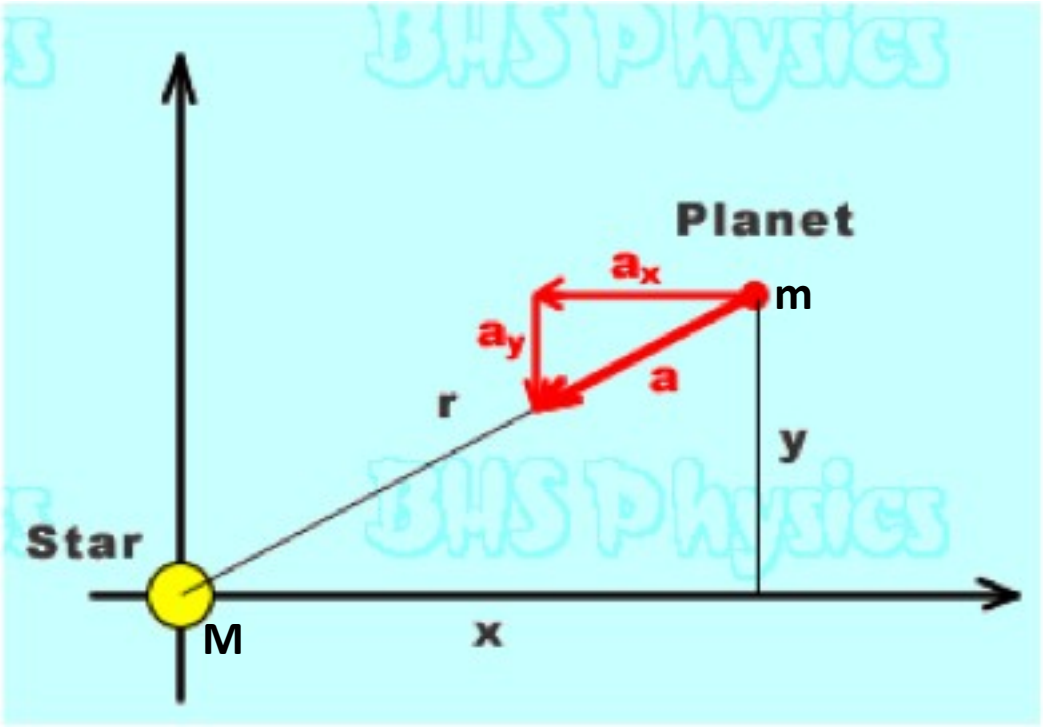

Fig. 1: Diagram of a planet orbiting around a star

http://www.batesville.k12.in.us/physics/phynet/mechanics/gravity/lab/excel_orbits.htm

Equations

$$F_g = -\frac{Gm\mu}{r^2}$$

$$F_t = \frac{2GM\mu r}{d^3} \Rightarrow F_g = F_t \Rightarrow d = R\left(2\frac{\rho_M}{\rho_m}\right)^{1/3}$$

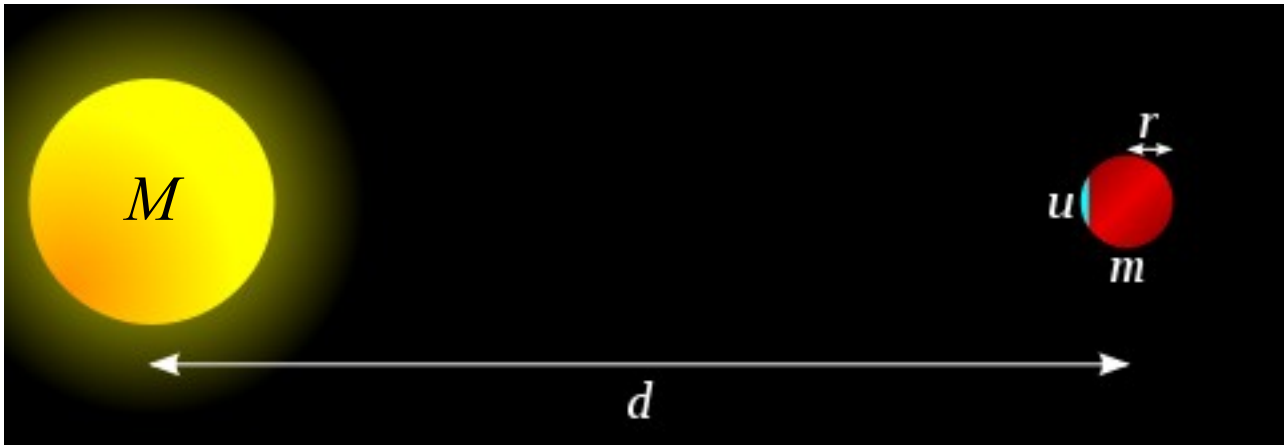

Fig 2: Gravitational forces between two masses
http://en.wikipedia.org/wiki/Roche_limit

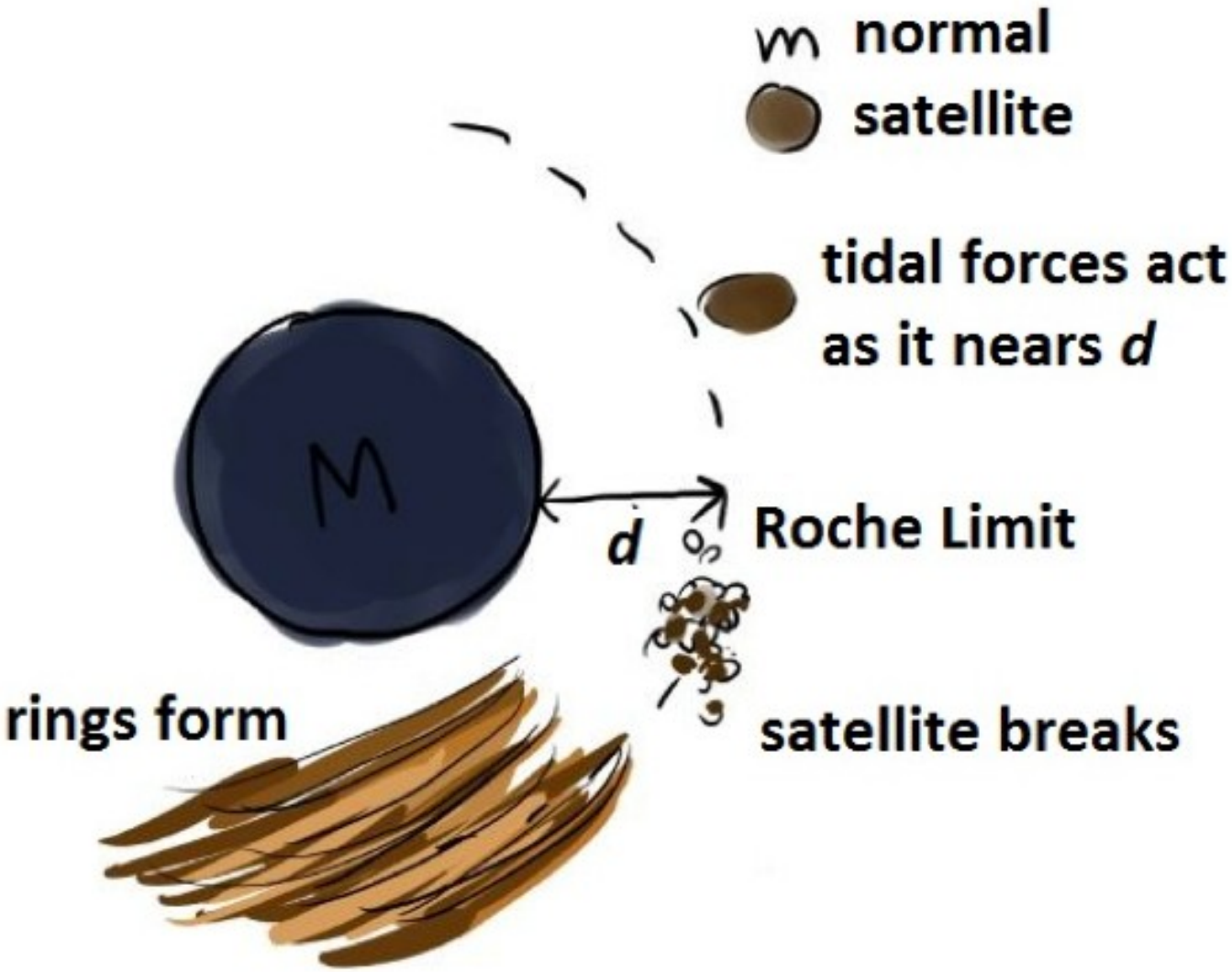

Fig. 3: Diagram describing the formation of rings due to Roche Limit

Program Algorithm

- Define constants for *G*, *M*, initial *x* and *y* positions, and initial *x* and *y* velocity components for a two body system (e.g. Titan and Saturn)

- Can increase/decrease initial constants by a percentage of the original

Program Algorithm

- Use Runge-Kutta second order (RK2) method to plot an orbit

$$\frac{dv}{dt} = -\frac{GM}{r^2}$$

$$\frac{dv_x}{dt} = -\frac{GMx}{r^3}$$

$$\frac{dv_y}{dt} = -\frac{GMy}{r^3}$$

- Plot results and compare

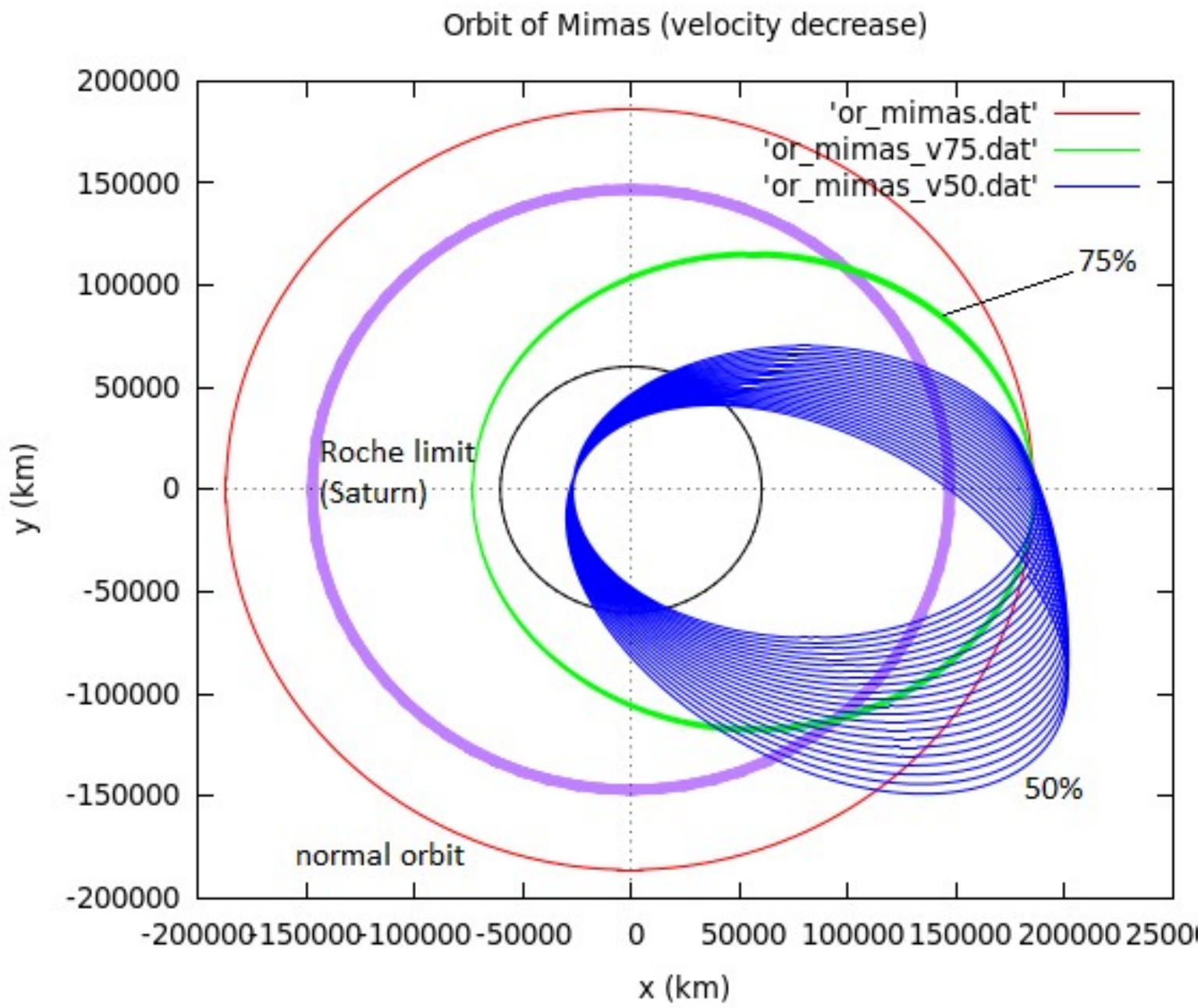

Fig 4: Orbit of Mimas with distance reduced by 75%, 50%

Mimas

- x0 = 185,520 km, v0 = 14.32 km/s [1]
- Density = 1148 kg/m^3 [1]
- Roche limit is 50787.68 km
- Too small and beyond Saturn's orbit
- Graph shows that Mimas will eventually crash into Saturn and probably won't break up to form rings

[1] http://alumnus.caltech.edu/~marcsulf/saturn/mimas.html

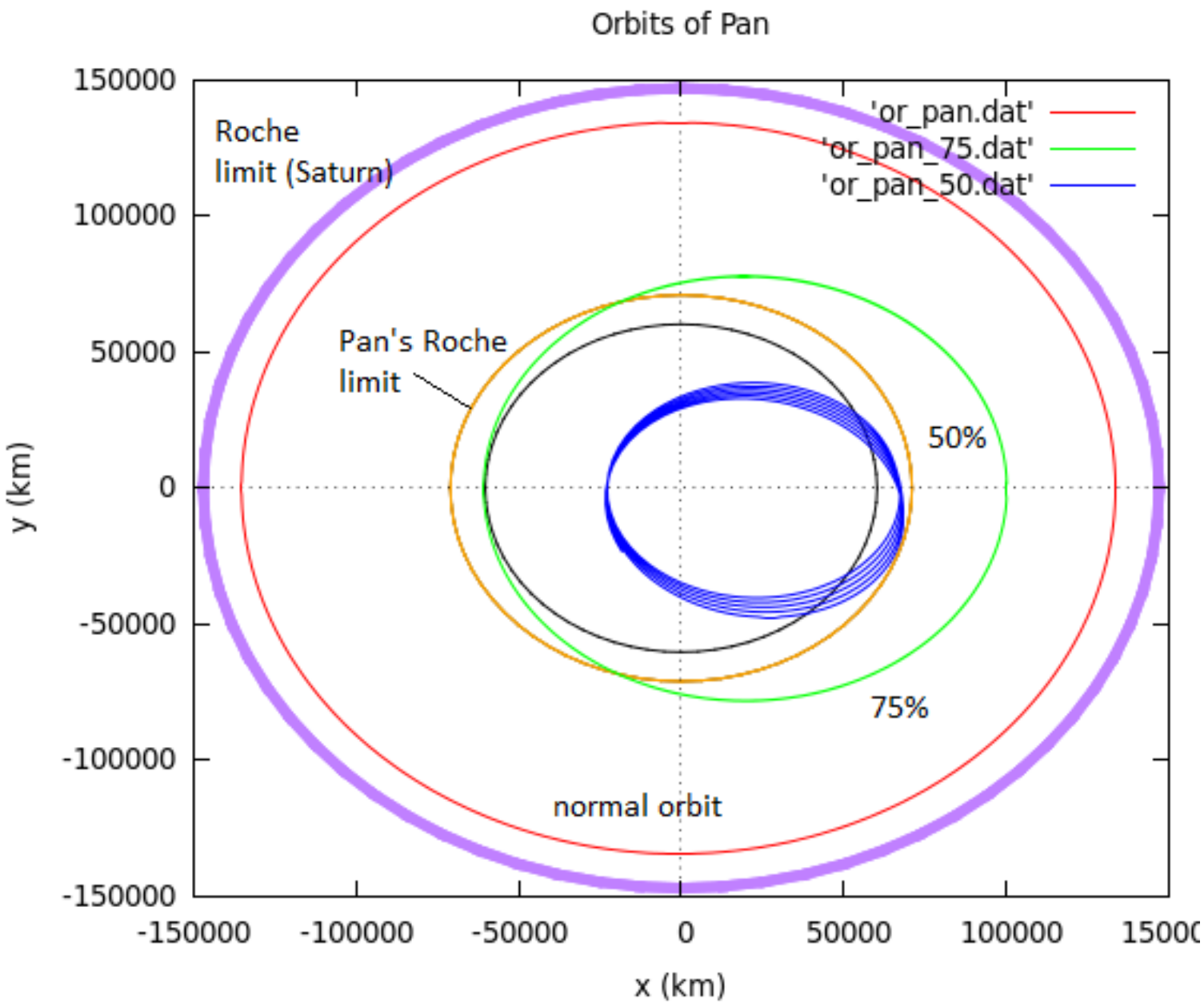

Fig 5: Orbits of Pan with distance reduced by 75%, 50%

Pan

- x0 = 133,583 km, v0 = 16.9 km/s [2]
- Density = 420 kg/m^3 ± 150 [2]
- Roche limit is 71,010.5 km
- If the maximum orbital distance is decreased by 75% at 100,187.3 km and keeping v0 at 16.9 km/s, Pan reaches its Roche limit and will break up
- It is known as a “ring shepard”

Thoughts/conclusion

- Gives quantitive data for orbital paths, which allows visualization of bodies relating to the Roche limit

- The Roche limit is dependent on the densities of the primary and the orbiting satellite