

An study of relativistic energy, momentum, and particle collisions

Natalie Nagata

Kernel Review

PHYS 305: Computational Physics

Overview

- Background
- Objective
- Project Approach
- Results (thus far)
- Discussion
- Further work

Background

- 1905 (“*Annus Mirabilis*”, the Miracle Year!)
- Albert Einstein—from patent office clerk to science rockstar!



Special Relativity

- Postulate 1
 - The laws of physics remain the same in all inertial frames
- Postulate 2
 - The speed of light is invariant in all inertial frames
- What does this mean???

Time Dilation and Length Contraction

- For a fixed frame S and a moving frame S' with velocity v:

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} = T_0 \gamma$$

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$

- Counterintuitive ideas

Objective

- Show the relationship between relativistic momentum and energy
 - Lab frame vs CM frame
 - Magnitude and scale comparison using different masses (protons, electrons, etc)

$$p_a' = \frac{p_a - \frac{v_{cm}}{c^2} E_a}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}};$$

$$p_b' = \frac{p_b - \frac{v_{cm}}{c^2} E_b}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}};$$

$$E_a' = \frac{E_a - p_a v_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad E_b' = \frac{E_b - p_b v_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}};$$

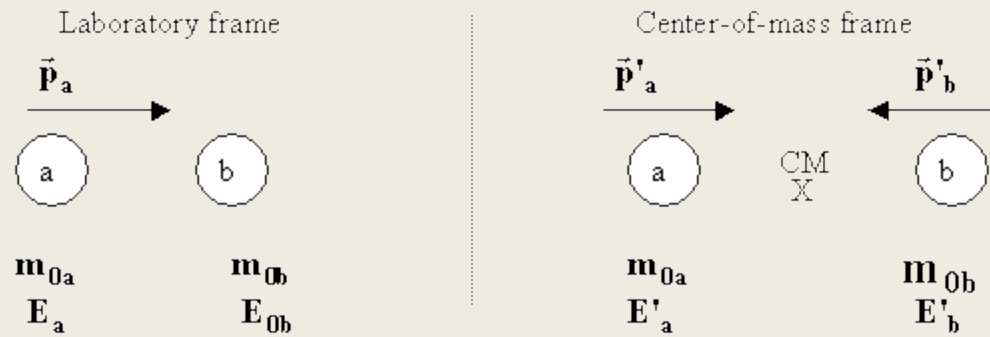


Fig. 1 Two-particle collision observed in lab and CM frames

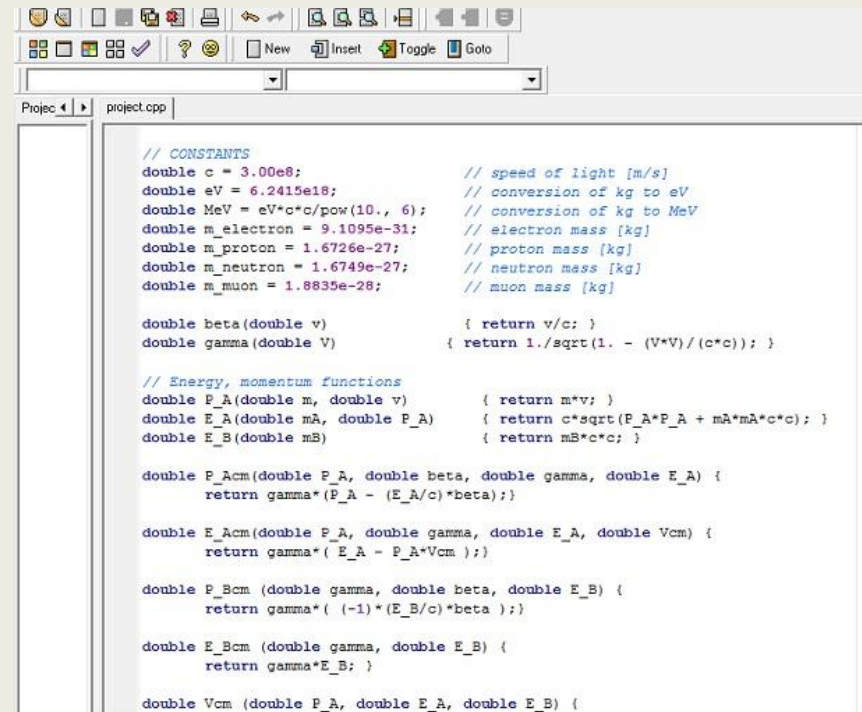
http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/applications_of_special_relativi.htm

Objective

- Graphically calculate and display the minimum threshold energies in some particle collisions
 - Necessary for the creation of certain particles

Project Approach

- Defining constants
- Defining functions for momentum and energy
 - Functions passing functions as parameters (BE CAREFUL!)



```
// CONSTANTS
double c = 3.00e8; // speed of light [m/s]
double eV = 6.2415e18; // conversion of kg to eV
double MeV = eV*c*c/pow(10., 6); // conversion of kg to MeV
double m_electron = 9.1095e-31; // electron mass [kg]
double m_proton = 1.6726e-27; // proton mass [kg]
double m_neutron = 1.6749e-27; // neutron mass [kg]
double m_muon = 1.8835e-28; // muon mass [kg]

double beta(double v) { return v/c; }
double gamma(double V) { return 1./sqrt(1. - (V*V)/(c*c)); }

// Energy, momentum functions
double P_A(double m, double v) { return m*v; }
double E_A(double mA, double P_A) { return c*sqrt(P_A*P_A + mA*mA*c*c); }
double E_B(double mB) { return mB*c*c; }

double P_Acm(double P_A, double beta, double gamma, double E_A) {
    return gamma*(P_A - (E_A/c)*beta);}

double E_Acm(double P_A, double gamma, double E_A, double Vcm) {
    return gamma*( E_A - P_A*Vcm );}

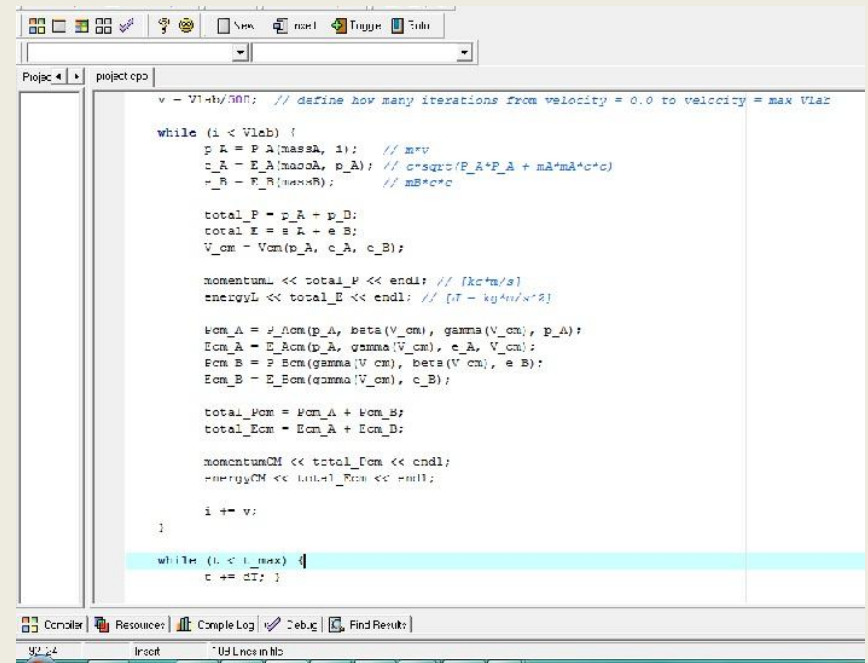
double P_Bcm (double gamma, double beta, double E_B) {
    return gamma*( (-1)*(E_B/c)*beta );}

double E_Bcm (double gamma, double E_B) {
    return gamma*E_B; }

double Vcm (double P_A, double E_A, double E_B) {
```

Project Approach

- While-loop
 - Calls and defines energy and momentum functions
 - Calculates total momentum and energy over a varying velocity from 0 to nearly the speed of light



```
v = 71414700; // define how many iterations from velocity = 0.0 to velocity = max Vmax

while (i < Vmax) {
    p_A = P_A(massA, i); // m*v
    e_A = E_A(massA, p_A); // c*sqrt(p_A^2 + m^2*c^2)
    p_B = P_B(massB); // mB*c*c

    total_P = p_A + p_B;
    total_E = e_A + e_B;
    V_cm = Vcm(p_A, e_A, e_B);

    momentum_CM << total_P << endl; // [kg*m/s]
    energy_CM << total_E << endl; // [J = kg^2m^2/s^2]

    fcm_A = f_cm(p_A, beta(V_cm), gamma(V_cm), p_A);
    fcm_B = f_cm(p_B, gamma(V_cm), e_A, V_cm);
    fcm_B = f_cm(gamma(V_cm), beta(V_cm), e_B);
    fcm_B = f_cm(gamma(V_cm), e_B);

    total_fcm = fcm_A + fcm_B;
    total_fcm = fcm_A + fcm_B;

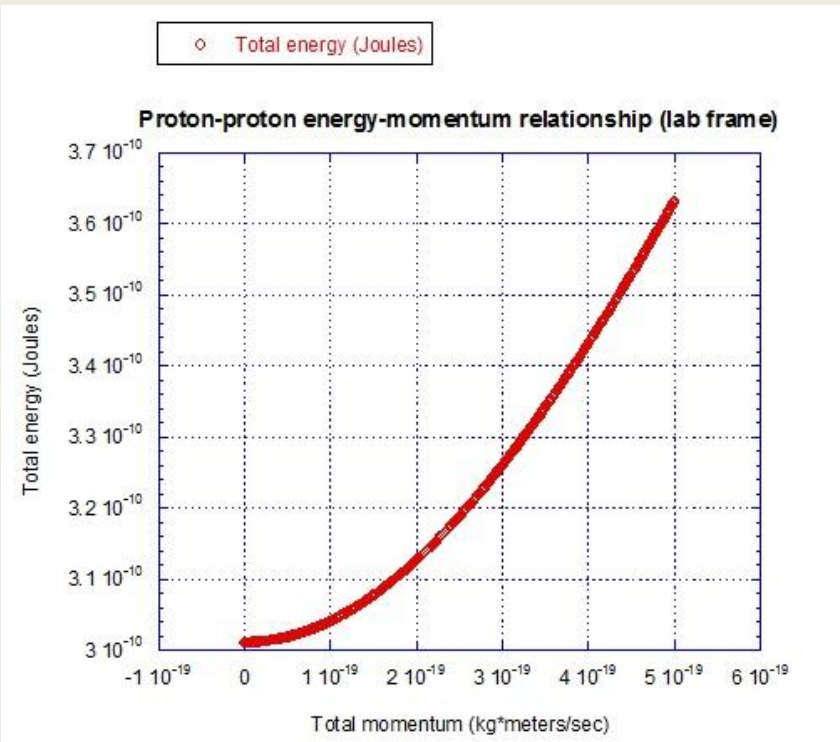
    momentum_CM << total_fcm << endl;
    energy_CM << total_fcm << endl;

    i += v;
}

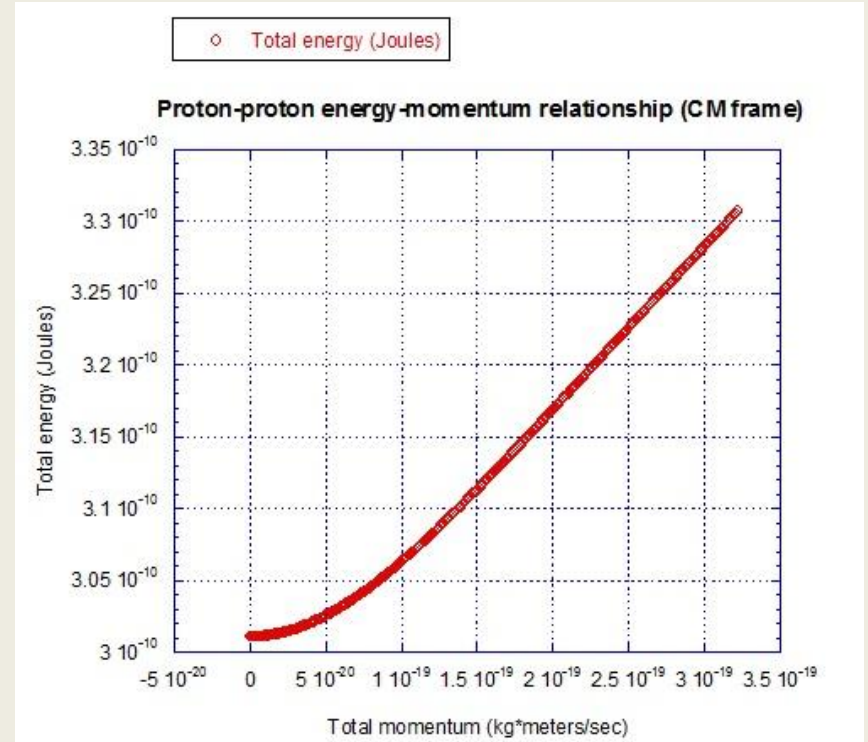
while (i < i_max) {
    c += dT; }
```

Results (thus far)

Lab frame ($0 < v < 2.99e8$)



CM frame ($0 < v < 2.99e8$)



Discussion

- Noticeable variation
 - $E_{\text{lab}} / E_{\text{CM}} \approx 1.098$
 - E_{lab} roughly 0.911 times greater than E_{CM}
- Demonstrates importance of particle colliders using colliding beams (CM frame) instead of colliding with a stationary particle (lab frame)

Discussion

- Some significant things to keep in mind:
 - Many constants—don't lost track!
 - *Many* functions!
 - Need better organization for clarity
 - Solution: stuctures? Arrays?
 - Don't forget—units, units, units!

Project Outlook

- Add asymptotic values of threshold energies needed to create new particles
- Continue with other scenarios of particle collision
- Experiment with adding new time and position functions

References

- Bitu, R. "Applications of Special Relativity on Particle Physics."
http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/applications_of_special_relativi.htm
- Taylor, John R. "Classical Mechanics". University Science Books. 2005