Modeling the Earth's Magnetic Field Interaction with the Solar Wind

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PHYS 305 /Professor Varner

Introduction

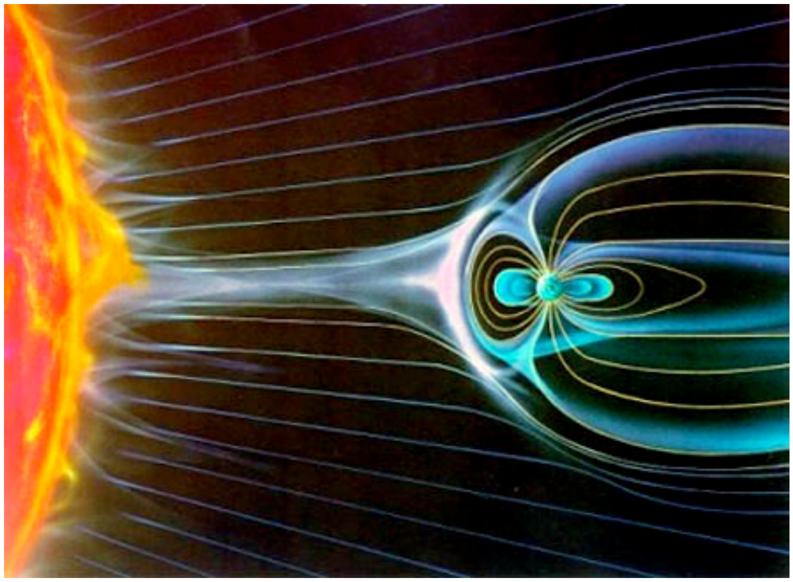

Figure 1: Illustration of the Earth's magnetic field interacting with the solar wind (http://www.teara.govt.nz/files/di6139pc.jpg)

The Earth is constantly being bombarded by cosmic particles travelling at nearly 400 km/s everyday from the Sun by the solar wind, which was found to originate from the Sun's corona [1]. Although the solar wind is an everyday occurrence that many have learned to live with everyday, a more dangerous solar occurrence occurs at the ending of the Sun's eleven-year cycle: solar flares. Solar flares are localized eruptions that occur on the surface of the Sun, usually around active regions such as sunspots [2]. Another type of solar occurrences are coronal mass ejections (CME's). CME's, unlike solar flares, are not localized, and instead are larger eruptions [3] that occur from the corona. Both solar flares and CME's can disrupt radio transmission, damage satellites and even knock out whole power grids, however more stress is put on solar flares due to the localization in which the particles are directed. With technology being very vital to society moreso now than ever, creating models to better predict these occurrences can help develop early warning systems and preventative measures.

The main goal of this project is to model the Earth's magnetic field as a simple dipole magnet, then to introduce an infinite wave of charged particles (the solar wind) to interact and reshape this field. Upon completing this main portion, creating other scenarios such as solar flares or CME's can be considered.

Main Problem

The main problem of this assignment is to first model the Earth's magnetic field as a simple dipole magnet, then to later introduce the solar wind and model the magnetic field's shape and interaction with it thereafter. After the main portion is completed, venturing into adding other variables such as rotation of the Earth, it's orbit around the Sun, as well as interactions with solar phenomena such as solar flares and coronal mass ejections can later be modeled.

Proposed Solution

Modeling the Earth's magnetic field requires generation of points at which the magnetic field as a certain magnetic flux density. Using C++ language one can generate the points where these magnetic flux densities occur. Creating a program to generate a sequence of points, plug them into the following [4] equations

$$B_r = -2B_0\left(\frac{R_E}{r}\right)^3 \cos\theta \tag{1}$$

$$B_\theta = -B_0\left(\frac{R_E}{r}\right)^3 \sin\theta \tag{2}$$

$$|B| = \sqrt{B_r^{\,2} - B_\theta^{\,2}} = B_0\left(\frac{R_E}{r}\right)^3 \sqrt{1+3\cos^2\theta} \tag{3}$$

and then select the points that correspond to a particular magnetic flux density |B| (magnitude); where $B_0 = 3.12 \times 10^{-5}$ Tesla is the average magnetic flux density of the Earth, r is the radial distance from the center, $R_E \sim 6370$ km is the Earth's radius, and θ is the azimuth measured from the north pole. A magnetic field can hopefully be plotted using gnuplot. Several shells of the magnetic field may also be obtained and plotted using the same process.

Upon completing this portion of the project, introducing the solar wind particles using and its interaction with the magnetic field using the Lorentz force equation

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{4}$$

the magnetic field can again be obtained with modifications of the same process to obtain the simple dipole model of the Earth's magnetic field.

Kernel Description

The main portion of this program will be able to generate points, as said above. One way of generating these points is to generate a vast array randomly using previous random number generator techniques learned previously in class. Setting an appropriate range can help generate a variety of points, which may help give a distribution of the field over a certain radial range. This method would be good for generating a wider range of possible magnetic fields, but the amount of points needed may prove cumbersome. After obtaining these points, they will be saved into a text-file, which can later be plotted in gnuplot. Using gnuplot, one can observe the contours (magnetic field density) and hopefully observe different levels of magnetic field lines.

Another technique to generating these points is to simply create three for-loops, each generating an x, y, and z point. The range can be initialized such that a complete scan of the radial range can give an overall range of points. This technique can narrow down at what radii particular magnetic flux densities occur, which can later be used to pick particular field lines. Again, similar to the above, the data would be saved into a text-file, then the contours plotted using gnuplot in hopes of observing magnetic field lines. This method may prove more useful in plotting field lines in a two-dimensional cross-section, if not three dimensionally.

The main altered parameters would include the range (radius) in which the field would be considered, the number of points (N_Total) generated for the loop/random numbers, and output (select points corresponding to magnetic field densities).

Status

As of now, work is still being done in generating the simple model of the Earth's magnetic field. The most recent has been to create a cross section of the field by plotting on the y and z points corresponding to a particular magnetic flux density. Although the shape is more round in comparison to previous trials, work still needs to be done to obtain the correct representation of a simple dipole magnet.

Previous trials have fallen short in comparison the most recent, where attempts at generating contours of the three dimensional points have proven unsuccessful. Graphs in the order of increasing progress have been included below.

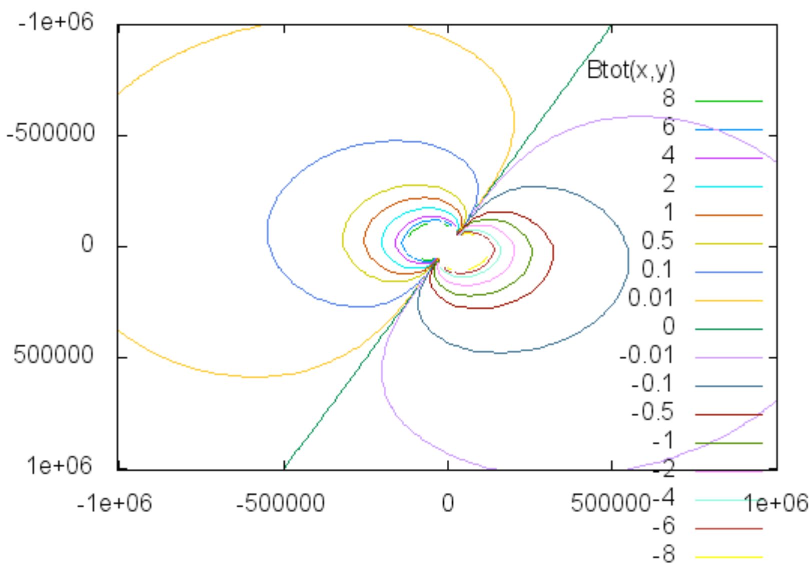

Figure 2: Earth's magnetic field using gnuplot contour feature. Although this looks very beautiful and similar to a magnetic dipole, it is titled which suggests a change in coordinates may be needed. Gnuplot only plots functions of up to two variables for some reason, which limits the three-variable scenario the magnetic flux density calculation.

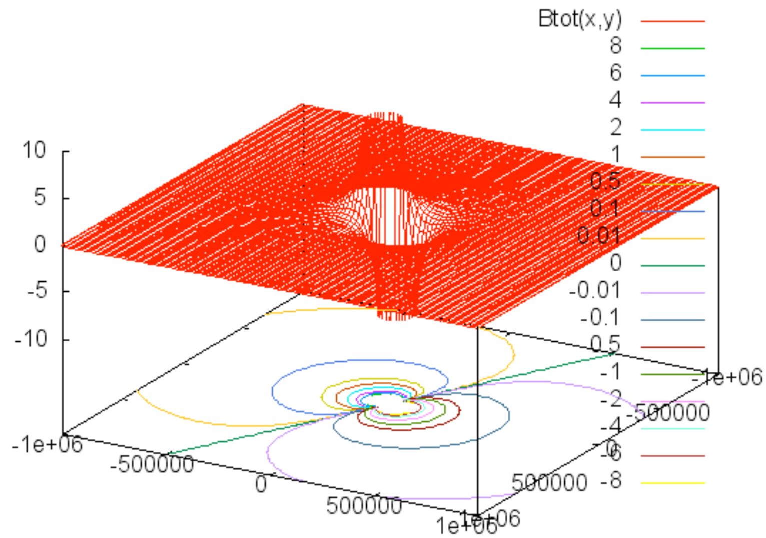

Figure 3: How gnuplot generates the contours in Figure 2

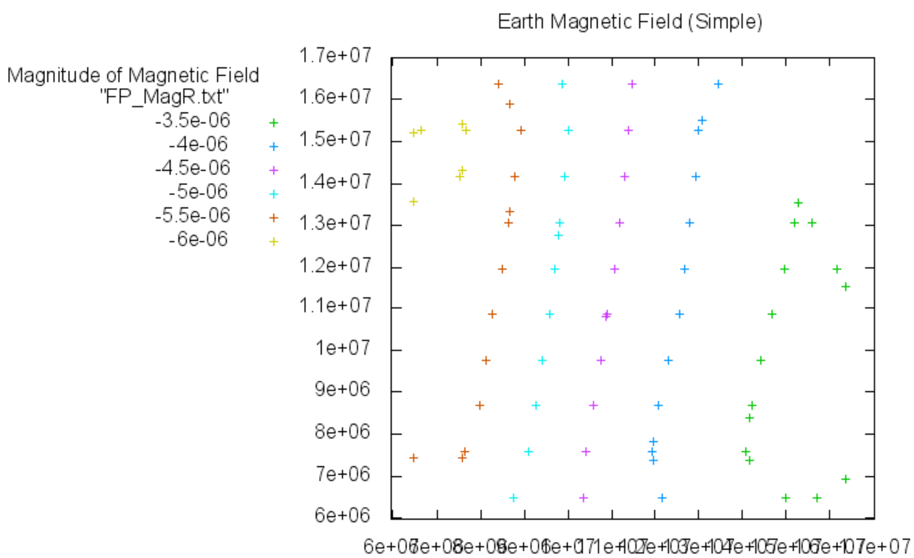

Figure 4: Contour of points generated by C++ code. Although some field line shapes may be seen, the downside to this is that the points are not centered at zero, suggesting the field lines do at the center are quite off from where they were expected to be

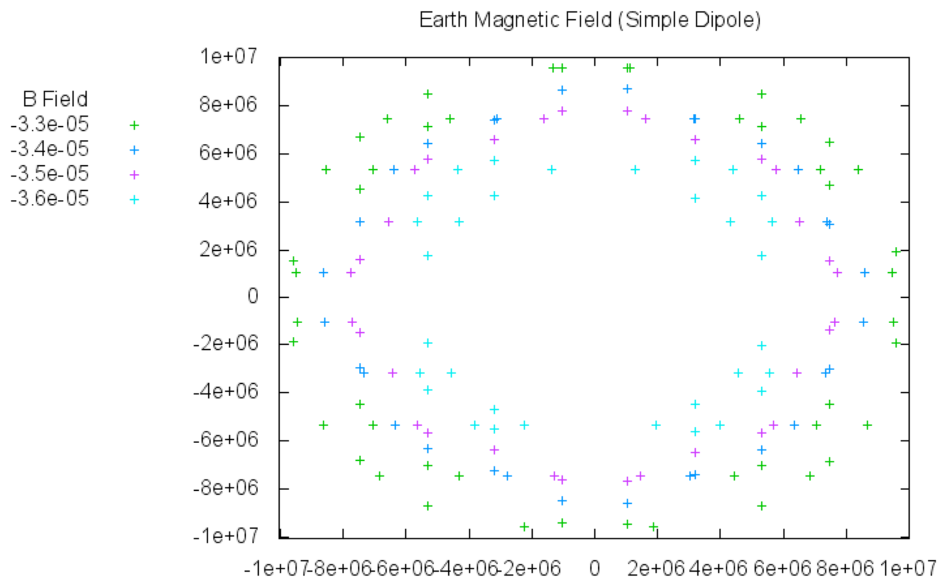

Figure 5: Similar to above, except centered. Although some shape is forming it was later found that the equations used to generate these points were not accurate (confused the radius in three-dimensions for the radius in two (used x-y, when I was suppose to use x,y,z)

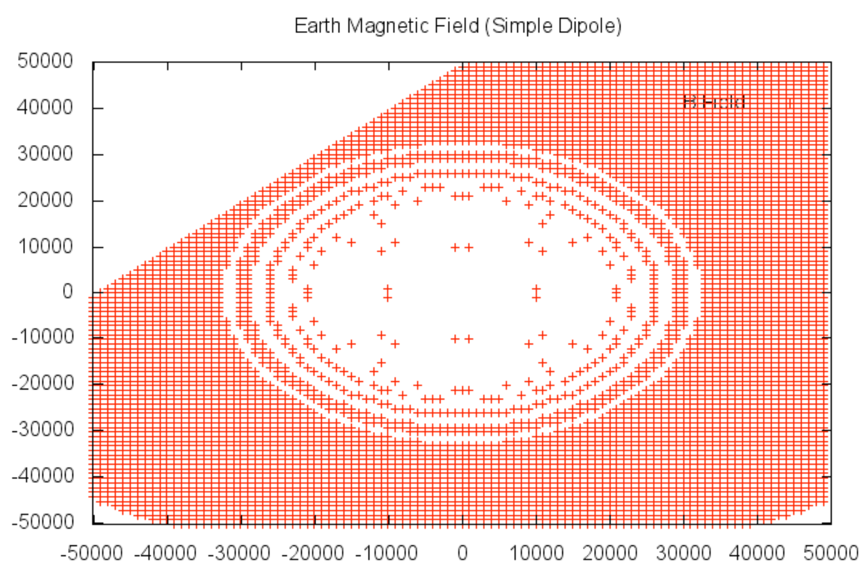

Figure 6: Top view of three-dimensional points (y,z, and magnitutde of B) plotted using gnuplot. Although some shape is there, it is not the contour view, which we would hope to expect to see. This surface is of a particular B-field.

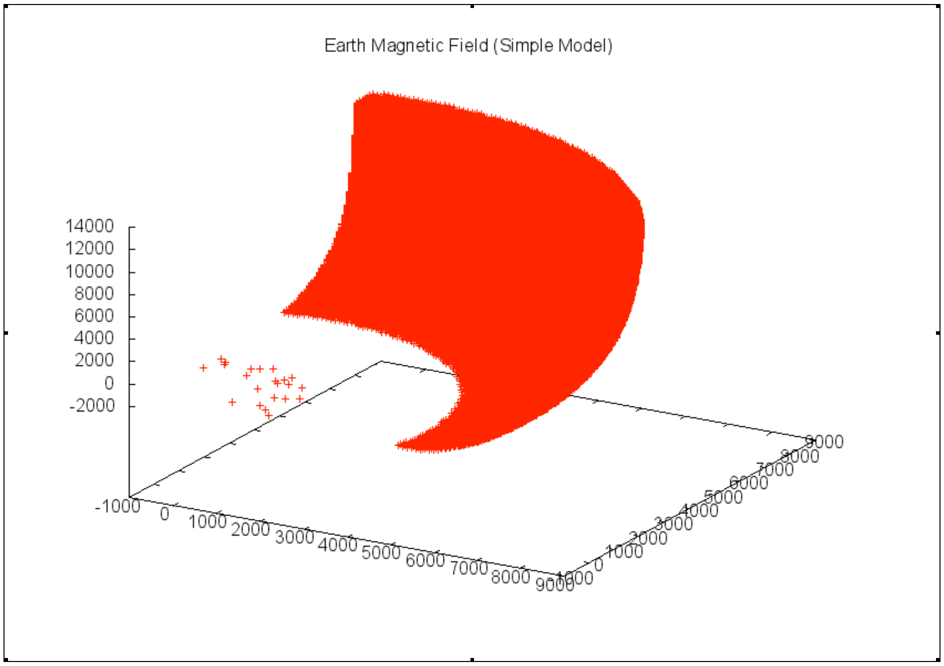

Figure 7: Three-dimensional magnetic field attempt. These points correspond to a particular magnetic field (plotted is x, y, z coordinates) using gnuplot.

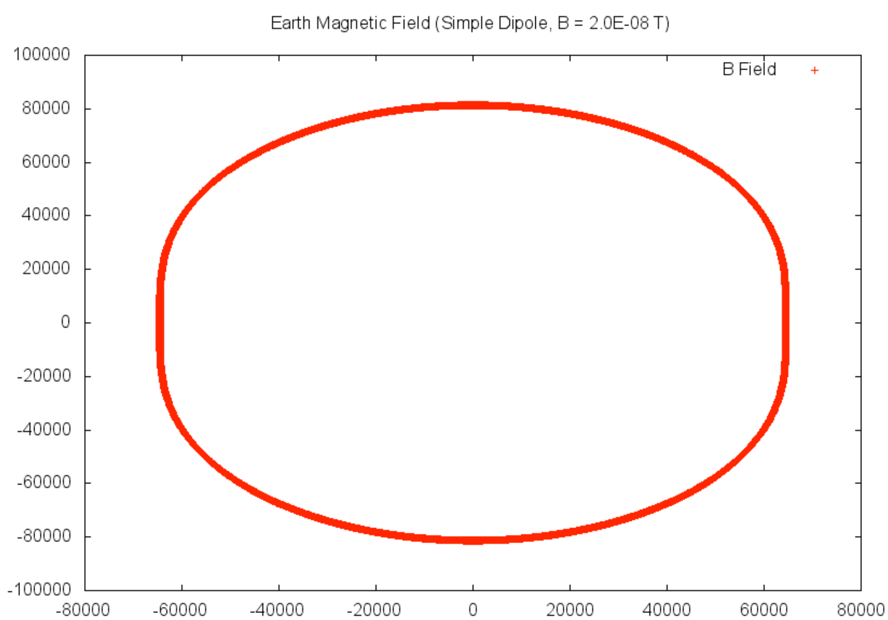

Figure 8: The most accurate representation of the magnetic field of Earth so far. Although the shape has yet to show a magnetic field, the equations used as well as the range and orientation are fairly correct (or at least to my knowledge they are). This is a two dimensional plot of a B-field where x = 0 and y, z were simply plotted as so.

More work needs to be done to perfect the shape of the magnetic field in the most simplest case. Once the techniques are refurbished, and a foundation is laid later steps into introducing the solar wind as well as other interactions/variables may occur. I am hopefully aiming to at least get the simple case of generating the simple magnetic dipole model working. If time permits, modeling the Earth’s magnetic field interaction with the solar wind may be possible. The main focus however is on the simplest case.

References

[1] http://solarscience.msfc.nasa.gov/SolarWind.shtml

[2] http://en.wikipedia.org/wiki/Solar_flare

[3] http://cse.ssl.berkeley.edu/coronalweather/CMEsFlares/

[4] http://en.wikipedia.org/wiki/Dipole_model_of_the_Earth's_magnetic_field