Theory of (Inclusive) Charmonium Production

Geoffrey Bodwin
Argonne National Lab

- Theoretical Framework
  - NRQCD Factorization of the Inclusive Production Cross Section
  - Status of a Proof of Factorization
  - The Problem of Large k Factors
- Comparisons of NRQCD Factorization with Experiment
  - Summary
  - First Complete NLO Calculations for Hadroproduction
  - Polarization
  - $e^+e^- \to J/\psi + X$(non-$c\bar{c}$)
  - J/ψ Production in $\gamma\gamma$ Scattering at LEP II
- Conclusions

Theoretical Framework

NRQCD Factorization of the Inclusive Production Cross Section

- Conjecture (GTB, Braaten, Lepage (1995)):
  The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as a sum of "short-distance" coefficients times long-distance matrix elements (LDMEs).

$$\sigma(H) = \sum_n F_n(\Lambda) \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle .$$

- The "short-distance" coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
- The LDMEs $\langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle$ are expressed in terms of the EFT nonrelativistic QCD (NRQCD). The LDMEs are the probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium.
- The LDMEs are matrix elements of four-fermion operators in NRQCD, but with a projection onto an intermediate state of the quarkonium H plus anything:

$$\mathcal{O}_n^H(\Lambda) = \langle 0 | \chi^\dagger \kappa_n \psi \left(\sum_X |H + X\rangle \langle H + X| \right) \psi^\dagger \kappa'_n \chi | 0 \rangle .$$

- κ_n and κ'_n are combinations of Pauli and Color matrices.

- The short-distance coefficients have expansions in powers of α_s.
- The LDMEs are nonperturbative, but they are conjectured to be universal (process independent).
  - Only the color-singlet production and decay LDMEs are simply related.
- The LDMEs have a known scaling with v.
  $v^2 \approx 0.23$ for the J/ψ.
- The current phenomenology of J/ψ, $\psi(2S)$, and Υ production uses LDMEs through relative order v^4:

$$\begin{aligned}
&\langle \mathcal{O}^H({}^3S_1^{[1]})\rangle \quad (O(v^0)),\\
&\langle \mathcal{O}^H({}^1S_0^{[8]})\rangle \quad (O(v^3)),\\
&\langle \mathcal{O}^H({}^3S_1^{[8]})\rangle \quad (O(v^4)),\\
&\langle \mathcal{O}^H({}^3P_J^{[8]})\rangle \quad (O(v^4)).
\end{aligned}$$

- A key feature of NRQCD factorization:
  Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  - Inconsistent at higher orders in v and for P-wave production: IR divergent.

Status of a Proof of Factorization

- A proof is complicated because gluons can dress the basic production process in ways that apparently violate factorization.
- A proof of factorization would involve a demonstration that diagrams in each order in α_s can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD LDMEs,
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.
- Nayak, Qiu, Sterman (2005, 2006): The color-octet NRQCD LDMEs must be modified by the inclusion of Wilson lines (path integrals of the gauge field) to make them gauge invariant.
  - Contributions involving the Wilson lines first appear in NNLO and are essential to allow soft contributions to be absorbed into the NRQCD LDMEs.
  - If they are to be universal, the NRQCD LDMEs must be independent of the direction of the Wilson lines.
  - At NNLO, a “miracle” occurs and the dependence on the direction of the Wilson lines cancels.
  - It is not known if this generalizes to all orders.
  - An all-orders proof is essential because the α_s associated with soft gluons is not small.

The Problem of Large k Factors

- Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large. (Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007))

NLO and NNLO* Color-Singlet J/ψ Production

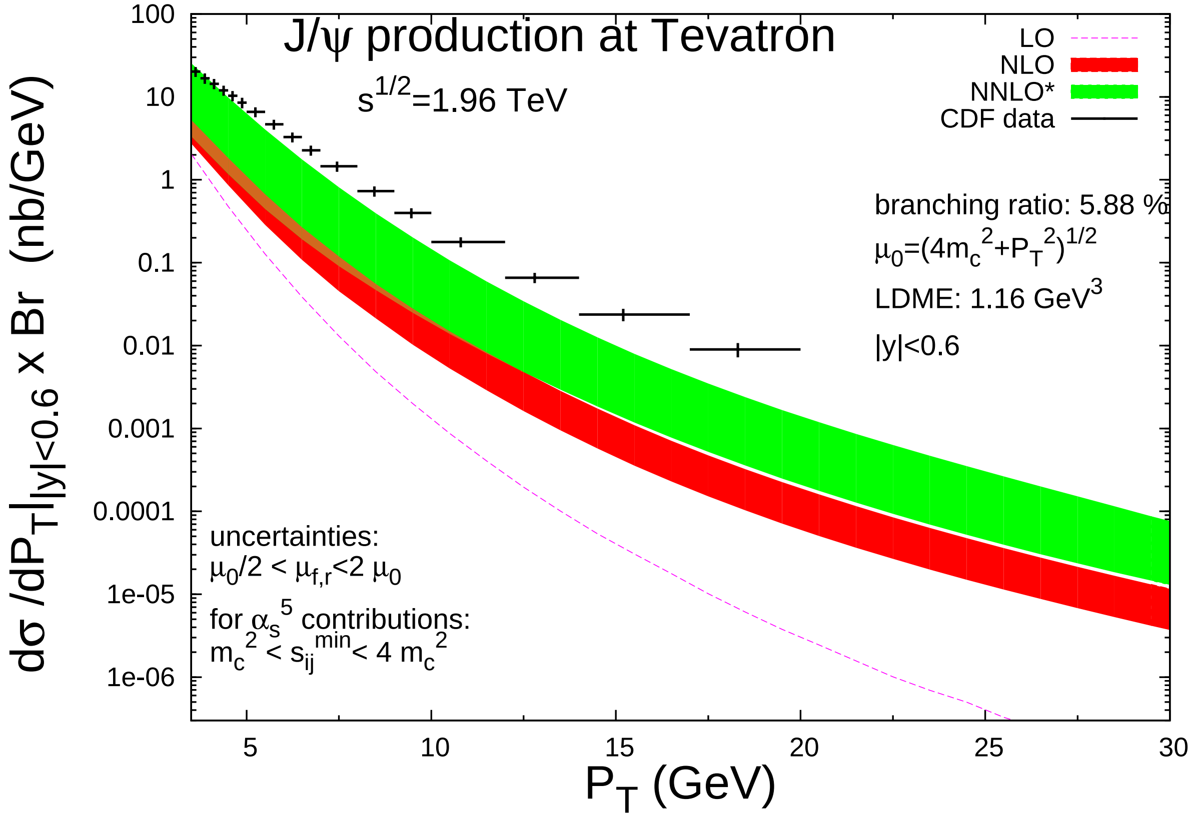


Figure courtesy of Pierre Artoisenet.

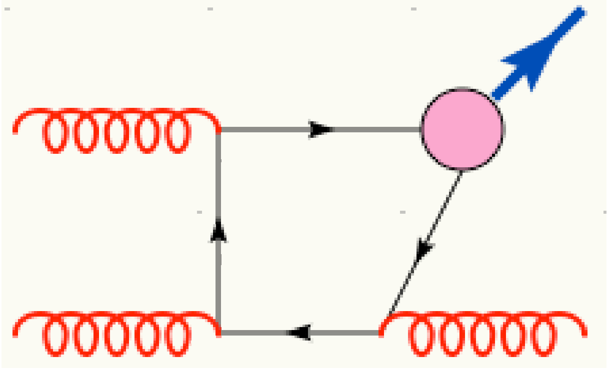
- The NNLO* calculation is an estimate based on real-emission contributions only.
- Ma, Wang, Chao (2011): The color-singlet NNLO* correction at large p_T seems be dominated by contributions proportional to $\log^2(p_T^2/p_{T\text{cut}}^2)$.
- Virtual corrections would cancel these logs, making the complete NNLO contribution smaller.
- The color-singlet contribution alone is insufficient to explain the data.

- A large k factor ~ -10 is also seen at NLO in the ${}^3P_J$ color-octet channel.
  (Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010))
- NLO corrections to the S-wave channels are small.
  (Gong, Li, and Wang (2008, 2010))
- Does the perturbation series converge?
- How do we understand the different k factors for different channels?

Explanation of Large k Factors

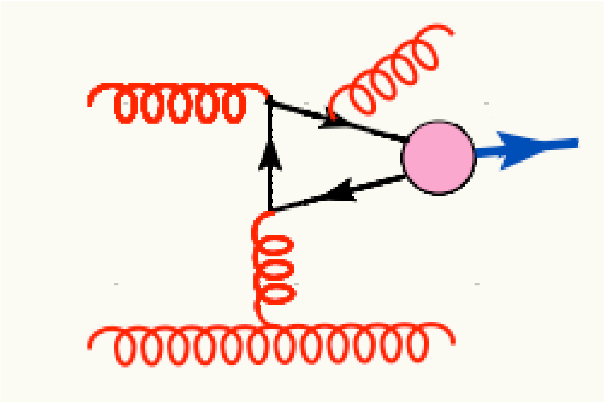
- At high p_T, higher powers of α_s can be offset by a less rapid fall-off with p_T.
  (Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007))

Color-singlet LO:

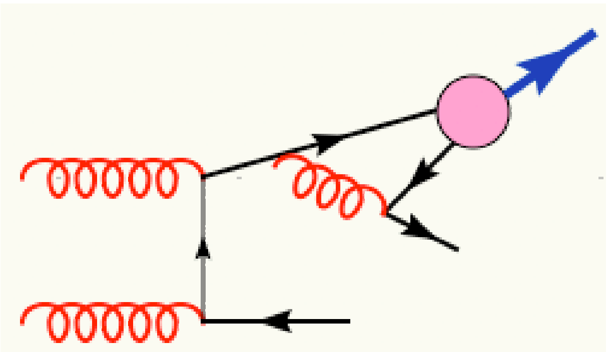


$$\sim \alpha_s^3 \frac{(2m_c)^4}{p_T^8}$$

Color-singlet NLO:

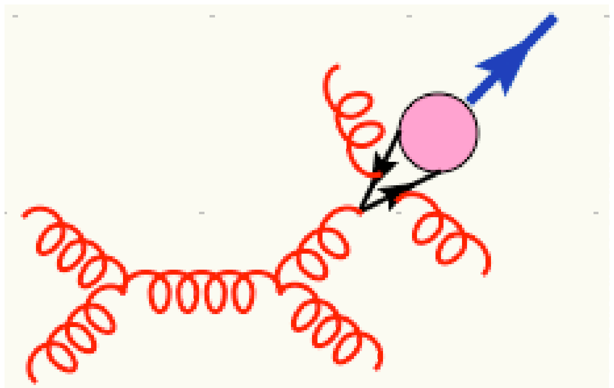


$$\sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6}$$



$$\sim \alpha_s^4 \frac{1}{p_T^4}$$

Color-singlet NNLO:



$$\sim \alpha_s^5 \frac{1}{p_T^4}$$

- Similar explanations account for the k factors in the color-octet channels.
  - The color-octet ${}^3S_1$ channel receives a small correction in NLO because if already has $1/p_T^4$ behavior in LO (gluon fragmentation).
  - The color-octet ${}^3P_J$ channel receives a large correction in NLO because it first shows $1/p_T^4$ behavior in NLO (gluon fragmentation).
  - The color-octet ${}^1S_0$ channel first shows $1/p_T^4$ behavior in NLO (gluon fragmentation). But the NLO correction is numerically small at moderate p_T because the fragmentation process has little support at $z \approx 1$.
- The perturbation series is expected to be stable beyond NLO for the color-octet channels. No further enhancement from the p_T behavior is possible.
- In LO, the $1/p_T^4$ behavior of the ${}^3S_1$ color-octet channel makes it dominant at large p_T.
- In NLO, all three color-octet channels can be important at large p_T.

The Fragmentation Approach of Kang, Qiu, and Sterman (2011)

- Provides a systematic way to understand and organize the p_T enhancements by writing the cross section in terms of
  - single-particle fragmentation functions times short-distance coefficients $(1/p_T^4)$,
  - two-particle $(Q\bar{Q})$ fragmentation functions times short-distance coefficients (m_c^2/p_T^6).
- Believed to hold to all orders in perturbation theory up to corrections of order m_c^4/p_T^8.
  - Holds independently of the validity of NRQCD factorization.
  - If NRQCD factorization holds, then the fragmentation functions can be written as a sum of NRQCD LDMEs times perturbatively calculable short-distance coefficients.
- Can be used to simplify higher-order corrections and resummation of logarithms for the most important processes at large p_T.
- Validity of the fragmentation picture confirmed for
  - the color-singlet NLO correction (Kang, Qiu, Sterman (2011)),
  - the ^3P_J color-octet NLO correction (GTB, Jungil Lee (in progress)).

Comparisons of NRQCD Factorization with Experiment

Summary

- NLO corrections have been computed for many quarkonium production processes:
  - J/ψ and $\psi(2S)$ cross sections at the Tevatron, RHIC, and the LHC;
  - J/ψ polarization, χ_{cJ} and $\Upsilon(1S)$cross sections at the Tevatron and the LHC;
  - J/ψ photoproduction cross sections and polarization at HERA;
  - $J/\psi + \eta_c$ production, $J/\psi + c\bar{c}$ production, and $J/\psi + X$(non-$c\bar{c}$) in e^+e^- annihilation at the B factories.
  - J/ψ production in $\gamma\gamma$ scattering at LEP II.
- Data and theory for charmonium production generally agree within errors.
  - See the global fit of NRQCD LDMEs of Butenschön and Kniehl (2011).
- There are three important exceptions:
  - Polarization of J/ψ at the Tevatron,
  - $J/\psi + X$(non-$c\bar{c}$) in e^+e^- annihilation at the B factories,
  - J/ψ production in $\gamma\gamma$ collisions at DELPHI.

First Complete NLO Calculations for Hadro-Production

(Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010))

- The results of the two groups for the short-distance coefficients agree.
- However, the fitted NRQCD LDMEs are very different.
- Using the CDF J/ψ data, Ma, Wang, and Chao were able fit only two linear combinations of LDMEs unambiguously:

$$M_{0,r_0} = \langle O^\psi({}^1S_0^{[8]})\rangle + (r_0/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (7.4 \pm 1.9) \times 10^{-2}\ \text{GeV}^3,$$
$$M_{1,r_1} = \langle O^\psi({}^3S_1^{[8]})\rangle + (r_1/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (0.05 \pm 0.02) \times 10^{-2}\ \text{GeV}^3.$$

  $r_0 = 3.9$ and $r_1 = -0.56$ chosen on the basis of approximate relations between the short-distance coefficients.

- Butenschön and Kniehl (2011) used their NLO calculations for ep, $\gamma\gamma$, and e^+e^- production to fit all three color-octet LDMEs, using data from the Tevatron, LHC, RHIC, HERA, LEP II, KEKB:

$$\langle O^\psi({}^1S_0^{[8]})\rangle = (4.76 \pm 0.06) \times 10^{-2}\ \text{GeV}^3,$$
$$\langle O^\psi({}^3S_1^{[8]})\rangle = (0.265 \pm 0.014) \times 10^{-2}\ \text{GeV}^3,$$
$$\langle O^\psi({}^3P_0^{[8]})\rangle/m_c^2 = (-0.716 \pm 0.089) \times 10^{-2}\ \text{GeV}^3,$$

  which implies that

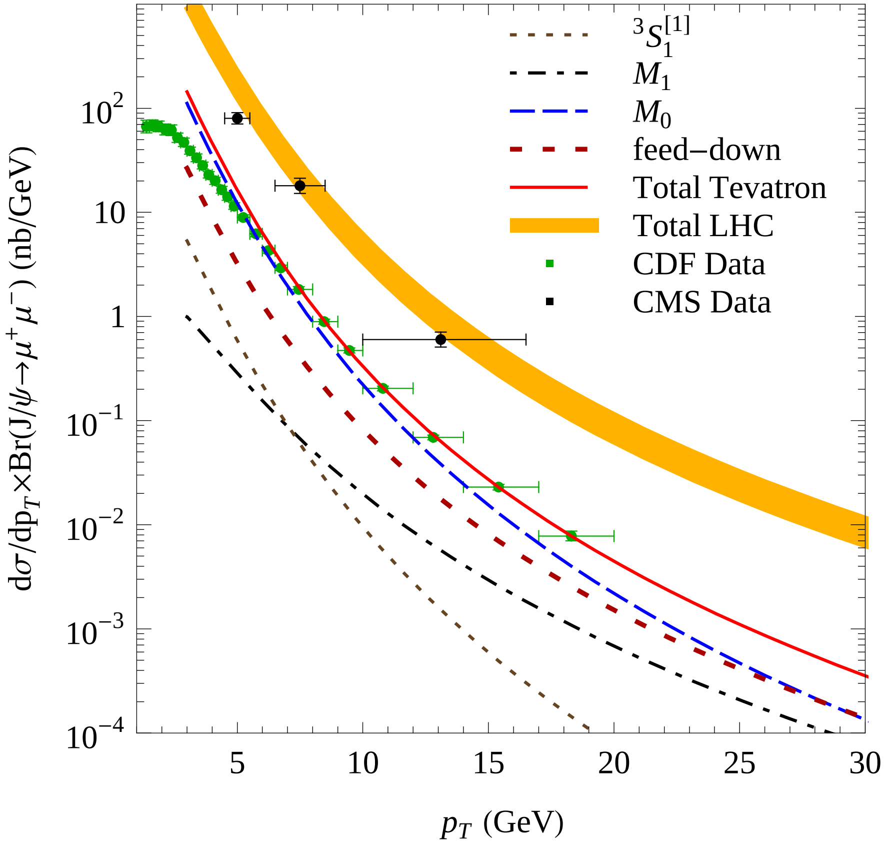
$$M_{0,r_0} = (2.17 \pm 0.56) \times 10^{-2}\ \text{GeV}^3,$$
$$M_{1,r_1} = (0.62 \pm 0.08) \times 10^{-2}\ \text{GeV}^3.$$

- There are many small differences in the fitting procedures.
  - An effect of about 30% from inclusion of feeddown from $\psi(2S)$ and χ_{cJ} states (Ma, Wang, and Chao),
  - Use of 2-parameter constrained fits (Ma, Wang, and Chao),
  - Different p_T cuts:
    Ma, Wang, and Chao: $p_T > 7$ GeV;
    Butenschön and Kniehl: $p_T > 3$ GeV for hadroproduction
    ($p_T > 1$ GeV for photoproduction and two-photon production.)
- The most important difference is the use of HERA (H1 (2002, 2005)) data by Butenschön and Kniehl.
  - Their fit to the Tevatron and HERA data alone gives a result that is very similar to that of the global fit:

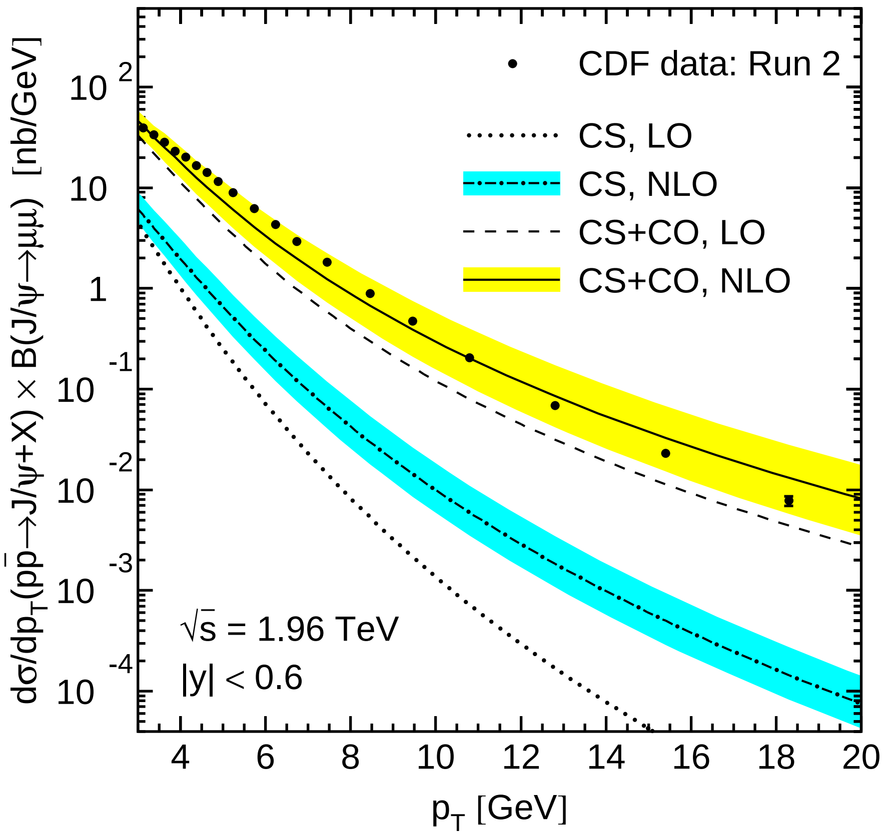
$$\begin{aligned} M_{0,r_0} &= (2.5 \pm 0.08) \times 10^{-2}\ \text{GeV}^3, \\ M_{1,r_1} &= (0.59 \pm 0.02) \times 10^{-2}\ \text{GeV}^3. \end{aligned}$$

    Can be used to predict the LHC data.
  - Most of the HERA data lies at $p_T \lesssim 3$ GeV.
  - Does factorization hold at such low values of p_T?

- Both predictions fit the data within errors, but the shape of the Ma, Wang, and Chao fit agrees with the CDF data better than the shape of the Butenschön and Kniehl (2011) global fit.

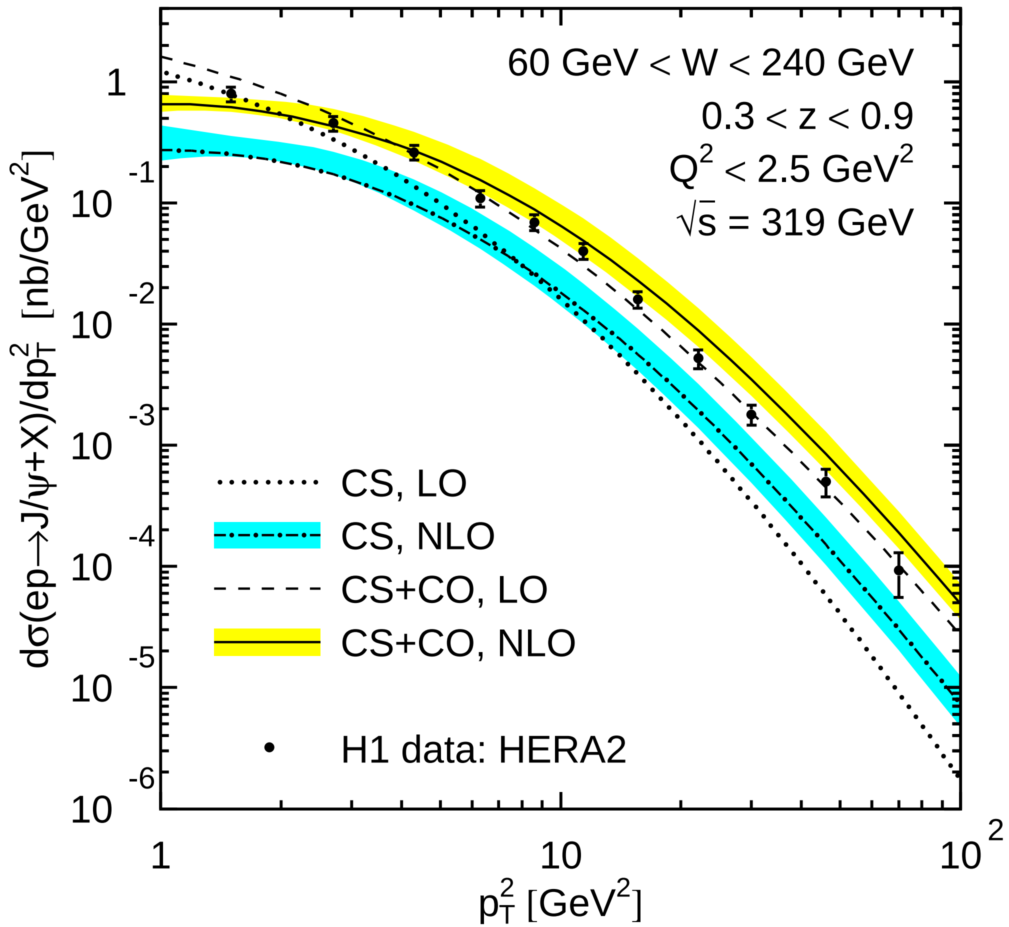


Ma, Wang, and Chao



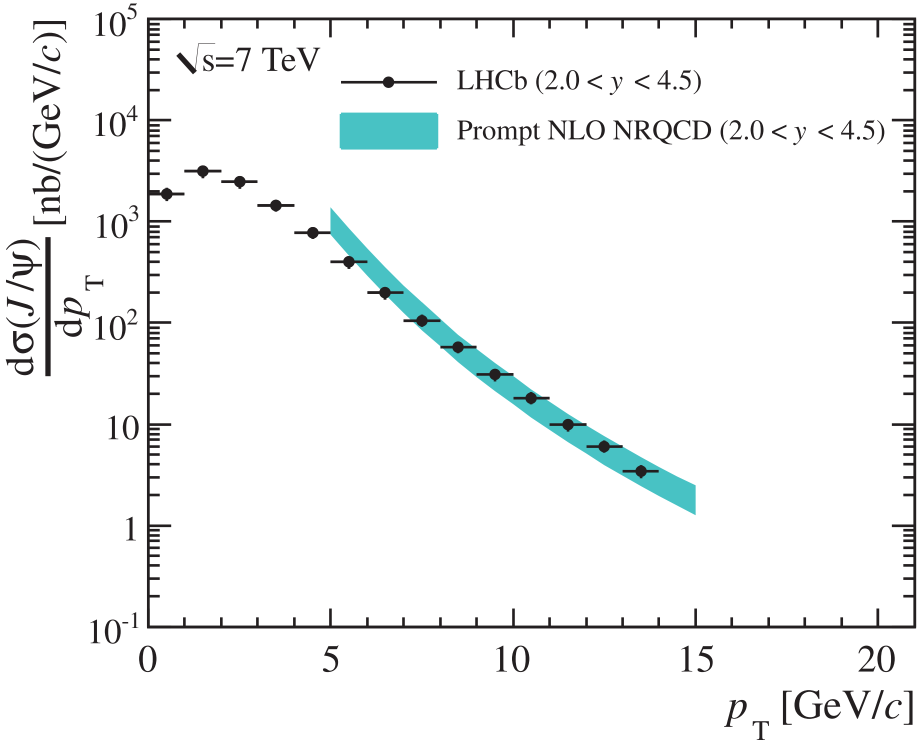
Butenschön and Kniehl

- At low p_T, does factorization break down?
- At high p_T, resummation of large logs of p_T^2/m_c^2 is needed.

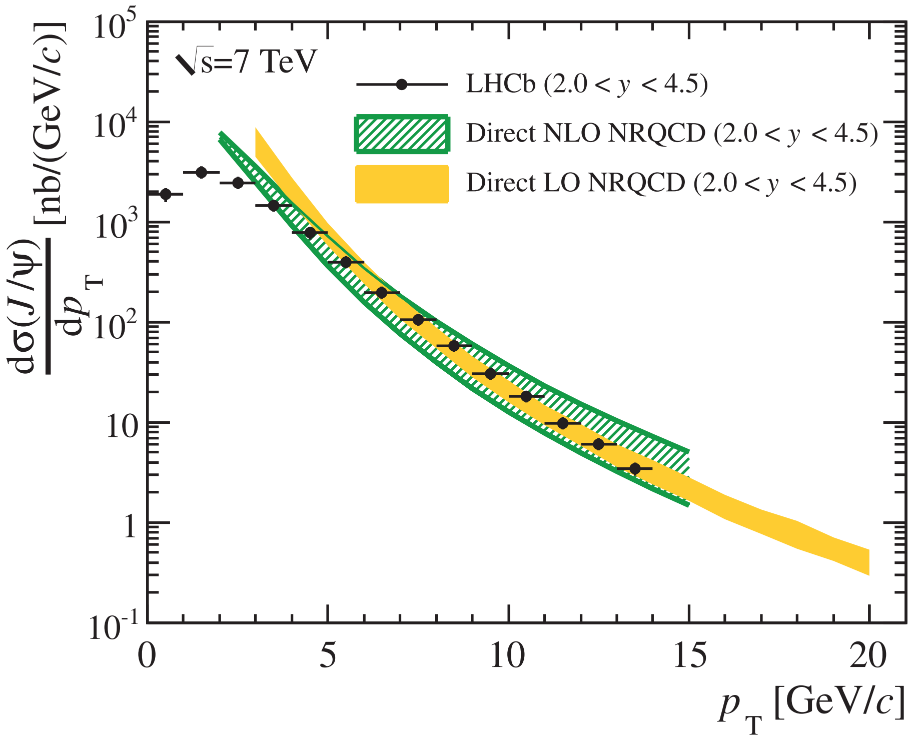


There is a slight discrepancy in shape between the Butenschön and Kniehl (2011) global fit and the H1 data.

- There is also a slight discrepancy in shape between the LHCb data and the Butenschön and Kniehl (2010) NLO fit to the Tevatron and HERA data.

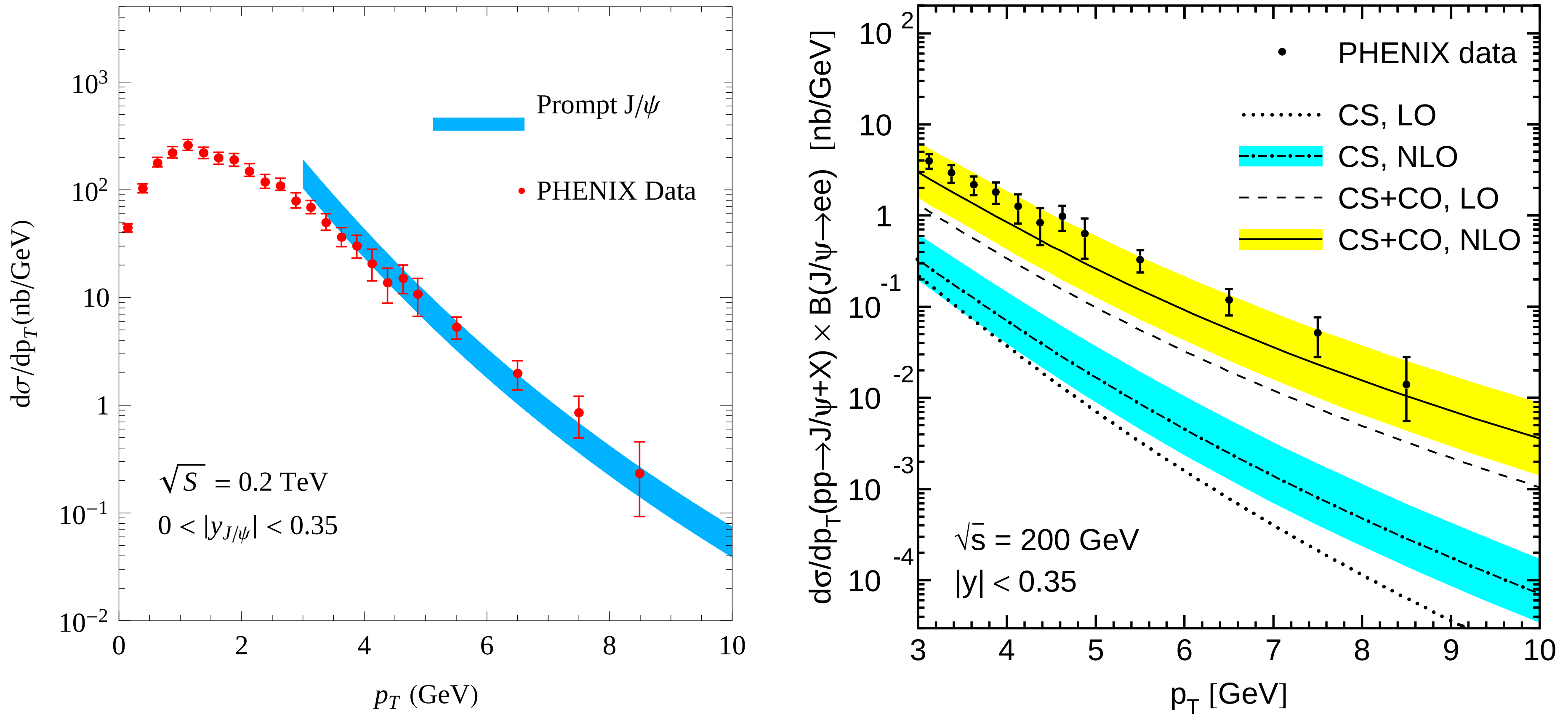


Ma, Wang, and Chao



Butenschön and Kniehl

- The Butenschön and Kniehl (2011) global fit lies slightly below the PHENIX (2011) data at high p_T.

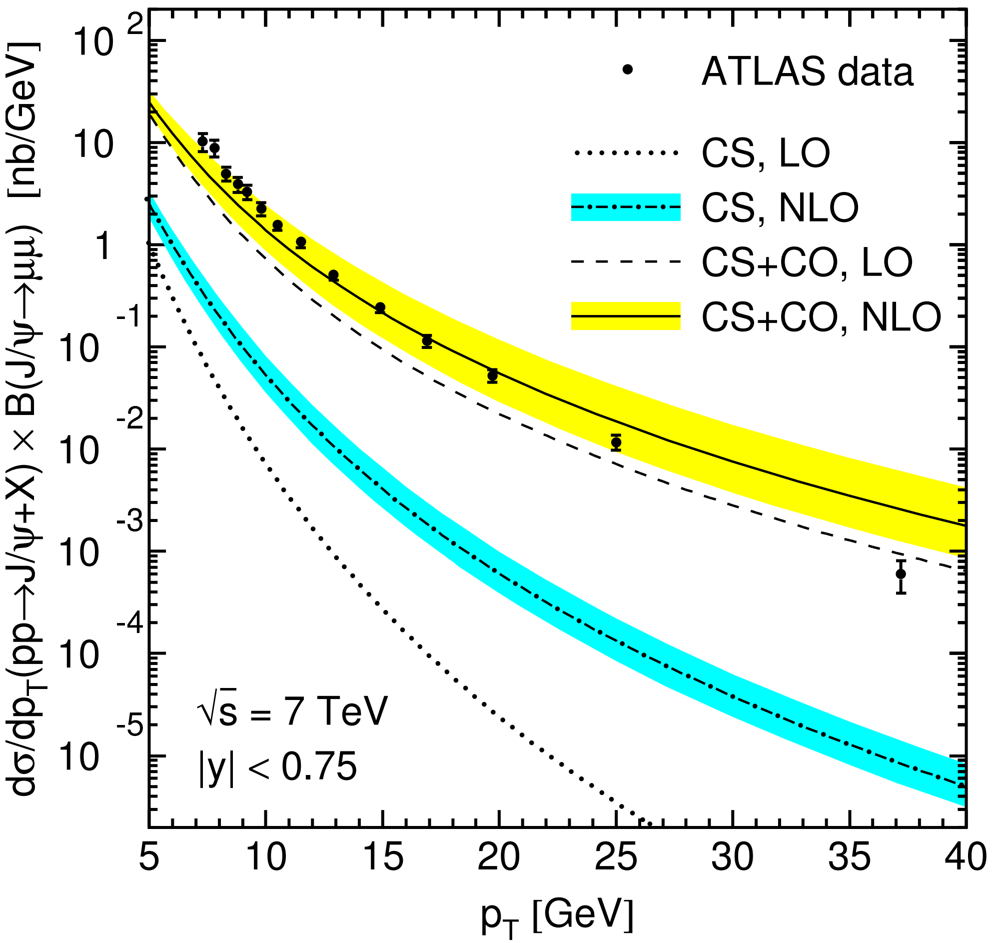


Ma, Wang, and Chao

Butenschön and Kniehl

- The fit and the prediction include feeddown.
- Feeddown is not included in the fit to the CDF data or in the prediction for PHENIX.

- The shape discrepancy between the Butenschön and Kniehl prediction and the data becomes more apparent at high p_T.



- ATLAS (2011) data.
- Not included in Butenschön and Kniehl global fit.

- Resummation of large logs of p_T^2/m_c^2 is needed at high p_T.
- The sign and magnitude of the resummation effect are consistent with the discrepancy.
- Resummation might make the shapes of the Ma, Wang, and Chao fit and predictions worse.

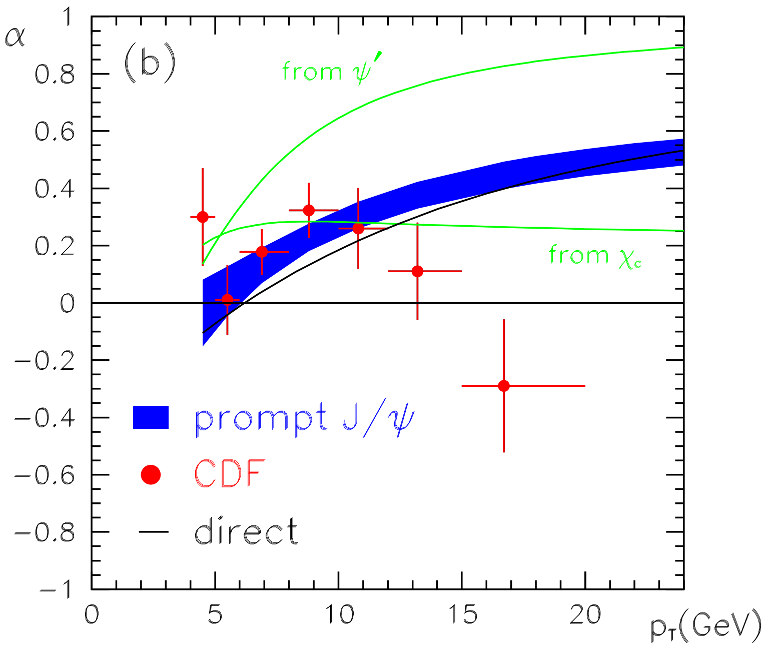
Polarization

Polarization in Leading Order

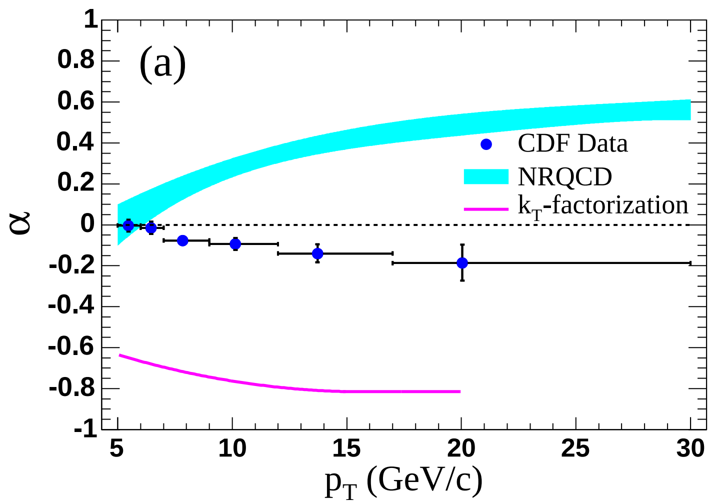
- In LO quarkonium production at large p_T ($p_T \gtrsim 4m_c$ for J/ψ), gluon fragmentation via the color-octet $^3S_1$ channel dominates.
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, most of the gluon's polarization is transferred to the quarkonium (Cho, Wise (1994)).
  - Spin-flip interactions are suppressed as v^3.
  - Verified in a lattice calculation of NRQCD decay LDMEs (GTB, Lee, Sinclair (2005)).

J/ψ Polarization in LO

Run I:



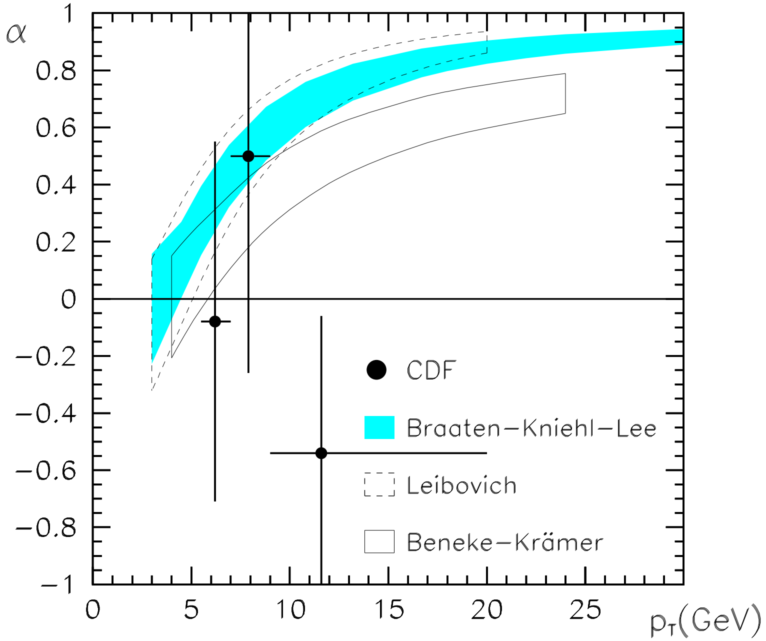
Run II:



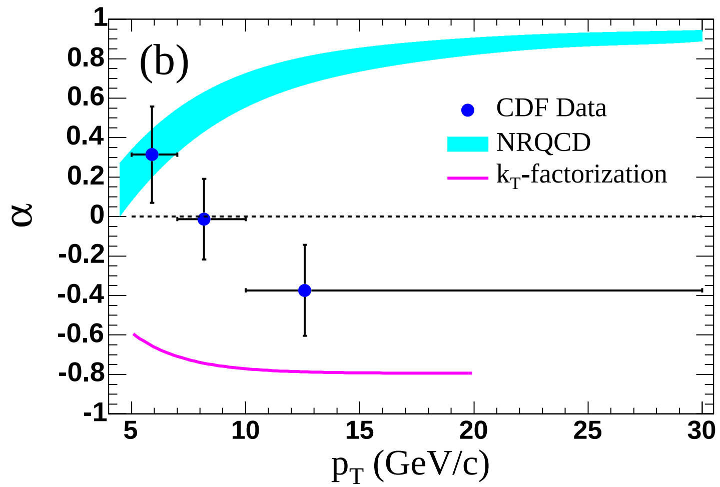
- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- NRQCD prediction: Braaten, Kniehl, Lee (1999).
- Feeddown from χ_c states ($\approx 30\%$) and $\psi(2S)$ ($\approx 10\%$) included in the theory and the data.
- Run I results are marginally compatible with the NRQCD prediction.
- The Run II results are inconsistent with the NRQCD prediction.
- The Run II results are also inconsistent with the Run I results.
  CDF was unable to track down the source of the Run I-Run II discrepancy.

$\psi(2S)$ Polarization in LO

Run: I



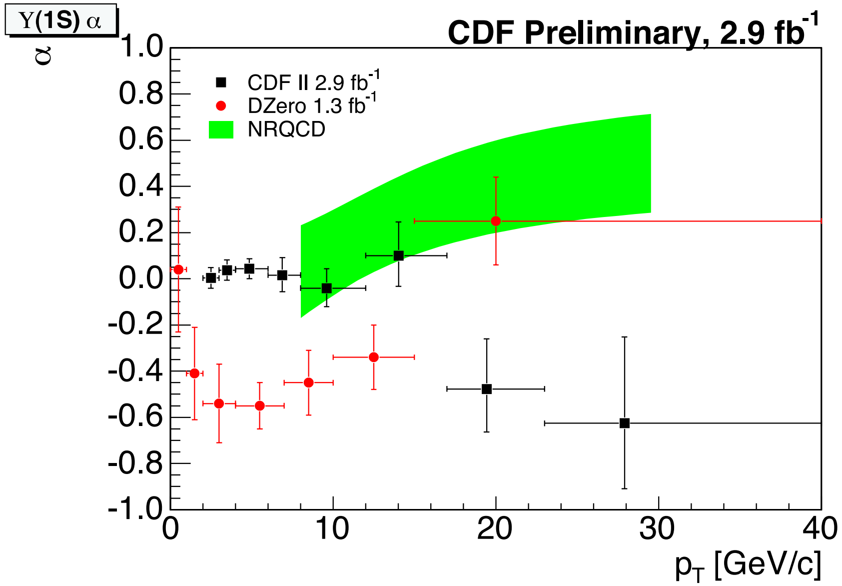
Run: II



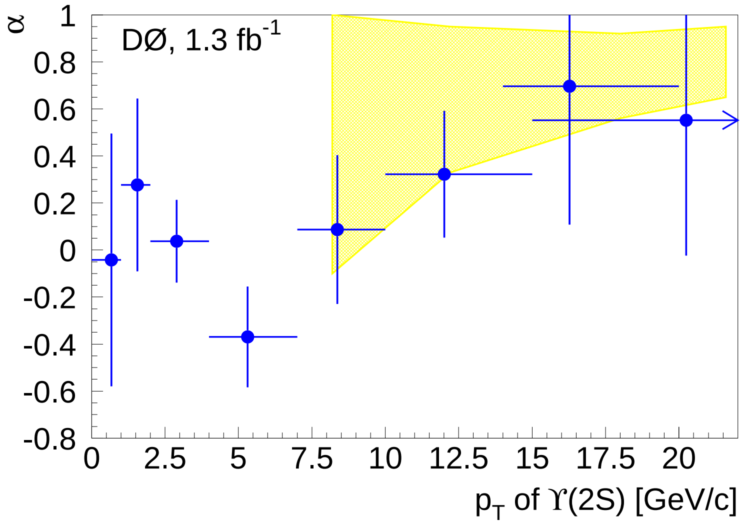
- The Run II data are incompatible with the LO NRQCD prediction.

Υ Polarization in LO

$\Upsilon(1S)$ Polarization:



$\Upsilon(2S)$ Polarization:



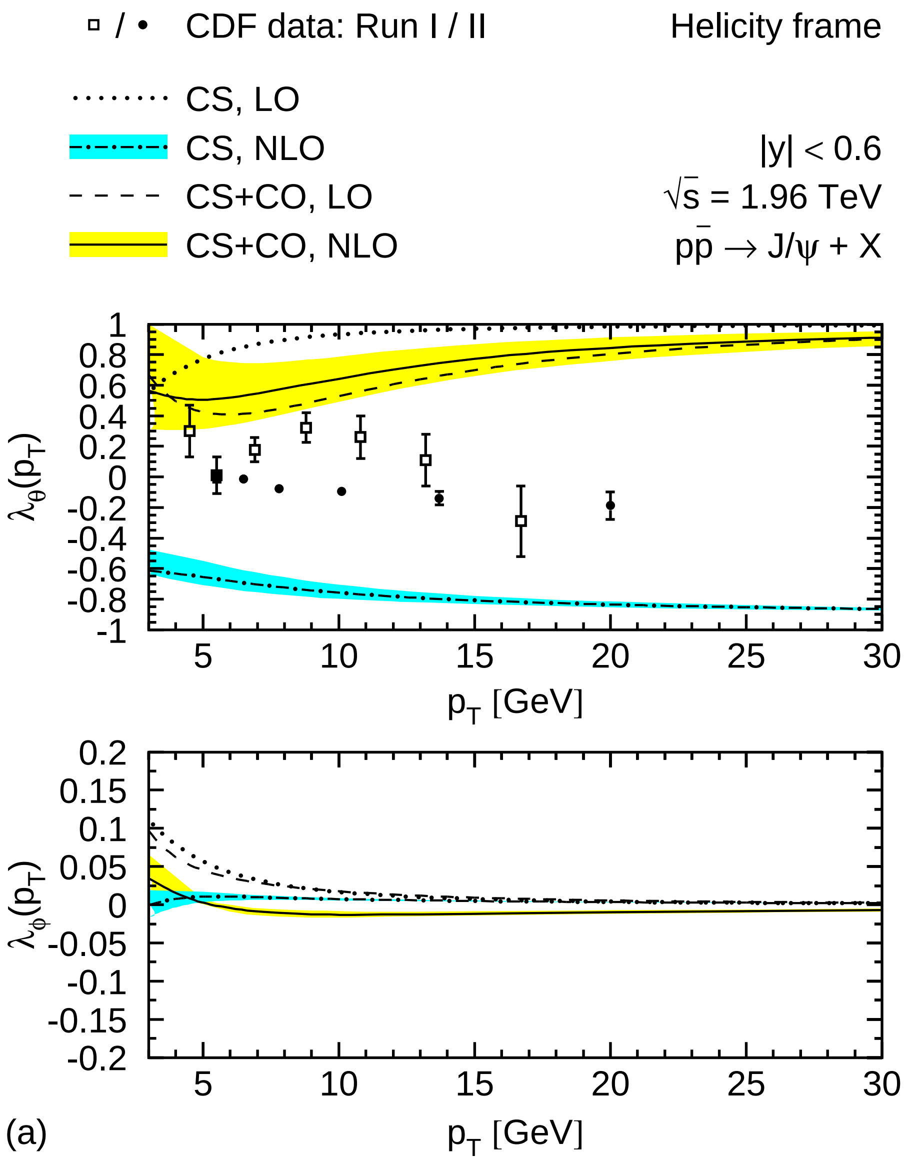
- In the $\Upsilon(1S)$ case, the D0 results (red) are incompatible with the CDF results (black).
- Both the CDF and D0 results are incompatible with the LO NRQCD prediction of Braaten and Lee (2000) (green), but in different regions of p_T.
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

First Complete NLO Calculations of Polarization

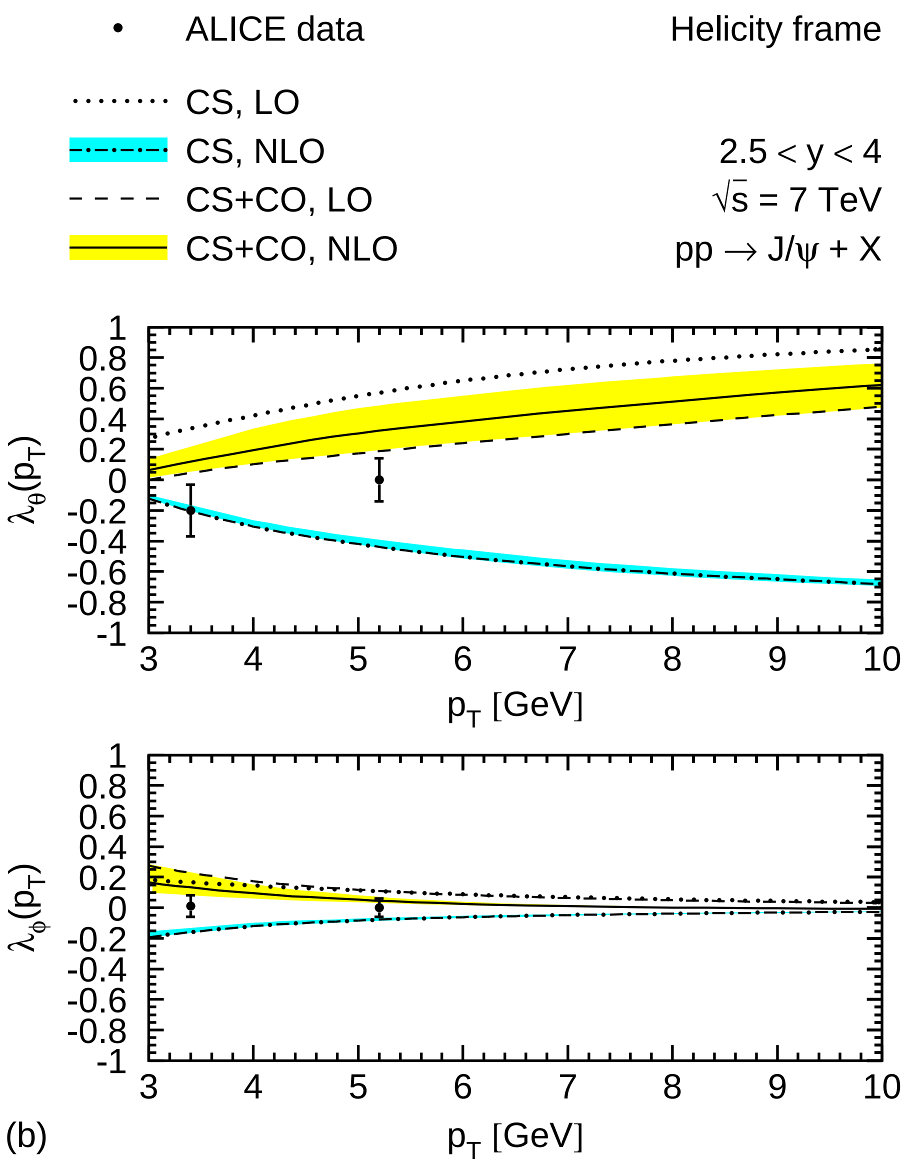
- Photoproduction at HERA: Butenschön and Kniehl (2011)
- Hadroproduction: Chao, Ma, Shao, Wang, Zhang (2012); Butenschön and Kniehl (2012)
  - Calculation for the ${}^3S_1$ color-singlet channel by Gong and Wang (2008) and Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
  - Calculations for the ${}^3S_1$ and ${}^1S_0$ color-octet channels by Gong, Li, Wang (2008) and Gong, Wang, and Zhang (2010)
- In NLO, there are large corrections to the ${}^3P_J$ color-octet channel at high p_T.
  - At high p_T, the ${}^3P_J$ color-octet channel is mostly transversely polarized.
- In NLO, the ${}^3S_1$ color-octet channel is no longer dominant at high p_T and is not the only source of transverse polarization.

Prediction Using LDMEs from the Global Fit of Butenschön and Kniehl (2011)

- The ${}^3P_J$ and ${}^3S_1$ color-octet channels add to produce substantial polarization at high p_T.

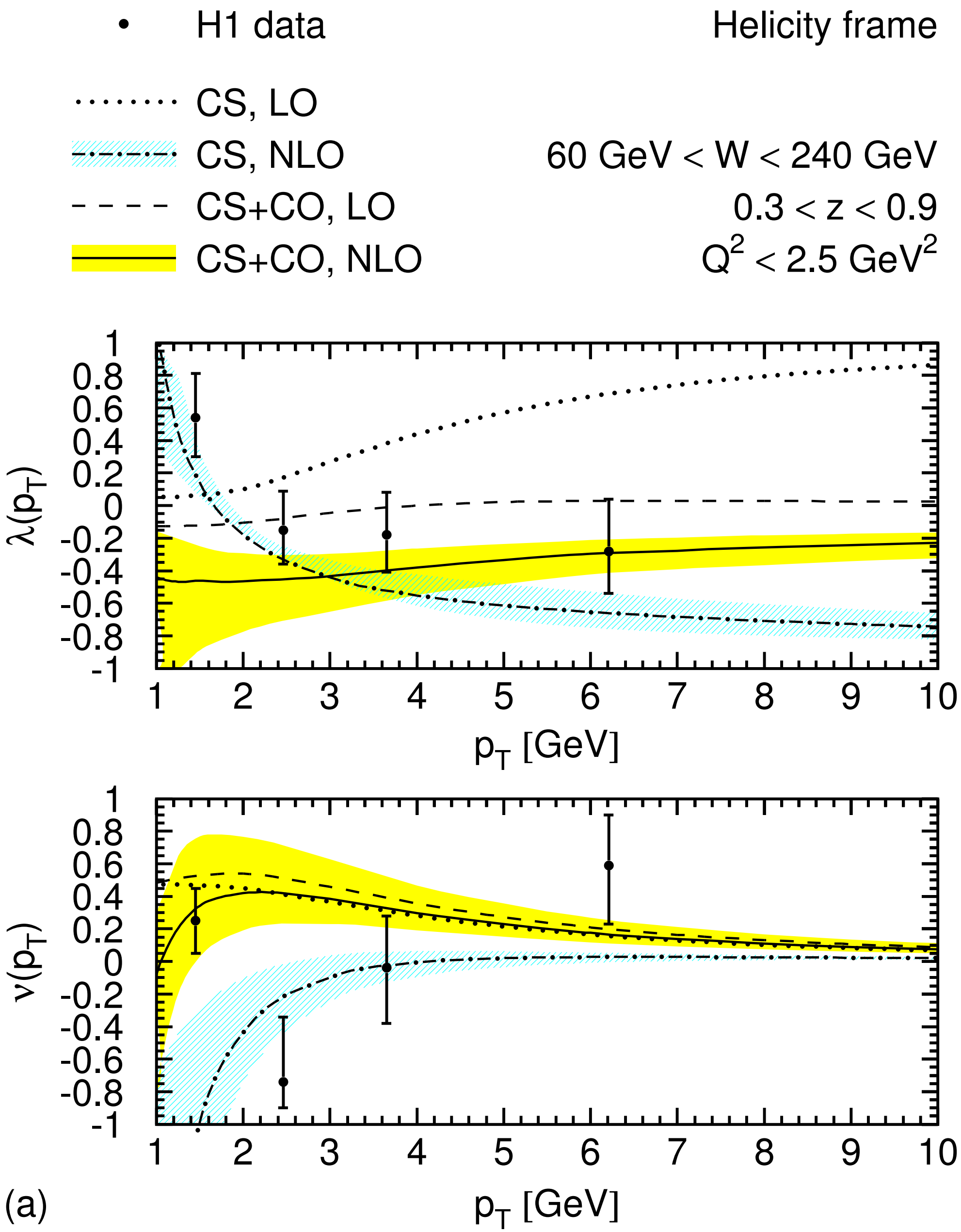


- The prediction from the Butenschön and Kniehl (2011) global fit is in disagreement with the CDF data.



- The prediction from the Butenschön and Kniehl (2011) global fit is in agreement with the ALICE (2012) data.
- But the theory is for direct production, while the ALICE data includes production in B-meson decays and feeddown from χ_{cJ} states and the $\psi(2S)$.

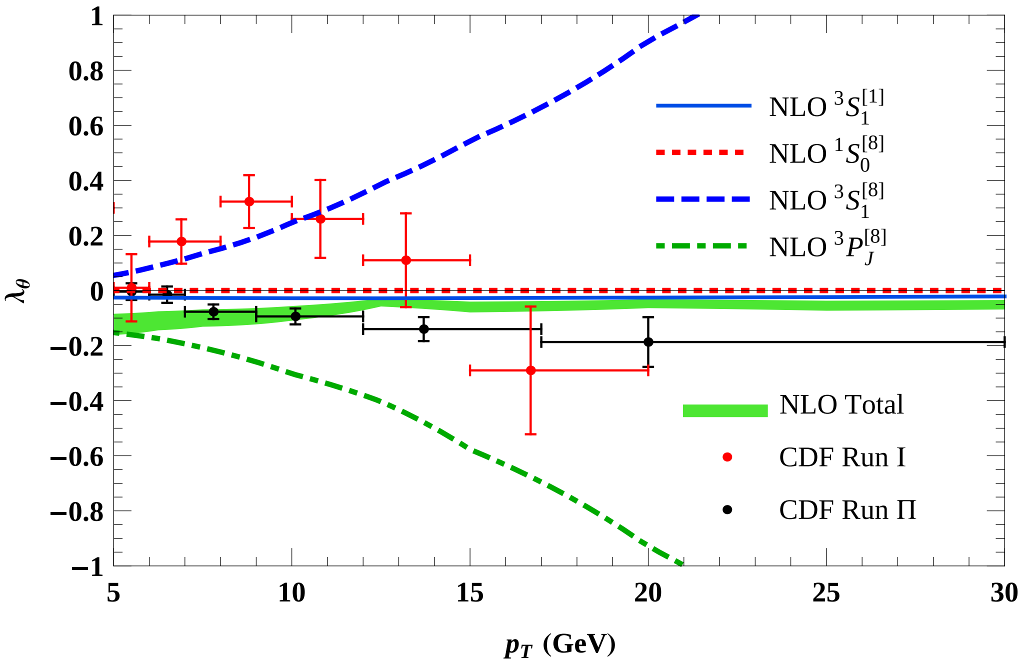
- The Butenschön and Kniehl (2011) global fit can also be used to predict the polarization in inelastic J/ψ photoproduction at HERA.



- The data are roughly compatible with the theory at large p_T, but the error bars are large.

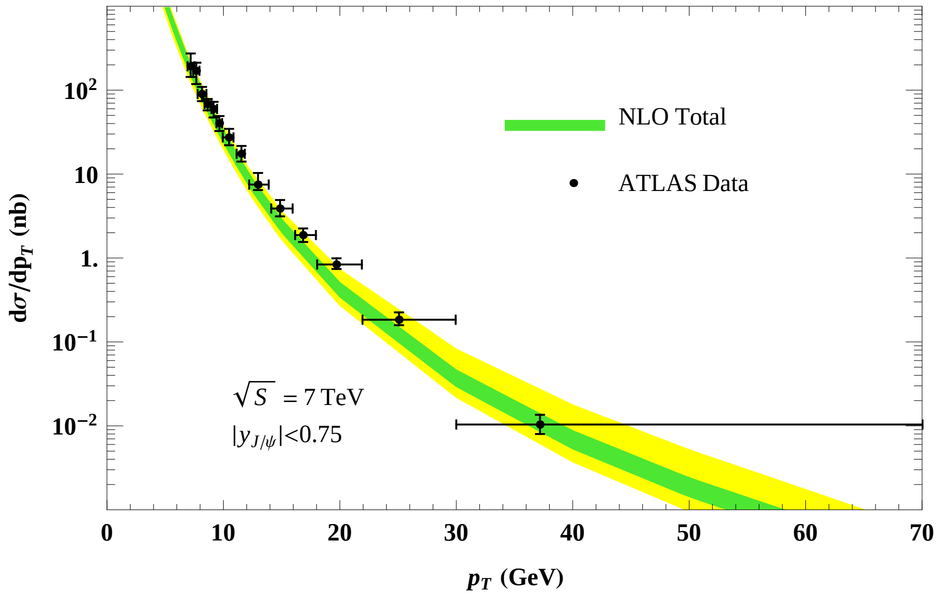
The Fit of Chao, Ma, Shao, Wang, and Zhang (2012)

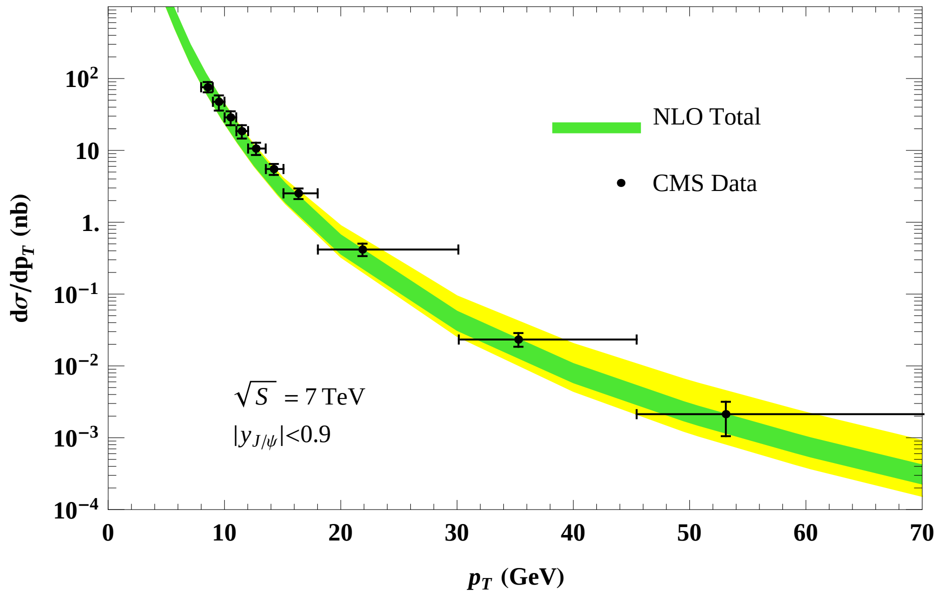
- Fix all three LDMEs by using the CDF Run II measurements of $d\sigma/dp_T$ and polarization for the J/ψ.
- The resulting LDMEs are compatible with the LDME constraints from the Ma, Wang, and Chao (2010) fit to $d\sigma/dp_T$ alone.

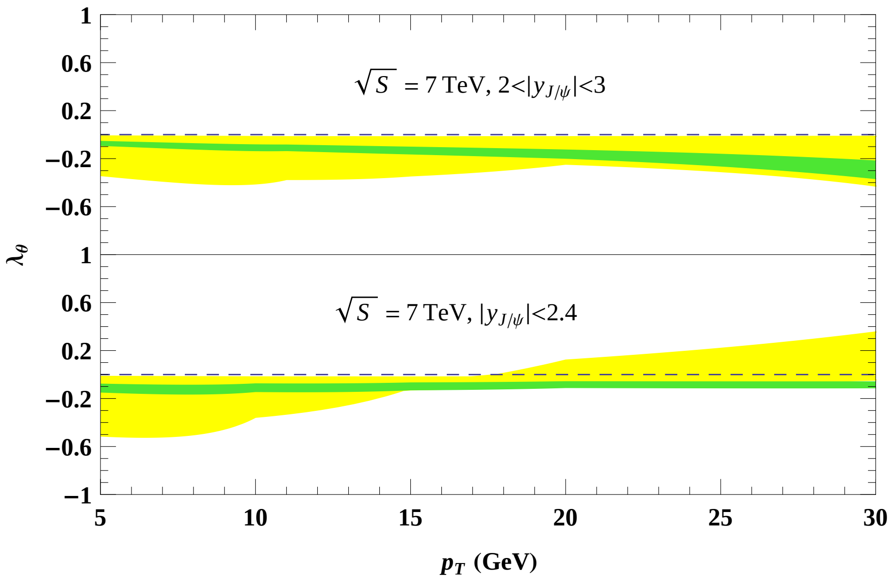


- In this fit, the color-octet $^3S_1$ and ^3P_J contributions to the transverse polarization largely cancel.

- The Chao et al. (2012) LDMEs still give reasonable predictions for the LHC p_T spectra.







- The Chao et al. (2012) LDMEs predict a slightly longitudinal polarization at the LHC.

- However, the Chao et al. (2012) LDMEs seem to be incompatible with the HERA data, even at high p_T.

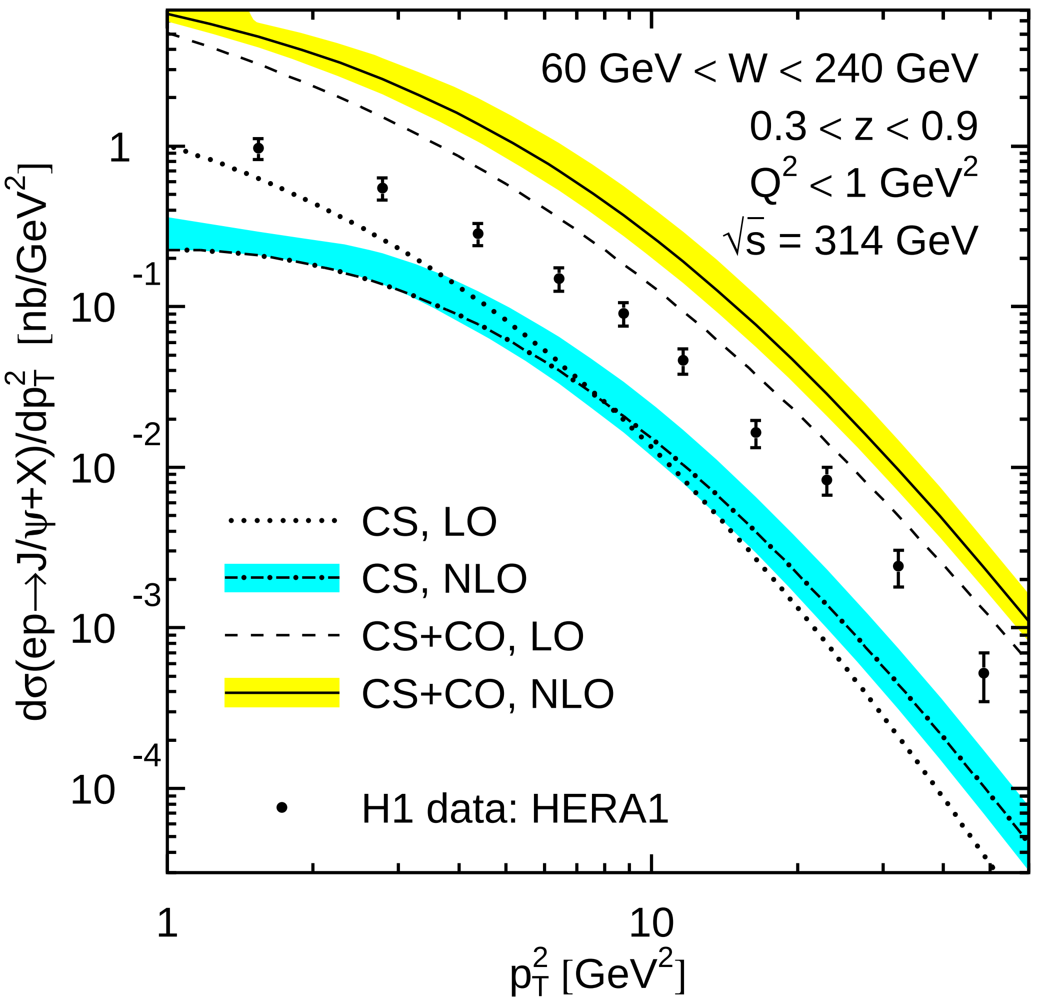


Figure courtesy of Mathias Butenschön.

$e^+e^- \to J/\psi + X(\text{non-}c\bar{c})$

- Belle (2009):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.43 \pm 0.09 \pm 0.09 \text{ pb}.$$

- NLO calculation (Zhang, Ma, Wang, Chao (2009), Butenschön and Kniehl (2011)):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.99^{+0.35}_{-0.17} \text{ pb} \qquad (\mu = \sqrt{s}/2).$$

  - NRQCD LDMEs from the Butenschön-Kniehl (2011) global fit.
  - Includes feeddown estimate of 0.29 pb from Zhang, Ma, Wang, Chao (2009).

- The comparison with the Belle data favors the Butenschön-Kniehl value of M_{0,r_0}.

Comments

- The most recent Belle (2009) measurements give

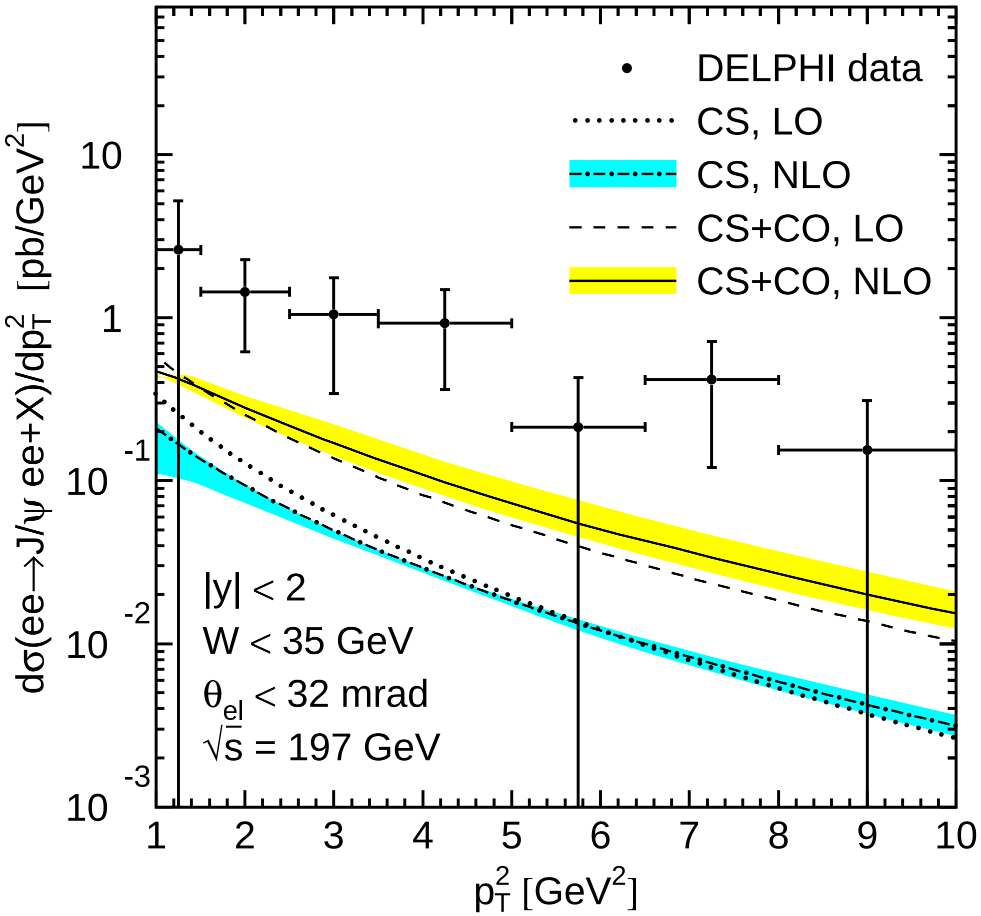
$$\sigma(e^+e^- \to J/\psi+X) = \sigma(e^+e^- \to J/\psi+c\bar{c}+X)+\sigma(e^+e^- \to J/\psi+X(\text{non-}c\bar{c})) = 1.17\pm0.12^{+0.13}_{-0.12} \text{ pb}.$$

- However, BaBar (2001) obtained

$$\sigma(e^+e^- \to J/\psi + X) = 2.52 \pm 0.21 \pm 0.21 \text{ pb}.$$

- Most of the data are at $p_T \lesssim 3$ GeV. Does factorization hold at such small values of p_T?

J/ψ Production in $\gamma\gamma$ Scattering at LEP II



- The DELPHI (2003) data are slightly incompatible with the prediction of the Butenschön and Kniehl (2011) global fit.
- The error bars are large, especially at high p_T.
- Factorization may not hold at such low values of p_T.

Conclusions

- NRQCD factorization provides a systematic framework for computing quarkonium production, but it is unproven beyond two loops.
- Large NLO corrections to inclusive quarkonium production are now believed to be understood.
  - For the color-octet channels, which are believed to be much more important than the color-singlet channels, there should be no large kinematic enhancements beyond NLO.
  - The NLO calculations should at least be correct qualitatively.
  - Resummation of large logs may be needed at high p_T.
- The predictions of the color-singlet model in NLO fail to describe the data.

- The predictions of NRQCD factorization are in agreement with most of the inclusive production data.

  Exceptions:

  - J/ψ and Υ polarization at the Tevatron (experimental issues)
  - $e^+e^- \to J/\psi + X(\text{non} - c\bar{c})$ (experimental issues)
  - J/ψ production in $\gamma\gamma$ scattering at LEP II (large error bars, low p_T)

- The measured J/ψ cross sections at the Tevatron and HERA, the measured J/ψ polarization at the Tevatron, and NRQCD factorization seem to be incompatible.

- Some possibilities:

  - The Run II J/ψ polarization measurement is wrong.
  - NRQCD factorization fails at the low values of p_T measured at HERA.
  - Additional corrections to the theory predictions are needed. Higher orders in v^2?
  - NRQCD factorization is incorrect.

- What we need from theory
  - Proof or disproof of NRQCD factorization
  - Higher-order corrections and resummation of large logs of p_T^2/m_c^2 for the production mechanisms that are dominant at high p_T
  - Calculations for additional processes (see below)
- What we need from experiment
  - J/ψ direct production cross sections at high p_T
  - all three J/ψ polarization parameters in different frames for direct production at high p_T
  - χ_{cJ} cross sections and polarizations at high p_T
    (Only two LDMEs enter at leading order in v^2.)
  - Υ direct production cross sections and polarization at high p_T
    (Lower value of v^2 means that LDMEs have different relative sizes.)
  - Measurements of additional high-p_T processes
    $J/\psi +$ jet cross section at high p_T?
- Measurements of polarizations and cross sections at high p_T at the LHC will be crucial in understanding quarkonium production mechanisms.

Backup Slides