# Dominant $1/m_c$ Contribution to $\Delta M_D$ in $D^0 - \bar{D}^0$ Mixing

Gagik Yeghiyan

Grand Valley State University, Allendale, MI

In collaboration with

E. Golowich and A.A. Petrov

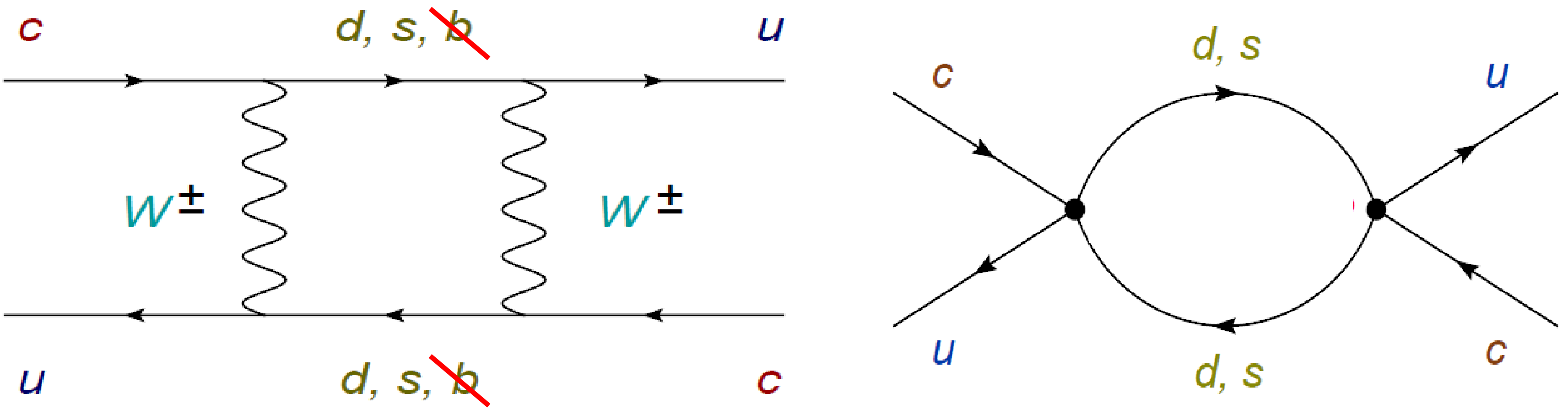
Charm 2012, Honolulu, Hawaii

A work in progress

(preliminary results)

# $D^0 - \overline{D^0}$ within the SM

**To the lowest order in perturbation theory:**




**One usually neglects the loops with b-quarks as**
**$|V_{cb}^{*} V_{ub}| << |V_{cs}^{*} V_{us}| \approx |V_{cd}^{*} V_{ud}|$**

**Within the SM $D^0 - \overline{D^0}$ mixing occurs by means of two consecutive (effective) $|\Delta C| = 1$ transitions.**

# $D^0 - \overline{D}^0$: Quantum Mechanical Description

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

$$M_{11} = M_{22} \text{ and } \Gamma_{11} = \Gamma_{22}$$

$$M_{12} = M_{21}^{\star} \text{ and } \Gamma_{12} = \Gamma_{21}^{\star}.$$

Non-diagonal elements control the mixing:

SM

$$M_{12} - \frac{i}{2}\Gamma_{12} = \frac{1}{2M_D}\langle\overline{D}^0|H_W^{\Delta C=2}|D^0\rangle + \frac{1}{2M_D}\sum_n \frac{\langle\overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle}{M_D - E_n + i\epsilon}$$

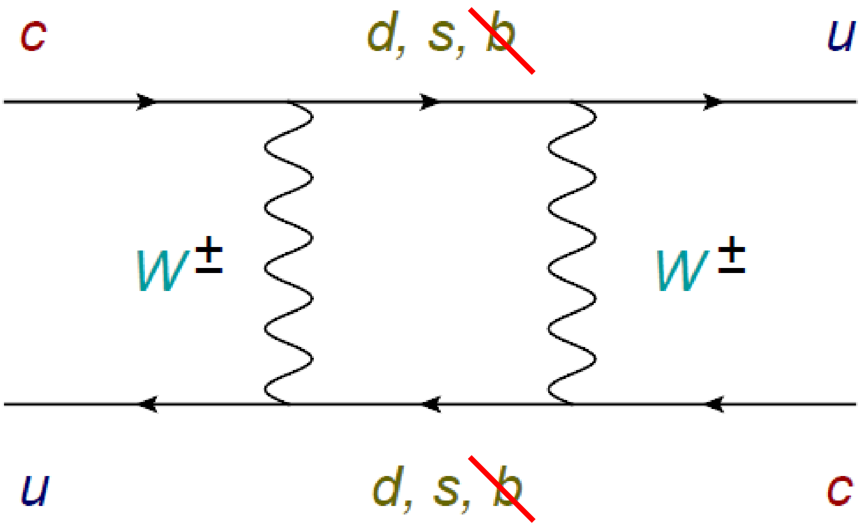
In the limit of CP-conservation:

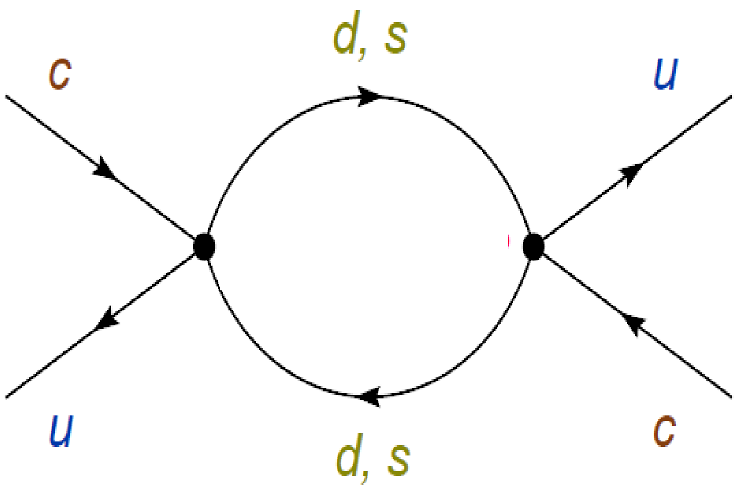
$$\Delta M \equiv M_+ - M_- = 2M_{12}$$

$$\Delta\Gamma \equiv \Gamma_+ - \Gamma_- = 2\Gamma_{12}$$

$$D_{\pm} = \frac{1}{\sqrt{2}}\left(D^0 \pm \overline{D}^0\right)$$

# $D^0 - \overline{D^0}$ Within the SM to the LO in Perturbation Theory




$\Delta M_D$ and $\Delta\Gamma_D$ vanish in the limit of exact SU(3) flavor symmetry.

In the real world flavor SU(3) is broken, so $\Delta M_D \neq 0$ and $\Delta\Gamma_D \neq 0$ however they are suppressed in powers of $m_s$ / $m_c$

$$x_D^{LO} \equiv \frac{\Delta M_D^{LO}}{\Gamma_D} = \frac{G_F^2 m_c^2}{3\pi^2} \frac{f_D^2 M_D}{\Gamma_D} \left( \frac{m_s^4}{m_c^4} \right) \sin^2\theta_C \cos^2\theta_C \left[ \frac{5}{4} \left( C_2^2 - \right.\right.$$

$$\left.\left. - 2C_1 C_2 - 3C_1^2 \right) \overline{B}_D^{(S)} - C_2^2 B_D \right] \approx 1.7 \times 10^{-6}$$

**HFAG**

$x_D^{exp} = (0.65 \pm 0.19)\%$

## The existing experimental data

$$x_D \equiv \Delta M_D / \Gamma_D = (0.65 \pm 0.19)\%$$

$$y_D \equiv \Delta \Gamma_D / (2\, \Gamma_D) = (0.74 \pm 0.12)\%$$

may be explained by

- (Short-distance) New Physics contribution to $\Delta M_D$ and (in certain SM extensions only) to $\Delta\Gamma_D$
- Long – distance SM contribution to $\Delta M_D$ and to $\Delta\Gamma_D$

# Long-Distance Contribution to $D^0 - \overline{D}^0$ Mixing

- Contribution to $D^0 - \overline{D}^0$ from exclusive channels, like

  $D^0 \to \pi\,\pi,\ K\,\pi,\ K\,K,\ \text{etc.} \to \overline{D}^0$

A. F. Falk et al., Phys. Rev. D 69, 114021 (2004),

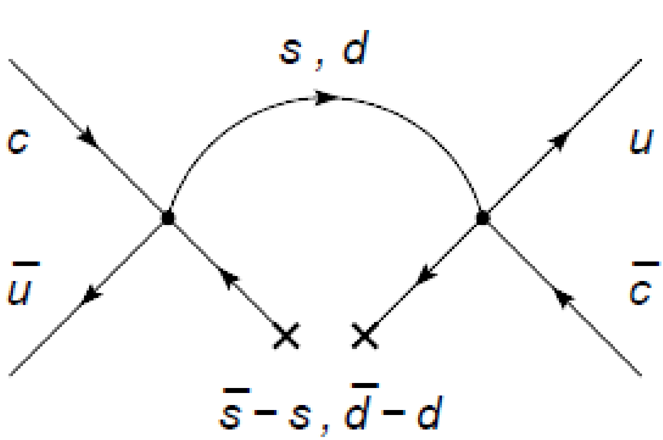
A.F. Falk et al., Phys. Rev. D 65, 054034 (2002):

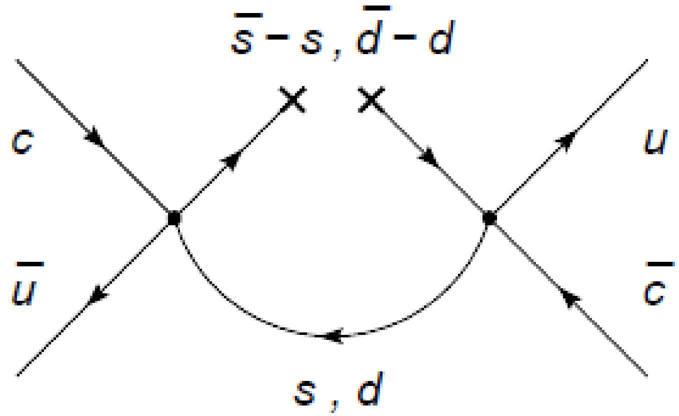
$x_{D,}\ y_D \sim \sin^2\theta_C\ (\text{flavor SU(3) breaking})^2 \sim 1\%$

- Contribution to $D^0 - \overline{D}^0$ from higher order terms in $1/m_c$ OPE (the inclusive approach)

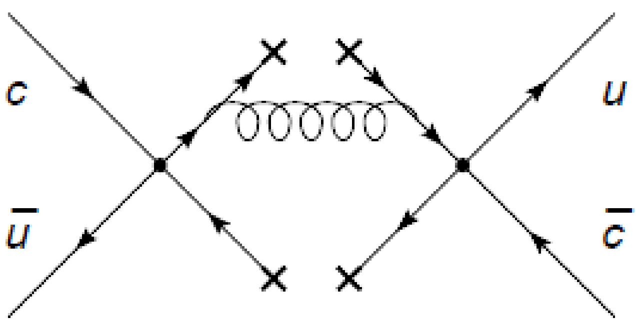
  - In particular from the terms corresponding to the diagrams containing low-energy intermediate down-type quark states (quark-antiquark condensates) or the diagrams with “hanging” quarks

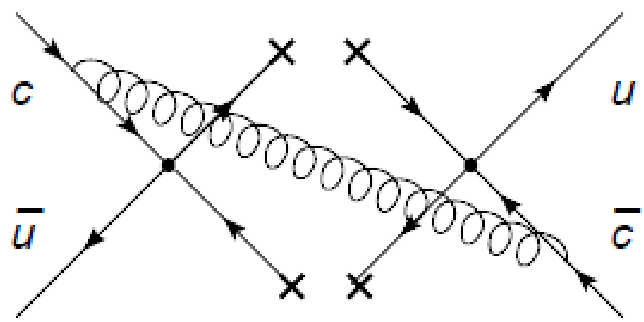
# Diagrams with "Hanging Quarks"




**Yields matrix elements of d=9 operators – $1/m_c^3$ terms in OPE**




\+ 14 other diagrams

**Yields matrix elements of d=12 operators – $1/m_c^6$ terms in OPE**

# Propagators vs. Hanging Quark Lines

H. Georgi, Phys. Lett.B297, 353 (1992):

- The U-spin symmetric structure of $\Delta C = 1$ weak effective Hamiltonian enforces the $D^0 - \bar{D}^0$ oscillation amplitude to vanish if any of the intermediate light quark states is assumed to be massless.

- Mass insertion in each propagator produces a factor $m_s^2 - m_d^2 \approx m_s^2$

- Mass insertion in each non-perturbative itermediate light quark-antiquark state produces a factor $m_s - m_d \approx m_s$

- Expect softer flavor SU(3) suppression (or softer GIM cancellations) in diagrams with hanging quarks!

# "The Rule of Thumb"

I.Bigi, N. Uraltsev, Nucl. Phys. B592, 92 (2001):

- **Cutting a quark line, we pay the price of a power suppression $\sim\mu_{had}^3/m_c^3$. Yet GIM (or flavor SU(3)) suppression (in this fermion line) is $m_s/\mu_{had}$ and there is no loop factor. Altogether we have the enhancement**

  $\sim 4\,\pi^2\,\mu_{had}^2/(m_s\,m_c) \sim 170 \Rightarrow (x_D)^{d=9} \sim 10^{-4}$

- **Cutting two quark lines… do the same math, but instead of the second loop factor $4\,\pi^2$ we have $4\,\pi\,\alpha_s$ – one must add a gluon to transfer a large momentum. Altogether, the enhancement compared to the LO**

  $\sim 4\,\pi^2\,4\,\pi\,\alpha_s\,\mu_{had}^4/(m_s^2\,m_c^2) \sim 3500 \Rightarrow (x_D)^{d=12} \sim \text{few} \times 10^{-3}$

- **May also work for $y_D$ but with caution: we are back to loop level ("dress" the diagrams by gluons) – $y_D \neq 0$ if only diagrams have an absorptive part.**

**More about $y_D$ - Bobrowski, Lenz (multiple talks)**

# The Purpose of Our Work

**To verify quantitatively the estimates of Bigi and Uraltsev (and other authors) for the (normalized) mass difference in $D^0 - \bar{D}^0$ mixing, $x_D = \Delta M_D / \Gamma_D$**

**Motivation: Calculation of the matrix elements may contain some surprises, like suppressing $1/N_c$ factors, etc.**

# Our Strategy

## Low-energy effective Hamiltonian:

$$H_W^{\Delta C=1} = \frac{4G_F}{\sqrt{2}} \sum_{q_1,q_2=s,d} V_{uq_1} V_{cq_2}^* \left[ C_1(m_c) \bar{u}\gamma^\mu P_L c \bar{q}_2 \gamma_\mu P_L q_1 + C_2(m_c) \bar{u}\gamma^\mu P_L q_1 \bar{q}_2 \gamma_\mu P_L c \right]$$

## SM-two consecutive |ΔC|=1 transitions:

$$x_D = -\frac{1}{2M_D} \left\langle \overline{D}^0 \right| i \int d^4x \, H_W^{\Delta C=1}(x) H_W^{\Delta C=1}(0) \left| D^0 \right\rangle$$

$$= \frac{C^{(3)}}{m_c^3} \left\langle \overline{D}^0 \right| \bar{u}\, \Gamma_1\, c\, \bar{u}\, \Gamma_2\, c \left( \bar{s}\, \Gamma_3\, s - \bar{d}\, \Gamma_3\, d \right) \left| D^0 \right\rangle +$$

$$+ \frac{C^{(6)}}{m_c^6} \left\langle \overline{D}^0 \right| \bar{u}\, \Gamma_1\, c\, \bar{u}\, \Gamma_2\, c \left( \bar{s}\, \Gamma_3\, s\, \bar{s}\, \Gamma_4\, s + .... \right) \left| D^0 \right\rangle + .....$$

# Factorization Approach

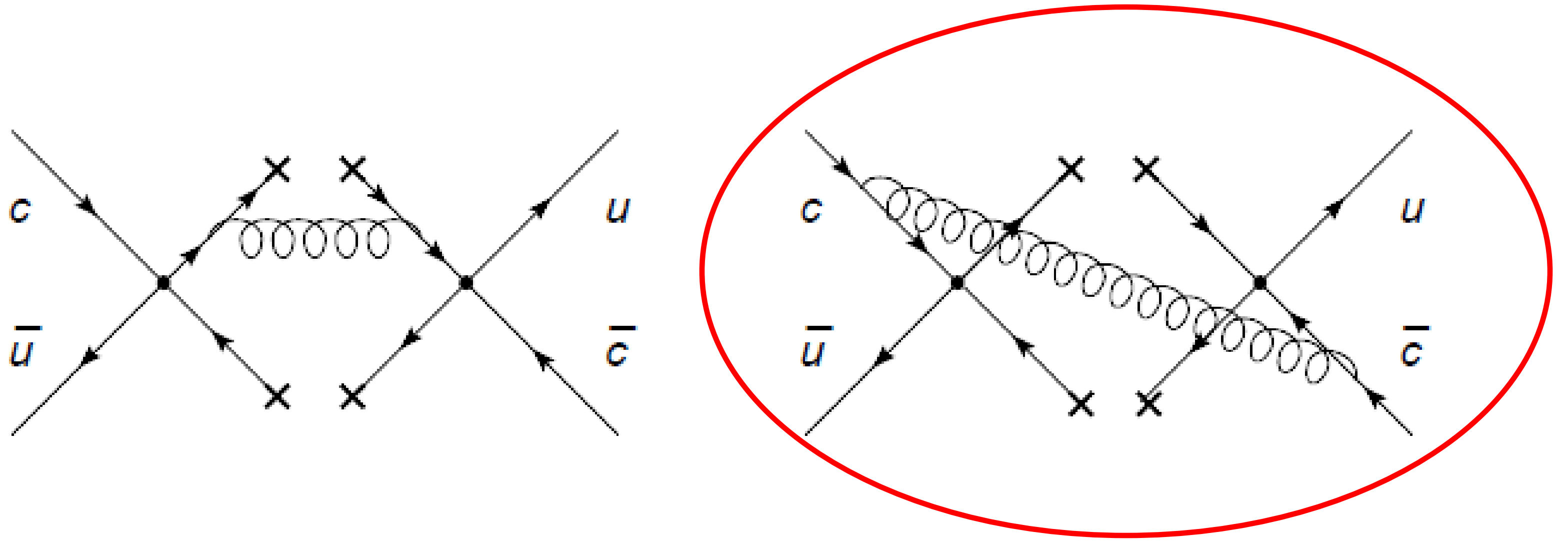
- **E.g. for d =12 operator matrix elements (dominant diagrams with 4 hanging quarks)**

$$\left\langle \overline{D}^0 \right| \bar{u}\,\Gamma_1\, c\, \bar{u}\,\Gamma_2\, c \left(\bar{s}\,\Gamma_3\, s\, \bar{s}\,\Gamma_4\, s + ....\right) \left| D^0 \right\rangle =$$

$$= \left\langle \overline{D}^0 \right| \bar{u}\,\Gamma_1\, c\, \bar{u}\,\Gamma_2\, c \left| D^0 \right\rangle \left\langle 0 \right| \left(\bar{s}\,\Gamma_3\, s\, \bar{s}\,\Gamma_4\, s + ....\right) \left| \, 0 \right\rangle$$

- **Some problems with this approach for d = 9 operator matrix elements - must include also d = 10 operators (Bobrowski, Lenz) or perhaps neglect diagrams with 2 hanging quarks as subdominant ones.**

# Dominant Diagrams




+ 14 other diagrams

In progress, showing the result for one diagram just for illustration.

$$(x_D)_{cc}^{d=12} = \frac{1}{9}\frac{4G_F^2 m_c^2}{3}(4\pi\alpha_s)\frac{f_D^2 M_D}{\Gamma_D}\left(\frac{M_D^2}{m_c^2}\right)\left(\frac{m_s^2}{m_c^2}\right)\left(\frac{\Lambda^4}{m_c^4}\right)\times$$
$$\times \sin^2\theta_c \cos^2\theta_c \left[C_2^2 + 4C_1C_2 + 6C_1^2\right]$$

where

$$m_s^2\Lambda^4 = \left\langle 0\left|\left(\bar{s}\gamma^\mu P_L s - \bar{d}\gamma^\mu P_L d\right)\left(\bar{s}\gamma_\mu P_L s - \bar{d}\gamma_\mu P_L d\right) - 2\bar{s}\gamma^\mu P_L d\bar{d}\gamma_\mu P_L s\right|0\right\rangle$$

**Compare**

$$(x_D)_{cc}^{d=12} = \frac{1}{9}\frac{4G_F^2 m_c^2}{3}(4\pi\alpha_s)\frac{f_D^2 M_D}{\Gamma_D}\left(\frac{M_D^2}{m_c^2}\right)\left(\frac{m_s^2}{m_c^2}\right)\left(\frac{\Lambda^4}{m_c^4}\right)\times$$

$$\times \sin^2\theta_c \cos^2\theta_c \left[C_2^2 + 4C_1C_2 + 6C_1^2\right]$$

**to**

$$x_D^{LO} \equiv \frac{\Delta M_D^{LO}}{\Gamma_D} = \frac{G_F^2 m_c^2}{3\pi^2}\frac{f_D^2 M_D}{\Gamma_D}\left(\frac{m_s^4}{m_c^4}\right)\sin^2\theta_C \cos^2\theta_C \left[\frac{5}{4}\left(C_2^2 - \right.\right.$$

$$\left.\left.- 2C_1C_2 - 3C_1^2\right)\overline{B}_D^{(S)} - C_2^2 B_D\right]$$

**Enhancements:**

$$\frac{m_s^4}{m_c^4} \rightarrow \frac{m_s^2}{m_c^2} \qquad \frac{1}{3\pi^2} \rightarrow \frac{4}{3}$$

**Factor** **$4\pi\alpha_s(m_c) \approx 4.8$**

**Suppressing factors:**

$$\frac{\Lambda^4}{m_c^4} \sim 0.1, \text{ if } \Lambda \sim 1\,\text{GeV}$$

**$1/N_c^2 = 1/9$**

$$\left[C_2^2 + 4C_1C_2 + 6C_1^2\right] = 0.48,$$

$$C_2^2 = 1.44,\ 4C_1C_2 = -1.92$$

# Numerical Result

$$(x_D)_{cc}^{d=12} = 0.33 \times 10^{-3}$$ - less than previous estimates

Other diagrams may yield $(1 - 1.5) \times 10^{-3}$, if there is no cancellation in the sum of the Wilson coefficient products.

However, it is still well below $x_D^{exp} = (6.5 \pm 1.9) \times 10^{-3}$

- Failure of OPE?
- Factorization is inappropriate at $1/m_c^6$ order?
- Or simply $x_D^{exp} = (6.5 \pm 1.9) \times 10^{-3}$ is due to New Physics contribution?
- No answer on these questions yet.

# Conclusions and Summary

- **We are examining the dominant $1/m_c$ contribution to the mass difference in $D^0 - \bar{D}^0$ mixing.**
- **Our goal is to verify quantitatively the estimates made for this contribution.**
- **The preliminary results show that the actual result seems to be slightly below the estimates and well below the experimental value of $\Delta M_D$ .**
- **The calculations are still in progress.**