$D \to K\ell\upsilon$ and $D \to \pi\ell\upsilon$ form factors from Lattice QCD

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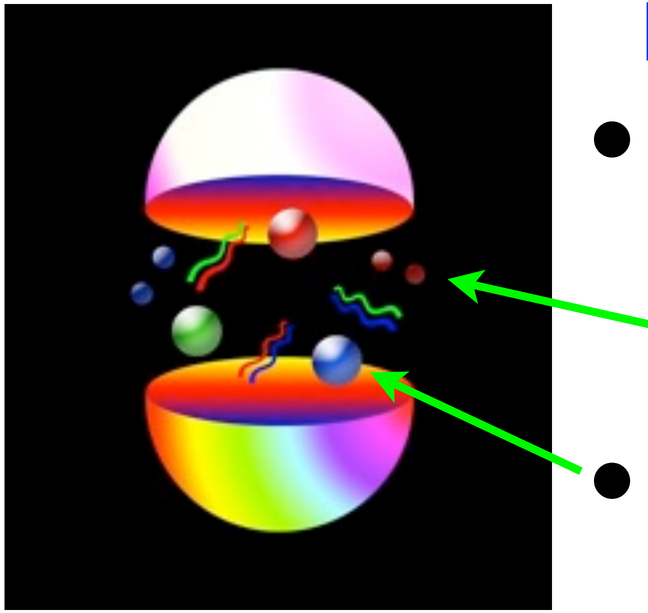
Motivation

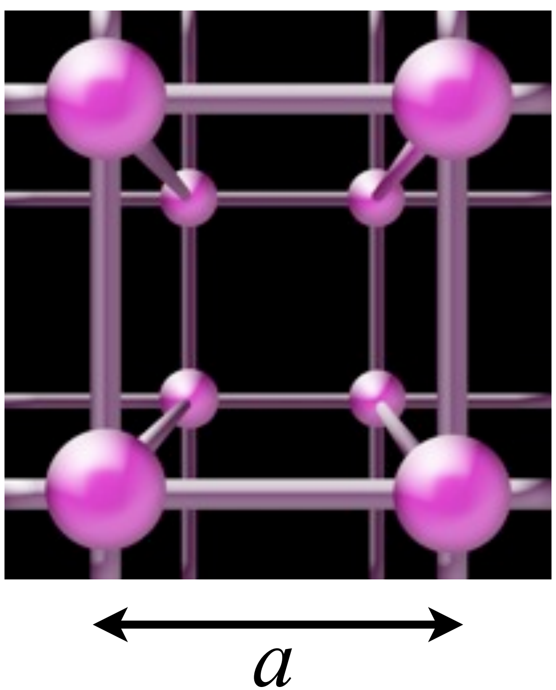
- A high precision calculation of D meson semileptonic form factors: the shape of f_0 and f_+ as a function of q^2
- This is part of the HPQCD program to calculate meson spectra, decay constants, form factors, and other QCD observables from first principles
  → use Lattice QCD
- Highly Improved Staggered Quark formalism (HISQ) enables us to treat charm quarks the same way as light quarks
- Combining a lattice calculation with experimental results allows us to extract CKM matrix elements

Lattice QCD = fully nonperturbative QCD calculation



RECIPE

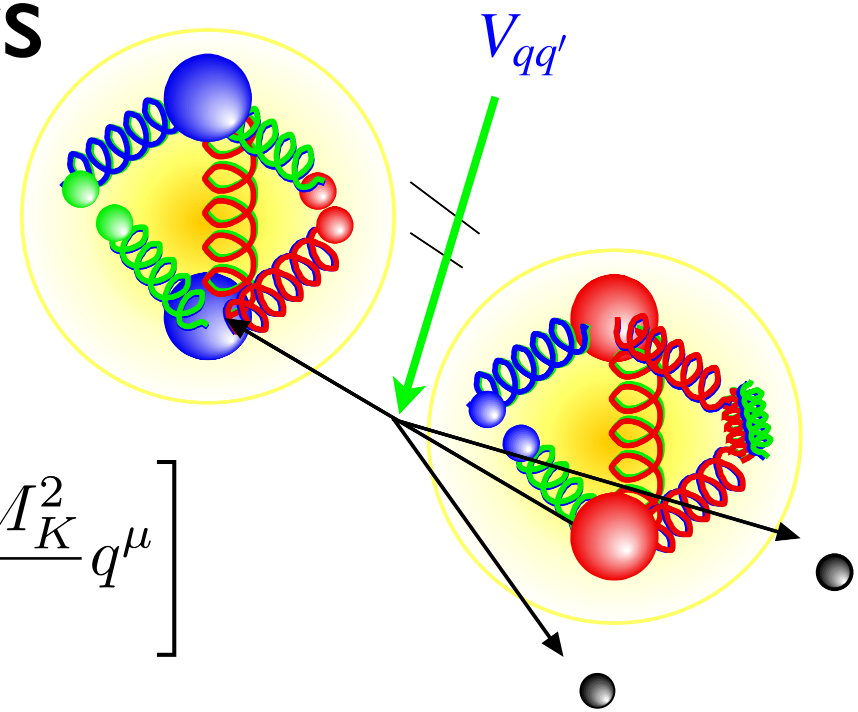
- Generate sets of gluon fields for Monte Carlo integration of Path Integral (incl. effect of u, d and s sea quarks)
- Calculate averaged "hadron correlators" from valence quark propagators



- Fit as a function of time to obtain masses and simple matrix elements
- Determine a and fix m_q to get results in physical units
- extrapolate to $a=0$, $m_{u,d}$ = phys for real world

Semileptonic form factors

= 3pt amplitudes



$$\langle K|S|D\rangle = f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}}$$

$$\langle K|V^\mu|D\rangle = f_+^{D\to K}(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right]$$

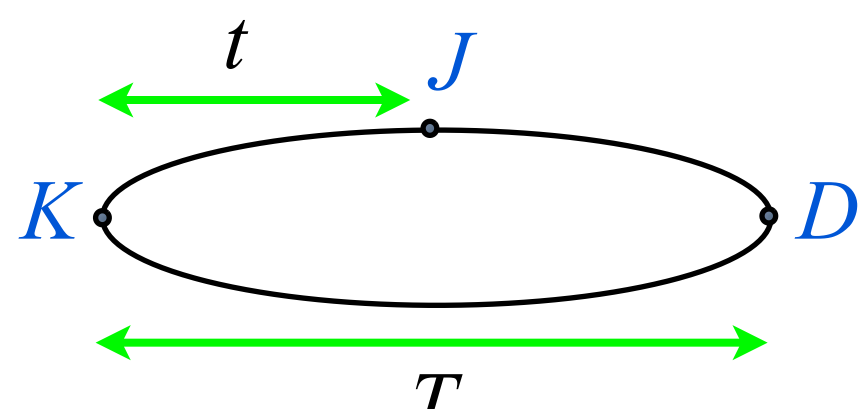
$$+f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu \quad \to \quad f_0(0) = f_+(0)$$

From experiments: differential decay rates

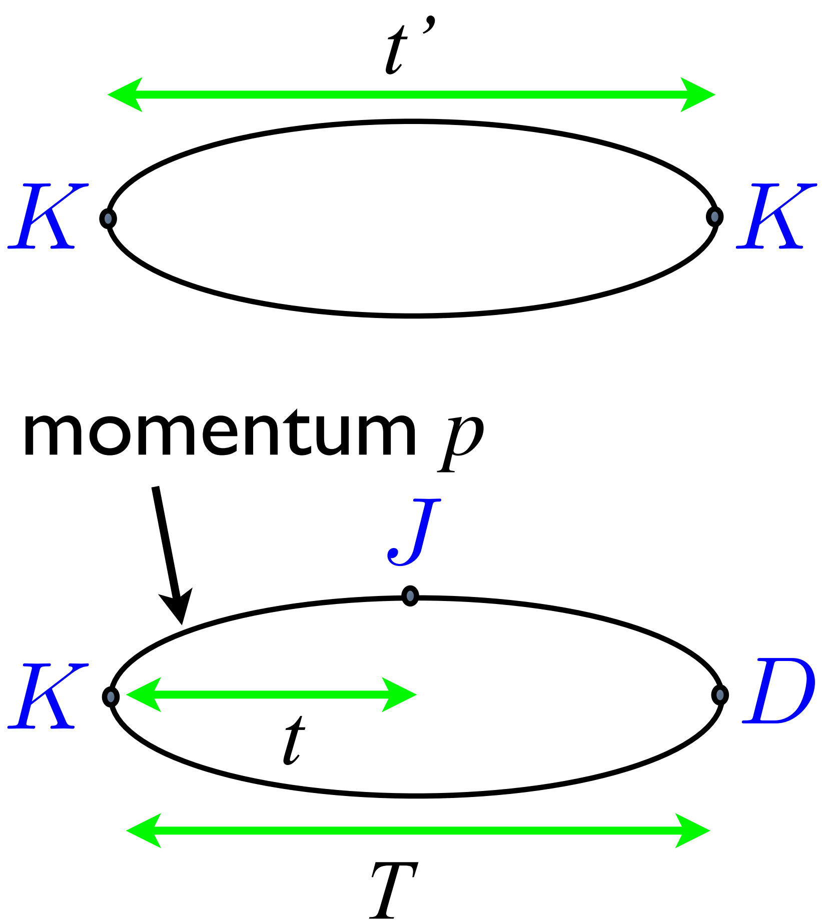
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 p_K^3}{24\pi^3}|V_{cs}|^2|f_+(q^2)|^2$$

$f_+(q^2)$ from theory or V_{cs} from unitarity

$$q^\mu = p_D^\mu - p_K^\mu$$

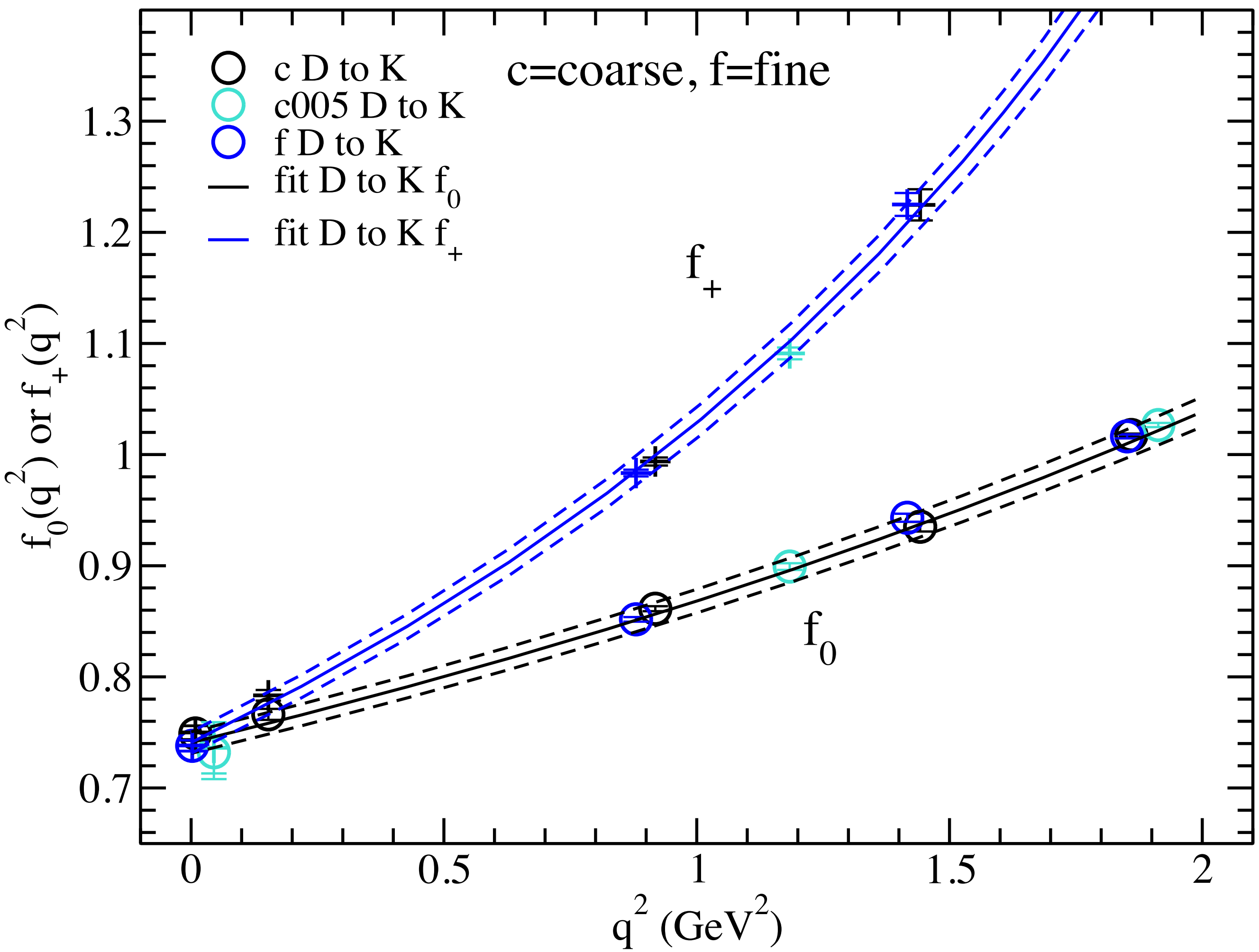


Fitting the Lattice QCD results



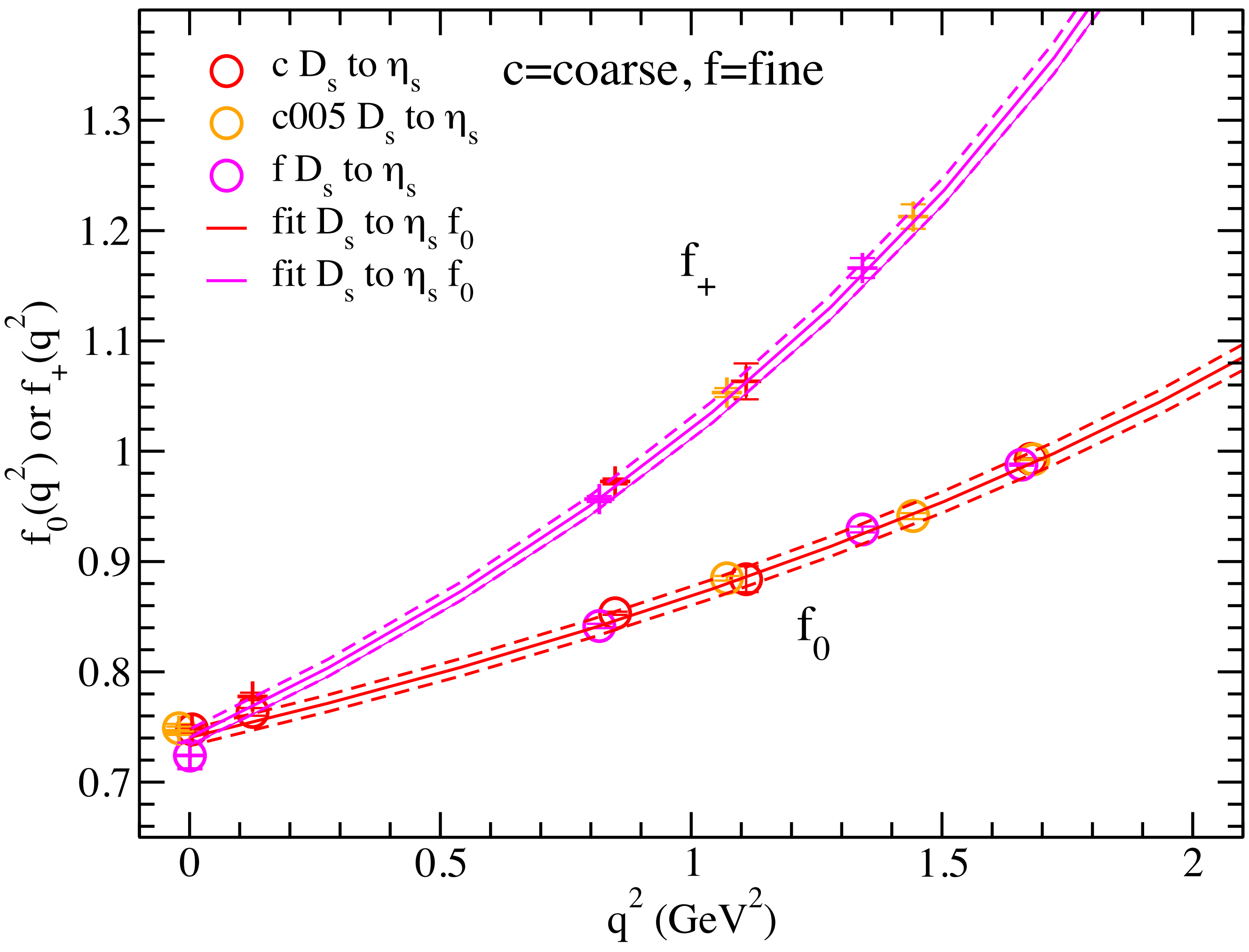
- Fit 2-point and 3-point correlators simultaneously
- Multi-exponential fits to reduce systematical errors from the excited states
- Use Bayesian priors to constrain fit parameters
- High statistics (~*100000* correlators) to get small statistical errors
- Can do any q^2 by giving one of the quarks a momentum p

Shape of the form factors: $D \to K \ell \nu$



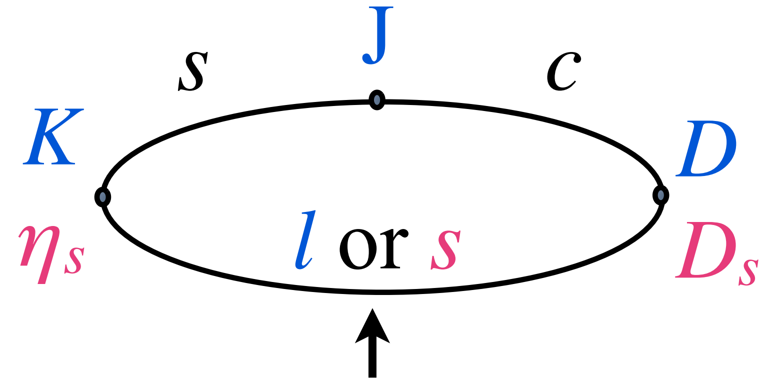
- Both scalar and vector form factor, f_0 and f_+, as a function of q^2
- q^2 dependence is well understood qualitatively

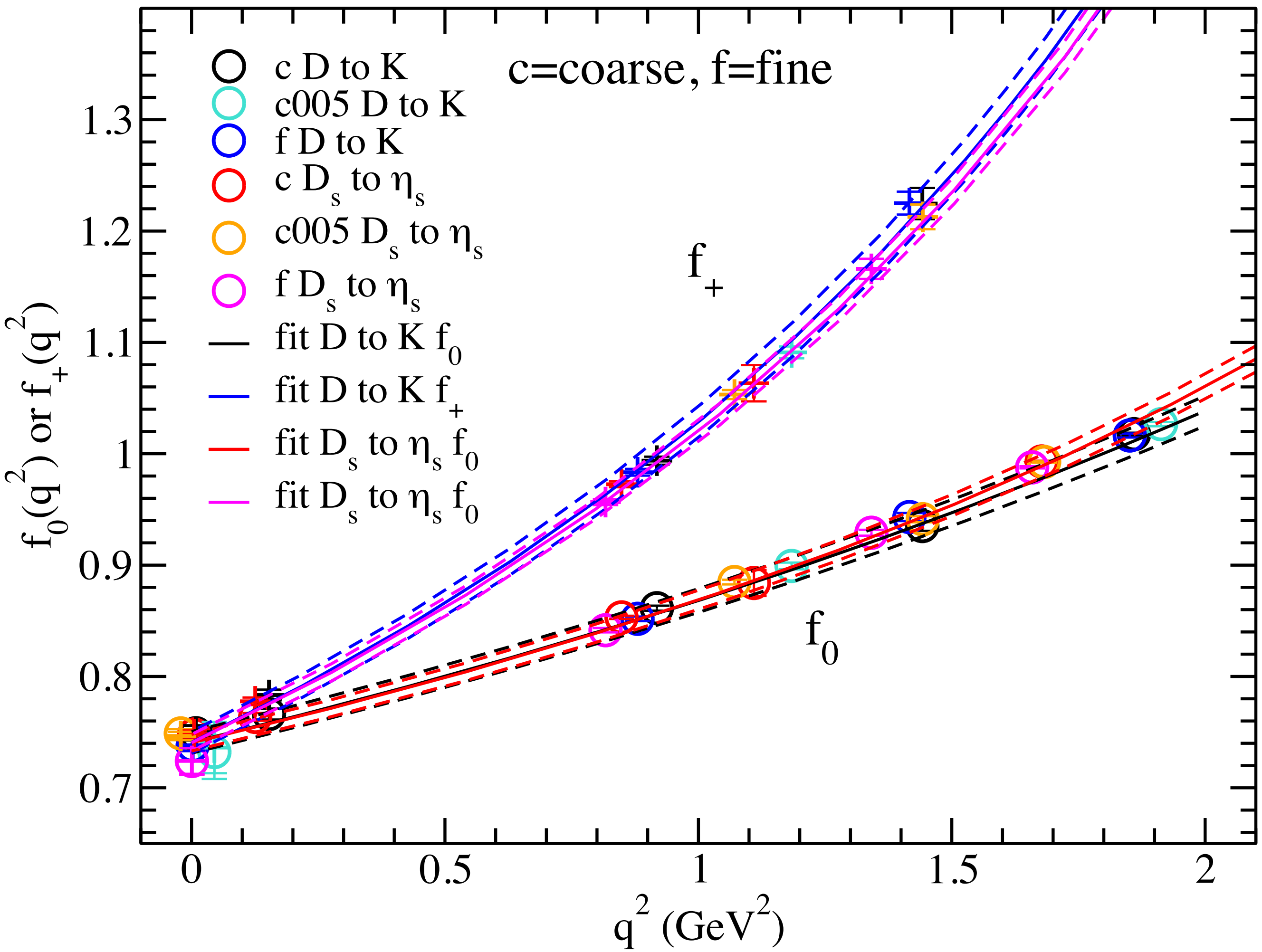
Shape of the form factors: $D_s \to \eta_s \ell \upsilon$



- η_s is the pseudoscalar $s\bar{s}$ meson (lattice only, not a physical meson)
- Same charm-strange current as in $D \to K\ell\upsilon$

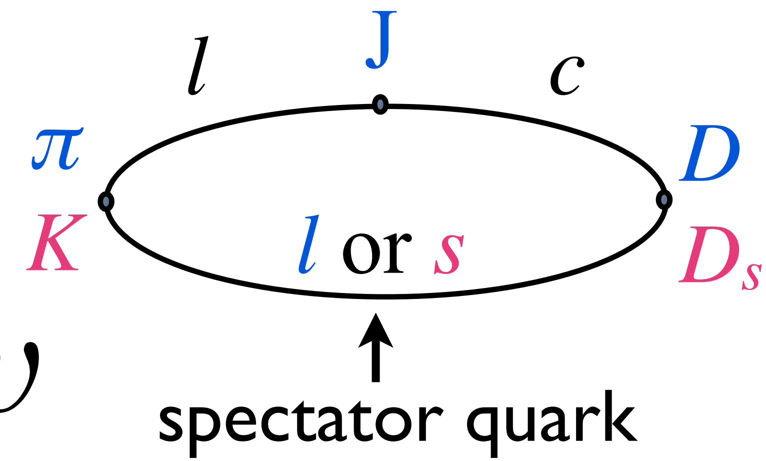
Shape of the FFs: spectator quark

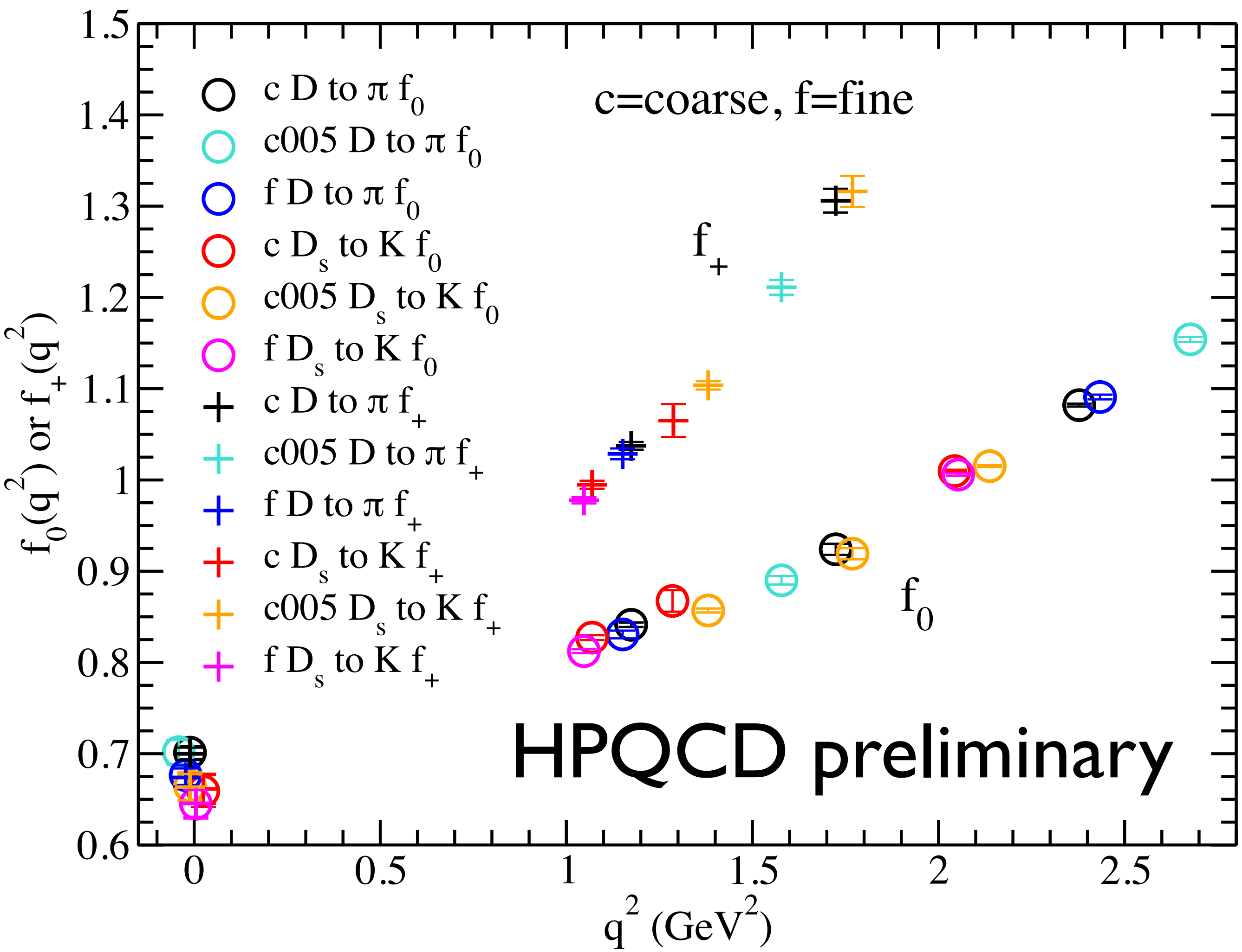




- The difference between $D \to K \ell \nu$ and $D_s \to \eta_s \ell \nu$ is the spectator quark - light vs. strange
- The shapes of the form factors are same within *3%*

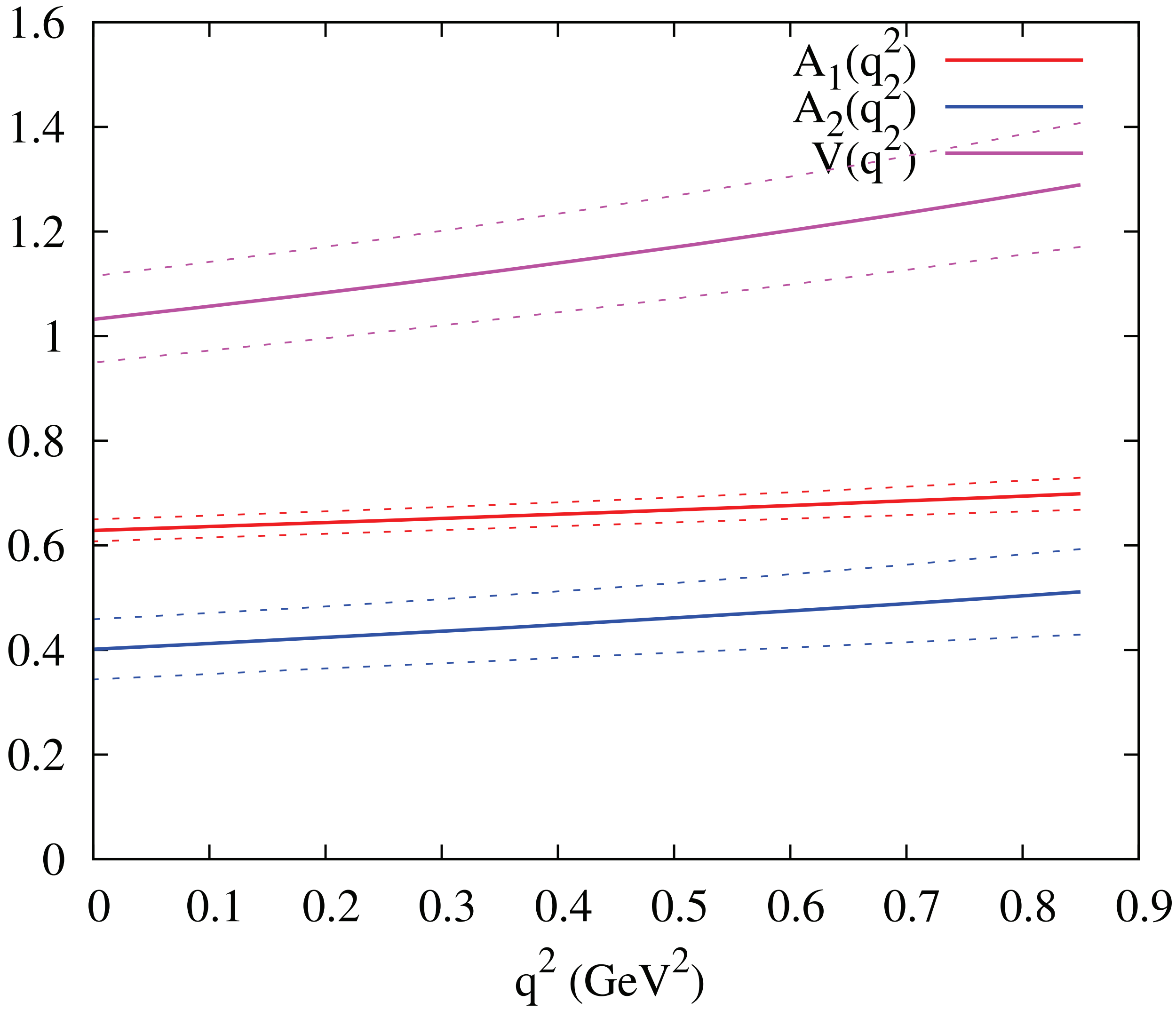
Shape of the FFs: $D \to \pi \ell \nu$ and $D_s \to K \ell \nu$





- Charm to light decay
- Both decays experimentally accessible
- spectator quark makes very little difference

Form factors: $D_s \to \phi \ell v$



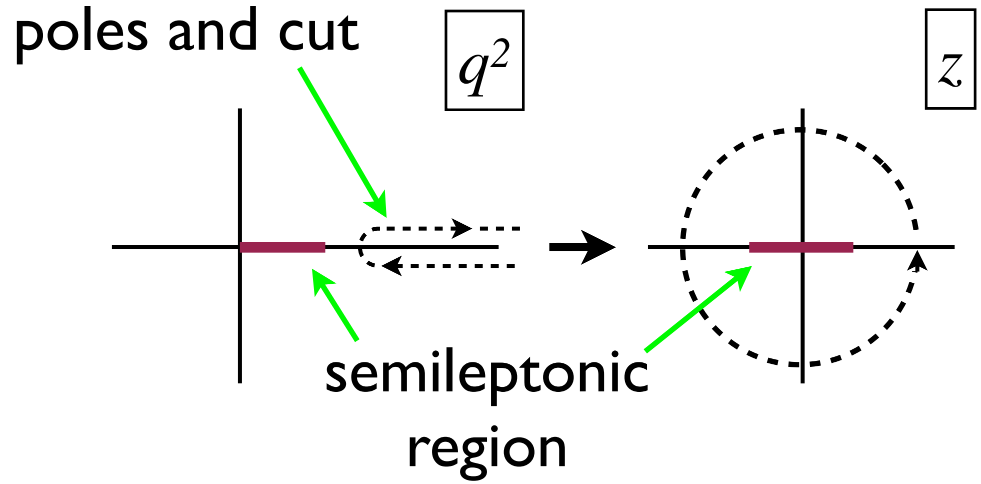
- On the lattice one can also calculate form factors for a D_s meson to a vector meson semileptonic decay
- $D_s \to \phi \ell v$ is charm to strange decay like $D_s \to \eta_s \ell v$ and $D \to K \ell v$

The z-expansion

- Remove the poles

$$\tilde{f}_0^{D\to K}(q^2) = \left(1 - \frac{q^2}{M^2_{D^*_{s0}}}\right) f_0^{D\to K}(q^2),$$

$$\tilde{f}_+^{D\to K}(q^2) = \left(1 - \frac{q^2}{M^2_{D^*_s}}\right) f_+^{D\to K}(q^2)$$



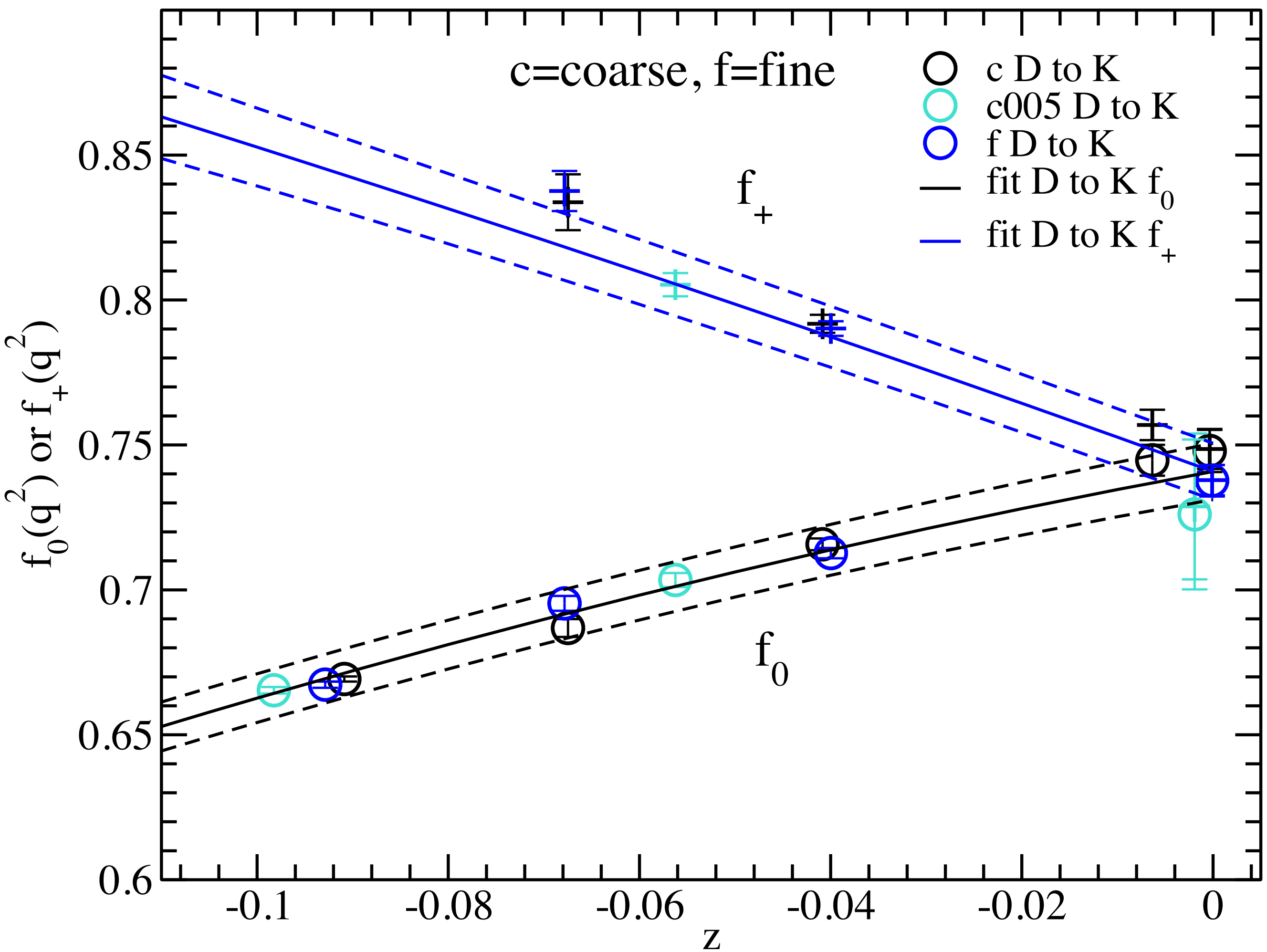
- Convert to z variable and fit $\tilde{f}$ as power series in z

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}, \quad t_+ = (m_D + m_K)^2,$$

$$\tilde{f}_0^{D\to K}(z) = \sum_{n\geq 0} b_n(a) z^n, \quad \tilde{f}_+^{D\to K}(z) = \sum_{n\geq 0} c_n(a) z^n, \quad c_0 = b_0$$

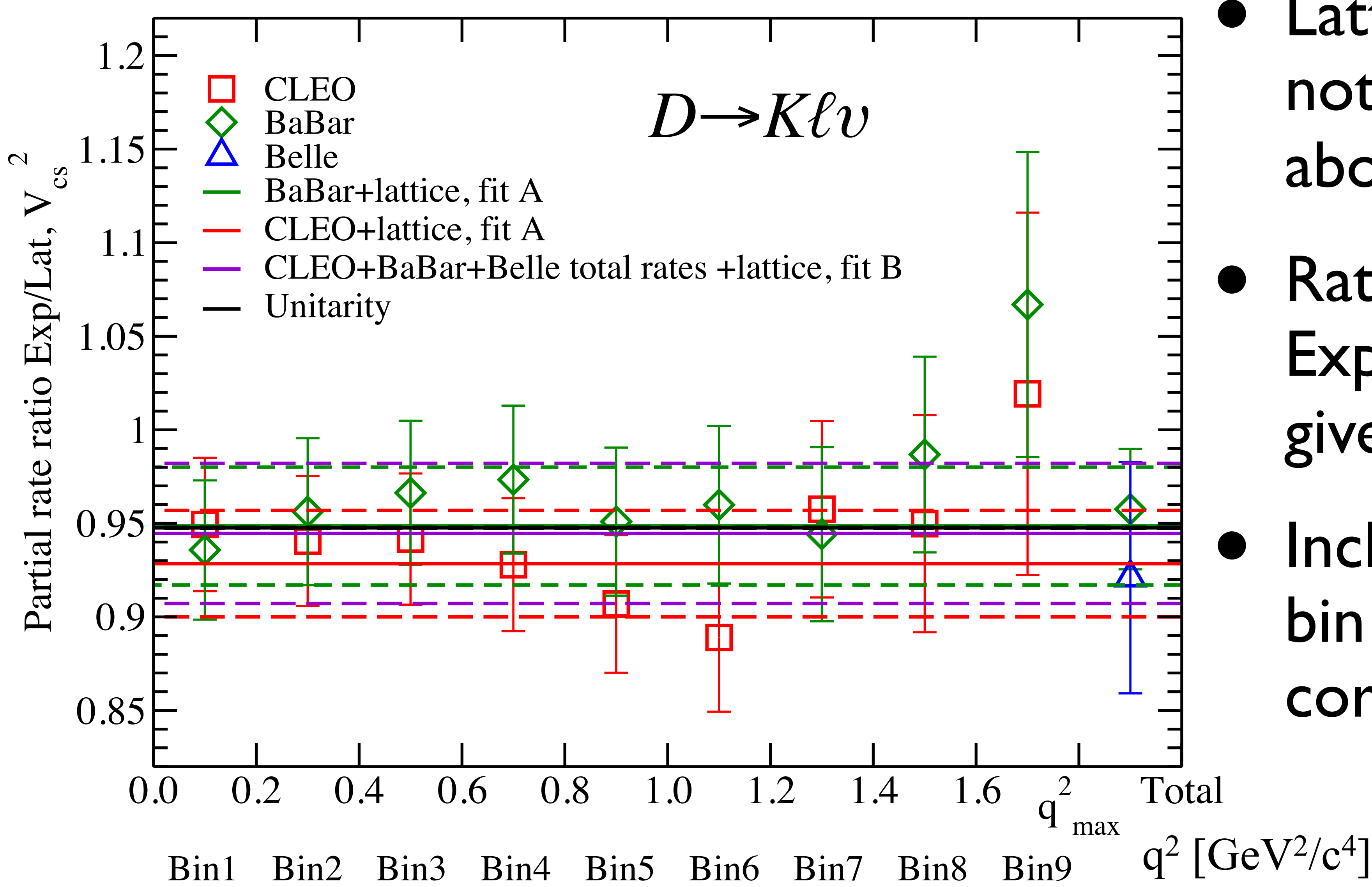
C. Bourrely, I. Caprini and L Lellouch, PRD 79, 013008 (2009)

Continuum and chiral extrapolation

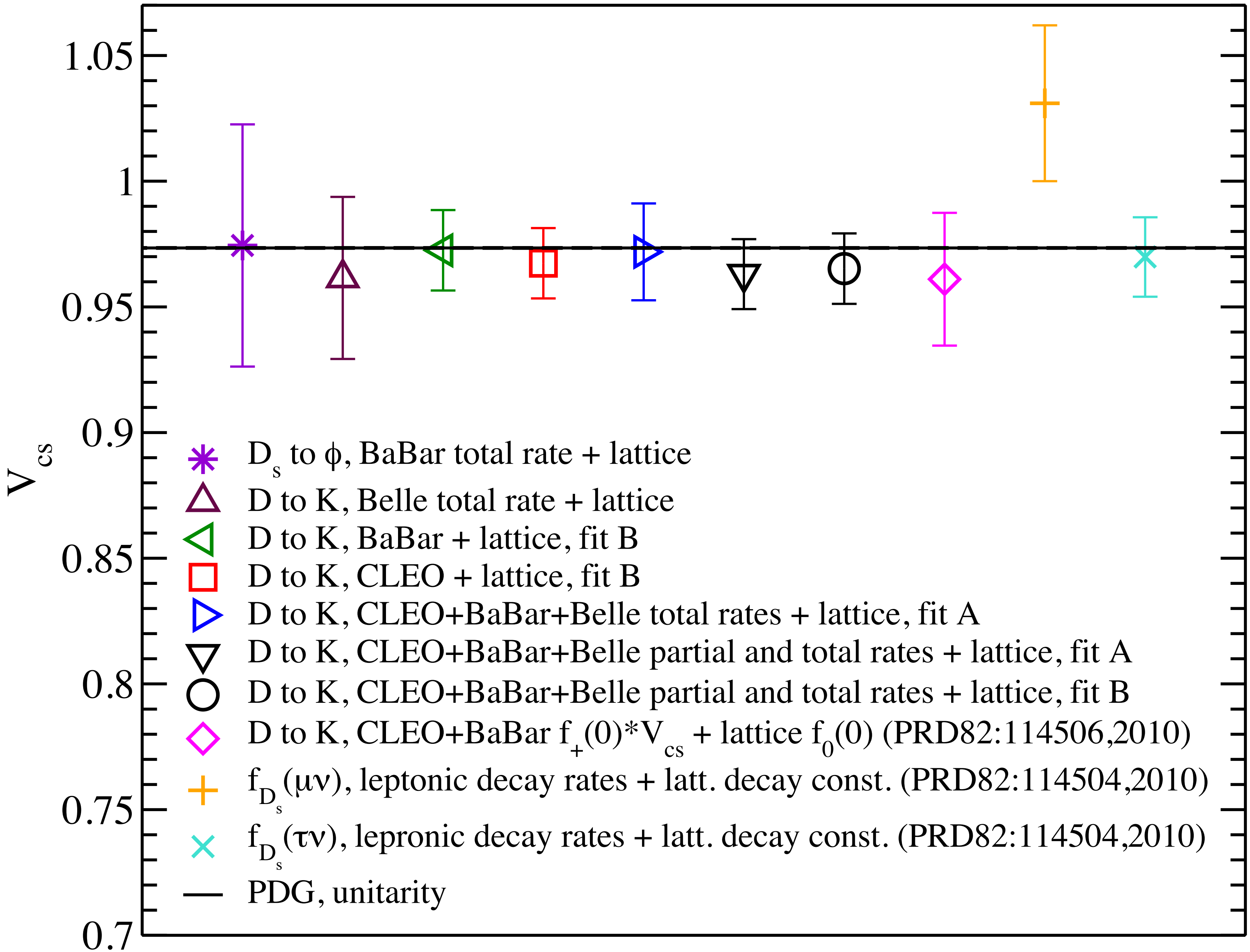


- Fit in z-space, including terms that depend on lattice spacing and quark masses
- Take $a=0$ and $m_q=m_q^{\text{phys}}$

Extracting V_{cs}

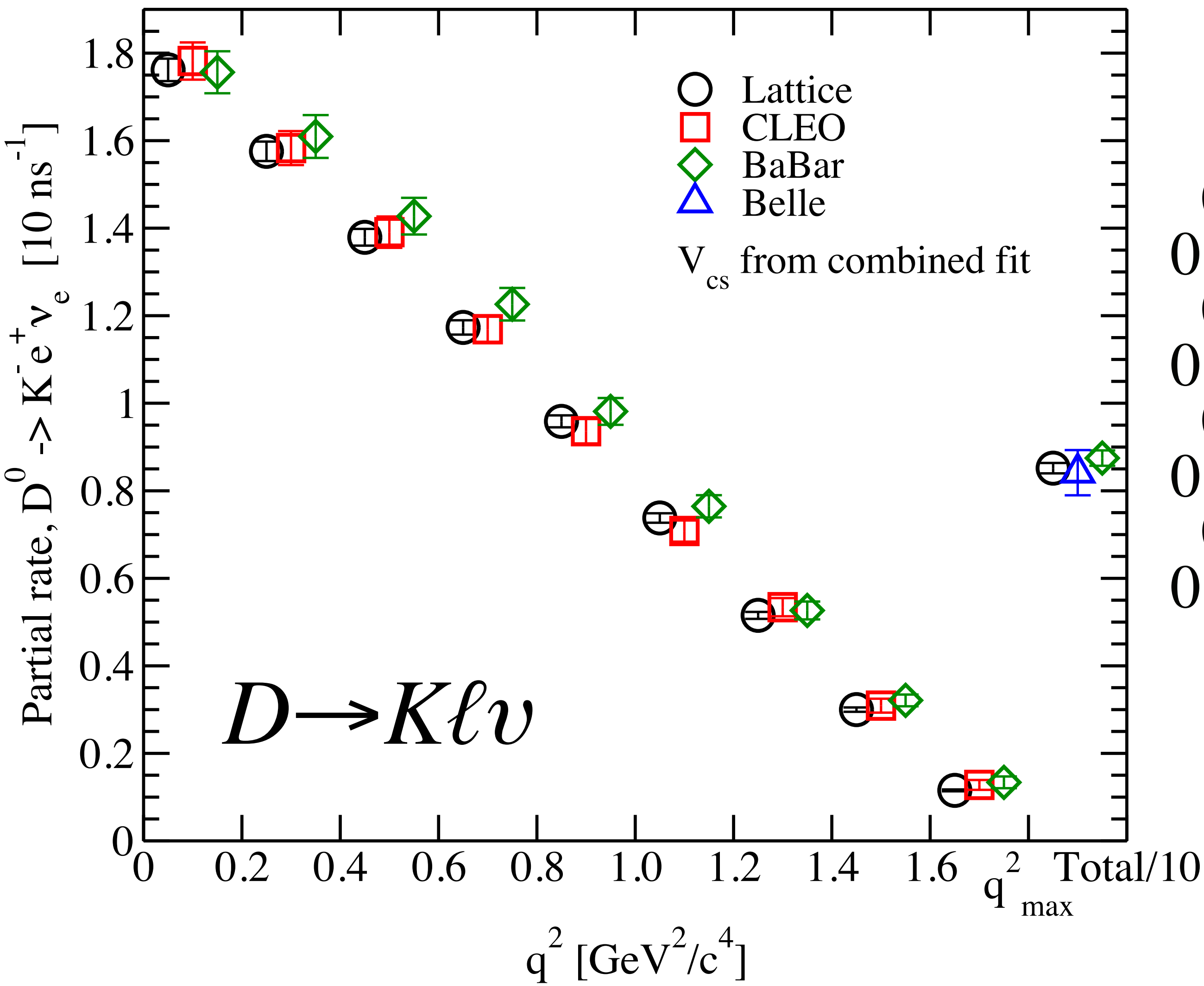


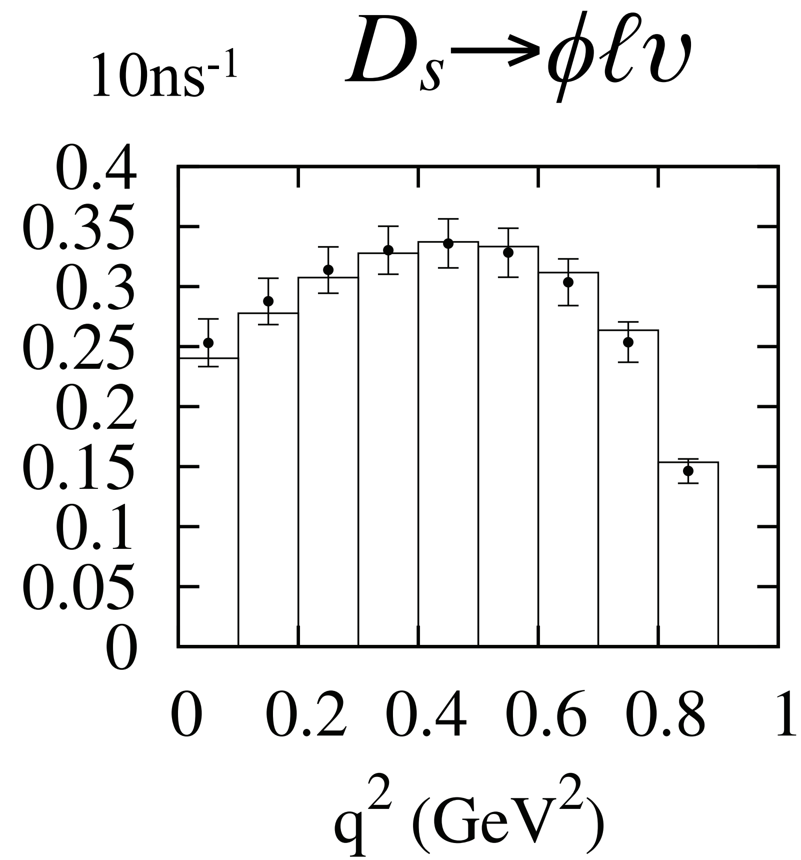
- Lattice does not know about V_{cs}
- Ratio Exp/Lat gives V_{cs}^2
- Include bin to bin correlations



Our best, PRELIMINARY value is V_{cs}=0.965(14)

Decay rates in q^2 bins

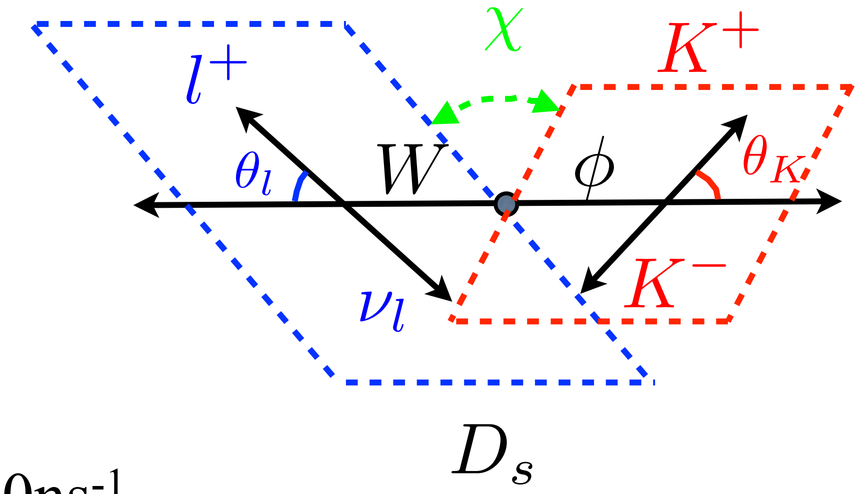


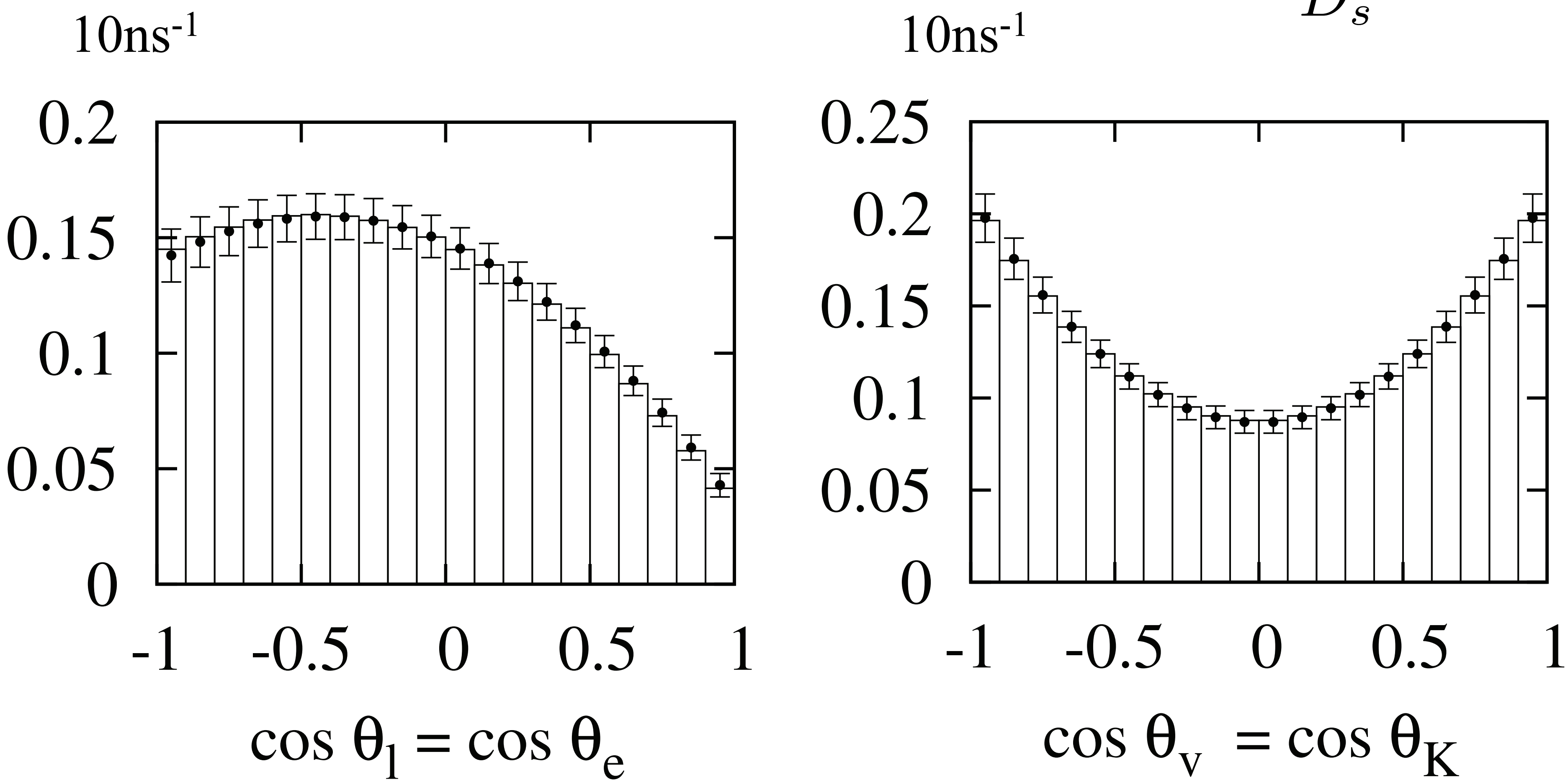


BaBar + lattice

Experimental data: CLEO, PRD 80, 032005 (2009); Belle, PRL 97, 061804 (2006); BaBar, PRD 76, 052005 (2007) and PRD 78, 051101(R) (2008)

$D_s \to \phi \ell \nu$ angular distributions





Experimental data from BaBar, PRD 78, 051101(R) (2008)

Summary

- High precision Lattice QCD calculation of D meson semileptonic decay form factors
  - full q^2 range
  - many different mesons
  - $D \to K\ell\nu$ form factors to 1.6% accuracy
- The D/D_s FFs are very insensitive to the spectator quark, and this is expected to be true for B/B_s as well
- Calculate decay rates in q^2 bins to compare with experiments - get very good agreement
- $D \to \pi\ell\nu$ and $D_s \to K\ell\nu$ form factors coming soon!

Thank you!

Spare slides

Form factors: $D_s \to \phi \ell \upsilon$

$$\langle \phi(p', \epsilon)|V^\mu - A^\mu|D(p)\rangle = \frac{2i\epsilon^{\mu\nu\alpha\beta}}{M + m_\phi}\epsilon^*_\nu p'_\alpha p_\beta V(q^2) + (M + m_\phi)\epsilon^{*\mu} A_1(q^2)$$

$$+\frac{\epsilon^* \cdot q}{M + m_\phi}(p + p')^\mu A_2(q^2) + 2m_\phi \frac{\epsilon^* \cdot q}{q^2} q^\mu A_3(q^2)$$

$$-2m_\phi \frac{\epsilon^* \cdot q}{q^2} q^\mu A_0(q^2),$$

where $V^\mu = \bar{q}'\gamma^\mu Q, A^\mu = \bar{q}'\gamma^\mu\gamma_5 Q,$

and $A_3(q^2) = \frac{M + m_\phi}{2m_\phi} A_1(q^2) - \frac{M - m_\phi}{2m_\phi} A_2(q^2)$ with $A_0(0) = A_3(0)$

$D_s \to \phi \ell v$ differential decay rate

$$
\begin{aligned}
\frac{\mathrm{d}\Gamma(P \to V\ell\nu, V \to P_1P_2)}{\mathrm{d}q^2\mathrm{d}\cos\theta_V\mathrm{d}\cos\theta_\ell\mathrm{d}\chi} = \; & \frac{3}{8(4\pi)^4}G_F^2|V_{q'Q}|^2\frac{p_V q^2}{M^2}\mathcal{B}(V \to P_1P_2) \\
& \times \{(1-\eta\cos\theta_\ell)^2\sin^2\theta_V|H_+(q^2)|^2 \\
& + (1+\eta\cos\theta_\ell)^2\sin^2\theta_V|H_-(q^2)|^2 \\
& + 4\sin^2\theta_\ell\cos^2\theta_V|H_0(q^2)|^2 \\
& - 4\eta\sin\theta_\ell(1-\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\theta_\chi H_+(q^2)H_0(q^2) \\
& + 4\eta\sin\theta_\ell(1+\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\theta_\chi H_+(q^2)H_0(q^2) \\
& - 2\sin^2\theta_\ell\sin^2\theta_V\cos 2\chi H_+(q^2)H_-(q^2)\},
\end{aligned}
$$

where the helicity amplitudes are

$$
H_0(q^2) = \frac{1}{2m_\phi\sqrt{q^2}}\left[(M^2-m_\phi^2-q^2)(M+m_\phi)A_1(q^2) - 4\frac{M^2p_\phi^2}{M+m_\phi}A_2(q^2)\right]
$$

$$
H_\pm(q^2) = (M+m_\phi)A_1(q^2) \mp \frac{2Mp_\phi}{M+m_\phi}V(q^2)
$$

Lattice configurations

- MILC $n_f=2+1$ asqtad lattice configurations
- Highly Improved Staggered Quarks (HISQ) as valence quarks
- coarse: $20^3\times64$ and $24^3\times64$, about $(2.4\text{ fm})^3$, $a\approx0.12$ fm
  valence m_s tuned, $m_l\approx m_s/3.5$ and $m_l\approx m_s/7$
- fine: $28^3\times96$, about $(2.4\text{ fm})^3$, $a\approx0.085$ fm
  valence m_s tuned, $m_l\approx m_s/4.2$
- High statistics calculation:
  - *2000* configurations in each ensemble
  - *8* (coarse) and *4* (fine) time sources per config.