Lattice results on Charmonium

Daniel Mohler

TRIUMF, Theory Group
Vancouver, B.C., Canada

Honolulu,
May 14 2012

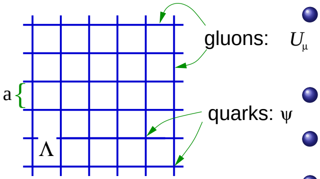
Outline

1. Lattice QCD - Basics and recent developments
    - Recent progress
    - Lattice systematics - a word of caution
    - Excited states, why are they difficult?
2. Recent charmonium simulations
    - Fermilab lattice and MILC collaborations
    - Hyperfine-splitting from HPQCD
    - Fermilab method with Wilson-Clover sea quarks
    - Hadron spectrum collaboration
    - BMW and QCDSF collaborations
3. Concluding remarks - Where will the lattice go?

Lattice QCD

- Regularization of QCD on a space-time grid

$$S_{QCD}[\psi,\bar{\psi},A] = \sum_{f=1}^{N_f} \int d^4x\, \bar{\psi}^{(f)}(x) \left(\not{D} + m^{(f)}\right) \psi^{(f)}(x) + \frac{1}{2g^2} Tr[F_{\mu\nu}(x) F_{\mu\nu}(x)]$$



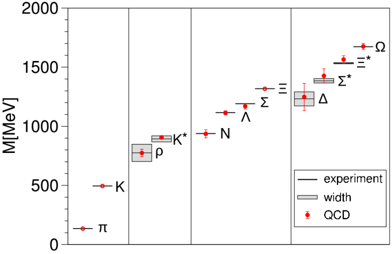
- Lattice $\Lambda = \{\vec{n} = (n_1, n_2, n_3, n_4) | n_i \in \{0, 1, \ldots, L_i - 1\}\}$
- $\vec{x} \quad \rightarrow \quad a\vec{n}$ with lattice spacing a
- $\int d^4x \ldots \quad \rightarrow \quad a^4 \sum_{\vec{n}\in\Lambda} \cdots$
- $D_\mu\psi(\vec{x})$ covariant lattice derivative

- Enables simulations using well established Monte Carlo methods

Recent developments

- Light mesons and baryons

Ground states have been calculated with full control of systematic uncertainties
Example from Dürr et al. Science 322 (2008)



- Calculations close to
  `PACS-CS Collaboration Phys. Rev. D79 (2009) 034503`
  or at the physical pion mass
  `PACS-CS Phys.Rev. D81 (2010) 074503`
  `BMW Collaboration Phys.Lett. B701 (2011) 265`
  We will soon see more (MILC,. . .)
- First dynamical 2+1+1 flavor simulations (ETMC, MILC)

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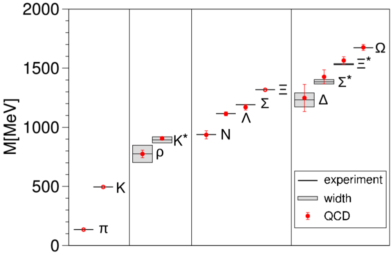

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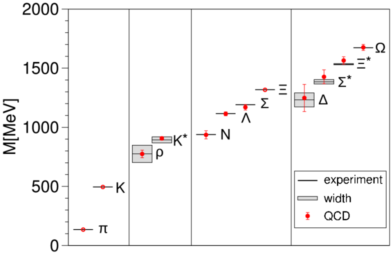

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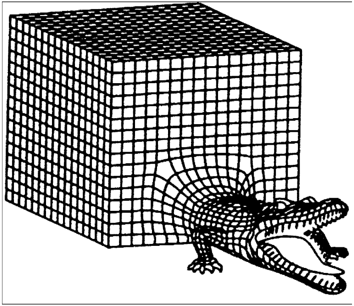
Needed: Multiple extrapolations

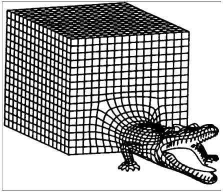
- *Continuum limit*: $a(g, m) \to 0$
  - Need simulations at multiple different lattice spacings
- *Thermodynamic limit*: $L \to \infty \quad (L \cdot a = const.)$
  - Hadron physics in a small box → finite volume effects
    Typical volume $\approx 2.5\mathrm{fm}$
- Calculation at physical quark masses or extrapolation to the *Chiral limit*: $m \to m_0 \quad (M_\pi \to M_{\pi,exp})$
  - Physical u, d quark masses small → Simulation very expensive!
  - Chiral Perturbation Theory (χPT) ↔ Lattice QCD

The dangerous animals of lattice QCD

- While progress in the field is fast, studies taking into account all sources of uncertainty are still lacking in many cases.
- Qualitative studies provide/implement crucial insights/improvements and pave the way



Identifying lattice crocodiles - an example



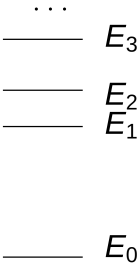
- 1S Hyperfine splitting $M_{HFS} = M_{J/\Psi} - M_{\eta_c}$
- traditionally spin-dependent quantities came out too small in simulations
- This has been blamed on many things: quenching/large pion masses, 2 flavor simulations, neglected annihilation diagrams, discretization effects
- I will argue that controlling discretization effects is crucial and that high precision calculations are possible

The problem with excited states

From the analysis of Euclidean correlators:

$$\left\langle \hat{O}_2(t)\hat{O}_1(0)\right\rangle_T \propto \sum_n e^{-tE_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

- The whole tower of states contributes
- Ground state is dominant at large t
- Exited states appear as sub-leading exponentials
- Noisy background from limited statistics



- For a single correlator, fit to several exponentials leads to poor results

  → Advanced methods needed for excited states!

Method of choice: The variational method

Matrix of correlators projected to fixed momentum (will assume 0)

$$C(t)_{ij} = \sum_n \mathrm{e}^{-tE_n} \langle 0|O_i|n\rangle \left\langle n|O_j^\dagger|0\right\rangle$$

Solve the generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$
$$\lambda^{(k)}(t) \propto \mathrm{e}^{-tE_k}\left(1 + \mathcal{O}\left(\mathrm{e}^{-t\Delta E_k}\right)\right)$$

At large time separation: only a single state in each eigenvalue. Eigenvectors can serve as a fingerprint.

`Michael Nucl. Phys. B259, 58 (1985)`

`Lüscher and Wolff Nucl. Phys. B339, 222 (1990)`

`Blossier et al. JHEP 04, 094 (2009)`

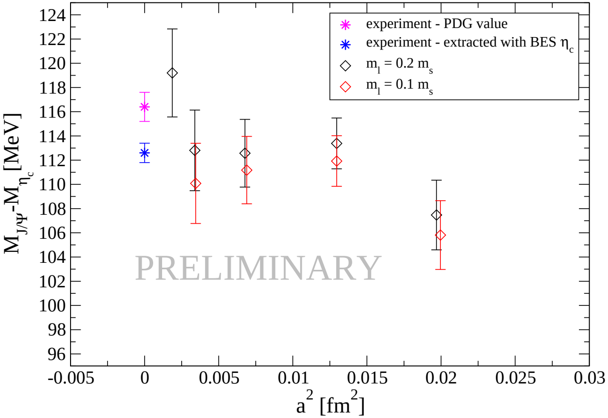
FNAL/MILC program overview

- **Main goal:** Precision study of the low-lying states including higher spin states with a full error budget
- **Analysis campaigns:**

Study	sea	Charm	Ops	States	Comment
2009 FNAL/MILC	$2+1$	FNAL	2	low S, P	$a \geq 0.09$ fm
2012 FNAL/MILC	$2+1$	FNAL	many	many	$a \geq 0.045$ fm
Current FNAL/MILC	$2+1+1$	HISQ	2	low S, P	$a \geq 0.06$ fm
Future FNAL/MILC	$2+1+1$	HISQ	many	many	$a \geq 0.06$ fm

- I will present selected *preliminary* results for 2+1 flavor asqtad
- Data for many other states (including higher spin) is available but not yet fully analyzed
- This is a progress report

Selected results: Charmonium hyperfine splitting



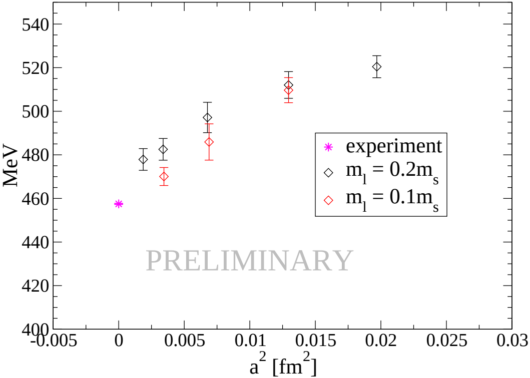
- Use of an improved heavy-quark action → mild discretization effects
- Quark annihilation contributions not yet included

Selected results: $\overline{1S}$ - $\overline{1P}$ splitting

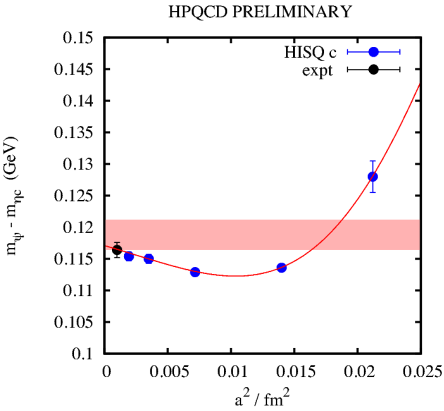
- Defining the mass centroids

$$M_{\overline{1S}} = \frac{1}{4}(M_{\eta_c} + 3M_{J/\Psi})$$

$$M_{\overline{1P}} = \frac{1}{9}(M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}})$$



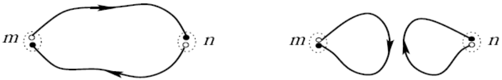
Hyperfine-splitting from HPQCD



`from E. Follana PoS (Lattice 2010) 305`

- The red band indicates the preliminary result: 118.8 ± 2.4 MeV

Disconnected contributions?



- For HPQCD hyperfine-splitting, contributions from disconnected diagrams are estimated from perturbation theory which leads to a shift enlarging the hyperfine splitting.
- MILC estimated the effects by direct calculations and find that disconnected diagrams reduce the hyperfine splitting by 1-4MeV `Levkova and DeTar, Phys.Rev. D83 (2011) 074504`
- Novel methods should lead to a more precise determination (see later) `Liu et al. arXiv:1204.5425`

Two exploratory studies

- $N_f = 2 + 1$ Wilson-Clover sea and Fermilab charm quarks at almost physical pion masses

$N_L^3 \times N_T$	a[fm]	L[fm]	#configs	m_π[MeV]
$32^3 \times 32$	0.0907(13)	2.9	≥ 198	$156 \leq m_\pi \leq 702$

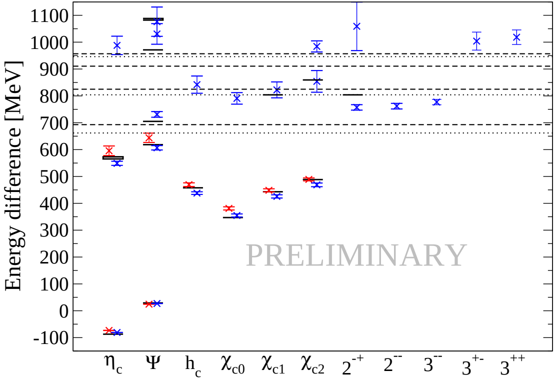
`DM and R. M. Woloshyn Phys.Rev. D84 (2011) 054505`

- $N_f = 2$ smeared Wilson-Clover sea and Fermilab charm quarks

$N_L^3 \times N_T$	a[fm]	L[fm]	#configs	m_π[MeV]
$16^3 \times 32$	0.1239(13)	1.98	280	266(3)(3)

`DM, S. Prelovsek and R. M. Woloshyn (to be published)`

Results from $N_f = 2$ smeared Wilson-Clover sea and Fermilab charm



`DM and R. M. Woloshyn Phys.Rev. D84 (2011) 054505`

`DM, S. Prelovsek and R. M. Woloshyn (to be published)`

- High statistical precision has been achieved through the *distillation* technique `Peardon et al. Phys.Rev. D80 (2009) 054506`

Good statistical precision for many quantities!

Mass difference	This study [MeV]	Experiment [MeV]
$\overline{1P} - \overline{1S}$	$441.7 \pm 4.0 \pm 4.6$	457.5 ± 0.3
$\overline{2S} - \overline{1S}$	$592.3 \pm 4.9 \pm 6.2$	606.1 ± 1.0
1S hyperfine	$107.9 \pm 0.3 \pm 1.1 \pm^{2.2}_{0}$	116.6 ± 1.2
1P spin-orbit	$39.7 \pm 2.1 \pm 0.4$	46.6 ± 0.1
1P tensor	$11.02 \pm 0.87 \pm 0.12$	16.25 ± 0.07
1P hyperfine	3.7 ± 2.7	-0.10 ± 0.22
2S hyperfine	57.9 ± 2.0	49 ± 4
2P spin-orbit	$24.6 \pm 15.7 \pm 0.3$	-
2P tensor	2.2 ± 4.3	-
$\overline{2P} - \overline{1S}$	$836.4 \pm 30.5 \pm 8.8$	-
$2M_{\overline{D_s}} - M_{\overline{\bar{c}c}}$	1065.6	1084.8 ± 0.8

Excited charmonium states from anisotropic Wilson-Clover lattices

- The *Hadron Spectrum Collaboration* has recently studied ground and excited charmonium states

  `Liu et al. arXiv:1204.5425`
- They use anisotropic Wilson-Clover lattices with volumes $16^3 \times 128$ and $24^3 \times 128$ at a single lattice spacing ($a_s \approx 0.12 fm$ and $\xi = 3.5$)
- They simulate at a single pion mass $m_\pi = 396 MeV$
- A large basis containing interpolating fields with up to three covariant derivatives is used

Excited charmonium states from anisotropic Wilson-Clover lattices

Notable features:

- Use of the *distillation* technique allows calculations with small statistical uncertainty

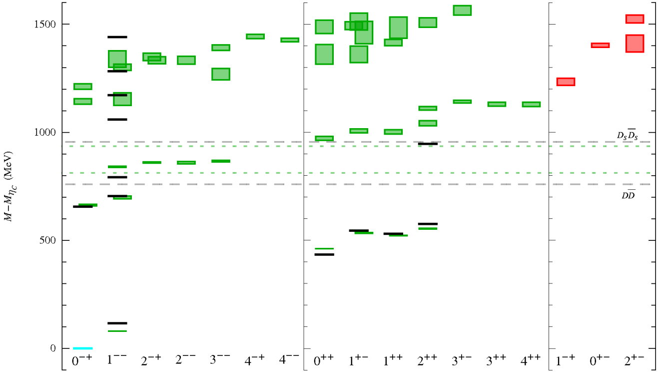
  `Peardon et al. Phys.Rev. D80 (2009) 054506`

- Advanced spin identification techniques allows unambiguous spin assignment.

  `Dudek et al. Phys.Rev. D82 (2010) 034508`

  This allows to identify Spin 4 states and states with exotic quantum numbers.

Excited charmonium states from anisotropic Wilson-Clover lattices



`from Liu et al.   arXiv:1204.5425`

What about systematics?

- Hyperfine splitting:

$$M_{HFS} = 80 \pm 1\mathrm{MeV}$$

  Illustrates the typical size of discretization effects.
- This can be improved by the use of an improved heavy quark action or by continuum extrapolation
- Higher states will be influenced by unphyscial thresholds. These can be included systematically within the distillation technique!

Charmonium using $N_f = 2 + 1$ BMW and CQDSF configurations

- **BMW configurations:** Smeared $N_f = 2 + 1$ Wilson Clover

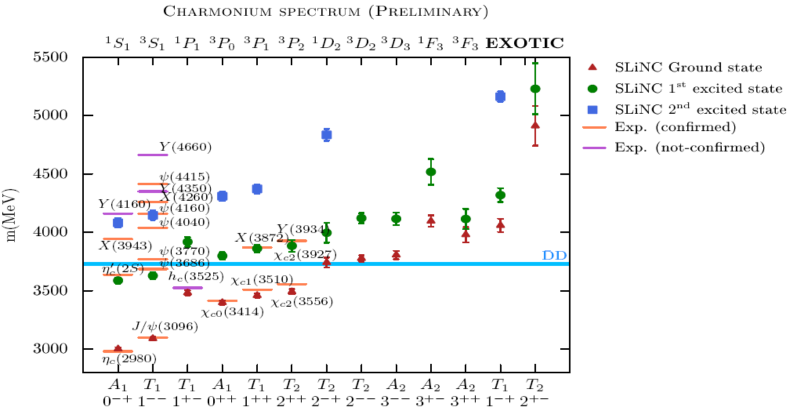
$$
\begin{aligned}
0.054\mathrm{fm} \le a &\le 0.092\mathrm{fm} \\
120\mathrm{MeV} \le M_\pi &\le 520\mathrm{MeV} \\
L^3 \times T &= 32^3 \times 64, 64^3 \times 144
\end{aligned}
$$

- **QCDSF configurations:** $N_f = 2 + 1$ SLiNC fermions
  So far preliminary results from the following ensembles:

$N_L^3 \times N_T$	a[fm]	L[fm]	m_π[MeV]
$24^3 \times 48/32^3 \times 64$	0.0795(3)	1.9/ 2.5	442
$24^3 \times 48/32^3 \times 64$	0.0795(3)	1.9/ 2.5	348

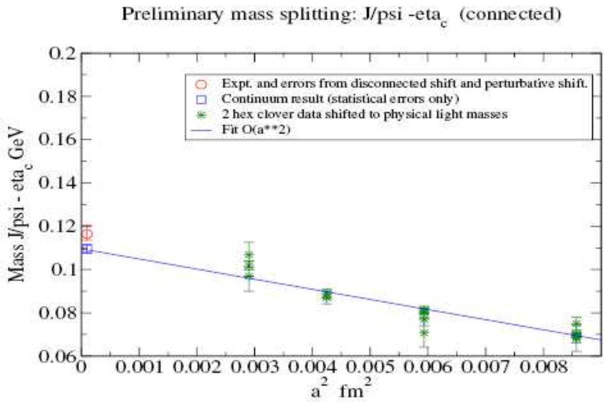
- In both cases the same action is used for light and charm quarks.

BMW/QCDSF - Preliminary results for the charmonium spectrum



`from Bali et al. arXiv:1108.6147`

BMW/QCDSF - Preliminary charmonium hyperfine splitting



`from Bali et al.  arXiv:1108.6147`

Concluding remarks - Where will the lattice go?

- The era of precision calculations using Lattice QCD has barely begun and there already is remarkable success
- Calculations at or around physical pion masses will become commonplace in the near future
- There are multiple collaborations generating gauge configurations with 2+1 or 2+1+1 dynamical flavors suitable for charmonium physics
- For states below inelastic thresholds, the extraction of resonance properties from the lattice is within reach
- This necessitates the inclusion of DD, DD*, etc. states. For a first step see

  `from Bali, Collins, Ehmann Phys.Rev. D84 (2011) 094506`
- There is much work ahead with regard to hadron excitations

Thank you!