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Charmonium potential from lattice QCD

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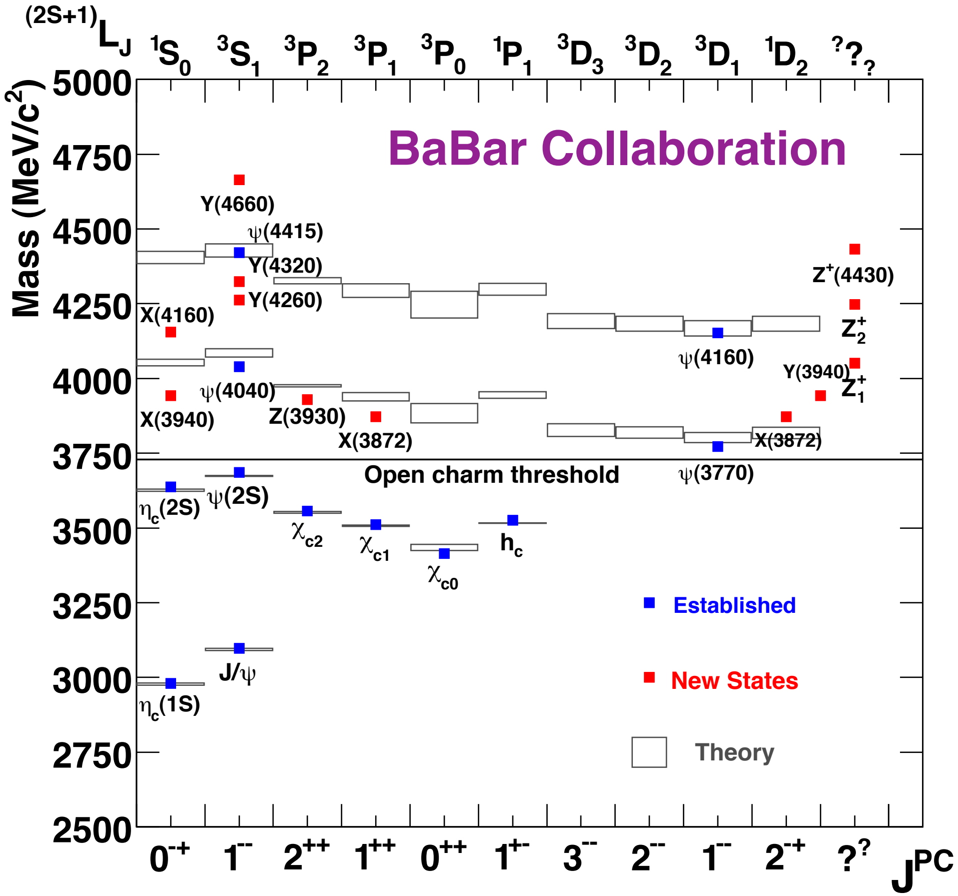
T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)
T. Kawanai and S. Sasaki, arXiv:1110.0888 accepted to PRD(R)

Why cc^{bar} potential？

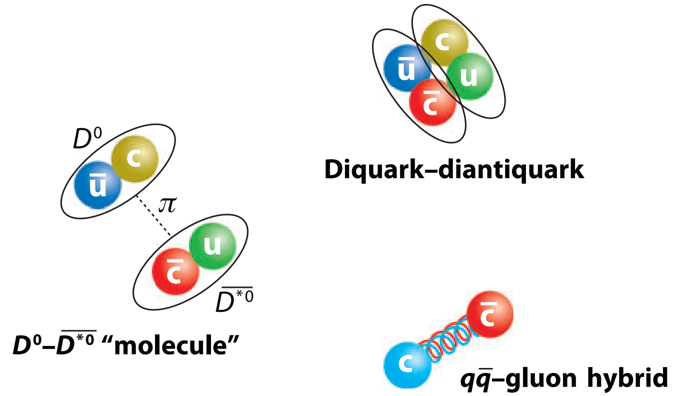
✦ Exotic XYZ charmonium-like mesons

“Standard” states can be defined in potential models



The XYZ mesons are expected to be good candidates for non-standard quarkonium mesons

S. Godfrey and S. L. Olsen,
Ann. Rev. Nucl. Part. Sci. 58, 51 (2008)



“Exotic” = “Non-standard”?

Why $c\bar{c}$ potential？

✦ $q\bar{q}$ interquark potential in quark models

S. Godfrey and N. Isgur, PRD 32, 189 (1985).
T. Barnes, S. Godfrey and E. S. Swanson, PRD 72, 054026 (2005)

$$V_{c\bar{c}} = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + \frac{32\pi\alpha_s}{9m_q^2}\delta(r)\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} + \frac{1}{m_q^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{4\alpha_s}{r^3}T\right]$$

Cornell potential　　　　spin-dependent potential

- Spin-spin, tensor and spin-orbit terms appear as corrections in the $1/m_q$ expansion.
- Functional forms of the spin-dependent terms are determined by one-gluon exchange.

→ Properties of higher charmonium states predicated in potential models may suffer from large uncertainties.

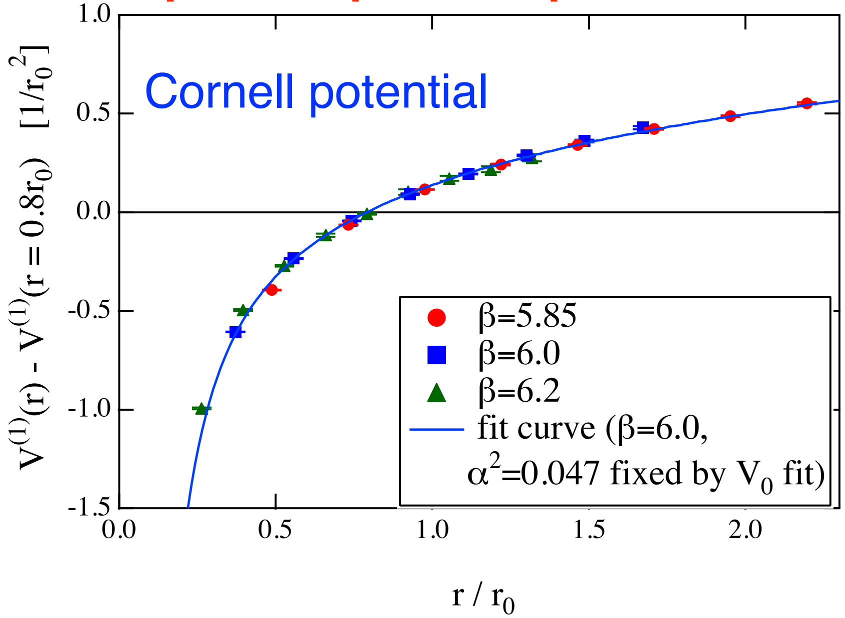
A reliable charmonium potential directly derived from first principles QCD is very important.

Why cc^{bar} potential？

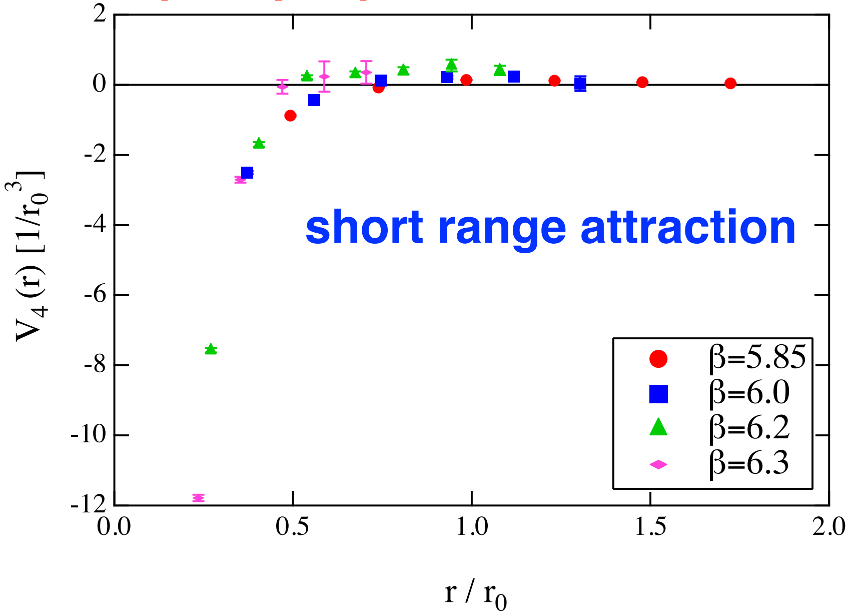
✦ Static interquark potential from Wilson loop

G. S. Bali, Phys. Rept. 343, 1 (2001).

Spin-independent potential



spin-spin potential



- The static potential have been precisely calculated by Wilson loop from lattice QCD.
- Relativistic corrections are classified in powers of $1/m_q$ within framework of pNRQCD.

N. Brambilla et al., Rev. Mod. Phys. 77, 1423 (2005).

→ spin-spin potential induced by $1/m_q^2$ correction exhibits short range attraction.
cf. short range repulsion is required in phenomenology.

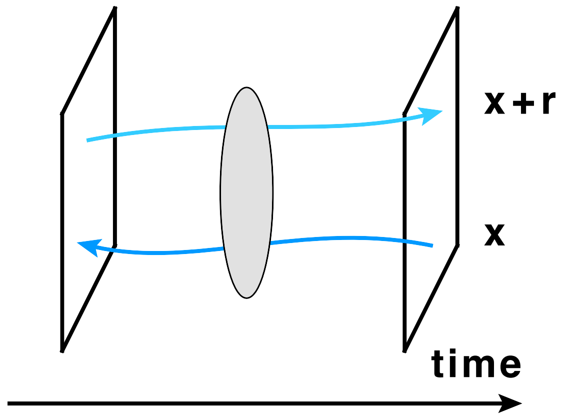
Koma et al., NPB769 (2007) 79
Koma et al., PRL97 (2006) 122003

How to calculate cc^{bar} potential？

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89.
Y. Ikeda and H. Iida, arXiv:1102.2097 [hep-lat].

1. Equal-time BS wavefunction

$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q}; J^{PC}\rangle$$



$$\sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{q}(\mathbf{x},t)\Gamma q(\mathbf{x}+\mathbf{r},t)\left(\bar{q}(\mathbf{x}',t_{\rm src})\Gamma q(\mathbf{y}',t_{\rm src})\right)^\dagger|0\rangle$$
$$= \sum_n A_n \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^\Gamma(t-t_{\rm src})}$$
$$\xrightarrow{t\gg t_0} A_0\phi_\Gamma(\mathbf{r})e^{-M_0^\Gamma(t-t_{\rm src})}$$

2. Schrödinger equation with non-local potential

$$-\frac{\nabla^2}{2\mu}\phi_\Gamma(\mathbf{r}) + \int dr' U(\mathbf{r},\mathbf{r}')\phi_\Gamma(\mathbf{r}') = E_\Gamma\phi_\Gamma(\mathbf{r})$$

3. Velocity expansion

$$U(\mathbf{r}',\mathbf{r}) = \{V(r) + V_{\rm S}(r)\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}} + V_{\rm T}(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\}\delta(\mathbf{r}'-\mathbf{r})$$

How to calculate cc^{bar} potential？

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89.
Y. Ikeda and H. Iida, arXiv:1102.2097 [hep-lat].

5. Projection to “S-wave” $\phi_\Gamma(\mathbf{r}) \rightarrow \phi_\Gamma(r; A_1^+)$

$$\left\{-\frac{\nabla^2}{m_q} + V(r) + \mathbf{S}_q \cdot \mathbf{S}_{\overline{q}} V_{\mathrm{S}}(r)\right\} \phi_\Gamma(r) = E_\Gamma \phi_\Gamma(r)$$

6. Linear combination

$$\begin{aligned} V(r) &= E_{\mathrm{ave}} + \frac{1}{m_q}\left\{\frac{1}{4}\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{3}{4}\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\} \\ V_{\mathrm{S}}(r) &= E_{\mathrm{hyp}} + \frac{1}{m_q}\left\{-\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\} \end{aligned}$$

The quark kinetic mass m_q is essentially involved in the definition of the potentials.
Under a simple, but reasonable assumption of $\lim_{r\to\infty} V_S(r) = 0$

T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011).

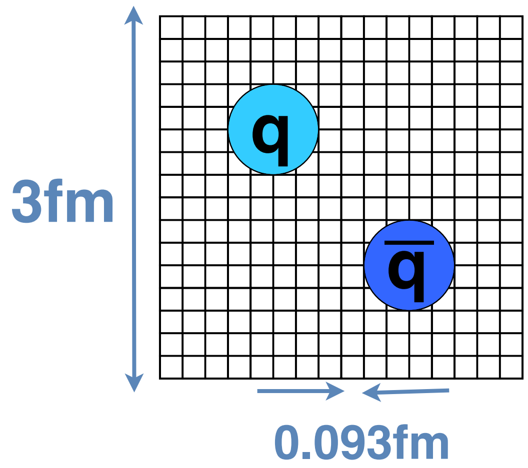
$$m_q = \lim_{r\to\infty} \frac{-1}{\Delta E_{\mathrm{hyp}}}\left(\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)} - \frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)}\right)$$

$$\Delta E_{\mathrm{hyp}} = M_{\mathrm{V}} - M_{\mathrm{PS}}$$

1. Quenched lattice QCD simulation

2. N_f =2+1 dynamical QCD simulation

Lattice Set up

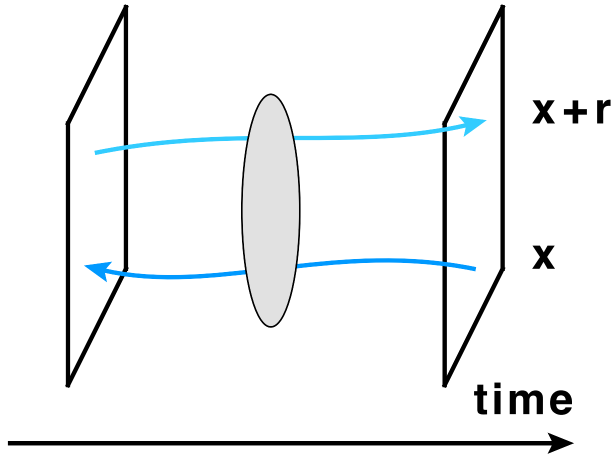


- Quenched QCD (No dynamical quarks)
- Lattice size : $L^3 \times T = 32^3 \times 48$ (~3fm^3)
- plaquette gauge action $\beta=6.0$ ($a=0.093$ fm, $a^{-1}=2.1$GeV)

  + RHQ action with tad-pole improved one-loop PT coefficients

  Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).
- 6 hopping parameters : $0.06667 \leq \kappa_Q \leq 0.11456$

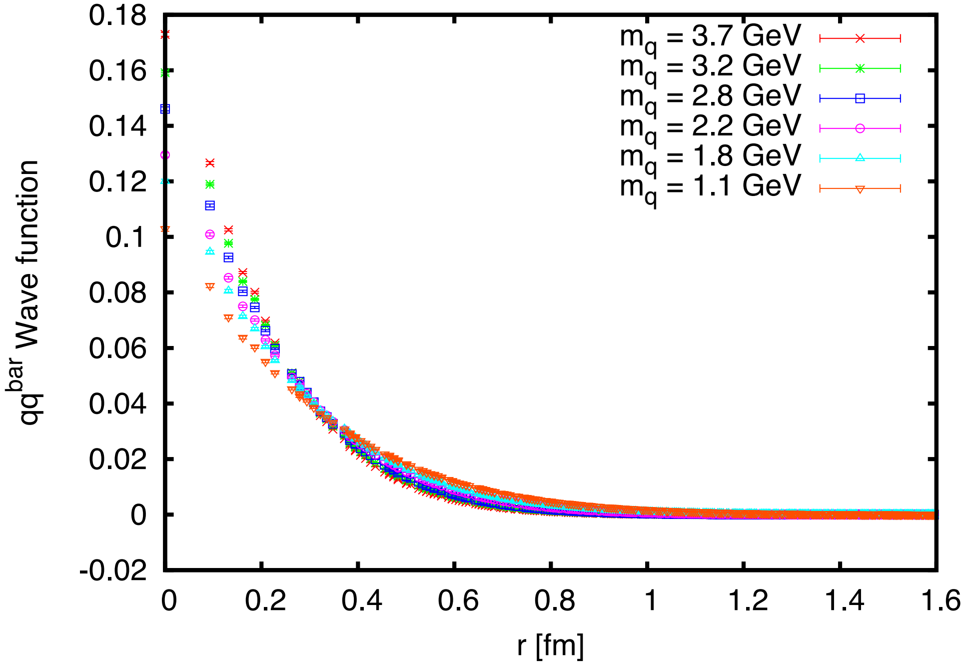
  $1.87 \text{ GeV} \leq m_{\text{pseudo}} \leq 5.83 \text{ GeV}$
- Statistics : 150 configs
- Wall source
- Coulomb gauge fixing



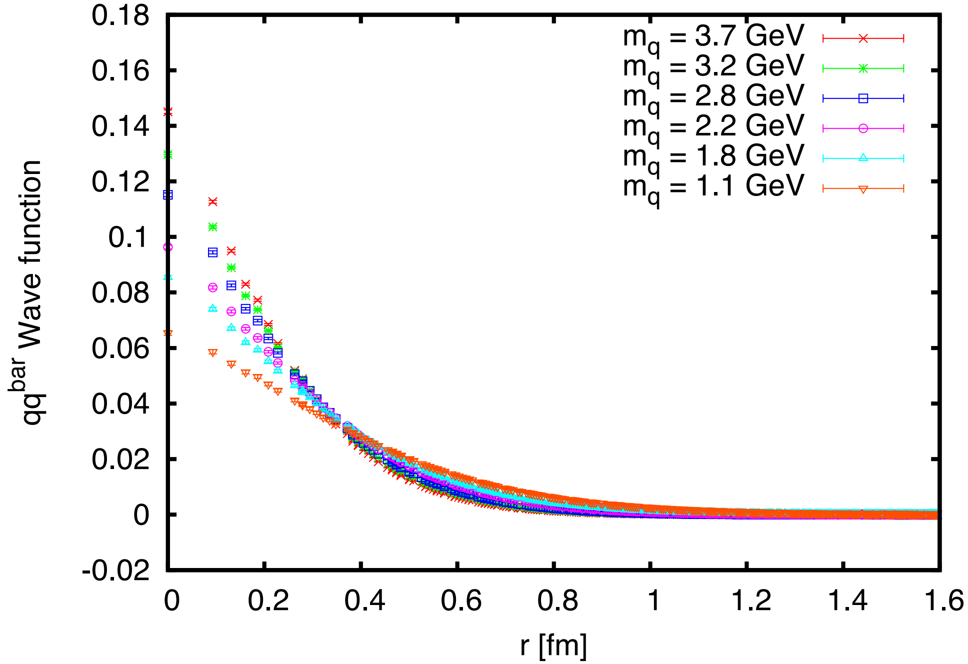
$q\bar{q}$ wave function

$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\overline{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q};J^{PC}\rangle$$

Pseudo scalar $J^P = 0^-$



Vector $J^P = 1^-$

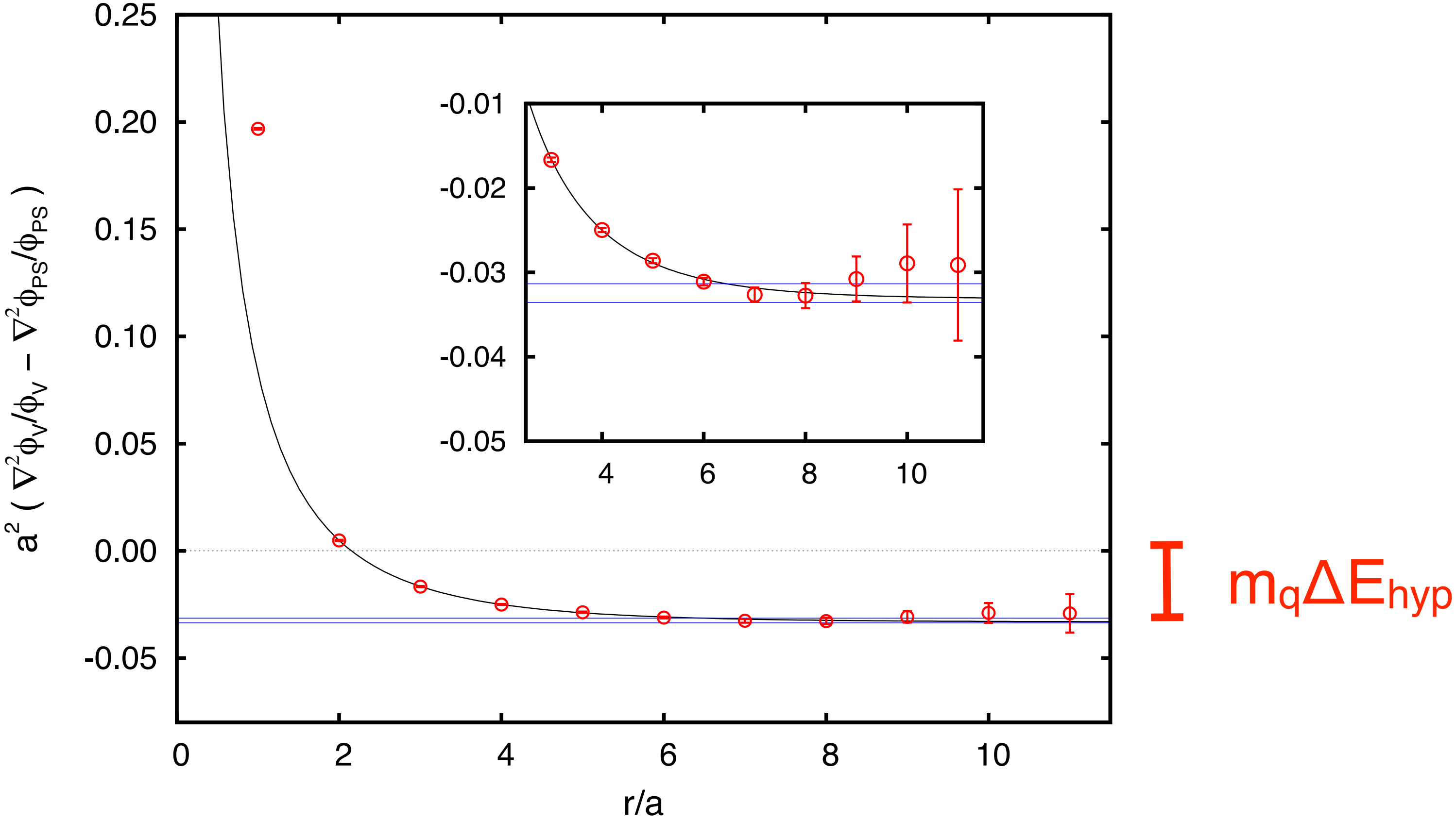


- Normalization $\int dr^3\, \psi^2(r) = 1$
- BS wave functions vanish at $r \sim 1$fm
- Size of wave function with heavier quark mass become smaller.

Determination of kinetic quark mass

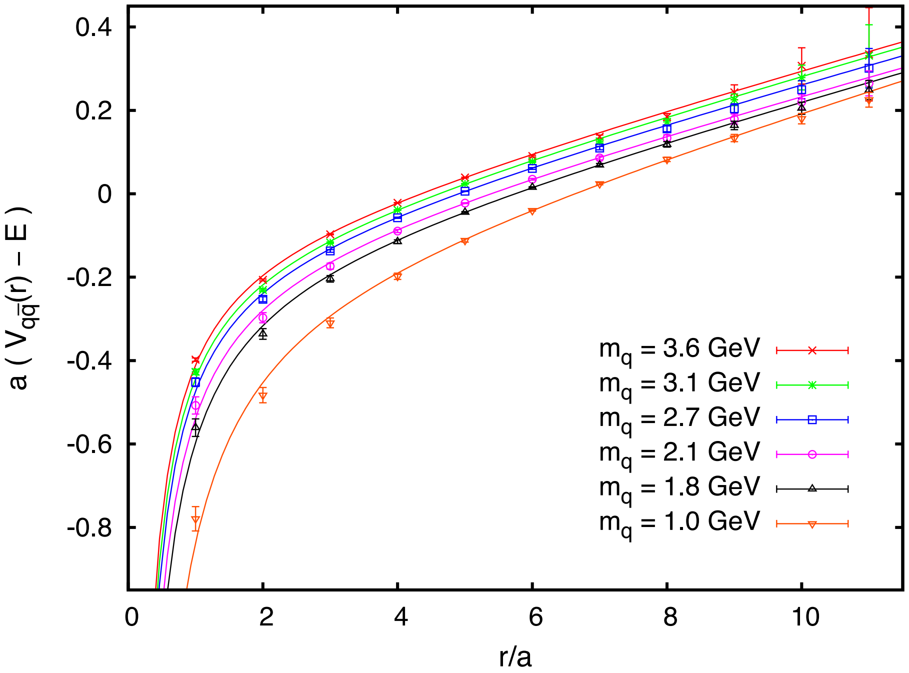
T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)

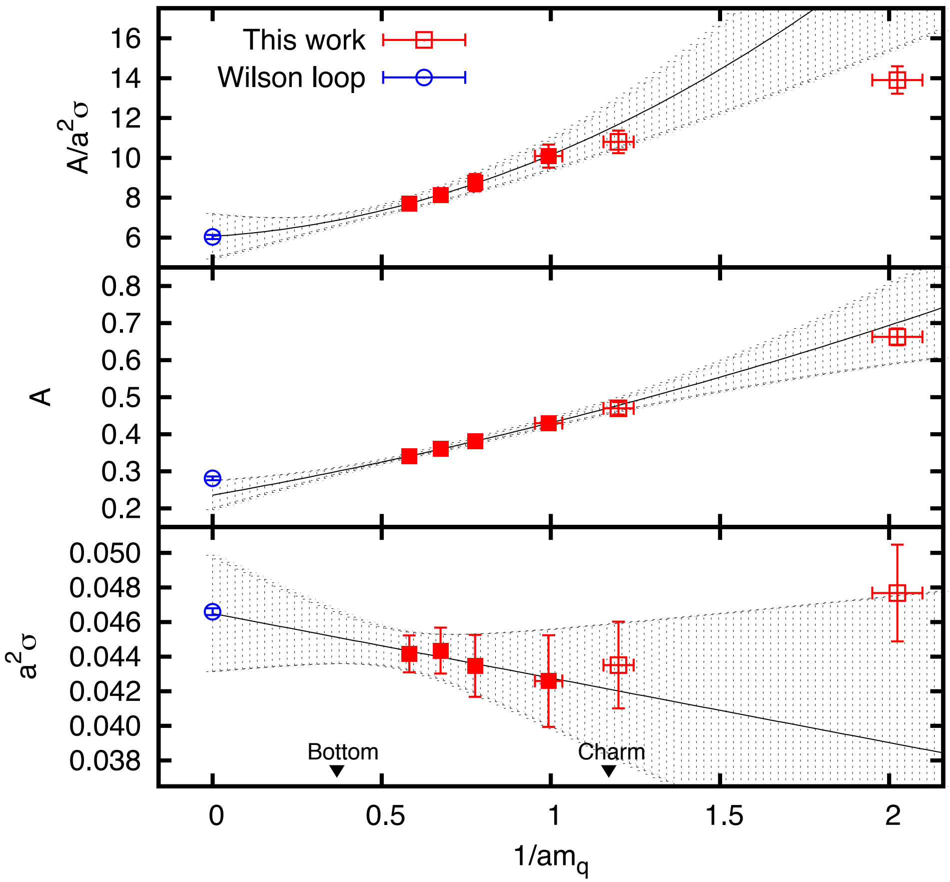
$$m_q = \lim_{r\to\infty} \frac{-1}{\Delta E_{\rm hyp}} \left(\frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} - \frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} \right)$$



spin-independent qq^{bar} potential

T. Kawanai and S. Sasaki, PRL. 107, 091601 (2011)





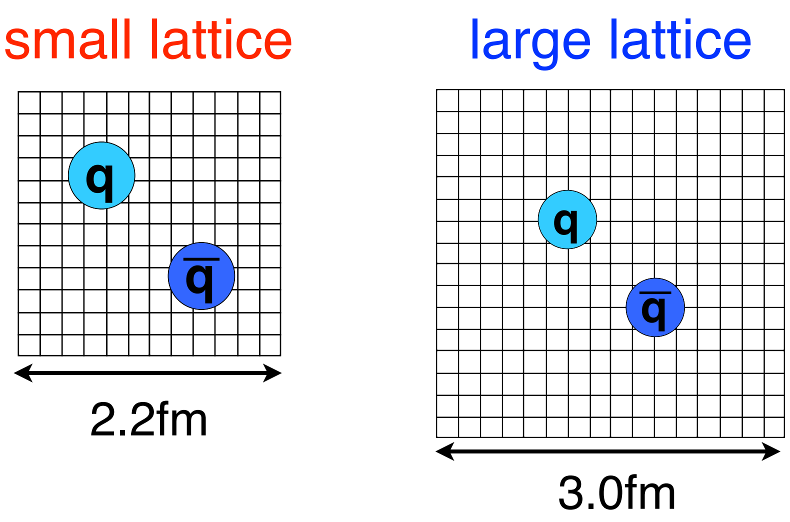
Cornell parameterization

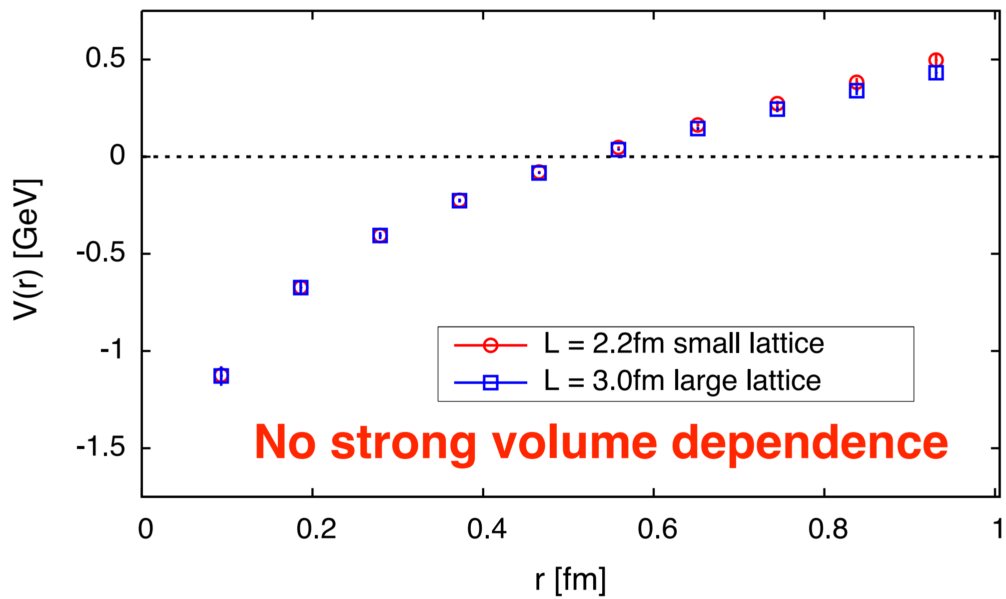
$$V(r) = -\frac{A}{r} + \sigma r + V_0$$

Consistent with the Wilson loops in the $m_q \to \infty$ limit

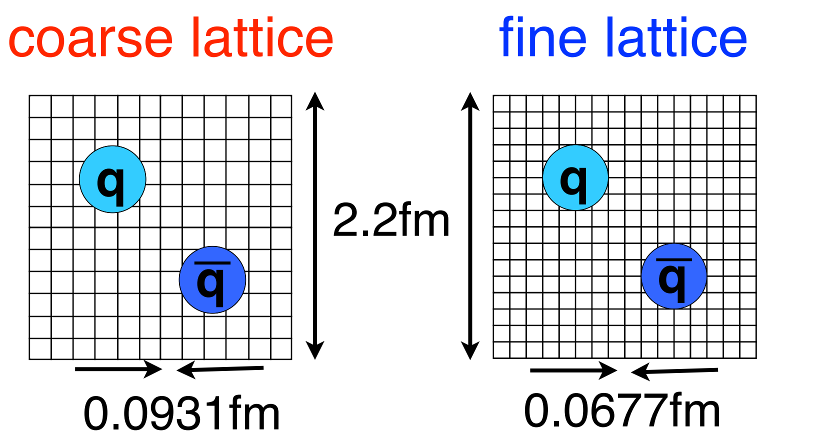
Study of the possible systematic uncertainties

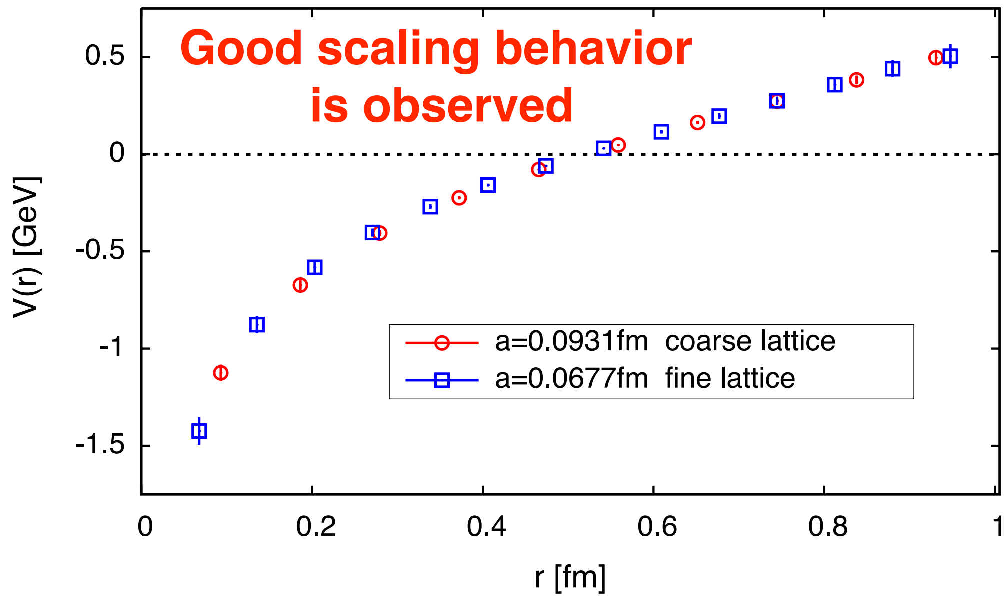
(1) Volume dependence





(2) finite lattice spacing effects

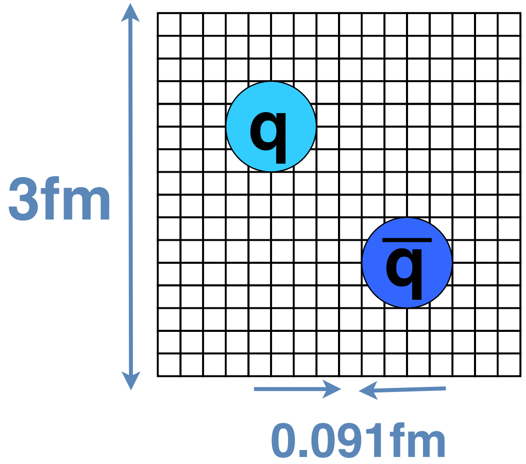




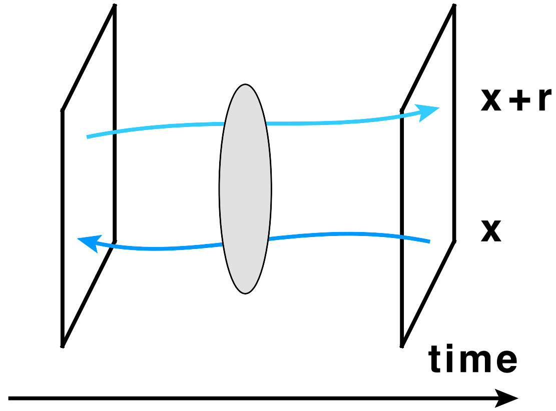
1. Quenched lattice QCD simulation

2. N_f =2+1 dynamical QCD simulation

Lattice Set up

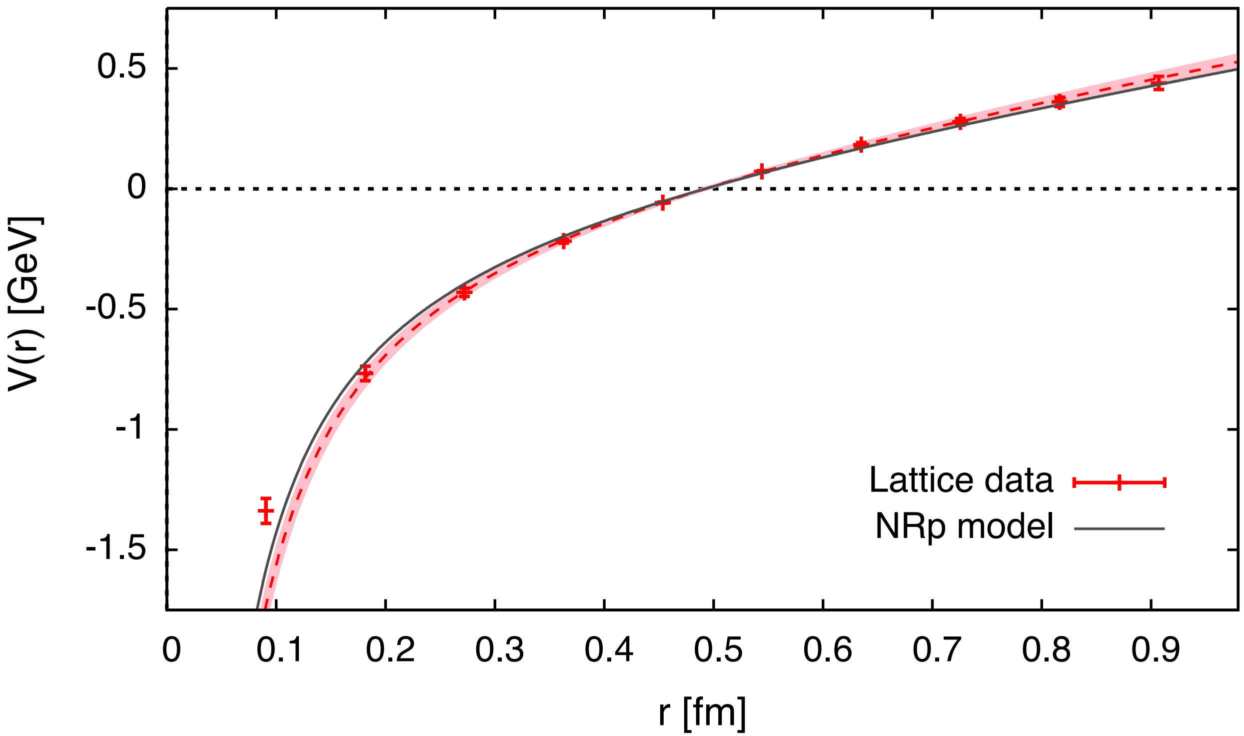


- 2+1 flavor dynamical gauge configurations generated by PACS-CS collaboration.
- Lattice size : $L^3 \times T = 32^3 \times 64$ (~$3fm^3$)
- Iwasaki gauge action β=1.9 (a≈0.091 fm, a^{-1}≈2.3GeV) + RHQ action with partially non-perturbative RHQ parameters.
- Light quark mass : m_π = 156(7) MeV, m_K = 553(2)MeV
  Charm quark mass : m_{ave}(1S) =3.069(2) GeV, m_{hyp}(1S)=111(2) MeV
- Statistics : 198 configs
- Wall source
- Coulomb gauge fixing



spin-independent cc^{bar} potential

T. Kawanai and S. Sasaki, arXiv:1110.0888



	This work	NRp model	Static
A	0.813(22)	0.7281	0.403(24)
$\sqrt{\sigma}$ [GeV]	0.394(7)	0.3775	0.462(4)
m_q [GeV]	1.74(7)	1.4794	∞

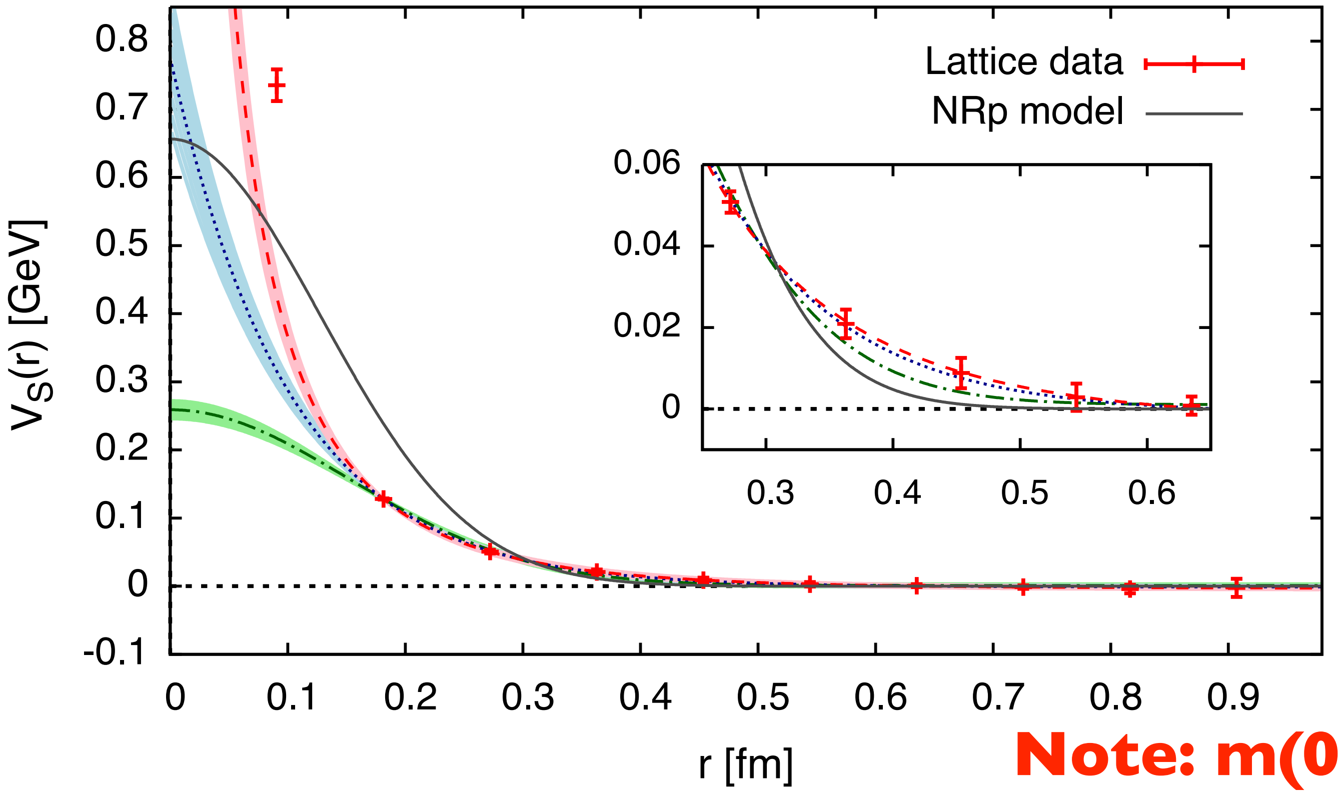
▶ The charmonium potential obtained from the BS wave function resembles the NRp model.

▶ String breaking is not observed

Non-relativistic potential (NRp) model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

spin-spin cc^{bar} potential

T. Kawanai and S. Sasaki, arXiv:1110.0888



Note: m(0$^-$) < m(1$^-$)

- Short range, but non-point like, repulsive interaction
- A difference appears in the spin-spin potential

Fitting function

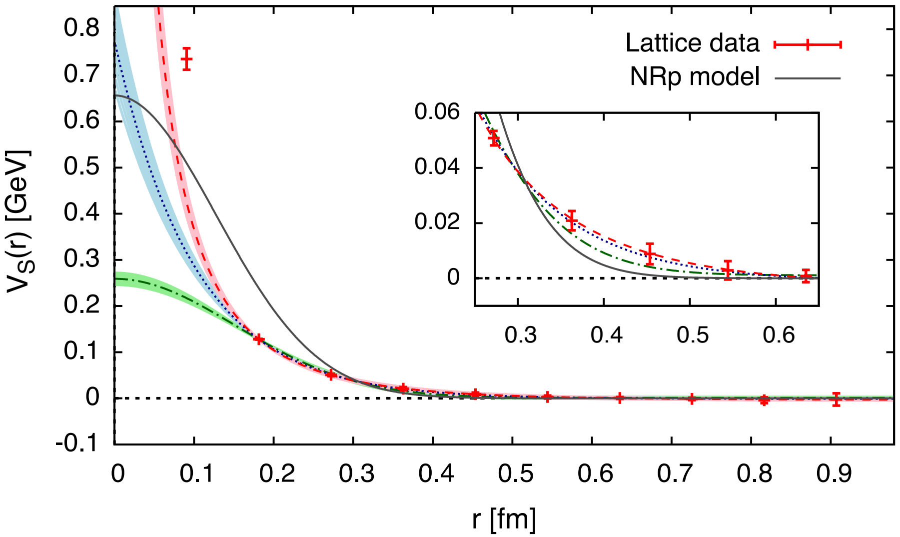
$$V_S(r) = \begin{cases} \alpha \exp(-\beta r)/r \\ \alpha \exp(-\beta r) \\ \alpha \exp(-\beta r^2) \end{cases}$$

	α	β	χ/ d.o.f
Yukawa	0.297(12)	0.982(47) GeV	0.89
exponential	0.866(29) GeV	2.067(37) GeV	0.45
Gaussian	0.309(7) GeV	1.069(17) GeV^2	12.40

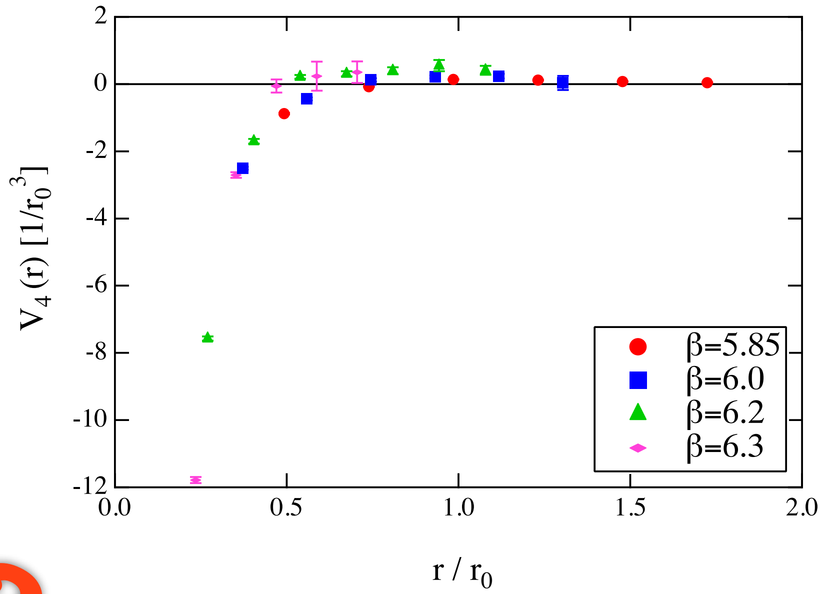
Comment on spin-spin potential

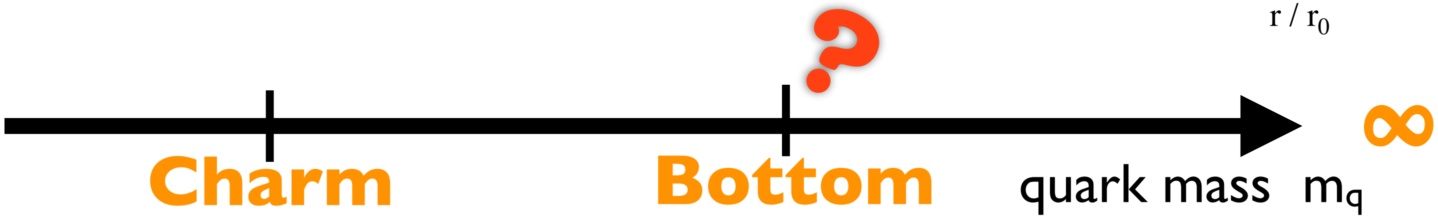
Spin-spin potentials calculated from lattice QCD show the quite different qualitative behavior between our approach and wilson loop approach.

Our approach



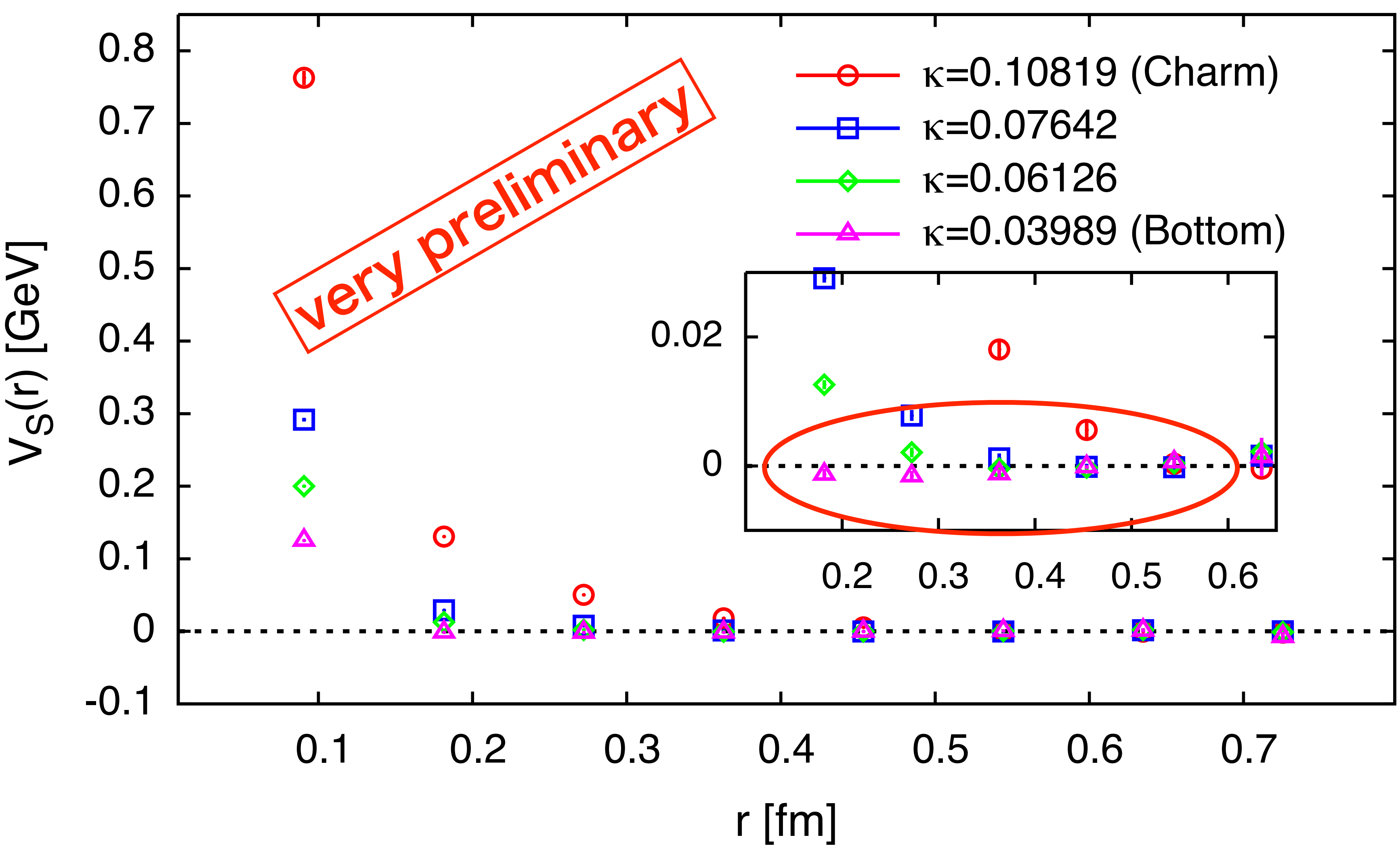
Wilson loop approach





Dose finite quark mass effect change the spin-spin potential from attractive to repulsive ?

spin-spin bb^{bar} potential



Spin-Spin potential have a expected tendency to switch the sign.

Summary

✦ We have derived both the spin-independent and -dependent part of the central $q\bar{q}$ interquark potential from the BS wave function in Quenched QCD simulation and 2+1 flavor dynamical lattice QCD simulation with almost physical quark masses.

✓ spin-independent $q\bar{q}$ potential from BS wave function smoothly approaches the static $q\bar{q}$ potential from Wilson loop.

✓ The spin-independent charmonium potential obtained from the BS wave function resembles the one used in the NRp model.

✓ Spin-spin potential from lattice QCD show the repulsive interaction.

✦ Future perspective

✓ Other spin-dependent potential: tensor and LS force.

✓ To extend the bottomonium (Now under way)