Dalitz Amplitude Analyses of D Decays

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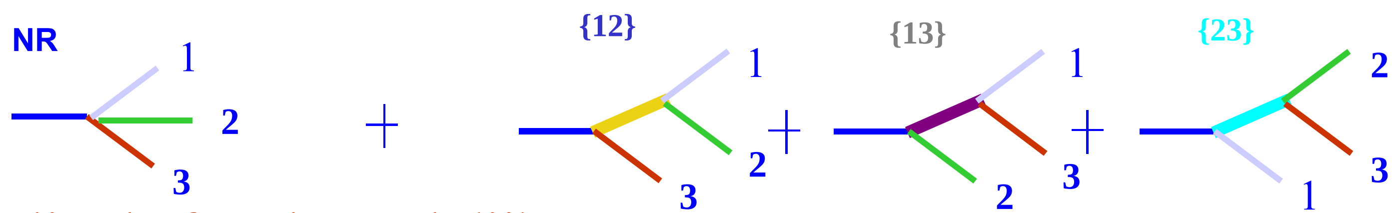
(from the BaBar Collaboration)

Outline of Dalitz Decays Talk

- How many D $\rightarrow$ PPP Dalitz plots are there?
- Early History by example: $D^+ \rightarrow K^-\pi^+\pi^+$ decays
  - Mark III
  - E791
  - FOCUS
- More recent results
  - $D_s^\pm \rightarrow K^+K^-\pi^\pm$ from BaBar, CLEO-c
  - $D_s^\pm \rightarrow \pi^+\pi^-\pi^\pm$ from BaBar, CLEO-c
- CP violation searches using Dalitz plots
  - SCS decays: $D^\pm \rightarrow K^+K^-\pi^\pm$ from CLEO-c, BaBar
  - Use in measurements of γ/ϕ_3
  - Use in D-mixing parameter measurements

Isobar Dalitz Plot Analyses

- The "isobar model", with Breit-Wigner resonant terms, has been widely used in studying 3-body decays of heavy quark mesons.



- Amplitude for channel {ij}:

$$\mathcal{A}_{ij} = d_0 e^{i\delta_0} + \sum_R d_R e^{i\delta_R} F_L^R(p, r_R)\, A_R(s_{ij}) \times F_L^D(q, r_D)\, M_L(p, q)$$

NR Constant — R form factor — D form factor — spin factor

- Each resonance "**R**" (mass M_R, width Γ_R) assumed to have form

$$A_R(s_{ij}) = \left[m_R^2 - s_{ij} - i m_R \Gamma(p, r_R)\right]^{-1}$$

p, q are momenta in ij rest frame

$r_D,\ r_R$ meson radii

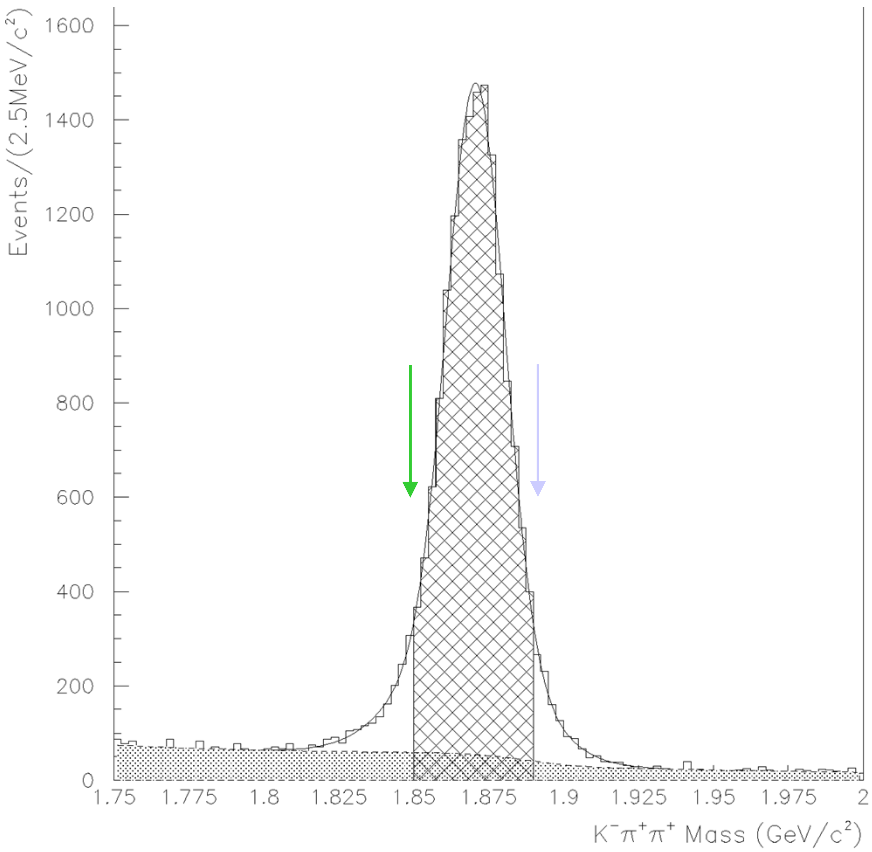
Introduction to $D^+ \rightarrow K^-\pi^+\pi^+$ decays

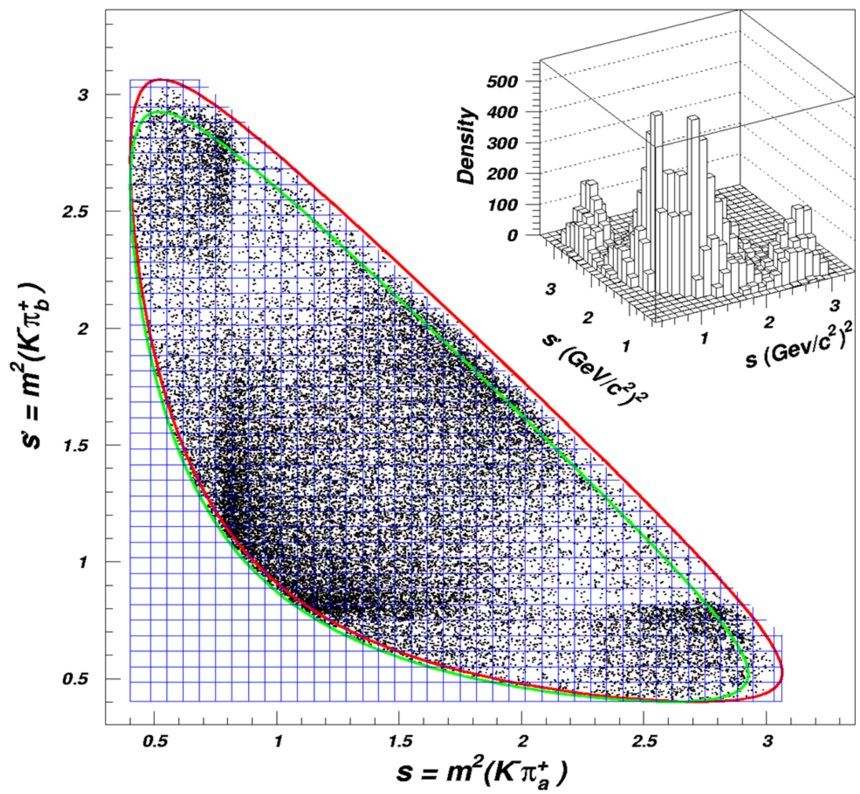
- In 1981 Mark II studied this Dalitz plot and observed "non-uniform" density.
- In 1987 Mark III published a Dalitz analysis
- In 1993, E691 confirmed the main features:
  - a strong non-resonant amplitude and
  - a poor fit.
- In 1988, LASS published $K^-\pi^+$ scattering results: D. Aston *et al.*, Nucl. Phys. **B296**, 493 (1988).
- Ca. 1996, Bill Dunwoodie suggested a model-independent way to compare to LASS results.

Introduction to $D^+ \to K^-\pi^+\pi^+$ decays, contd.

- E791 found evidence for a $\sigma(500)$ in $D^+ \to \pi^-\pi^+\pi^+$ decays [E. M. Aitala *et al.*, Phys. Rev. Lett. **86**, 770 (2001)].
- E791 found evidence for a $\kappa(500)$ in $D^+ \to K^-\pi^+\pi^+$ decays [E. M. Aitala *et al.*, Phys. Rev. Lett. **89**, 121 801 (2002)].
- A new MIPWA analysis was done by Brian Meadows et al., (E791) and published as E. M. Aitala *et al.*, Phys. Rev. D 73, 032004 (2006).
- FOCUS found an acceptable fit using a *K*-matrix description of the *S*-wave with no $\sigma(500)$ pole [J.M. Link *et al.*, Phys. Lett. B**585**, 200 (2004).].

$E791\ D^+ \rightarrow K^-\pi^+\pi^+$





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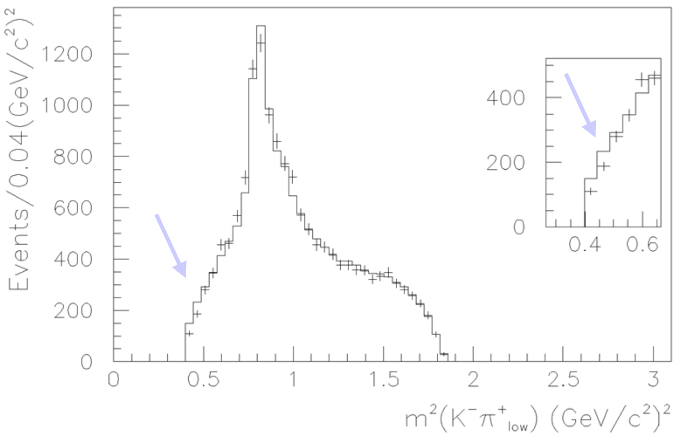
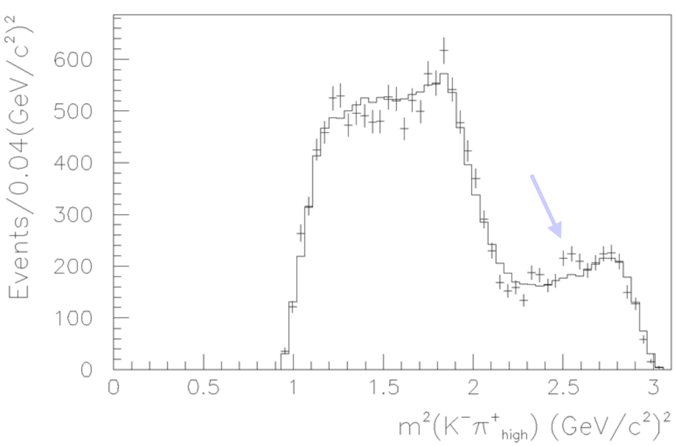
E791 $D^+ \to K^-\pi^+\pi^+$





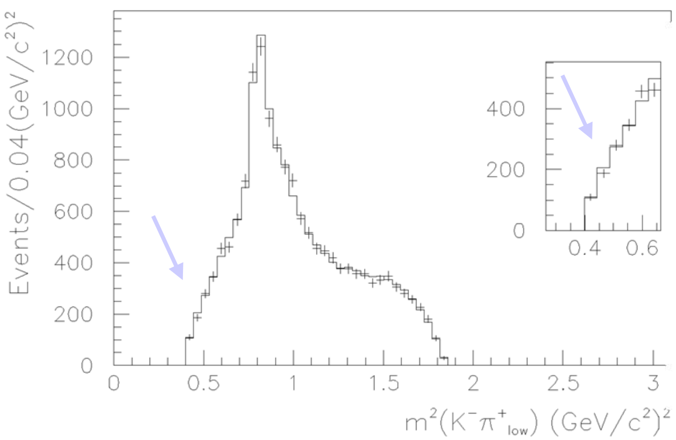
D_s^+ →

non resonant	90.0±2.6%	0° (fixed)
$K^*(890)\pi^+$	13.8±0.5%	54±2°
$K_0^*(1430)\pi^+$	30.6±1.6%	109±2°
$K_2^*(1430)\pi^+$	0.4±0.1%	33±8°
$K_1^*(1680)\pi^+$	3.2±0.3%	66±3°
Total	~138 %	

χ^2/d.o.f. = 2.7

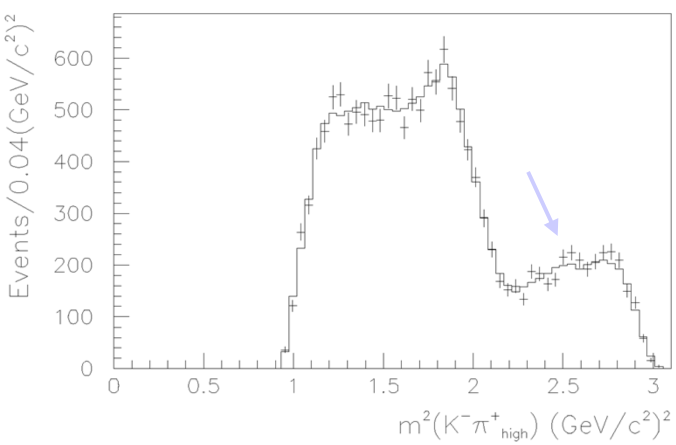
Flat "NR" term does not give good description of data.

E791 κ Model for S-wave



D^+ →

non resonant	13.0±5.8±2.6%	349±14±8°
"κ"π^+	47.8±12.1±3.7%	187±8±17°
$K^*(890)\pi^+$	12.3±1.0±0.9%	0° (fixed)
$K_0^*(1430)\pi^+$	12.5±1.4±0.4%	48±7±10°
$K_2^*(1430)\pi^+$	0.5±0.1±0.2%	306±8±6°
$K_1^*(1680)\pi^+$	2.5±0.7±0.2%	28±13±15°
Total	~89 %	



E. Aitala, et al, PRL 89 121801 (2002)

χ^2/d.o.f. = 0.73

(95 %) ← Probability

M_κ = 797 ± 19 ± 42 MeV/c^2

Γ_κ = 410 ± 43 ± 85 MeV/c^2

Model-Independent Partial-Wave Analysis

- Make partial-wave expansion of decay amplitude in angular momentum L of produced $K^-\pi^+$ system

$$\mathcal{A}_{ij} = \sum_{L=0}^{2} C_L(s_{K\pi}) \times F_L^D(q, r_D) \times (-2pq)^L P_L(\cos\theta)$$

D form factor

spin factor

- $C_L(s_{K\pi})$ describes scattering of produced $K^-\pi^+$.
  - Related to amplitudes $T_L(s_{K\pi})$ measured by LASS

Model-Independent Partial-Wave Analysis, continued

- Define S–wave amplitude at discrete points $s_{K\pi}=s_j$. Interpolate elsewhere.

$$S(s_j) \equiv C_0(s_j) = c_j e^{i\gamma_j}$$

→ model-independent - two parameters (c_j, γ_j) per point

- P- and D-waves are defined by known K^* resonances

$$\begin{aligned} P(s_{K\pi}) &\equiv C_1(s_{K\pi}) \\ &= F_1^R(p, r_R)\left[BW_{890}(s_{K\pi}) + d_{1680}e^{i\delta_{1680}}\ BW_{1680}(s_{K\pi})\right] \\ D(s_{K\pi}) &\equiv C_2(s_{K\pi}) \\ &= d_{1430}e^{i\delta_{1430}} F_2^R(p, r_R)\ BW_{1430}(s_{K\pi}) \end{aligned}$$

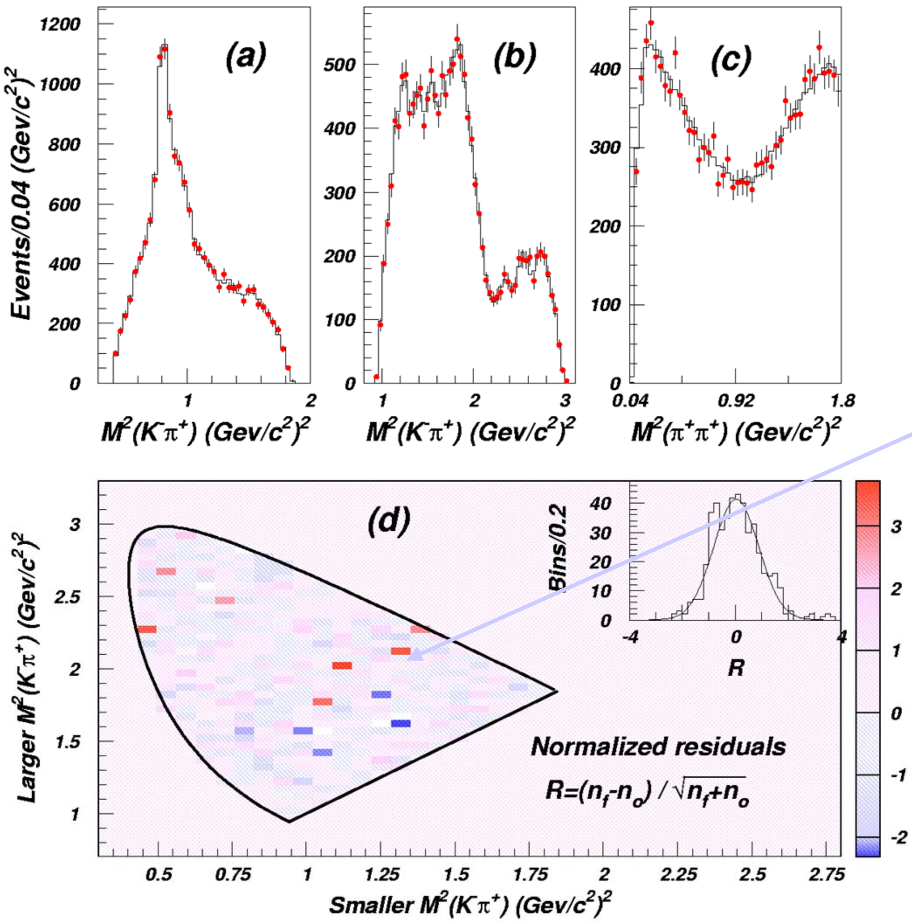
and act as analyzers for the S-wave.

Model-Independent Partial-Wave Analysis, continued

- Phases are relative to $K^*(890)$ resonance.
- Un-binned maximum likelihood fit:
  - Use 40 (c_j, γ_j) points for S
  - Float complex coefficients of $K^*(1680)$ and $K_2^*(1430)$ resonances
    - 4 parameters $(d_{1680}, \delta_{1680})$ and $(d_{1430}, \delta_{1430})$

! 40 x 2 + 4 = 84 free parameters.

Comparison with Data: Mass Distributions



χ^2/NDF = 272/277 (48%)

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Comparison with $K^-\pi^+$ Scattering (LASS)

- S ($s_{K\pi}$) is related to $K^-\pi^+$ scattering amplitude T ($s_{K\pi}$) :

2-body phase space — Production factor for $K\pi$ system — Measured by LASS

$$S(s_{K\pi})F_D^0 = \frac{\sqrt{s_{K\pi}}}{p} \times \Theta_0(s_{K\pi}) \times T(s_{K\pi})$$

- In elastic scattering $K^-\pi^+ \rightarrow K^-\pi^+$ the amplitude is unitary

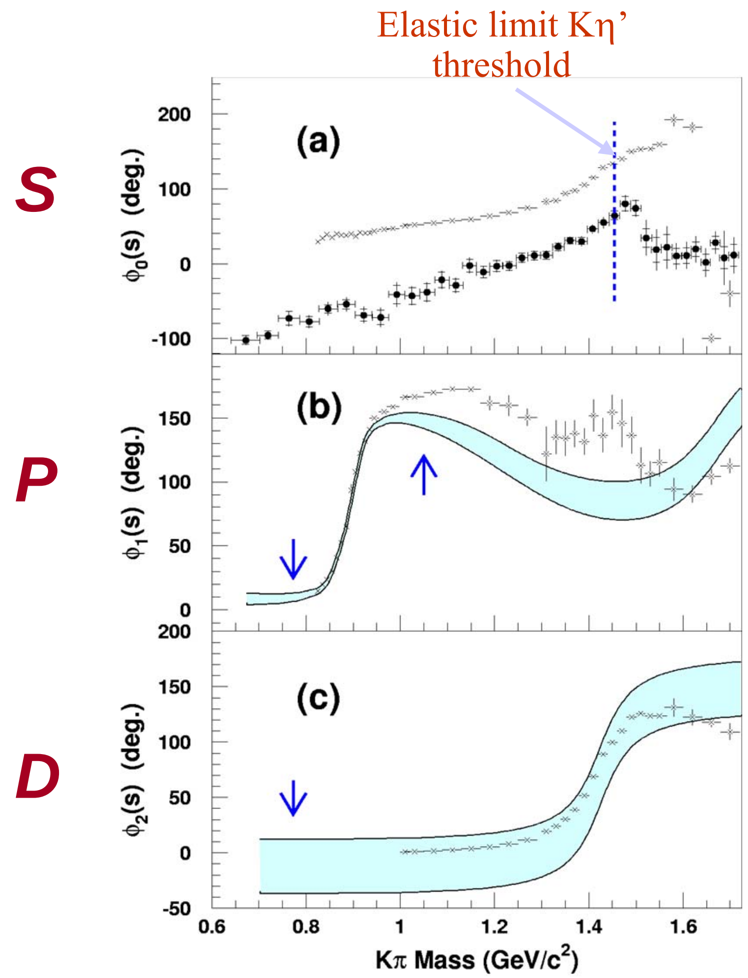
$$T(s_{K\pi}) = e^{i[\gamma(s_{K\pi})-\gamma_0]}\sin[\gamma(s_{K\pi})-\gamma_0]$$

- Watson theorem requires that $\Theta_0(s_{K\pi})$ be real:
  - Phase of $T_L(s_{K\pi})$ should match that of $C_L(s_{K\pi})$.
  - Applies to each partial wave (L=0, 1, 2, …)

K.M. Watson, Phys. Rev. 88, 1163 (1952)

Watson Theorem - a direct test

- Phases for *S*-, *P*- and *D*-waves are compared with measurements from LASS.
  - S-wave phase ϕ_s for E791 is shifted by -75^0 wrt LASS.
    - ϕ_s energy dependence differs below 1100 MeV/c^2.
  - P-wave phase does not match well above $K^*(892)$
    - Lower arrow is at $K\pi\pi$ threshold
    - Upper arrow at effective limit of elastic scattering observed by LASS.



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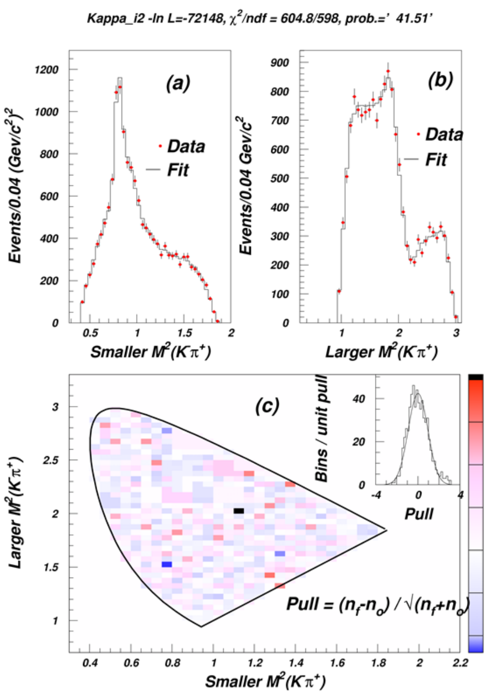
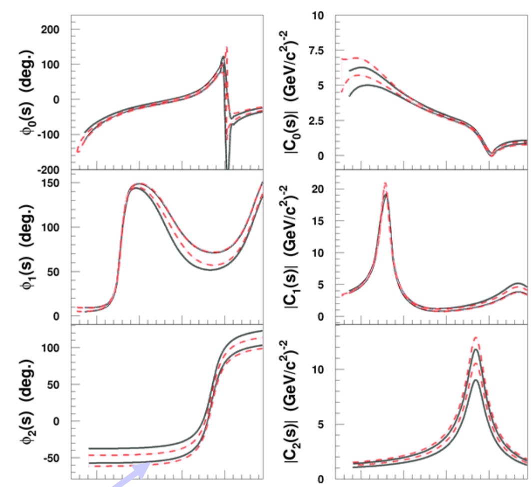
$\pi^+\pi^+$ (I=2) vs. $K^-\pi^+$ S-wave?

- Add $I=2$ amplitude, A_2 to best isobar model fit. An enhancement is known at high mass. (Fit includes a κ isobar):
  - Interpolate phases, $\delta_2(s)$, from Hoogland, *et al.*, *Nucl.Phys.B126:109,1977*
  - Assume amplitude is elastic [$A_2 = a_2 e^{i\alpha_2} \sin\delta_2(s)\, e^{I\delta_2(s)}$]
  - Fit for complex coefficient $a_2 e^{i\alpha_2}$ → Excellent fit



Channel	Fraction %	Amplitude	Phase (degrees)
NR	27.4 ± 8.6	1.58 ± 0.3	-1.1 ± 7.5
$\kappa\pi^+$	32.2 ± 12.7	1.53 ± 0.3	174.6 ± 11.8
$K_0^*(1430)\pi^+$	13.7 ± 1.7	0.58 ± 0.1	50.2 ± 6.4
$K^*(890)\pi^+$	12.2 ± 1.9	1.00 (fixed)	0.0 (fixed)
$K_1^*(1688)\pi^+$	2.7 ± 0.6	2.36 ± 0.4	26.8 ± 8.3
$K_2^*(1420)\pi^+$	0.5 ± 0.2	5.79 ± 0.8	-47.4 ± 9.8
I = 2:	0.7 ± 0.7	a_2 = 1.14 ± 0.61	α_2 = 120.2 ± 27.7



- S-wave $K^-\pi^+$ dominates over I=2
- $K^-\pi^+$ amplitudes and "isobar parameters virtually unchanged

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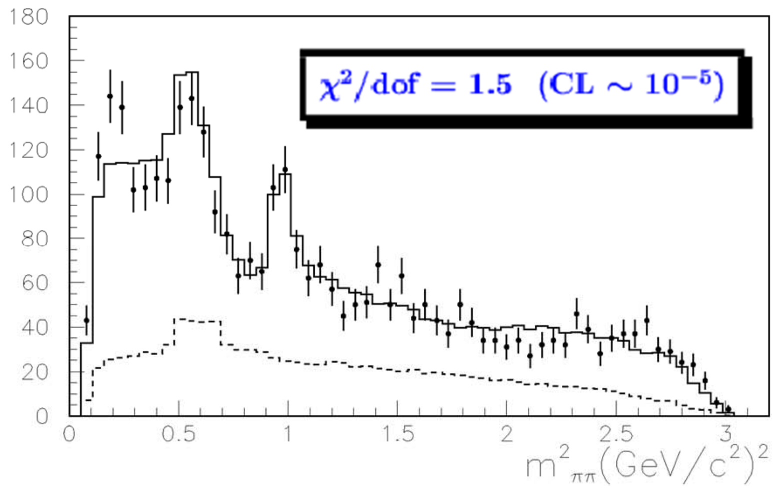
Summary of E791 D^+ to $K^-\pi^+\pi^+$ analysis

- A new technique for analyzing the amplitude describing a Dalitz plot distribution is used in D^+ decays to $K^-\pi^+\pi^+$.
- Could provide model-independent measurements of the complex amplitude of the $K^-\pi^+$ S-wave system, provided a good model for the P- and D-waves is used.
- New measurements for invariant masses below 825 MeV/c^2, down to threshold, are presented.
- No new information on κ(800) from sample this size
- The Watson theorem does not work well with $D^+ \rightarrow K^-\pi^+\pi^+$ decays (or there is an I=3/2 admixture).
- I=2 component not needed by fit.
- Better parameterization of P-wave is needed: perhaps the B-factories can do a model-independent measurement of S, P and D waves using their high-statistics data.

E791 $D^+ \rightarrow \pi^-\pi^+\pi^+$



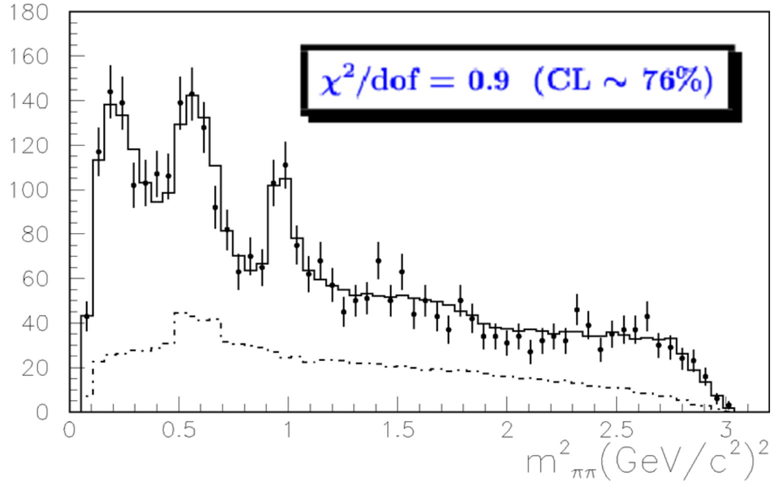
$D^+ \rightarrow$

non resonant	38.6±1.4%	150±12°
$\rho(770)\pi^+$	20.8±2.3%	0° (fixed)
$f_0(980)\pi^+$	7.4±4.3%	152±16°
$f_2(1270)\pi^+$	6.3±3.3%	103±16°
$f_0(1370)\pi^+$	10.7±7.7%	143±10°
$\rho(1450)\pi^+$	22.6±2.1%	46±15°

- Non resonant decay is dominant.
- $\rho(1450)$ and $\rho(770)$ next and equally strong.
- Bad fit quality in low mass $\pi^+\pi^-$ region.

No "σ(500)"

E791 $D^+ \rightarrow \pi^-\pi^+\pi^+$



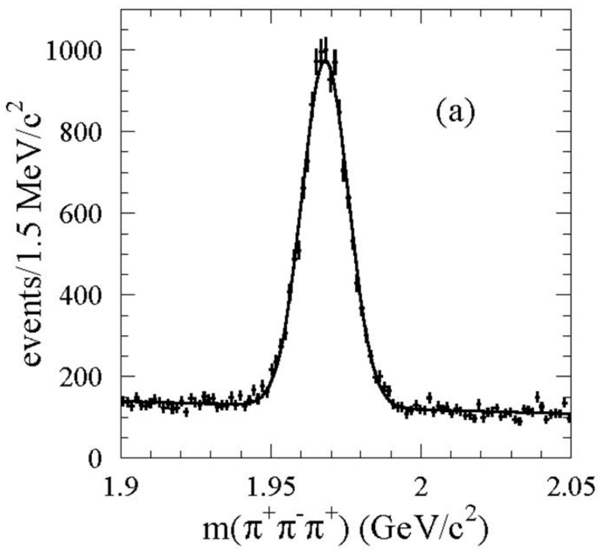
$D^+ \rightarrow$

non resonant	$7.8 \pm 6.0 \pm 2.7\%$	$57 \pm 20 \pm 6°$
$\rho(770)\pi^+$	$33.6 \pm 3.2 \pm 2.2\%$	0° (fixed)
$f_0(980)\pi^+$	$6.2 \pm 1.3 \pm 0.4\%$	$165 \pm 11 \pm 3°$
$f_2(1270)\pi^+$	$19.4 \pm 2.5 \pm 0.4\%$	$57 \pm 8 \pm 3°$
$f_0(1370)\pi^+$	$2.3 \pm 1.5 \pm 0.8\%$	$105 \pm 18 \pm 1°$
$\rho(1450)\pi^+$	$0.7 \pm 0.7 \pm 0.3\%$	$319 \pm 39 \pm 11°$
$\sigma\pi^+$	$46.3 \pm 9.0 \pm 2.1\%$	$206 \pm 8 \pm 5°$

- Model with $\sigma\pi$ has good fit quality.
- $\sigma\pi$ mode dominates the decay - but NR is small.
- $\rho(1450)\pi$ amplitude becomes negligible.

$$m_\sigma = (478^{+24}_{-23} \pm 17)\ \mathrm{MeV}/c^2$$

$$\Gamma_\sigma = (324^{+42}_{-40} \pm 21)\ \mathrm{MeV}/c^2$$



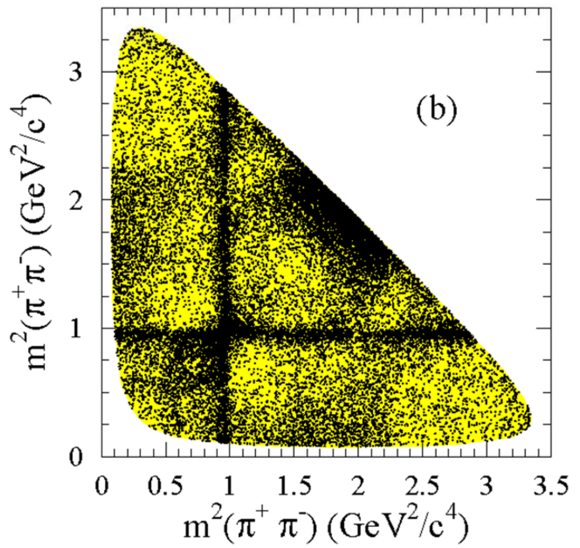


FIG. 1: (a) $\pi^+\pi^-\pi^+$ invariant mass distribution for the D_s^+ analysis sample. The line is the result of the fit described in the text. (b) Symmetrized $D_s^+ \to \pi^+\pi^-\pi^+$ Dalitz plot (two entries per event).

BaBar $D^+ \to \pi^-\pi^+\pi^+$

- B. Aubert *et al*., Phys. Rev. D79 032003 (2009).
- ~ 10,000 events
- Events with $D^0 \to \pi^-\pi^+$ where the D^0 is from D^{*+} decays are removed; same for $K \to \pi$ misid

- Very active Dalitz plot

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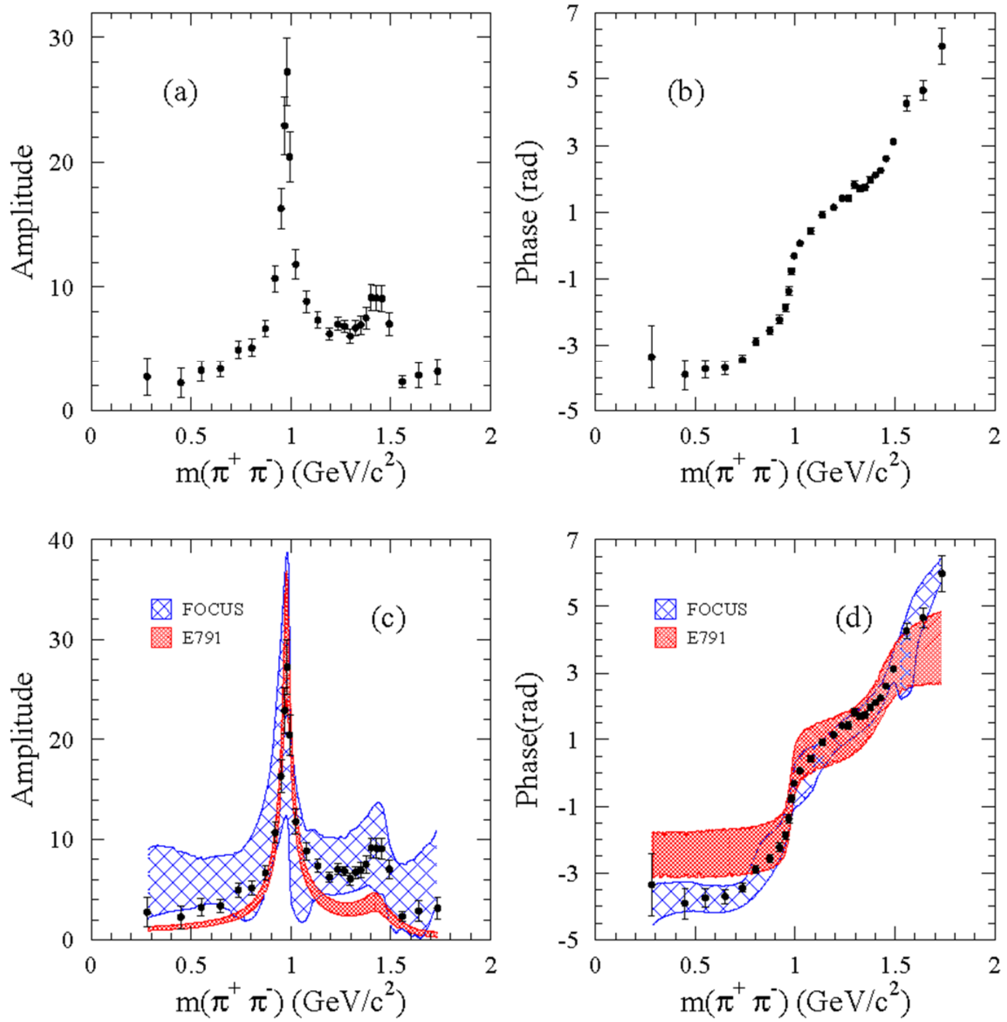
BaBar $D^+ \to \pi^-\pi^+\pi^+$



- 30-point interpolated S-wave amplitude
- Magnitude and Phase plotted vs $m(\pi^-\pi^+)$
- Both are compared to E791, FOCUS

FIG. 2: (a) $\mathcal{S}$-wave amplitude extracted from the best fit, (b) corresponding $\mathcal{S}$-wave phase, (c) $\mathcal{S}$-wave amplitude compared to the FOCUS and E791 amplitudes, (d) $\mathcal{S}$-wave phase compared to the FOCUS and E791 phases. Errors are statistical only.

BaBar $D^+ \to \pi^-\pi^+\pi^+$

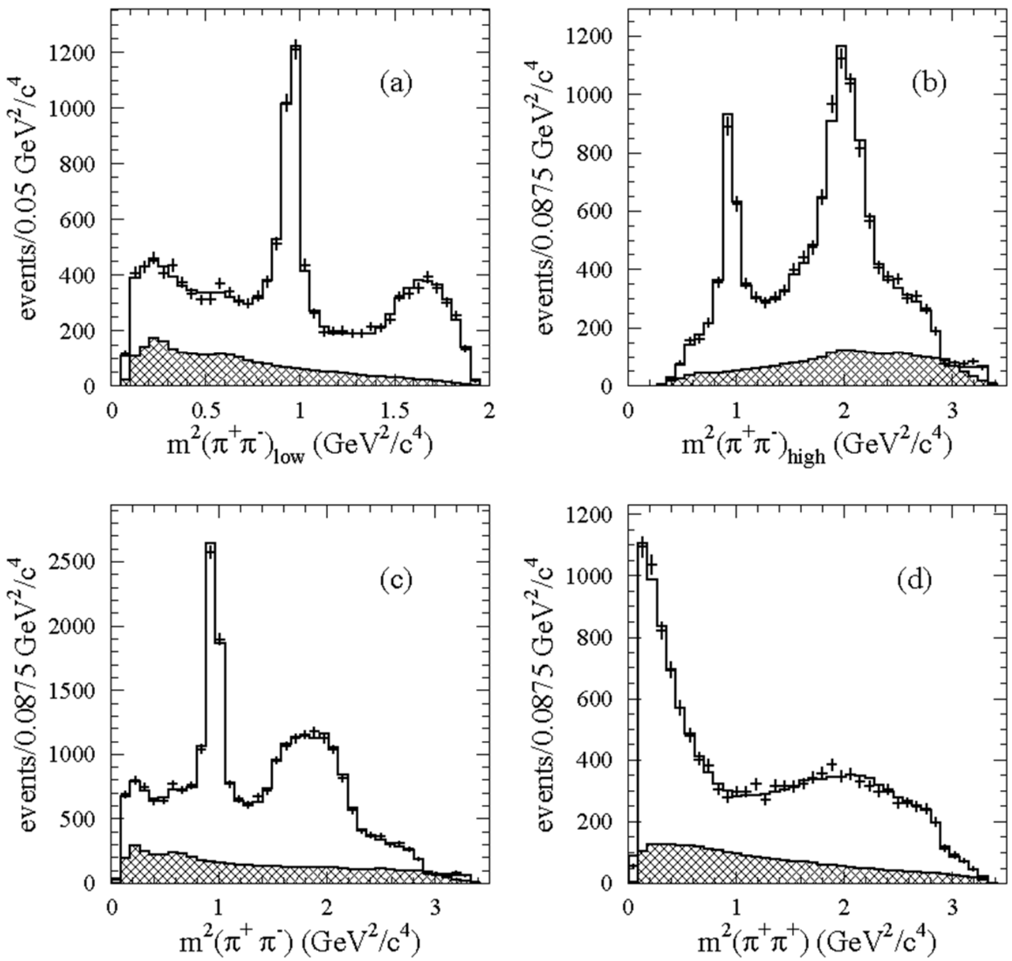


FIG. 3: Dalitz plot projections (points with error bars) and fit results (solid histogram). (a) $m^2(\pi^+\pi^-)_{\text{low}}$, (b) $m^2(\pi^+\pi^-)_{\text{high}}$, (c) total $m^2(\pi^+\pi^-)$, (d) $m^2(\pi^+\pi^+)$. The hatched histograms show the background distribution.

- Good χ^2: $\chi^2/\text{df} = 365/327$
- S-wave peaks at $f_0(980)$, as well as some activity at $f_0(1370)$ and $f_0(1500)$
- Spin-2 resonance $f_2(1270)$ is significant

CLEO $D_s^+ \rightarrow K^-K^+\pi^+$ analysis

R. E. Mitchell et al., Phys .Rev. D79 072008 (2009)

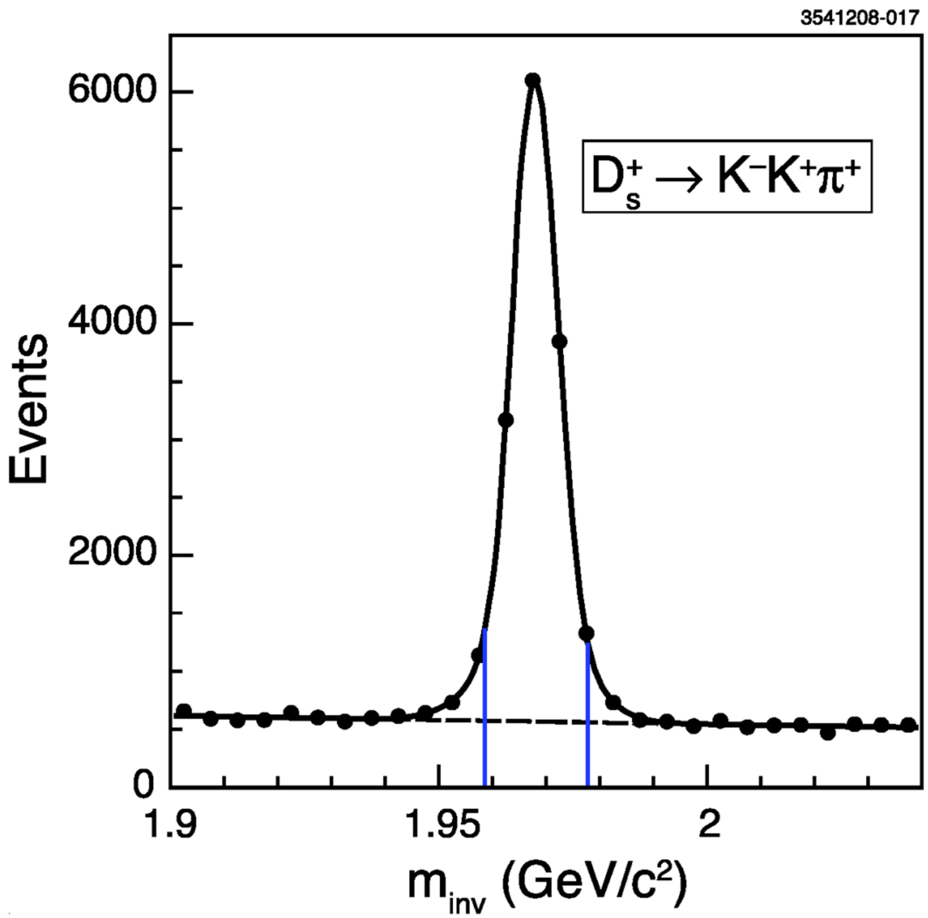


FIG. 1: The m_{inv} distribution. The vertical (online blue) lines show the $\pm 2\sigma$ signal region.

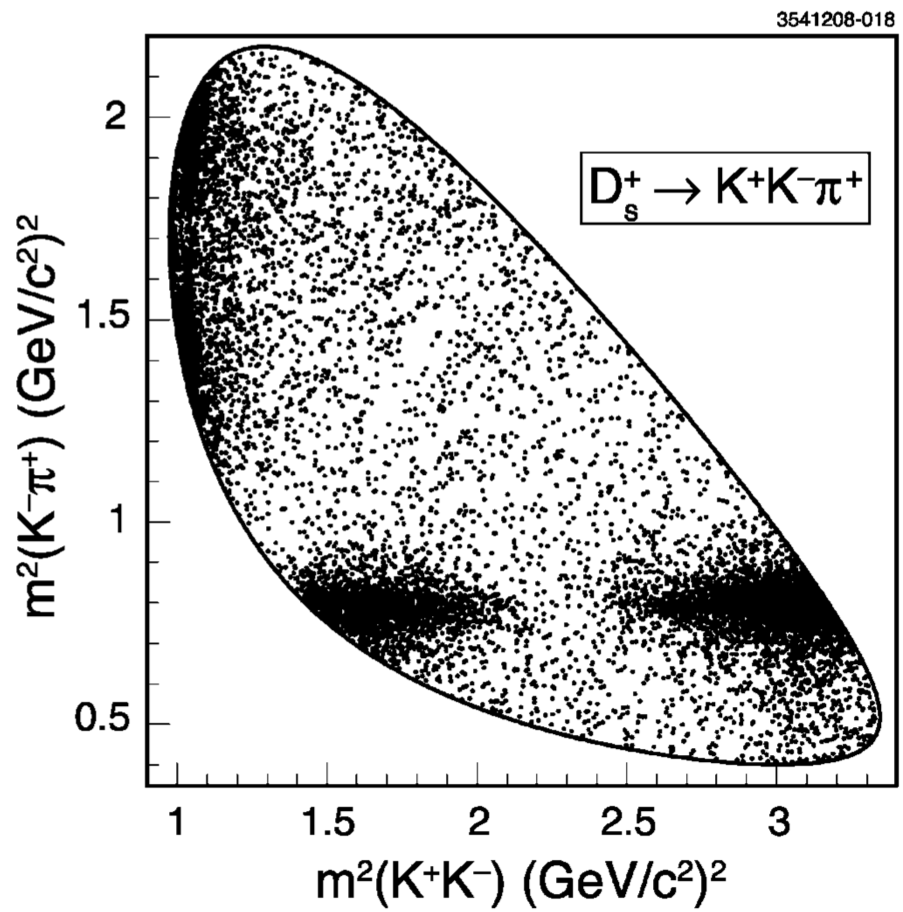


FIG. 2: The Dalitz plot for the data.

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CLEO $D_s^+ \rightarrow K^-K^+\pi^+$ analysis

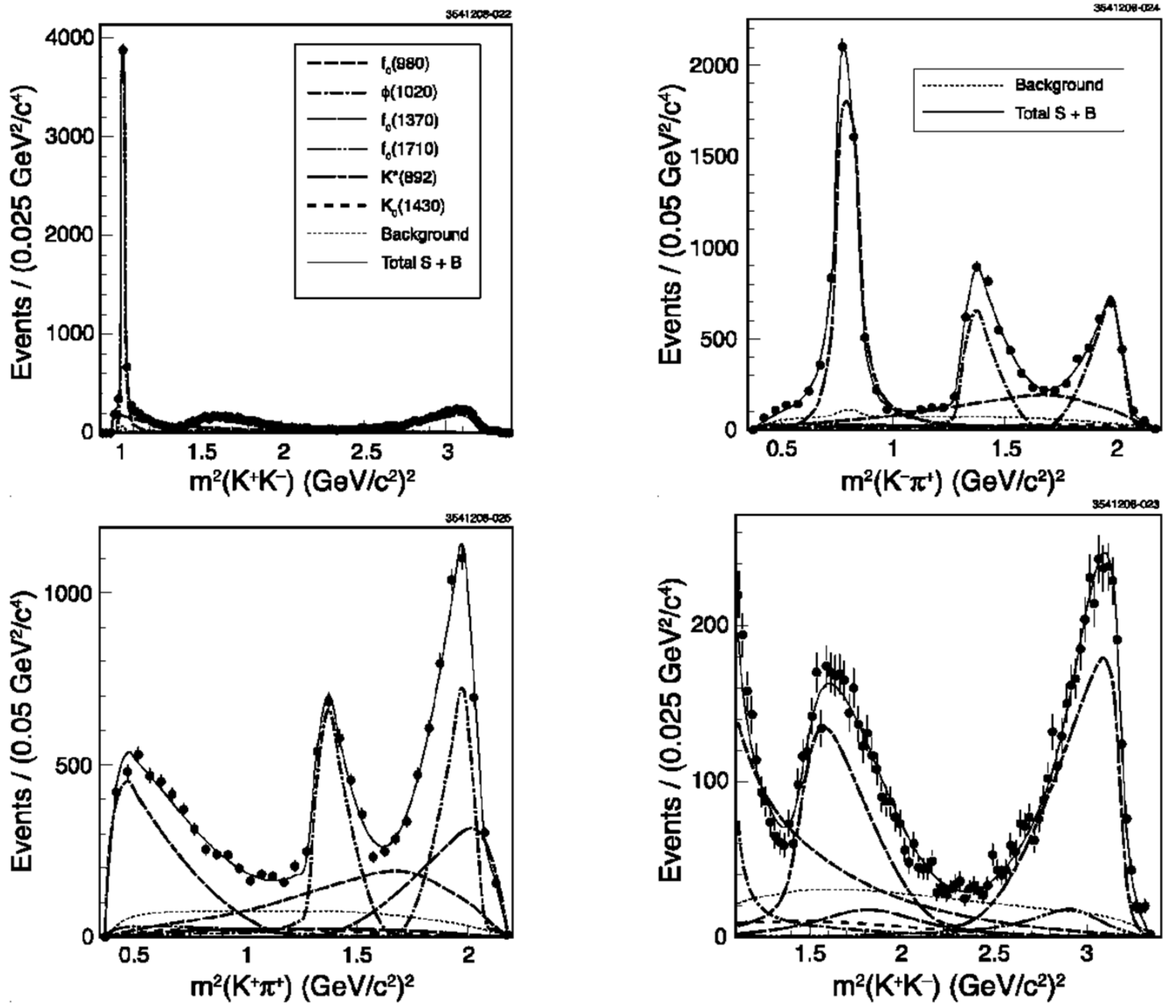

FIG. 4: Fit to data for Model A, and projections of the Dalitz plot. The final plot shows the $m^2(KK)$ projection of Dalitz plot for values of $m^2(KK)$ larger than the contribution from the $\phi(1020)$.

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CLEO $D_s^+ \to K^- K^+ \pi^+$ results

- Fit confirms E687 resonances:

  $K^*(892)$, $K_0^*(1430)$, $f_0(980)$, $\phi(1020)$

- Fit adds $f_0(1370)$ and $f_0(1710)$ at the ~4% and ~3% levels, respectively

- $\chi^2/\mathrm{df} = 178/117$

BaBar $D_s^+ \rightarrow K^-K^+\pi^+$ analysis
B. Aubert et al., Phys. Rev. D83, 052001 (2011)

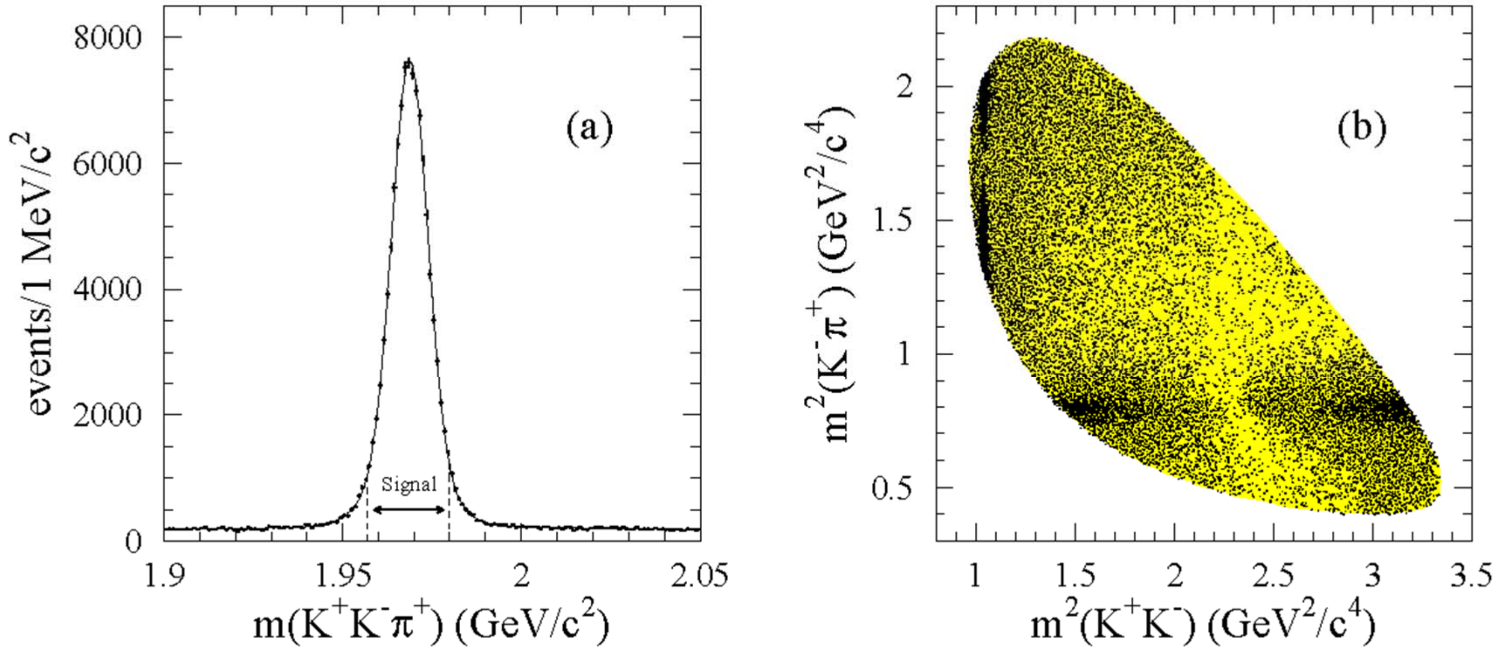


FIG. 2: (a) $K^+K^-\pi^+$ mass distribution for the D_s^+ analysis sample; the signal region is as indicated. (b) $D_s^+ \to K^+K^-\pi^+$ Dalitz plot.

BaBar $D_s^+ \to K^-K^+\pi^+$ analysis

- Using moments of Legendre polynomials of helicity angles, BaBar extracted the S, P magnitudes and relative phase in the region $(0.99 < m_{K^-K^+} < 1.15)$ GeV/c^2
- The P-wave is essentially pure $\phi(1020)$, and is therefore described as such to extract a binned S-wave amplitude.

BaBar $D_s^+ \to K^-K^+\pi^+$ S-wave intensities, compared

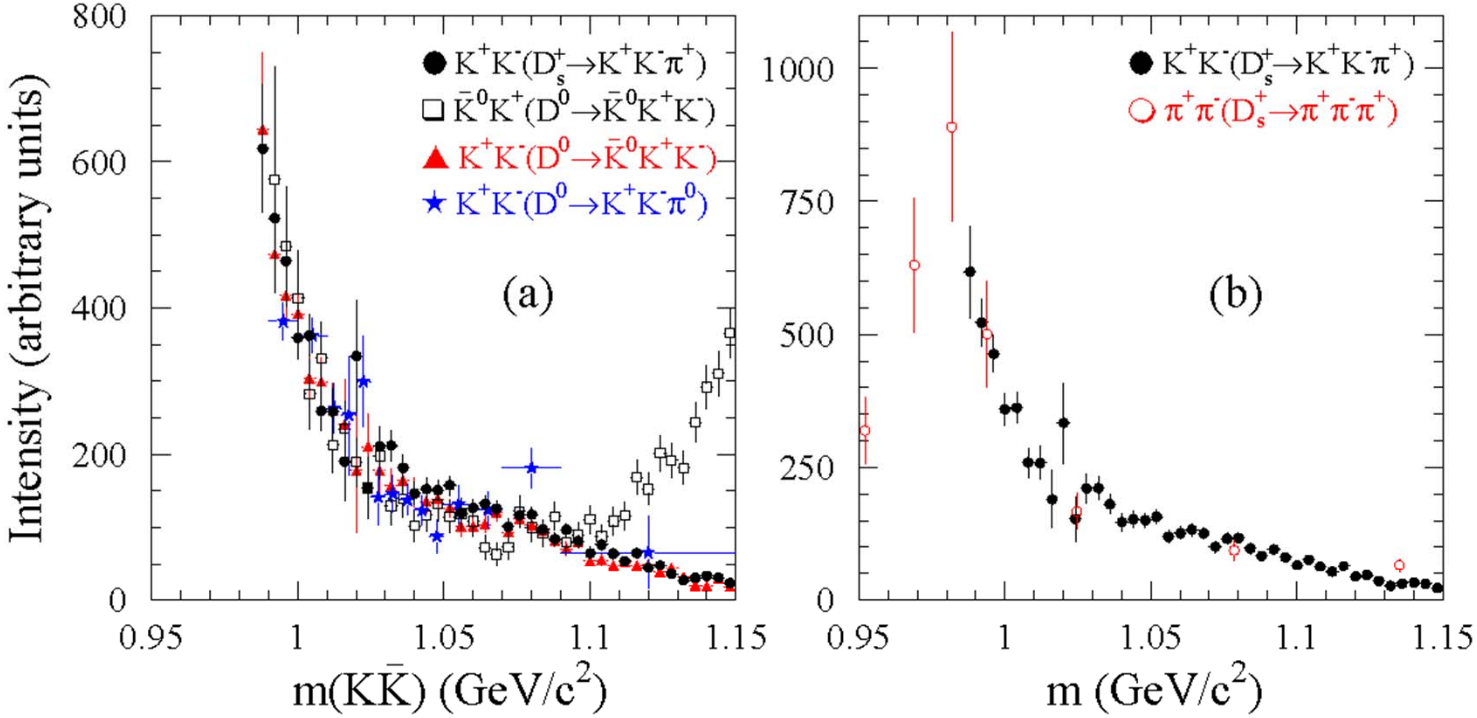


FIG. 6: (a) Comparison between $K\overline{K}$ S-wave intensities from different charmed meson Dalitz plot analyses. (b) Comparison of the $K\overline{K}$ S-wave intensity from $D_s^+ \to K^+K^-\pi^+$ with the $\pi^+\pi^-$ S-wave intensity from $D_s^+ \to \pi^+\pi^-\pi^+$.

BaBar $D_s^+ \to K^- K^+ \pi^+$ results

- BaBar performs a MIPWA S-wave analysis
- Fit confirms E687, CLEO-c resonances:

  K*(892), f_0(980), ϕ(1020), K_0*(1430),

  f_0(1370) and f_0(1710)
- BaBar result dominated by first 3 of above

TABLE VII: Comparison of the fitted decay fractions with the Dalitz plot analyses performed by E687 and CLEO-c collaborations.

Decay mode	Decay fraction (%) BABAR	E687	CLEO-c
$\overline{K}^*(892)^0 K^+$	$47.9 \pm 0.5 \pm 0.5$	$47.8 \pm 4.6 \pm 4.0$	$47.4 \pm 1.5 \pm 0.4$
$\phi(1020)\,\pi^+$	$41.4 \pm 0.8 \pm 0.5$	$39.6 \pm 3.3 \pm 4.7$	$42.2 \pm 1.6 \pm 0.3$
$f_0(980)\,\pi^+$	$16.4 \pm 0.7 \pm 2.0$	$11.0 \pm 3.5 \pm 2.6$	$28.2 \pm 1.9 \pm 1.8$
$\overline{K}_0^*(1430)^0 K^+$	$2.4 \pm 0.3 \pm 1.0$	$9.3 \pm 3.2 \pm 3.2$	$3.9 \pm 0.5 \pm 0.5$
$f_0(1710)\,\pi^+$	$1.1 \pm 0.1 \pm 0.1$	$3.4 \pm 2.3 \pm 3.5$	$3.4 \pm 0.5 \pm 0.3$
$f_0(1370)\,\pi^+$	$1.1 \pm 0.1 \pm 0.2$	—	$4.3 \pm 0.6 \pm 0.5$
Sum	$110.2 \pm 0.6 \pm 2.0$	111.1	$129.5 \pm 4.4 \pm 2.0$
χ^2/NDF	$\frac{2843}{(2305-14)} = 1.2$	$\frac{50.2}{33} = 1.5$	$\frac{178}{117} = 1.5$
Events	96307 ± 369	701 ± 36	12226 ± 22

Searching for CP Violation in SCS Charm Decays: $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$

- CP violation in charm is expected to manifest itself in Singly Cabibbo-suppressed (SCS) decays.
- Within the Standard Model, one expects CP violation asymmetries in SCS Decays $\sim 10^{-3}$

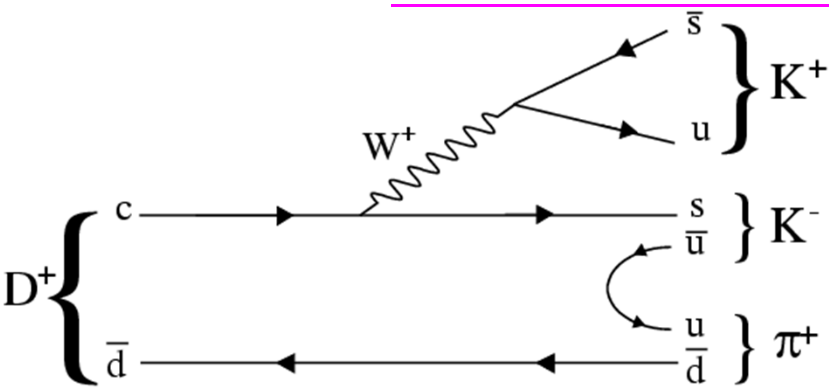
  [B. Bhattacharya, M. Gronau, and J. Rosner, arXiv:1201.2351,

  H. Cheng and C. Chiang, arXiv:1201.0785].
- New Physics can give CP violation asymmetries $\sim 10^{-2}$.

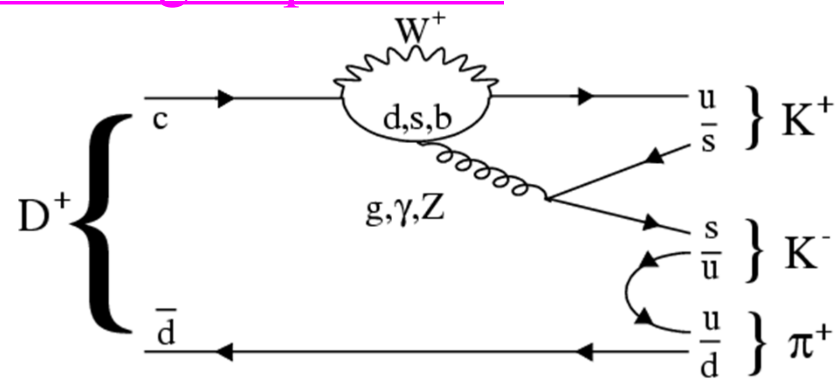
  [I. I. Bigi, A. I. Sanda, CP Violation, Cambridge (2000).

  I. I. Bigi, arXiv:hep-ph/0107102v1 (2001)].

Standard Model interfering amplitudes:



$A \sim V_{cs}^{*} V_{us}$



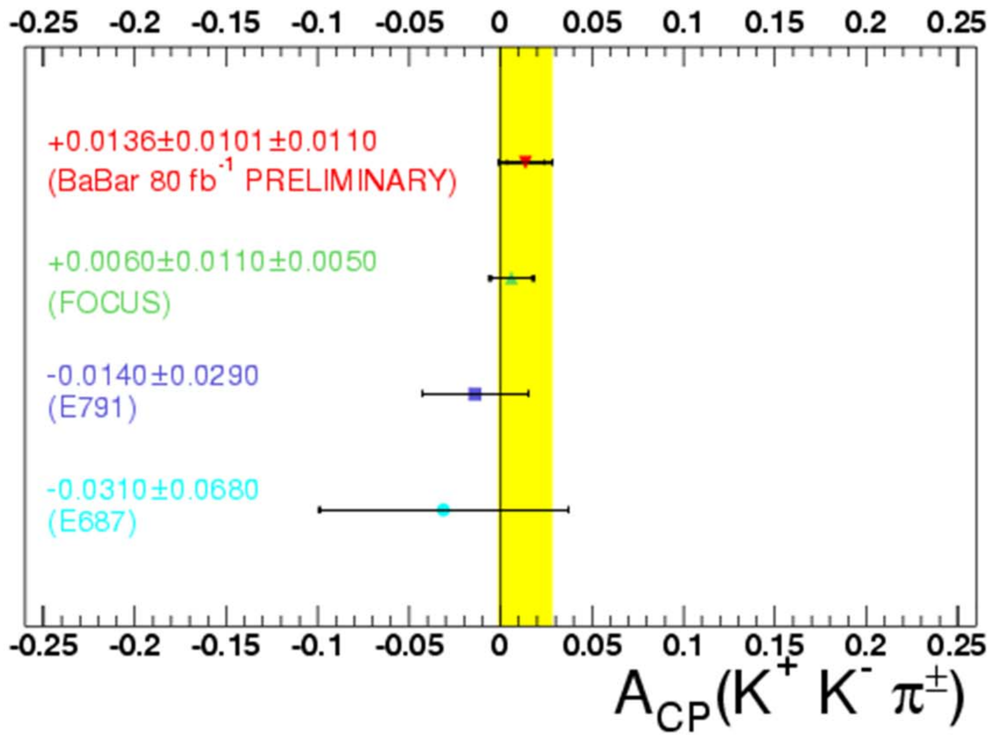
$A \sim V_{cs}^{*} V_{us}, V_{cd}^{*} V_{ud}, V_{cb}^{*} V_{ub}$

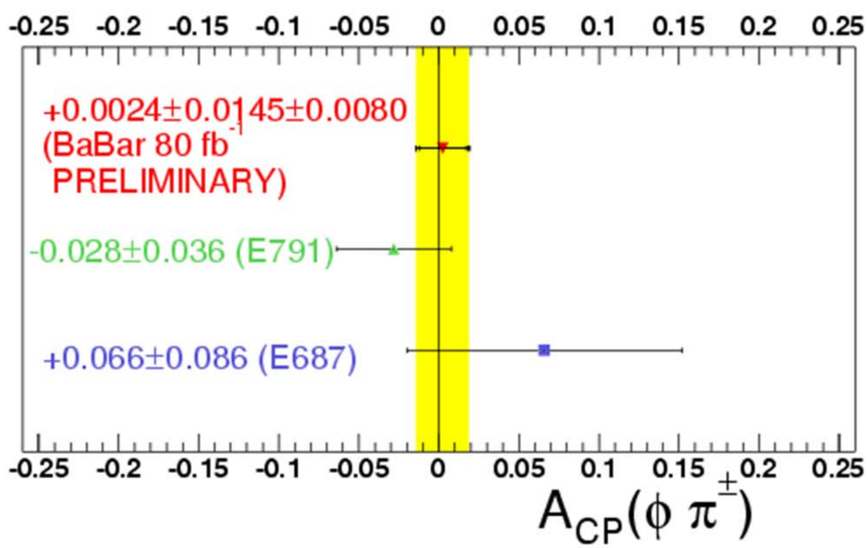
•In the Wolfenstein parameterization, the CKM matrix (below) clearly gives only a small CP violating asymmetry.

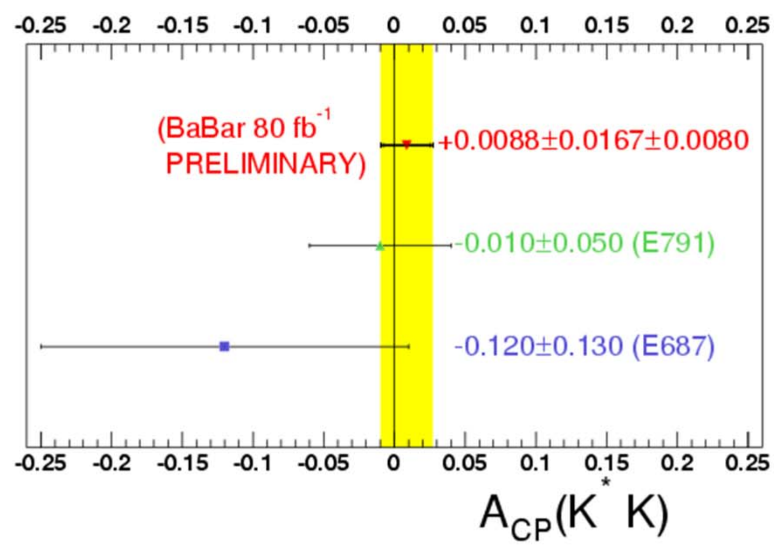
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad \begin{pmatrix} \text{where } \lambda \sim 0.22, \\ \text{and } A \sim 0.86 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Older A_{CP} Results, ca. 2005







CLEO-c $D^\pm \rightarrow K^+K^-\pi^\pm$

- P. Rubin *et al.*, Phys. Rev. D 78, 072003 (2008).

- Three different fits to the Dalitz plot.

- FitB is best

- D^+ and D^- have magnitudes $a_r + b_r$ and $a_r - b_r$, they have phases $\delta_r + \phi_r$ and $\delta_r - \phi_r$

- A simultaneous fit is done

CLEO-c $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$

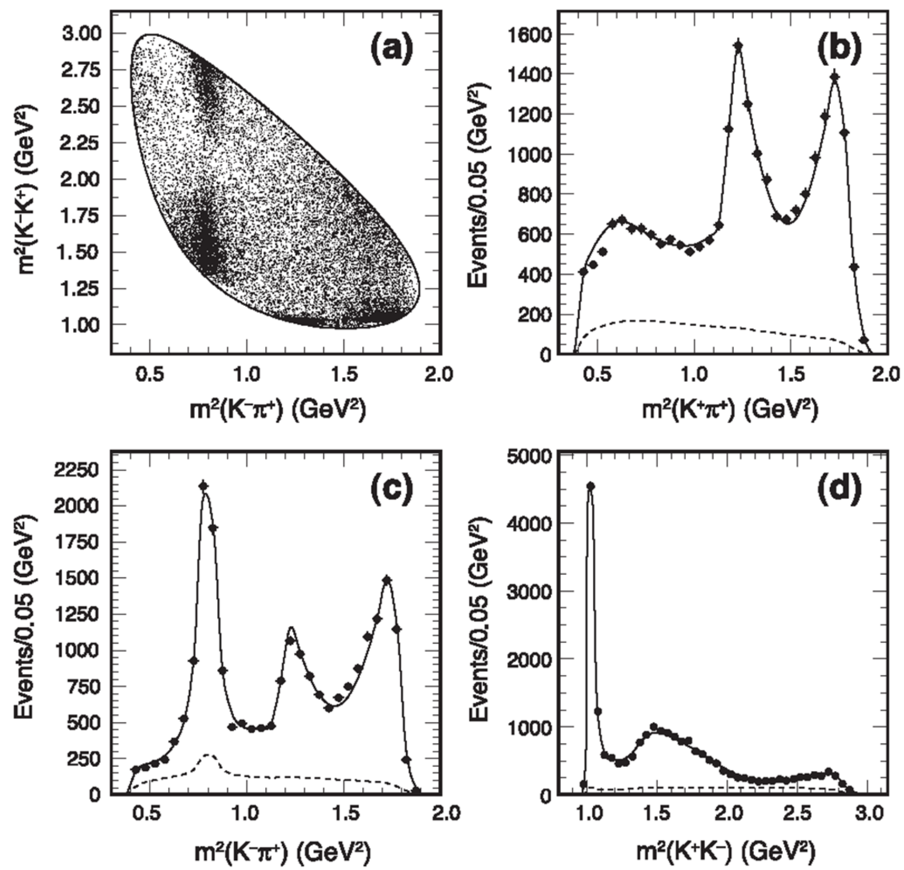


FIG. 3. (a) The Dalitz plot for $D^{+} \rightarrow K^{+}K^{-}\pi^{+}$ candidates. (b)–(d) Projections of the results of fit B (line) and the data (points). The dashed line shows the background contribution.

CLEO-c $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$

TABLE II. The magnitude asymmetries b_r/a_r, phase differences ϕ_r and asymmetries on the D^{+} and D fit fractions from fit B. The errors are statistical, experimental systematic, and decay-model systematic, respectively.

r	b/a (%)	ϕ (°)	FF asymmetry (%)
$\bar{K}^{*0}$	0(fixed)	0(fixed)	$-0.4 \pm 2.0^{+0.2+0.6}_{\ 0.5\ \ 0.3}$
$\bar{K}_0^*(1430)^0$	$4 \pm 3^{+1+2}_{\ 0\ \ 1}$	$-1 \pm 6^{+0+6}_{\ 3\ \ 1}$	$8 \pm 6^{+1+4}_{\ 1\ \ 1}$
ϕ	$-0.7 \pm 1.3^{+0.2+0.3}_{\ 0.1\ \ 0.2}$	$3 \pm 3^{+0+3}_{\ 1\ \ 1}$	$-1.8 \pm 1.6^{+0.0+0.2}_{\ 0.4\ \ 0.1}$
$a_0(1450)^0$	$-10 \pm 7 \pm 2^{+6}_{\ 3}$	$4 \pm 3^{+1+2}_{\ 2\ \ 1}$	$-19 \pm 12^{+5+6}_{\ 3\ \ 11}$
$\phi(1680)$	$-4 \pm 11^{+5+6}_{\ 4\ \ 4}$	$3 \pm 6 \pm 2^{+3}_{\ 2}$	$-9 \pm 22^{+10+9}_{\ 7\ \ 12}$
$\bar{K}_2^*(1430)^0$	$23^{+12+1+3}_{\ 11\ \ 7\ \ 7}$	$5^{+5+1+3}_{\ 4\ \ 3\ \ 1}$	$43 \pm 19^{+1+5}_{\ 13\ \ 12}$
$\kappa(800)$	$-6 \pm 6^{+3+1}_{\ 1\ \ 5}$	$3 \pm 6^{+4+1}_{\ 2\ \ 4}$	$-12 \pm 11^{+0+14}_{\ 6\ \ 2}$

We calculate an integrated CP asymmetry across the Dalitz plot, defined as

$$\mathcal{A}_{CP} = \int \frac{|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2} dm_+^2 dm_-^2 \Big/ \int dm_+^2 dm_-^2. \quad (18)$$

We obtain $\mathcal{A}_{CP} = (-0.4 \pm 2.0^{+0.2+0.6}_{-0.5-0.3})\%$, where the errors are statistical, experimental systematic, and decay-model systematic, respectively.

LHCb $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$

- R. Aaij *et al*., Phys. Rev. D 84, 112008 (2011).
- LHCb use a comparison of binned Dalitz data
- The overall normalization is corrected for

$$S^{i}_{CP} = \frac{N^{i}(D^{+}) - \alpha N^{i}(D^{-})}{\sqrt{N^{i}(D^{+}) + \alpha^{2} N^{i}(D^{-})}}, \quad \alpha = \frac{N_{\mathrm{tot}}(D^{+})}{N_{\mathrm{tot}}(D^{-})},$$

LHCb $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$

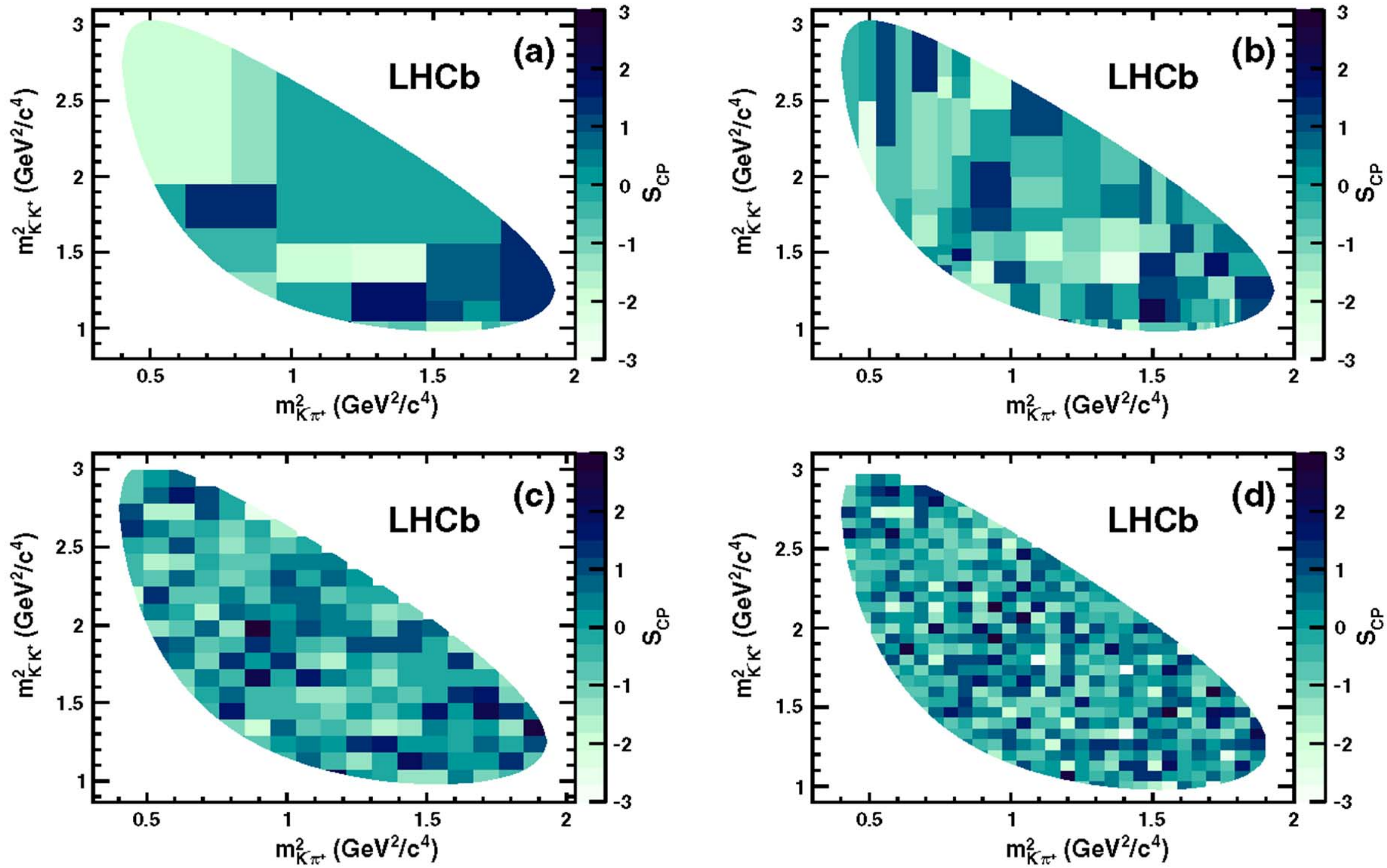


FIG. 7 (color online). Distribution of $\mathcal{S}^i_{CP}$ in the Dalitz plot for (a) "Adaptive I," (b) "Adaptive II," (c) "Uniform I" and (d) "Uniform II." In (c) and (d) bins at the edges are not shown if the number of entries is not above a threshold of 50 (see Sec. III).

LHCb $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$

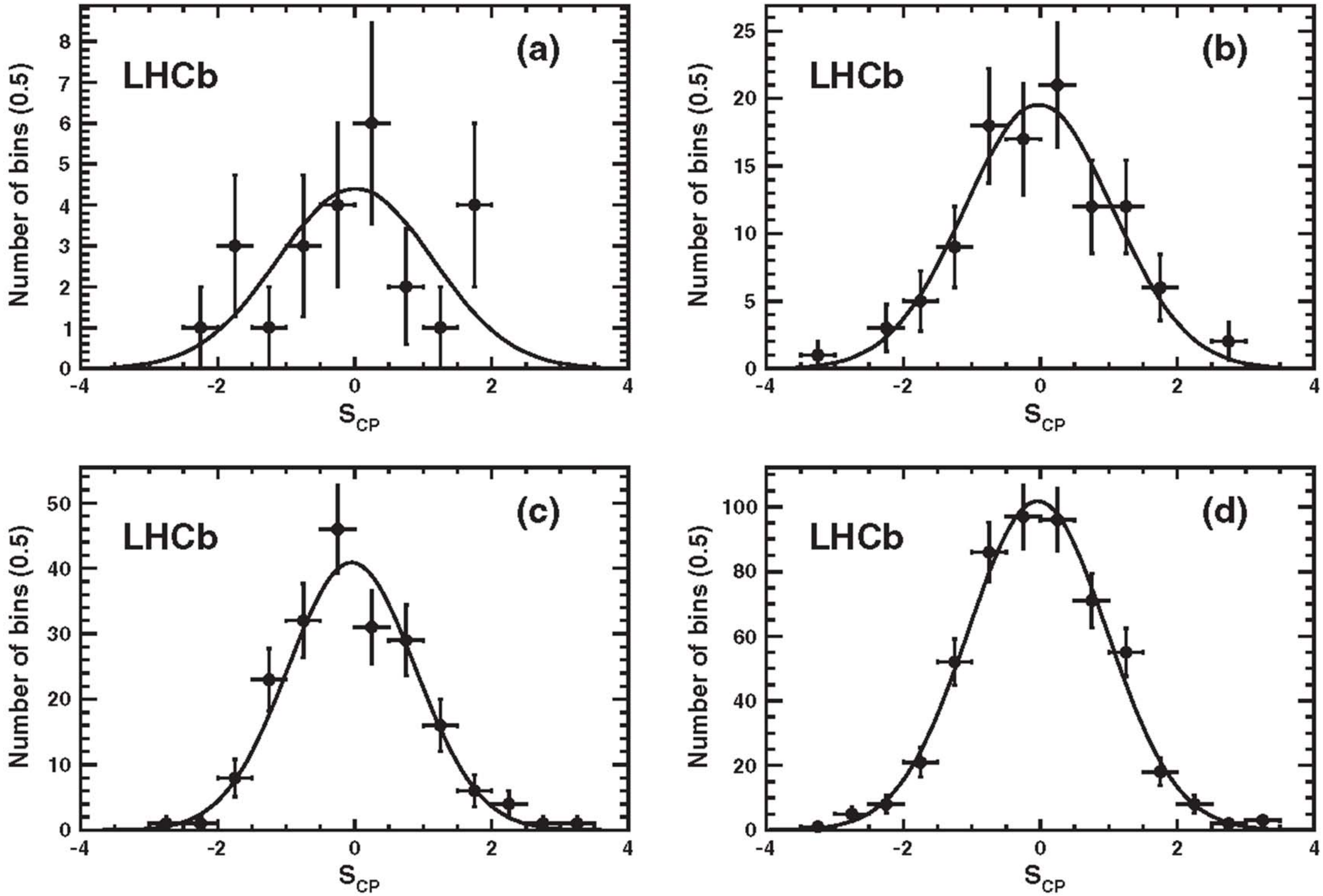


FIG. 8. Distribution of $\mathcal{S}^{i}_{CP}$ fitted to Gaussian functions, for (a) "Adaptive I," (b) "Adaptive II," (c) "Uniform I" and (d) "Uniform II." The fit results are given in Table IX.

Recent BaBar $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$ (Mass Plot, Efficiency vs $\cos\theta_{CM}$)

[BaBar Preliminary]

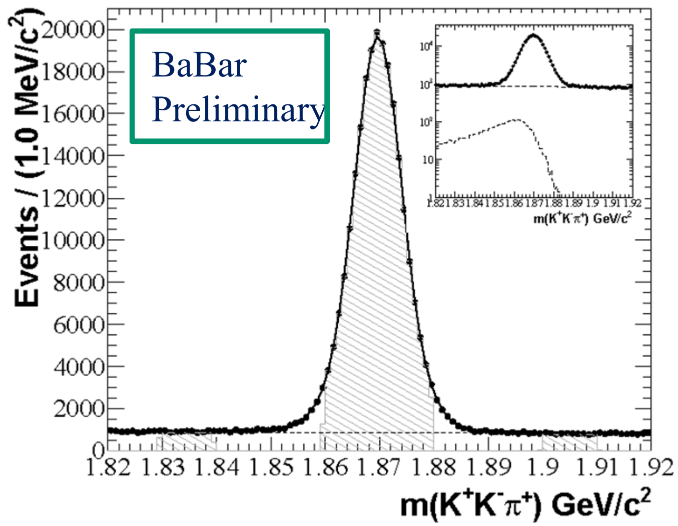
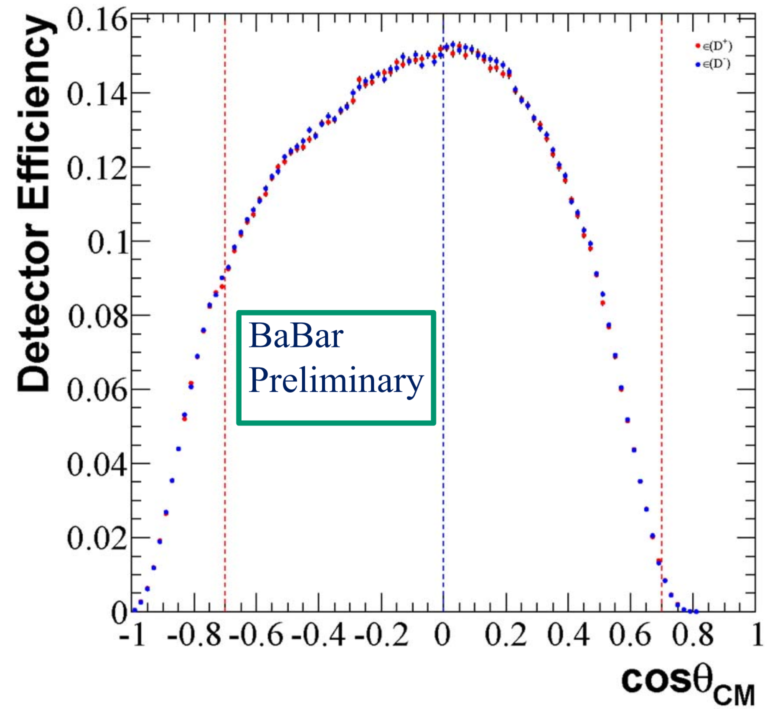


FIG. 2: $m(K^+K^-\pi^+)$ distribution and projection of the fit result. The points are the data, the solid line is the fit model, the dashed line is the background PDF, and the dotted curve the radiative decay PDF. The inset plot is the fit on a log scale, where the shape of the radiative PDF is apparent. The signal and background regions are shaded; the background regions are useful for background subtraction as well as for various background studies.



Recent BaBar $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$ (vs $\cos\theta_{CM}$)

$$A \equiv \frac{N_{D+}/\epsilon_{D+} - N_{D-}/\epsilon_{D-}}{N_{D+}/\epsilon_{D+} + N_{D-}/\epsilon_{D-}}$$

[BaBar

Preliminary]

Integrated Asymmetry:

(0.35 ± 0.30 ± 0.15)%

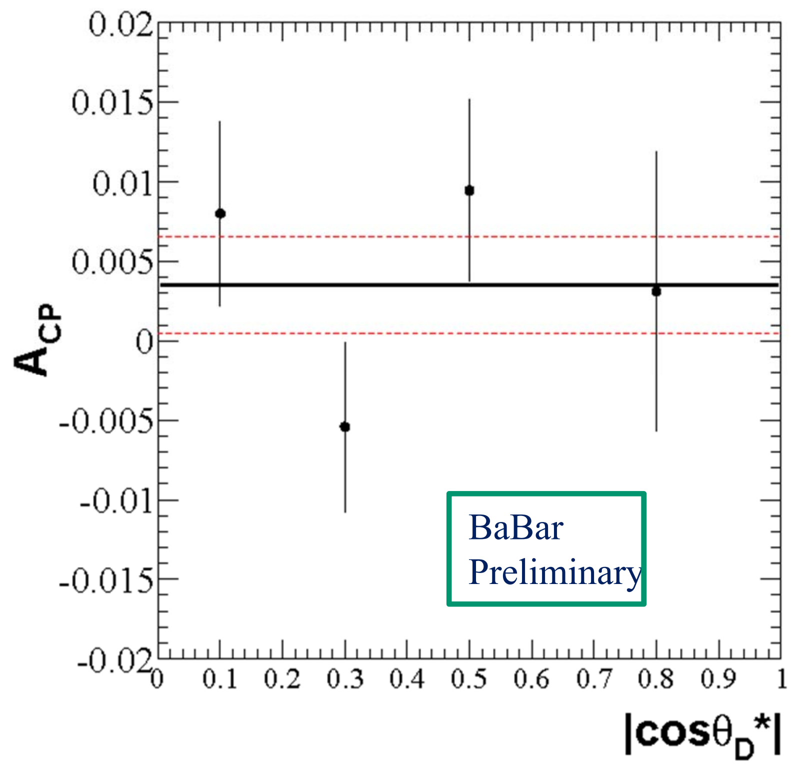
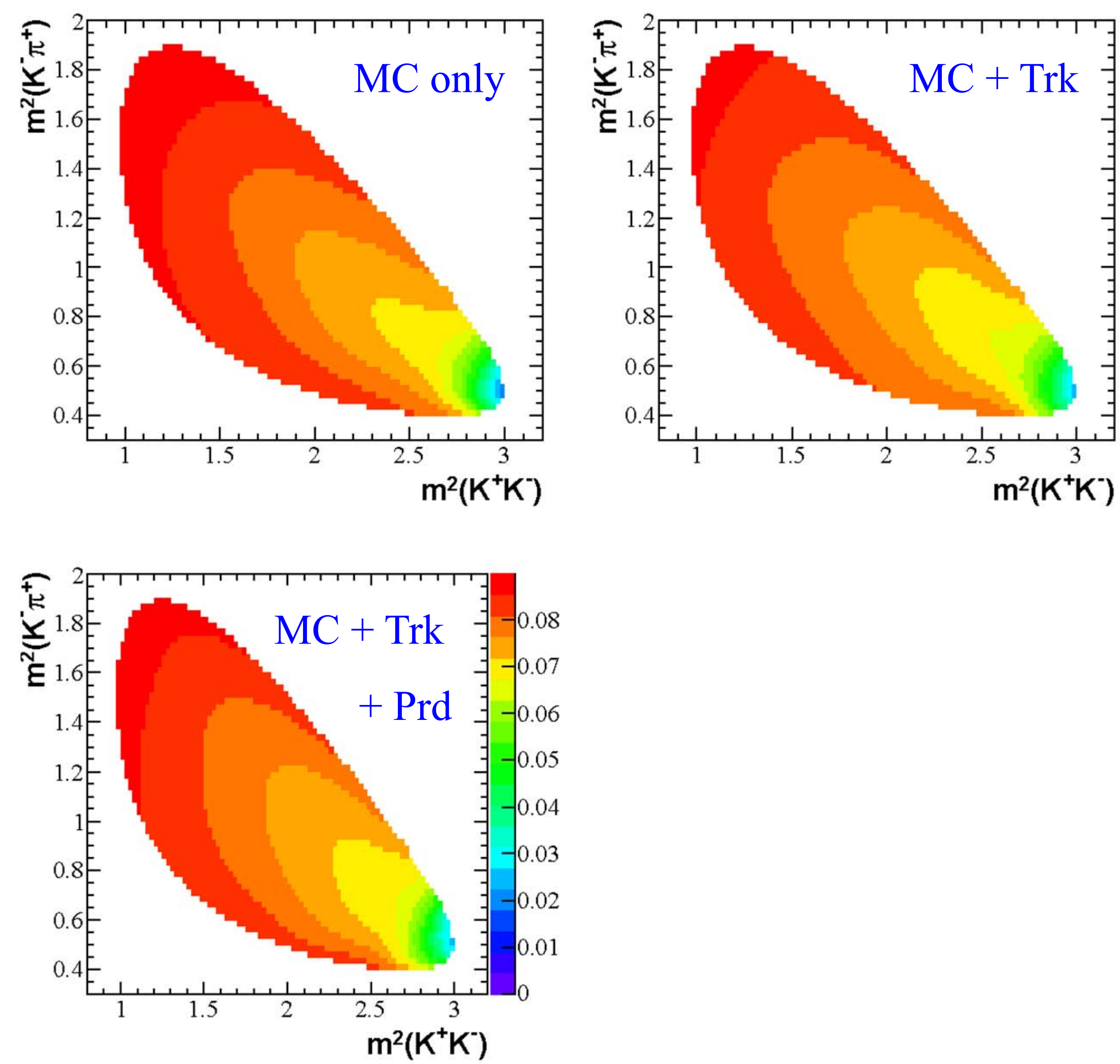


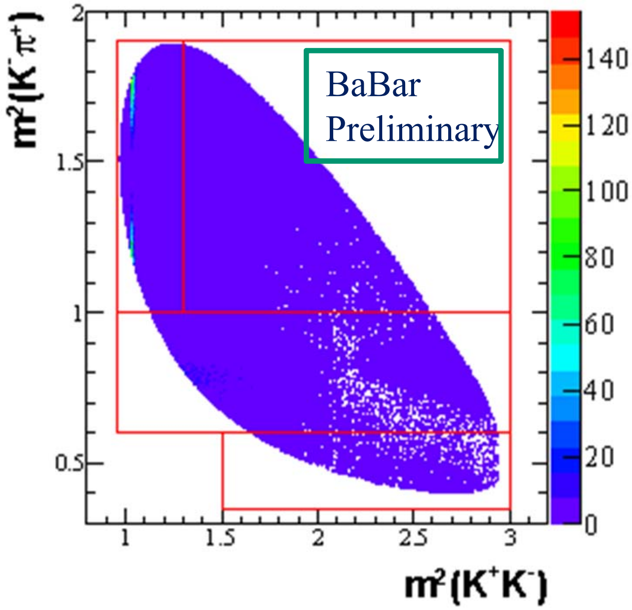
FIG. 3: CP asymmetry as a function of $|\cos\theta_{CM}|$ in the data sample. The solid line represents the central value of A_{CP} and the dashed lines represent the $\pm 1\sigma$ interval, determined from a χ^2 minimization assuming no dependence on $|\cos\theta_{CM}|$.

Recent BaBar $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$ (Efficiency Across Dalitz Plot)

[BaBar Preliminary]



Recent BaBar $D^{\pm} \rightarrow K^+K^-\pi^{\pm}$ (CPV By Regions)



[BaBar

Preliminary]

Dalitz plot region	$N(D^+)$	$\epsilon(D^+)[\%]$	$N(D^-)$	$\epsilon(D^-)[\%]$	$A_{CP}[\%]$
Below $\bar{K}^*(892)^0$	1882 ± 70	7.00	1859 ± 90	6.97	$-0.65 \pm 1.64 \pm 1.73$
$\bar{K}^*(892)^0$	36770 ± 251	7.53	36262 ± 257	7.53	$-0.28 \pm 0.37 \pm 0.21$
$\phi(1020)$	48856 ± 289	8.57	48009 ± 289	8.54	$-0.26 \pm 0.32 \pm 0.45$
Above $\bar{K}^*(892)^0$ and $\phi(1020)$	25616 ± 244	8.01	24560 ± 242	8.00	$1.05 \pm 0.45 \pm 0.31$

TABLE I: Yields, efficiencies, and CP asymmetry in regions of the Dalitz plot shown in Fig. 5. For the CP asymmetry the first error is statistical and the second is systematic.

Recent BaBar $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$ (LHCb Style)

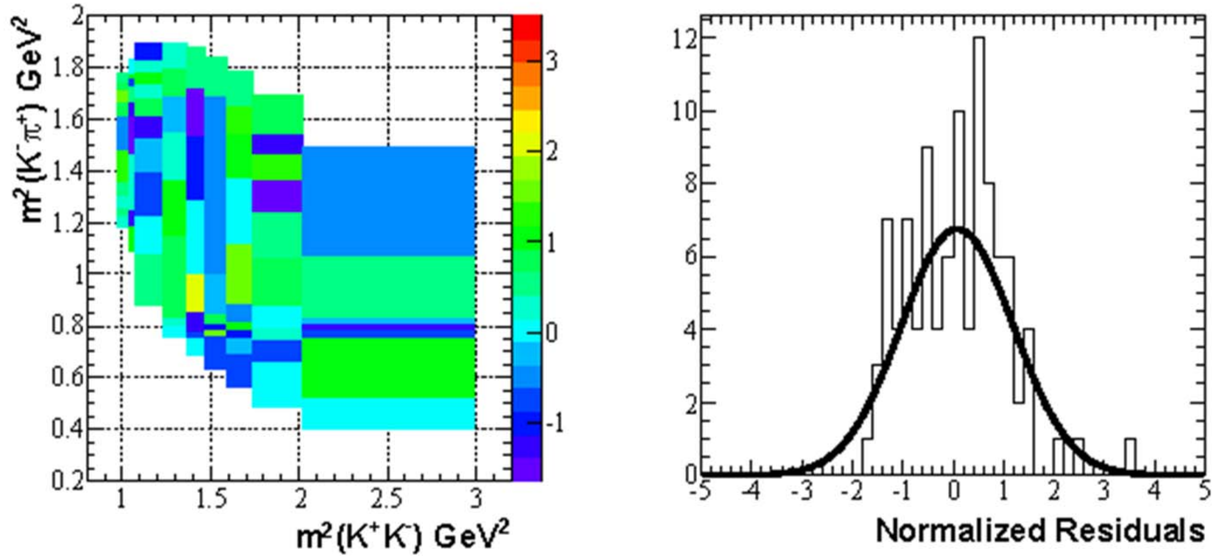


FIG. 4: Normalized residuals of the D^{+} and D^{-} Dalitz plots in equally populated bins (left) and their distribution fitted with a Gaussian. We find the distribution to have a mean of 0.08 ± 0.15 and a width of 1.11 ± 0.15 and a probability of 72% that the two Dalitz plots are consistent with no CP asymmetry.

Recent BaBar $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$ (Dalitz Fit)

[BaBar Preliminary]

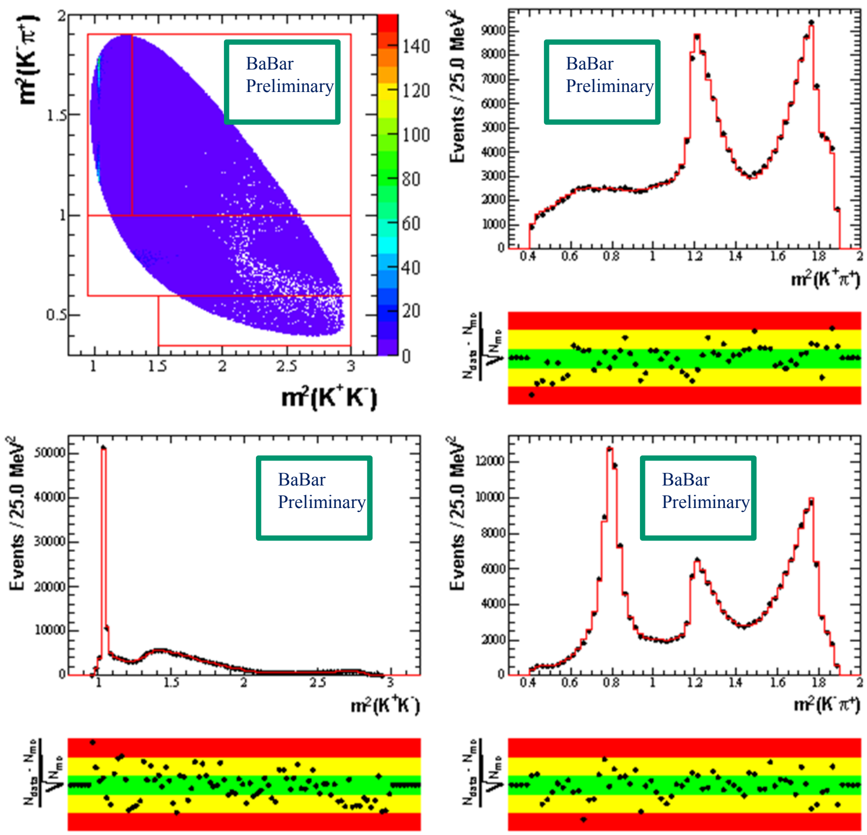


FIG. 5: $D^+ \rightarrow K^+K^-\pi^+$ Dalitz plot fit projections assuming no CPV, with the regions used for model-independent comparisons also indicatedas boxes. The data are represented by points with errors, the fit results by the histogram. The normalized residuals below, defined as $(N_{Data} - N_{MC})/\sqrt{N_{MC}}$, lie between $\pm 5\sigma$. The horizontal divisions correspond to 1σ (green), 3σ(yellow), and 5σ(red).

Recent BaBar $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$ (Results for combined fit)

[BaBar Preliminary]

Resonance	Fraction (%)
$\bar{K}^{*}(892)^{0}$	21.15 ± 0.20
$\phi(1020)$	28.42 ± 0.13
$\bar{K}^{*}_{0}(1430)^{0}$	25.32 ± 2.24
NR	6.38 ± 1.82
$\kappa(800)$	7.08 ± 0.63
$a_0(1450)^0$	3.84 ± 0.69
$f_0(980)$	2.47 ± 0.30
$f_0(1370)$	1.17 ± 0.21
$\phi(1680)$	0.82 ± 0.12
$\bar{K}^{*}_{1}(1410)$	0.47 ± 0.37
$f_0(1500)$	0.36 ± 0.08
$a_2(1320)$	0.16 ± 0.03
$f_2(1270)$	0.13 ± 0.03
$\bar{K}^{*}_{2}(1430)$	0.06 ± 0.02
$\bar{K}^{*}(1680)$	0.05 ± 0.16
$f_0(1710)$	0.04 ± 0.03
$f'_2(1525)$	0.02 ± 0.008
Sum	97.92 ± 3.09

Recent BaBar $D^{\pm} \rightarrow K^{+}K^{-}\pi^{\pm}$ (Simultaneous Fit to D^{+}, D^{-})

[BaBar Preliminary]

Resonance	r (%)	$\Delta\phi$ (°)
$\bar{K}^*(892)^0$	0. (FIXED)	0. (FIXED)
$\bar{K}_0^*(1430)^0$	$-9.40^{+5.65}_{-5.36} \pm 4.42$	$-6.11^{+3.29}_{-3.24} \pm 1.39$
$\phi(1020)$	$0.35^{+0.82}_{-0.82} \pm 0.60$	$7.43^{+3.55}_{-3.50} \pm 2.35$
NR	$-14.30^{+11.67}_{-12.57} \pm 5.98$	$-2.56^{+7.01}_{-6.17} \pm 8.91$
$\kappa(800)$	$2.00^{+5.09}_{-4.96} \pm 1.85$	$2.10^{+2.42}_{-2.45} \pm 1.01$
$a_0(1450)^0$	$5.07^{+6.86}_{-6.54} \pm 9.39$	$4.00^{+4.04}_{-3.96} \pm 3.83$
	Δx	Δy
$f_0(980)$	$-0.199^{+0.106}_{-0.110} \pm 0.084$	$-0.231^{+0.100}_{-0.105} \pm 0.079$
$f_0(1370)$	$0.019^{+0.049}_{-0.048} \pm 0.022$	$-0.0045^{+0.037}_{-0.039} \pm 0.016$

$$r = \frac{|\mathcal{M}_r|^2 - |\bar{\mathcal{M}}_r|^2}{|\mathcal{M}_r|^2 + |\bar{\mathcal{M}}_r|^2}$$

$$\Delta\phi = \phi_r - \bar{\phi}_r$$

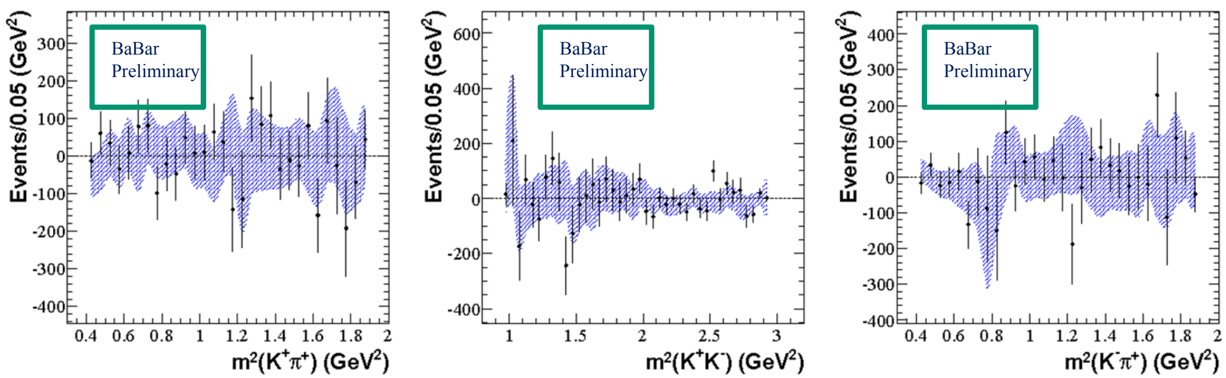


FIG. 6: The difference of the Dalitz plot projections of data (points) and the fit (blue curve) between the D^+ and D^- decays. The width of the curve represents the $\pm 1\sigma$ error of the fit.

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Conclusions

- High statistics are here to stay. Much work done over the years with increasing statistics by Mark II, Mark III, E691, E687, E791, FOCUS, BaBar, Belle, CLEO, LHCb, …
- The isobar model needs updating. We may also need better descriptions of resonances, especially when they overlap (for a given l-wave).
- Some changes already tried by one or more collaborations:
  - K-matrix approach (requires ad-hoc parameterization of NR amplitude).
  - Model-independent partial wave amplitudes (MIPWA)
  - Threshold effects
  - Experimental resolution may need more attention
- MIPWA obviates questions such as
  - Kappa, Sigma: do they exist or not?
  - Unitarity problems of overlapping resonances (for a given l-wave)[?]
  - Threshold effects
- CP Violation:
  - SCS Dalitz decays investigated by CLEO-c, LHCb and BaBar
  - Dalitz plots have proved useful in extracting CP parameters in neutral D-mixing
  - Dalitz plots also used in extracting γ/ϕ_3

EXTRA SLIDES

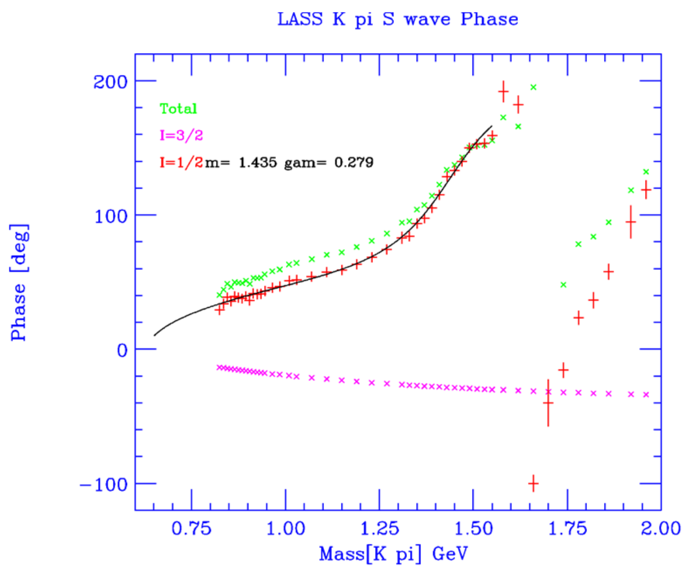
The K-matrix approach

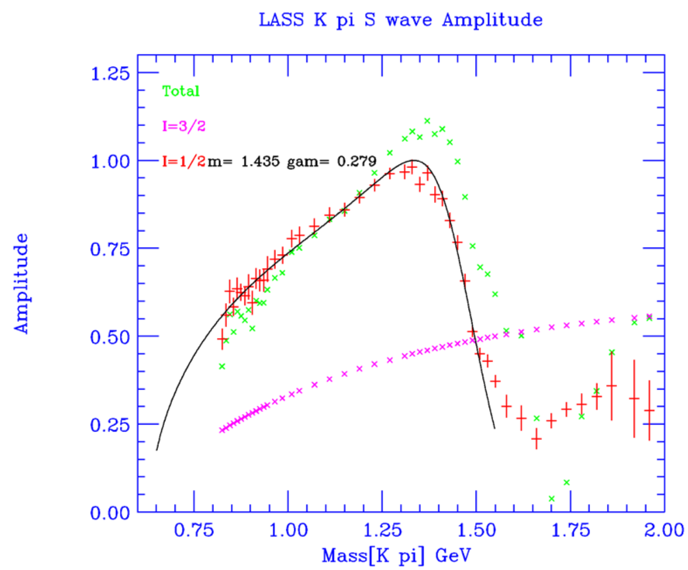
- Most recently championed by FOCUS
- Gives as good result as isobar fit with κ (CL = 7.7% vs. 7.5% for isobar fit. Without a κ, CL = 10^{-6}. See Edera's talk, Daphne '04).
- Respects unitarity in $K\pi$ scattering, but is this also true in D decays?
- Further, the K-matrix also requires an ad-hoc parameterization of the non-resonant amplitude.
- Does this mean that the conclusion from the K-matrix work (no broad new scalars are required) is correct?

Kπ Scattering

- Most information on $K^-\pi^+$ scattering comes from the LASS experiment (*SLAC*, E135)

No data from E135 below 825 MeV/c^2





Data from:

K^-p! $K^-\pi^+n$

and

K^-p! $K^0\pi^-p$

NPB 296, 493 (1988)

Parametrize s-wave (I=1/2) by

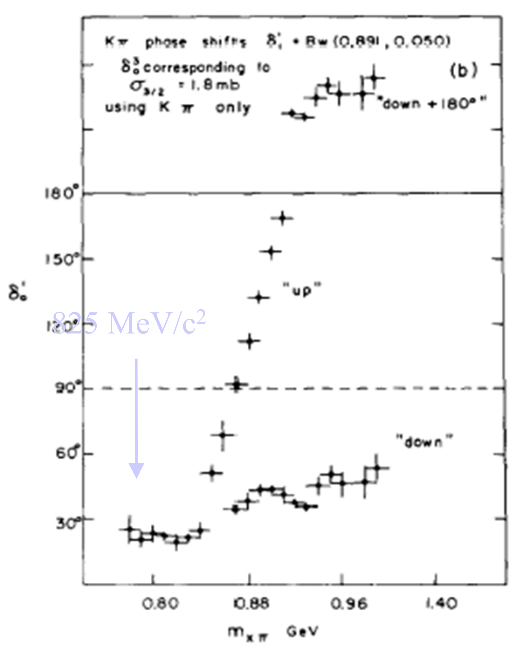
$$\left\{ \begin{array}{rcl} T & = & \sin(\delta_R + \delta_B) e^{i(\delta_R + \delta_b)} \\ \cot \delta_R & = & m_{1430}\Gamma_{1430}(p)/(m_{1430}^2 - s) \\ \cot \delta_B & = & 1/a_{1/2}p + b_{1/2}p^2 \end{array} \right\}$$

a – scattering length
b – effective range
p – momentum in CM

Kπ Scattering

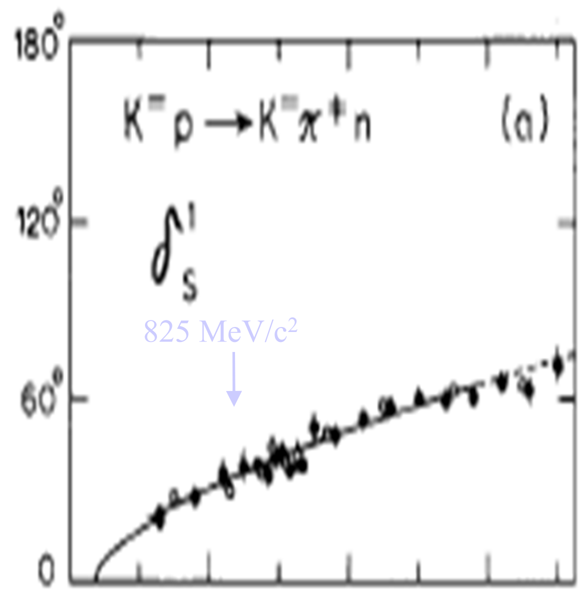
- Relatively poor data is available below 825 MeV/c^2.

I = 1/2

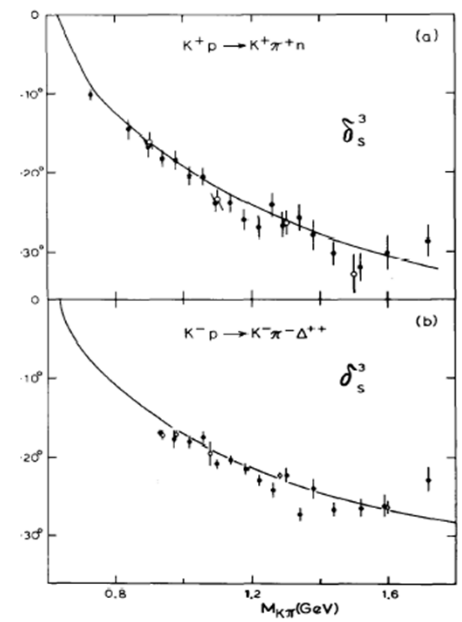


H. Bingham, et al,
NP B41, 1-34 (1972)

I = 1/2



I = 3/2



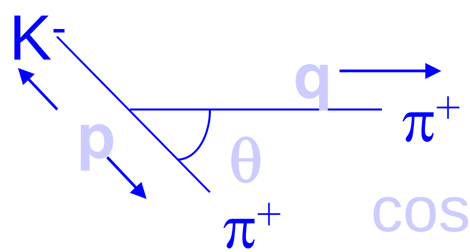
P. Estabrooks, *et al.*, NP B133, 490 (1978)

$K\pi$ Scattering in Heavy Quark Decays

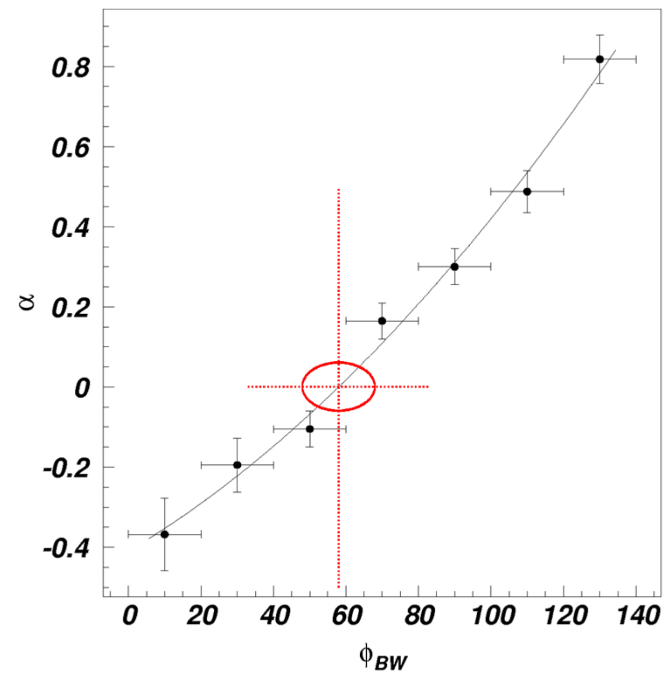
- Precise knowledge of the S-wave $K\pi$ system, particularly in the low mass region, is of vital interest to an understanding of the spectroscopy of scalar mesons.
- It may be possible to learn more from the large amounts of data on D and B decays now becoming available.
- The applicability of the Watson theorem can also be tested.
- E791 is first to use, in this report, a Model-Independent Partial Wave Analysis of the S-wave in these decays to investigate these issues.

Asymmetry in $K^*(892)$

- Helicity angle θ in $K^-\pi^+$ system



$\cos\theta = \hat{p} \cdot \hat{q}$



$= \tan^{-1} m_0\Gamma_0/(m_0^2 - s_{K\pi})$ →

- Asymmetry:

$$\alpha = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N(\cos\theta > 0) + N(\cos\theta < 0)}$$

$$= 0 \quad \text{when } \phi_P - \phi_S = 90^\circ$$

LASS finds $\alpha = 0$ when $\phi_{BW} \approx 135^\circ$

⇒ $\phi_P - \phi_s$ is -75^0 relative to elastic scattering

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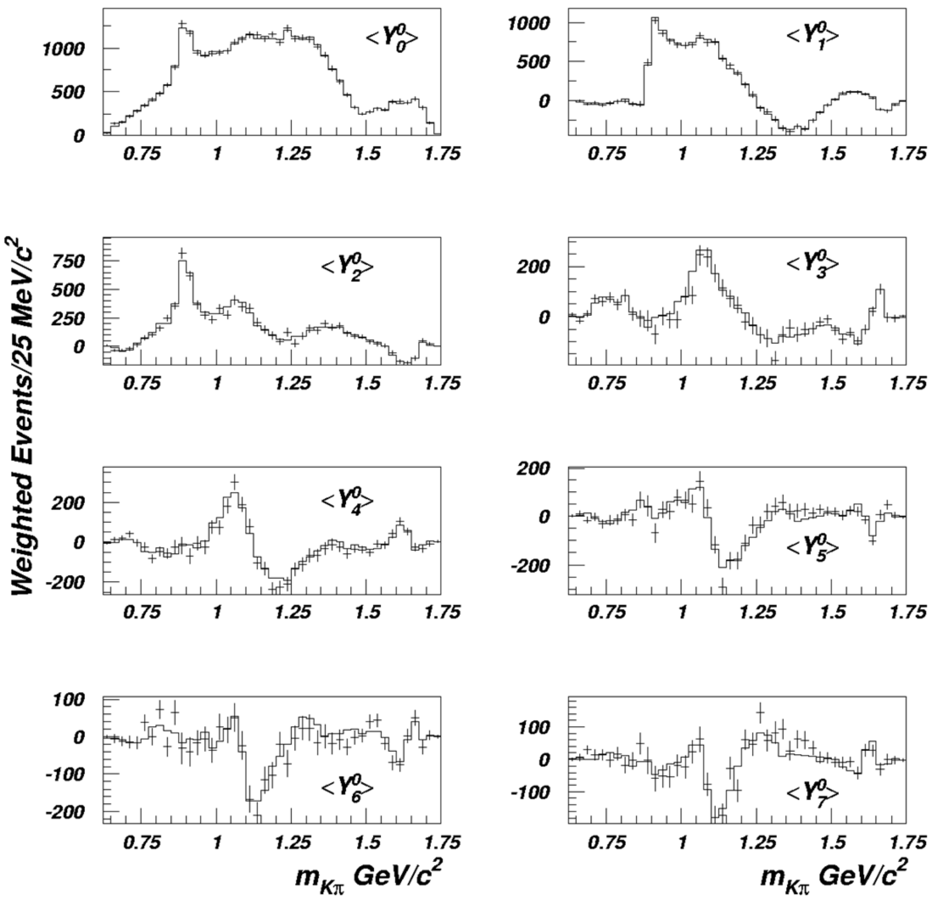
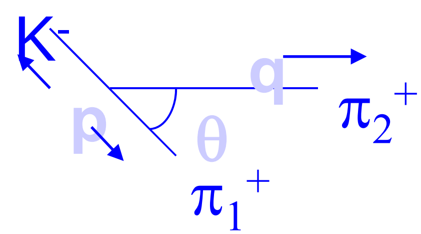
Comparison with Data - Moments





- Mean values of $Y_L^0(\cos\theta)$
  - Exclude K*(890) in $K^-\pi_2^+$

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Production of $K^{-}\pi^{+}$ Systems

- Production factor $\Theta_0\ (s_{K\pi})$ is

$$|\Theta_0(s_{K\pi})| \quad = \quad \frac{p}{\sqrt{s_{K\pi}}} \frac{\left|S(s_{K\pi})F_D^0\right|}{\sin[\gamma(s_{K\pi}) - \gamma_0]}$$

- Value for γ_0 found by minimizing

$$\chi^2 \quad = \quad \sum_{j=1}^{N_{elastic}} \left(\frac{|\Theta_0(s_j)| - Q}{\sigma(\Theta_0)} \right)^2$$

Summed over measured γ_j's

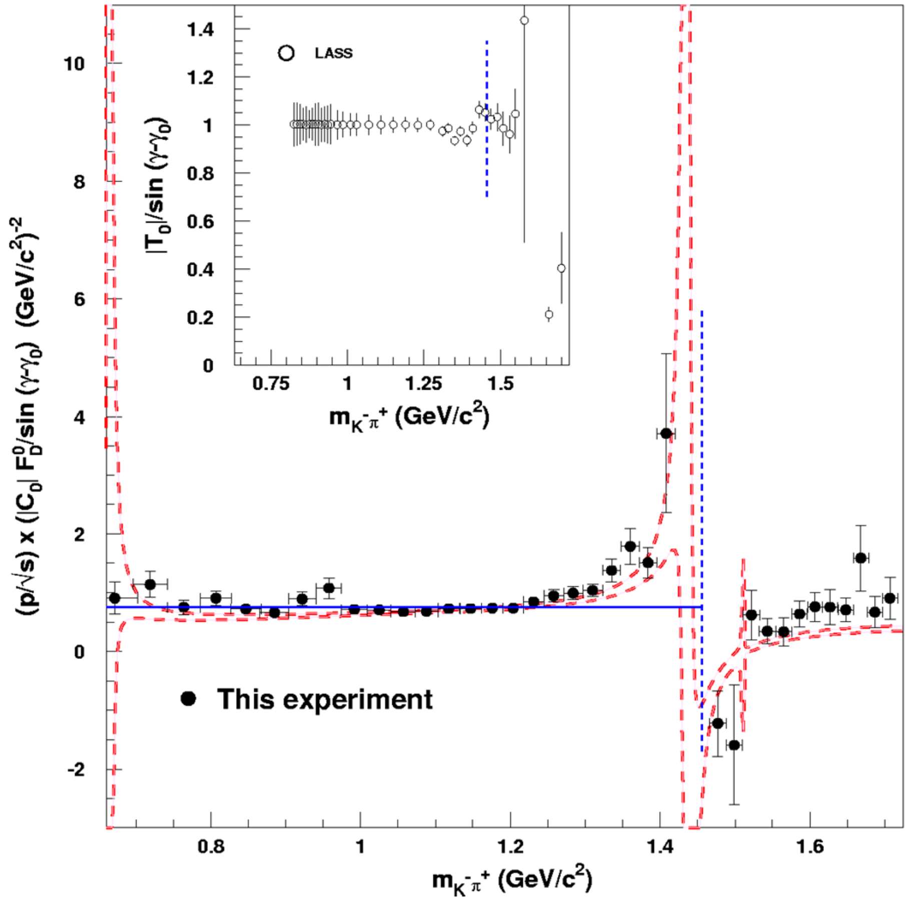
- $\gamma_0 = (123.3 \pm 3.0)^0$; $Q = 0.74 \pm 0.01$

Production of $K^-\pi^+$ Systems

Plot quantities $\Theta(s_j)$, evaluated at each s_j value, using measured γ_j there.

- Roughly constant up to about 1.250 GeV/c^2

Constant = 0.74§ 0.01 $(GeV/c^2)^2$.



Fit E791 Data for S-wave

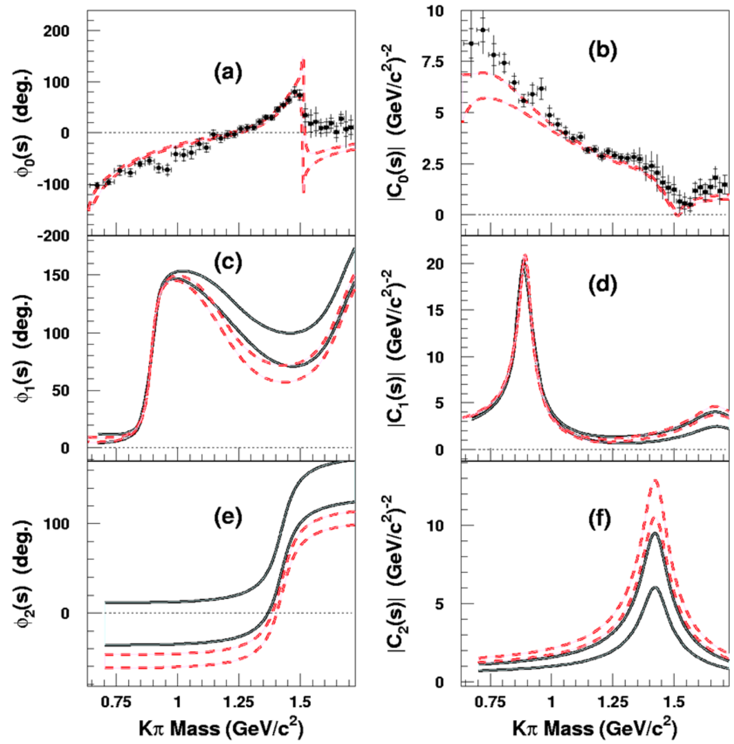
- Find S. Allow P and D parameters to float
  - General appearance of all three waves very similar to isobar model fit.
  - Contribution of P-wave in region between $K^*(892)$ and $K^*(1680)$ differs slightly – balanced by shift in low mass S-wave.

Phase Magnitude

S

P

D



Observing CPV in D Dalitz Decays

$$ {}^{\mathrm{Dp}}S_{\mathrm{CP}}(i) \doteq \frac{N(i) - \overline{N}(i)}{\sqrt{N(i) + \overline{N}(i)}} $$

$$ N, \overline{N} = \alpha + \beta \,\mathrm{Re}\left\{ \frac{e^{\pm i\delta_{\mathrm{CP}}}}{s - M_{\mathrm{res}}^2 + iM_{\mathrm{res}}\Gamma_{\mathrm{res}}} \right\} \quad \Rightarrow \quad N - \overline{N} = \frac{\sin\delta_{\mathrm{CP}} \times 2\beta M_{\mathrm{res}}\Gamma_{\mathrm{res}}}{(s - M_{\mathrm{res}}^2)^2 + (M_{\mathrm{res}}\Gamma_{\mathrm{res}})^2} $$

Final State Interactions: What's Interesting

- Final State Interactions can affect / produce
  - the amplitudes of resonances
  - threshold effects etc.
  - strong phases
- Important for CP Violation studies
  - small (BSM) CP phases can be enhanced in regions
  - differential observables can help shed light on the theory / mechanisms at play
- See, e.g., B. Kubis, arXiv:1108.5866