More Theory of XYZ States

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Theory of XYZ States

Introduction: see previous talks by

- W. Gradl
- L. Pillonen
- A.A. Alves Jr
- A. Polosa

XYZ States

- dependence on QCD parameters
- dependence on heavy quark mass
- recent results from Lattice QCD
- Born-Oppenheimer expansion?

Understanding the nature of the XYZ states

requires understanding how their properties depend on the parameters of QCD:

QCD coupling constant: α_s

light quark masses: m_u, m_d, m_s (or m_π, m_K)

heavy quark masses: M_c, M_b (or M_D, M_B)

number of colors?

XYZ states are particularly sensitive to
pion mass: m_π
heavy quark mass: M_Q

- Experiments can measure their properties
  only at m_π = 140 MeV
  $M_Q = M_c$ and M_b

- Lattice QCD will calculate their properties
  only at $M_Q = M_c$ and M_b
  and as function of m_π in the range
  $140\ \text{MeV} < m_\pi < 500\ \text{MeV}$

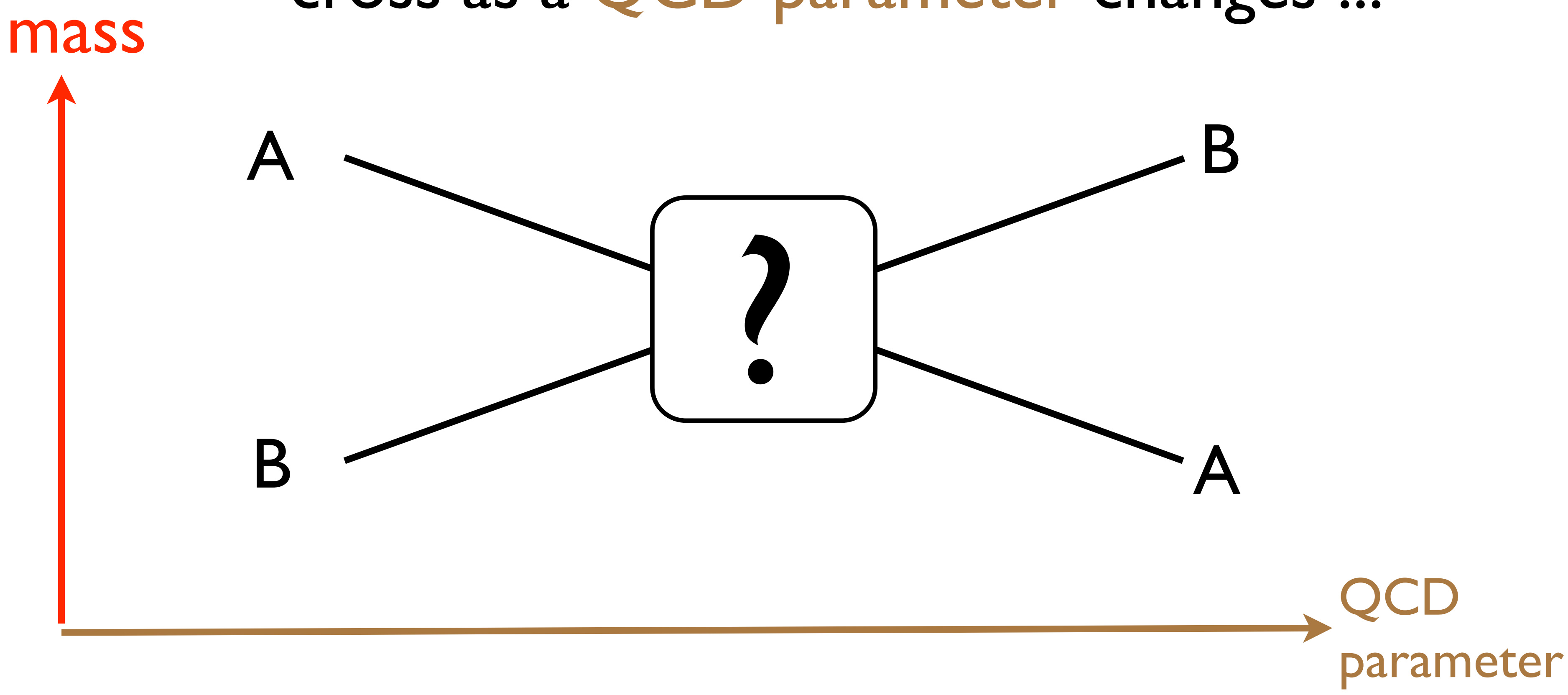
Challenge: Use experiment and lattice QCD
to understand XYZ states

As a QCD parameter is varied,
masses of different hadrons
change at different rates.

- The masses of two hadrons can cross.
- The mass of a hadron can cross thresholds for other hadrons.

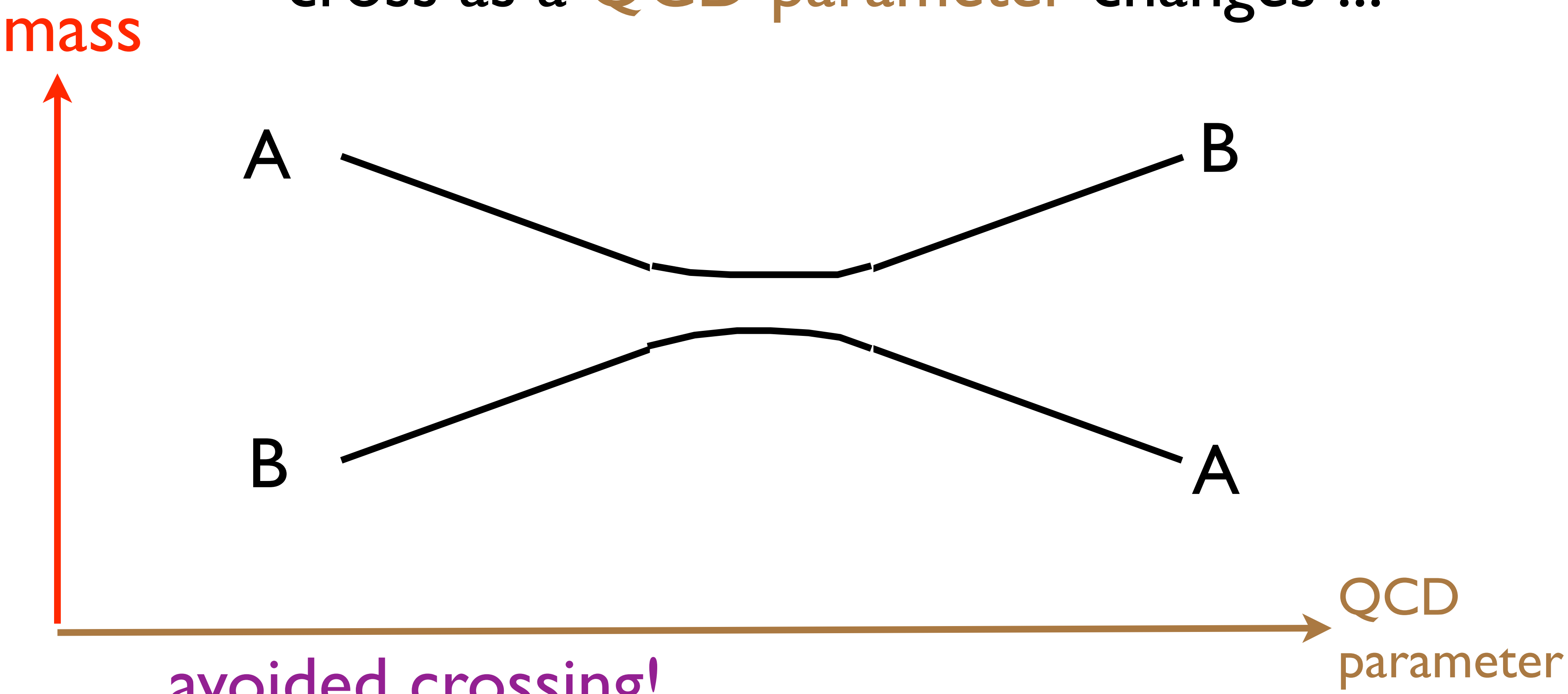
The properties of a hadron
can depend dramatically on the parameter
near the crossing.

Suppose masses of two hadrons A, B with same J^{PC} cross as a QCD parameter changes ...



How do they behave near the crossing?

Suppose masses of two hadrons A, B
with same J^{PC}
cross as a QCD parameter changes ...



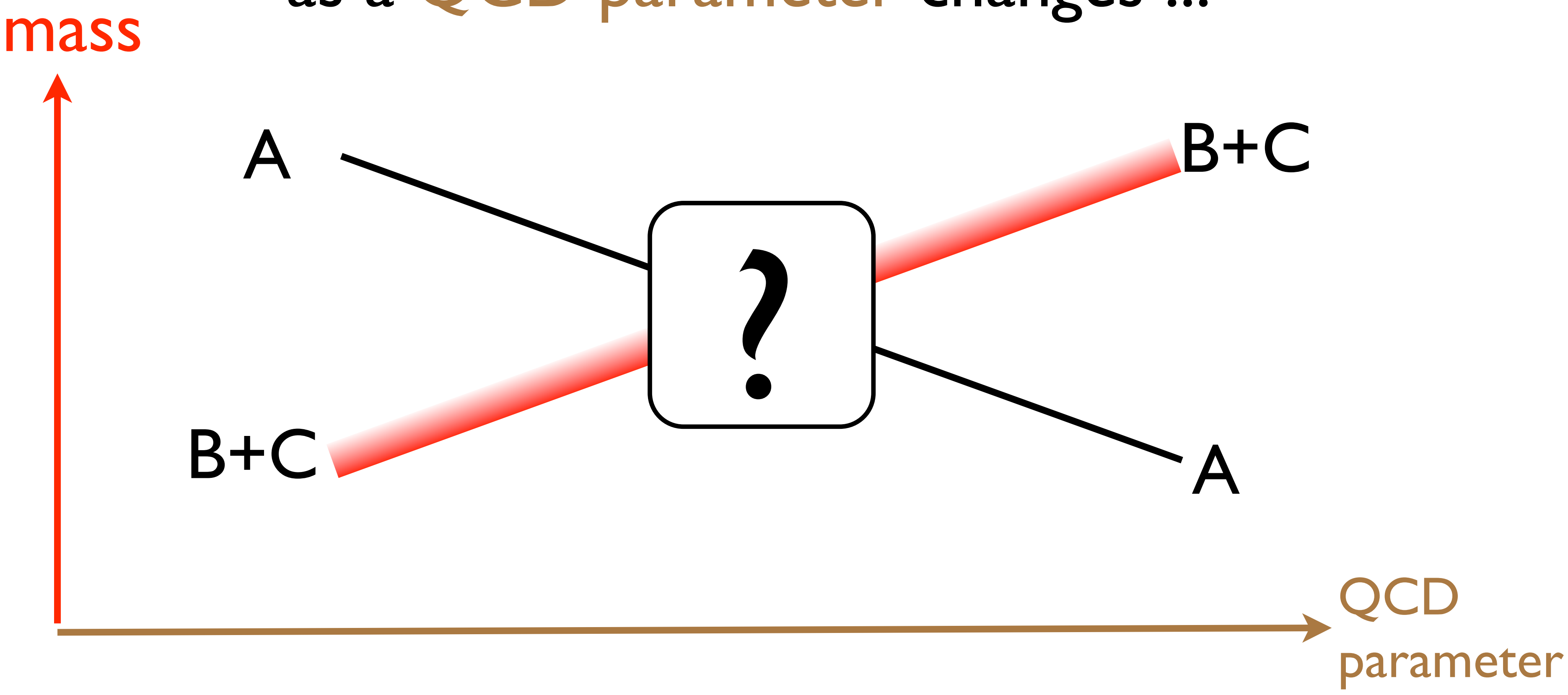
avoided crossing!

hadrons described by mixing of A and B:

$\cos\vartheta\, A + \sin\vartheta\, B$

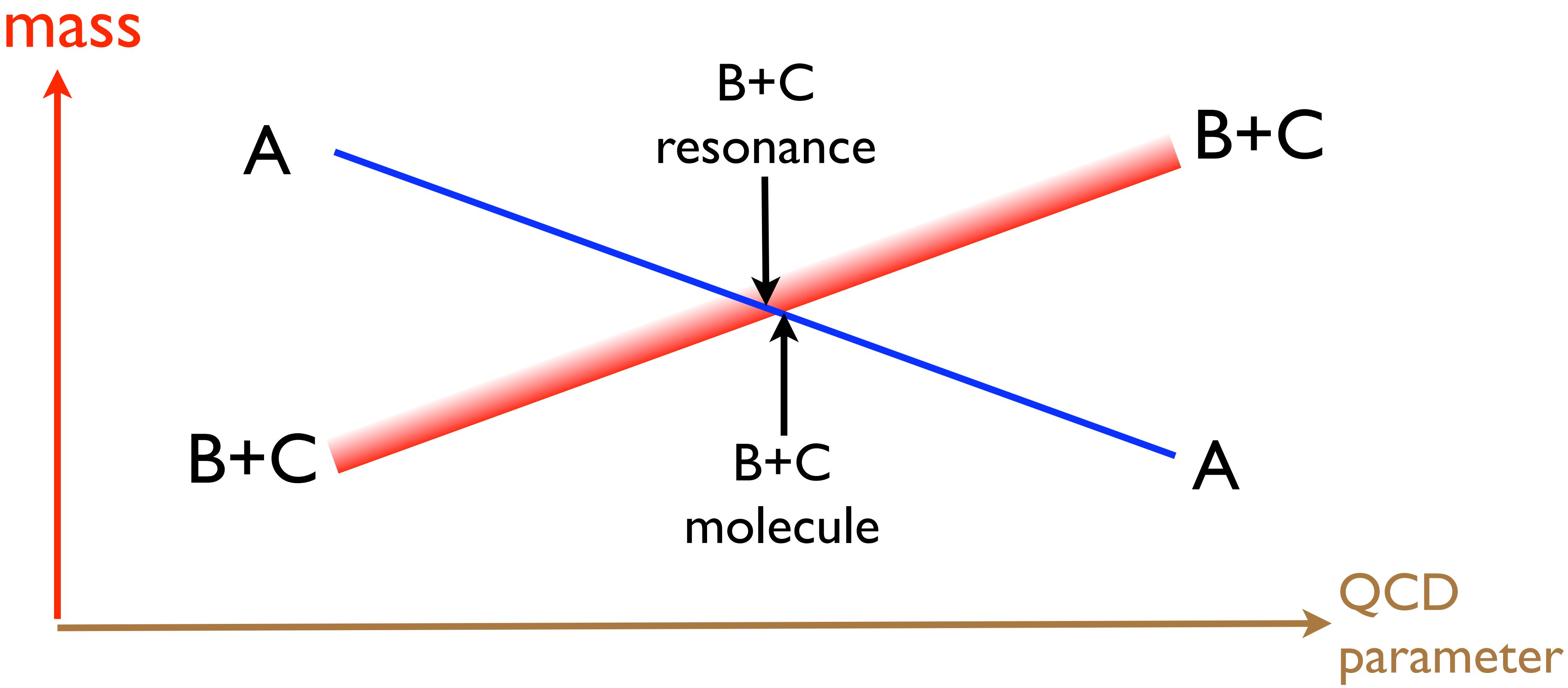
$\cos\vartheta\, B - \sin\vartheta\, A$

Suppose mass of hadron A
crosses threshold for pair of hadrons B+C
as a QCD parameter changes ...



What happens near the crossing?

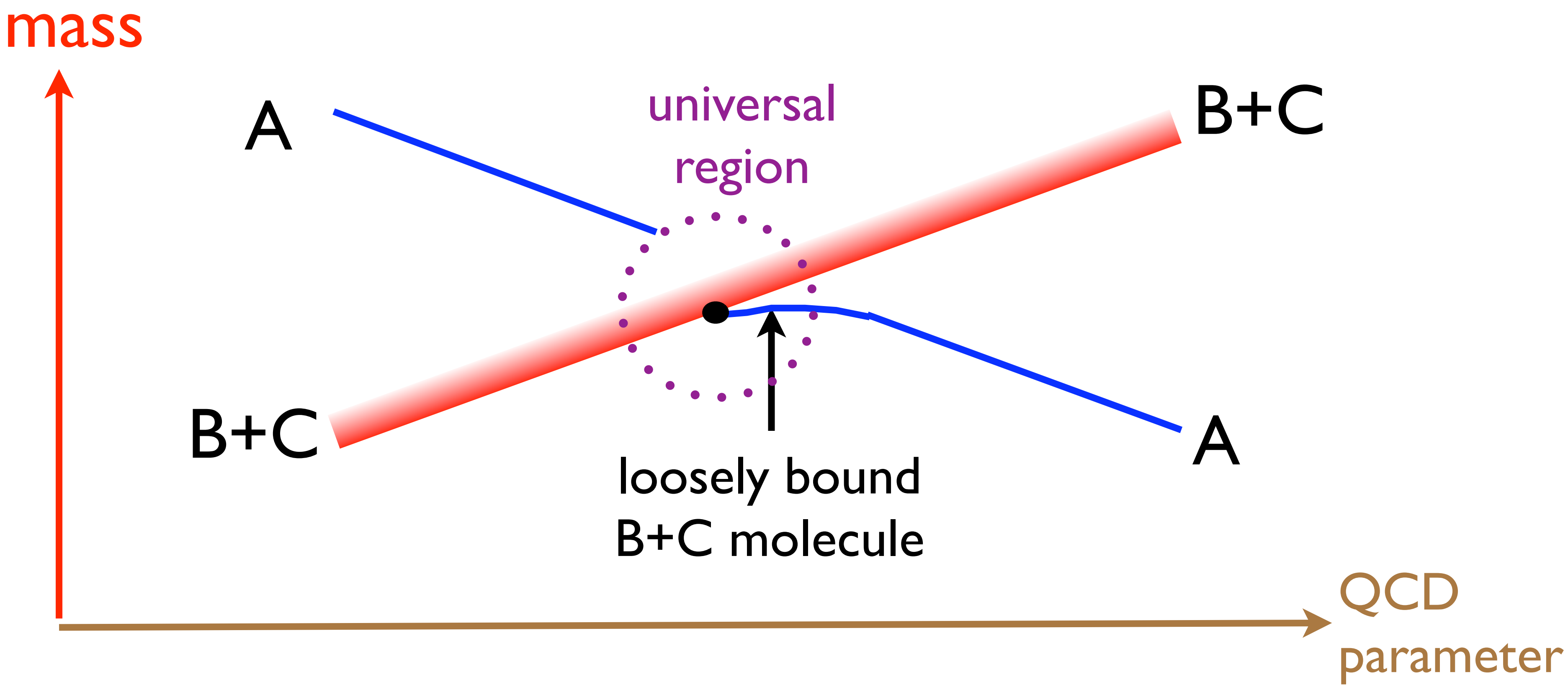
Suppose mass of hadron A crosses threshold for pair of hadrons B+C as a QCD parameter changes



Feshbach resonance!

If coupling of A to B+C is P-wave, D-wave, ...
properties of A are modified only very close to crossing

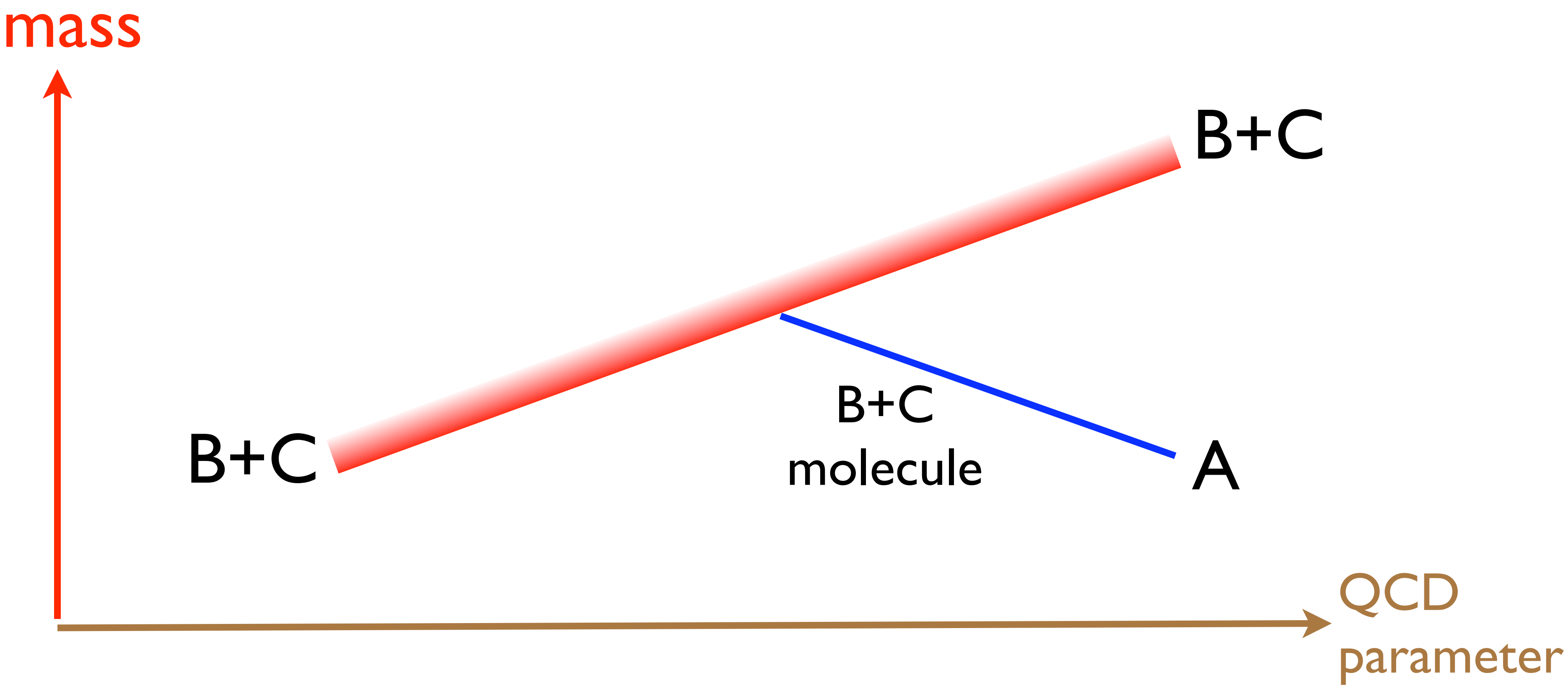
Suppose mass of hadron A crosses threshold for hadrons B+C as a QCD parameter changes



S-wave Feshbach resonance

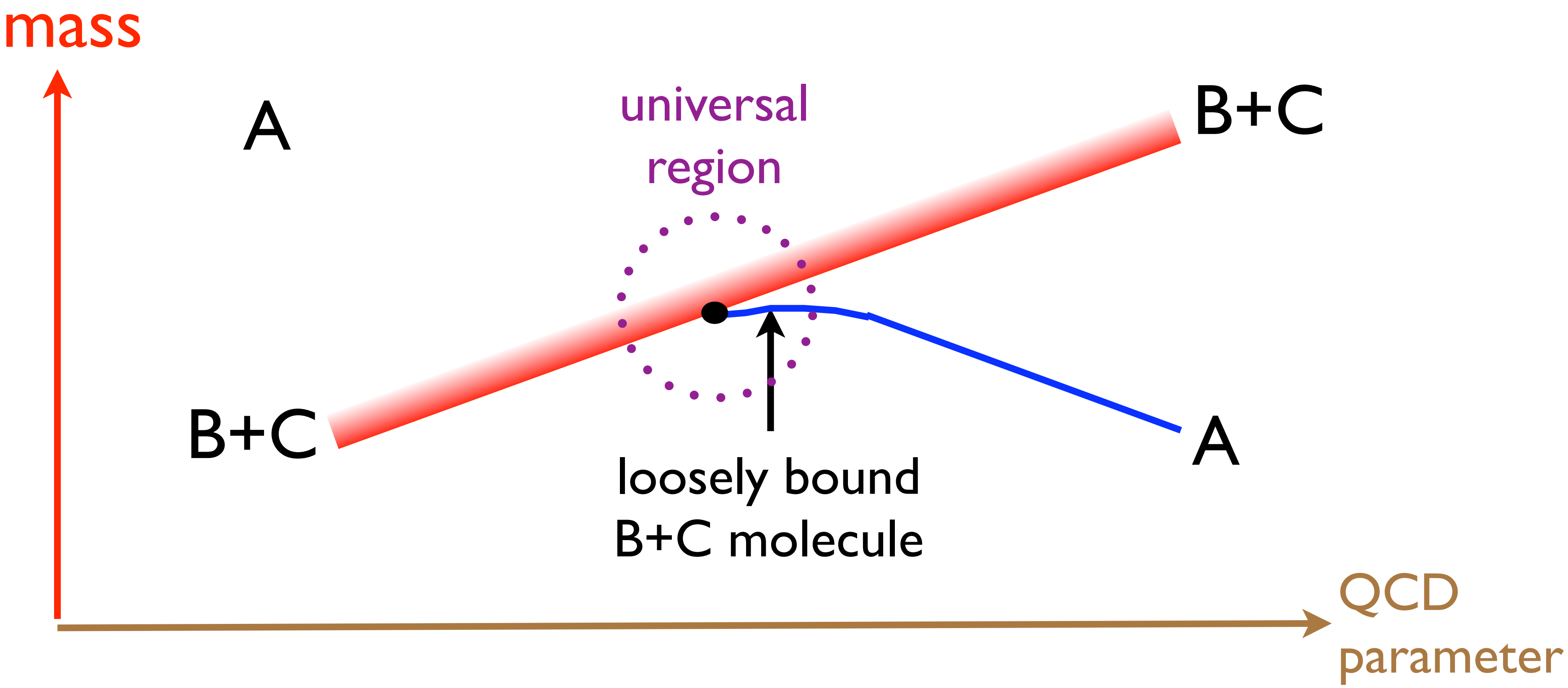
If coupling of A to B+C is S-wave,
A has universal properties near the crossing

Suppose an ordinary bound state of B+C crosses threshold for B+C



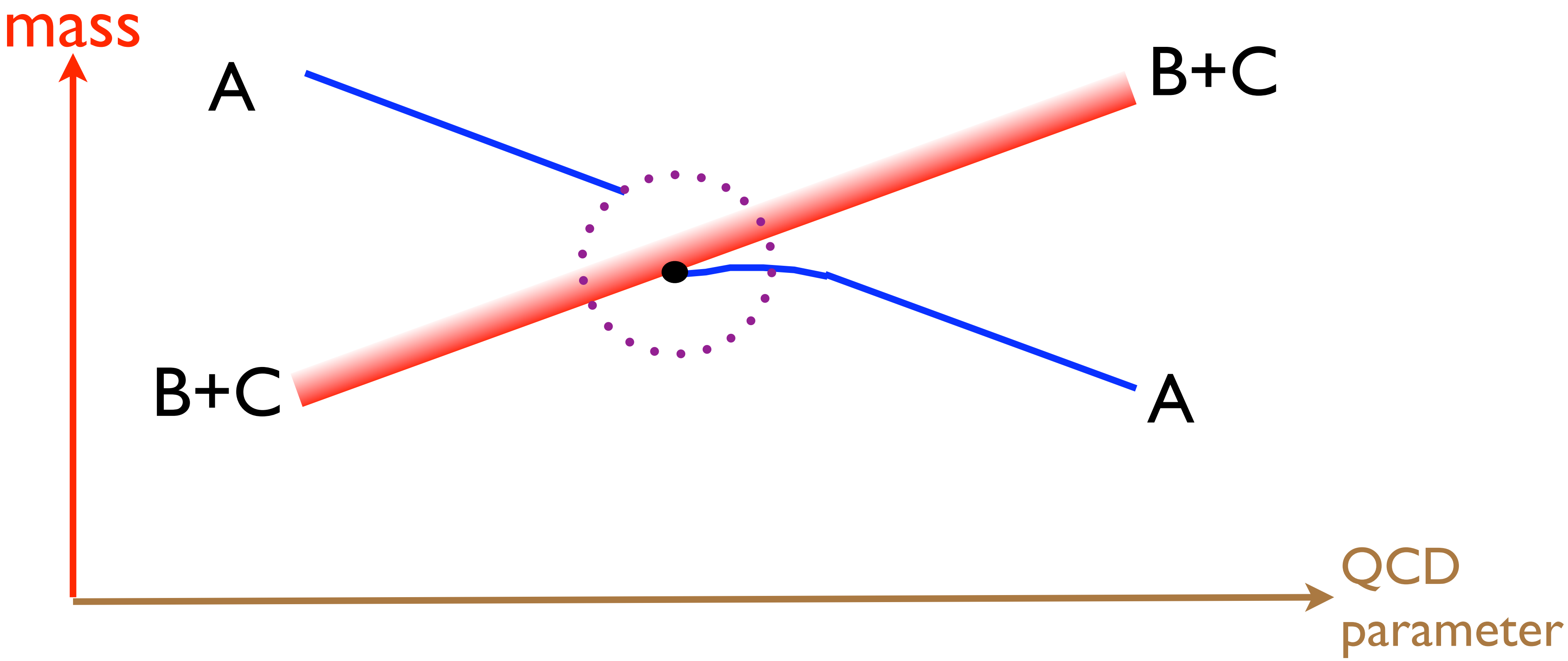
B+C molecule disappears at the threshold

Suppose an ordinary S-wave bound state of B+C crosses threshold for B+C



S-wave B+C molecule disappears at the threshold but it has universal properties near the crossing

S-wave Feshbach resonance
and ordinary S-wave bound state
have same universal behavior near crossing



In universal region,

A is a loosely-bound molecule

with large mean separation ‹r›

determined by its small binding energy E_b: $\langle r\rangle^2 = 1/(4\,\mu\,E_b)$

What is the X(3872)?

Universal properties
of an S-wave near-threshold resonance with *a>0*

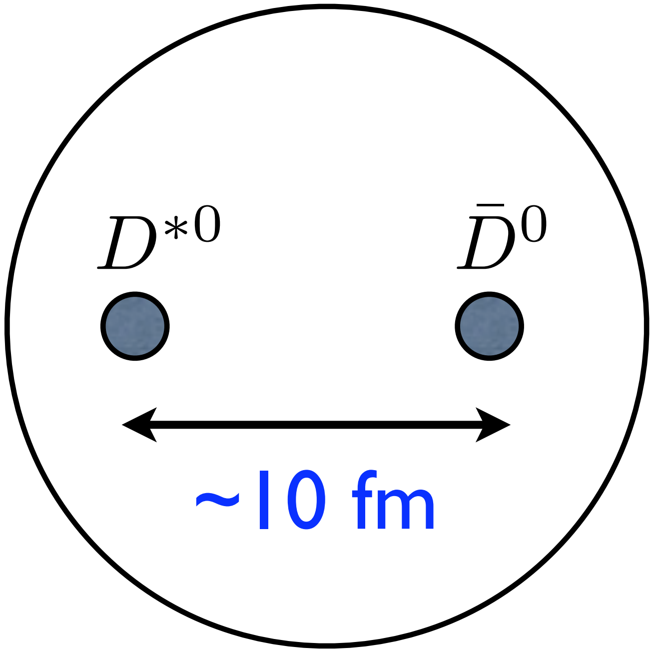
a) binding energy: $E_X = \hbar^2/(2\mu a^2)$

b) rms separation: $r_X = a/\sqrt{2}$

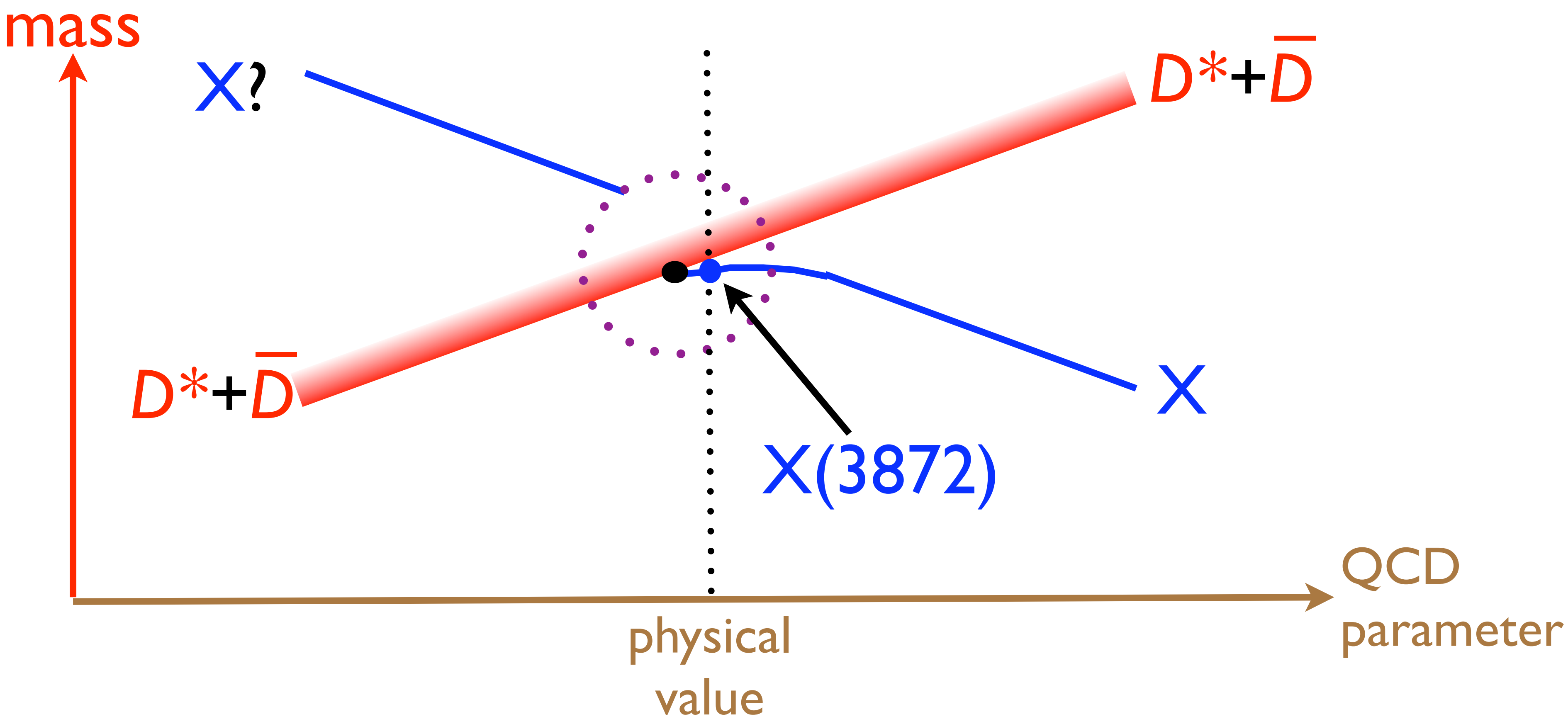
Apply to *X(3872)*:

E_X = *0.11* ± *0.33* MeV

$J^{PC} = 1^{++} \Rightarrow r_X = 10^{+\infty}_{-5}$ fm



(if $J^{PC} = 1^{++}$) X(3872) has the same universal properties whether it is a $\chi_{c1}(2P)$ Feshbach resonance or an ordinary $D^*\bar{D}$ molecule



What is the nature of the X(3872)?

How does it depend on QCD parameters outside universal region?

- For a hadron to be a constituent
  in a hadronic molecule,
  its width must be smaller than its binding energy.

  ρ meson (Γ=150 MeV) can never be
  a constituent in a hadronic molecule

- For a threshold to dramatically affect properties
  of hadrons crossing it,
  the hadrons forming the threshold must be narrow

  nothing interesting can happen
  near ρ J/ψ threshold

Narrow Hadrons: $\Gamma < 20$ MeV

light mesons

π K η η'

ω? ($\Gamma = 8.5$ MeV)

φ? ($\Gamma = 4.3$ MeV)

light baryons

N Λ Σ Ξ Ω^-

charm mesons

D D^*

D_s D_s^* D_{s0} D_{s1}^* D_{s1}'

28 narrow
charm-charm
thresholds!

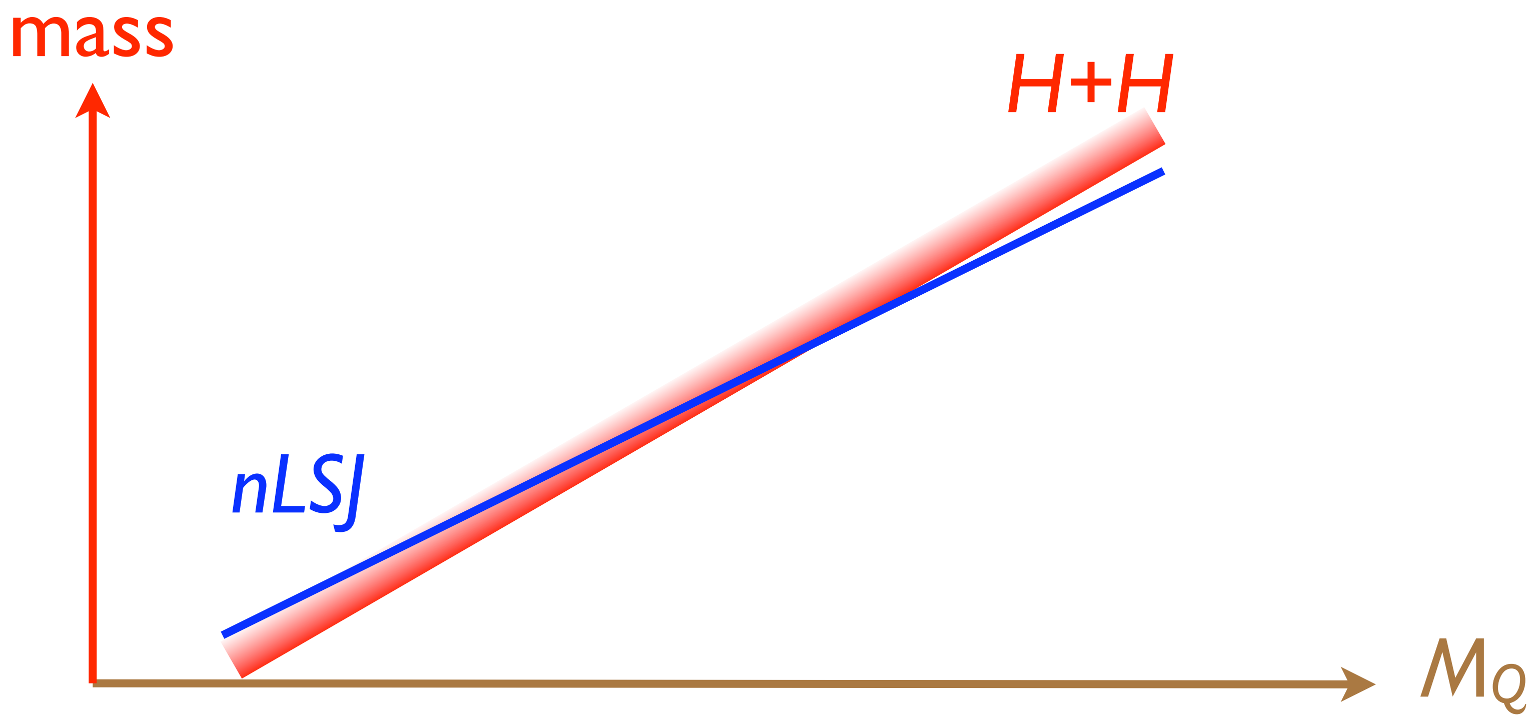
Dependence on heavy quark mass

Heavy-light meson mass: expansion in $1/M_Q$

$$M_H = M_Q + a + b/M_Q + \ldots$$

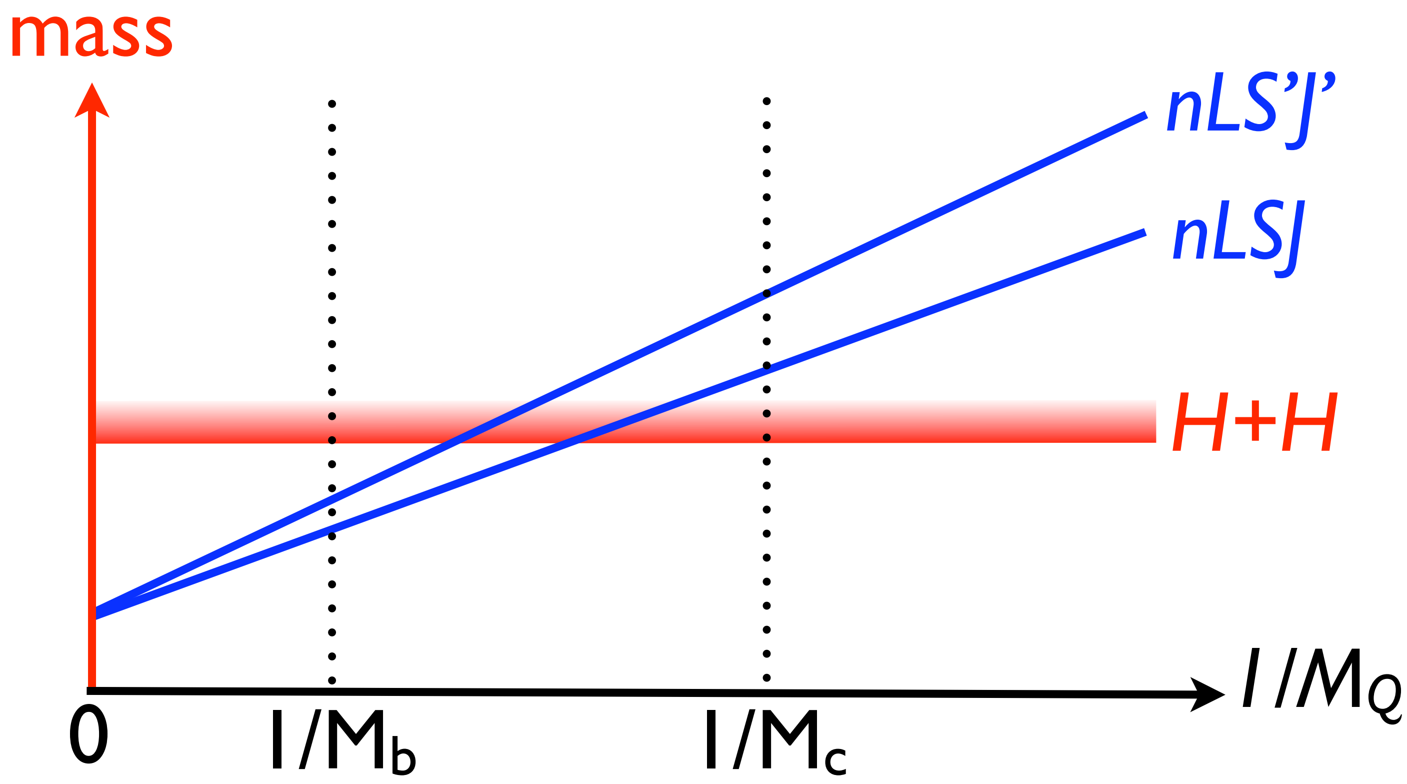
Quarkonium mass: expansion in $v_Q^2 \sim 1/M_Q$

$$M_{nLSJ} = 2\, M_Q + A_{nL}\, M_Q\, v_Q^2 + B_{nLSJ}\, M_Q\, v_Q^4 + \ldots$$



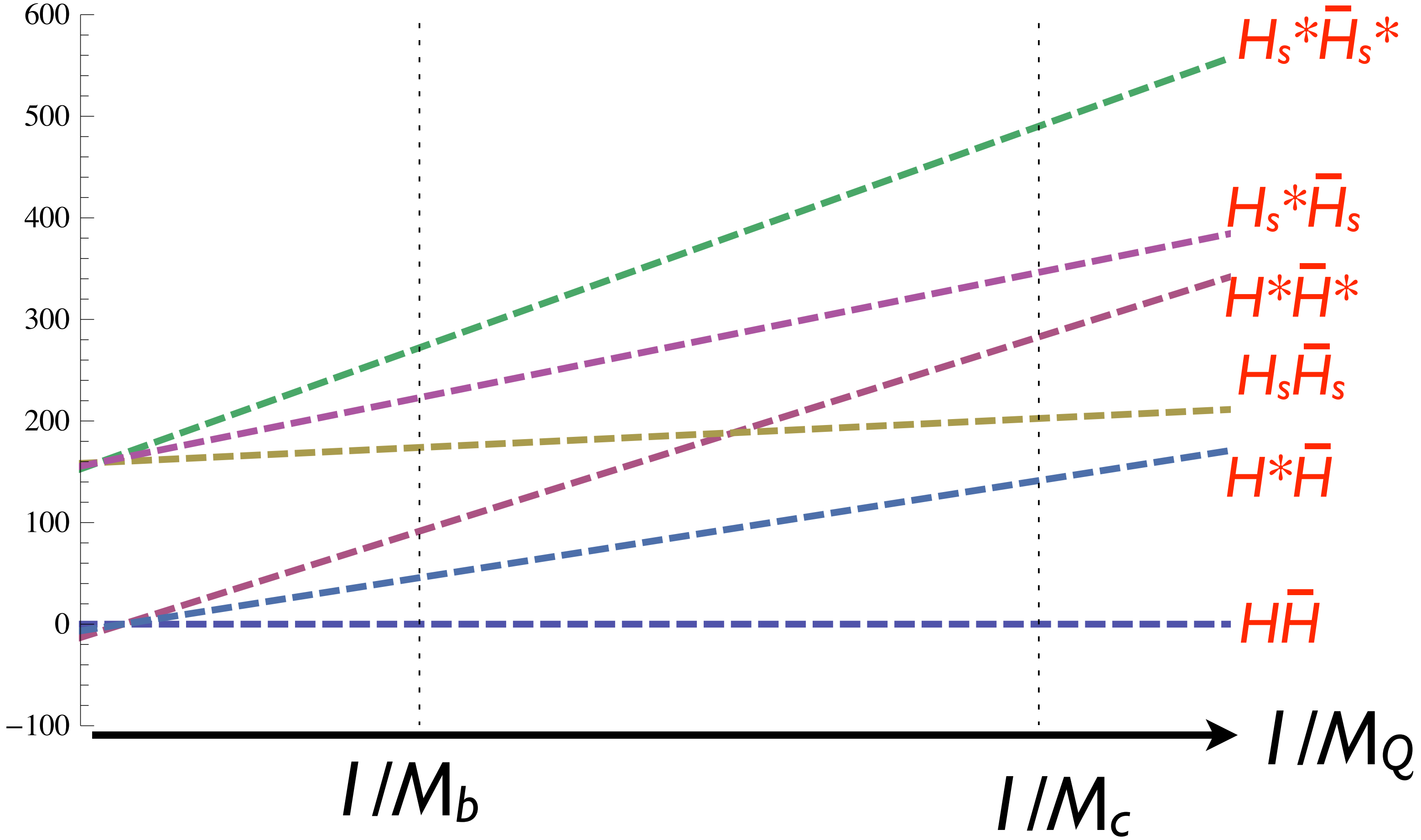
Dependence on heavy quark mass

- subtract *H+H* threshold
  to make it horizontal
- use $1/M_Q$ as parameter instead of M_Q
  exploits heavy-quark spin symmetry:
  spin splittings → 0 as $M_Q \to \infty$



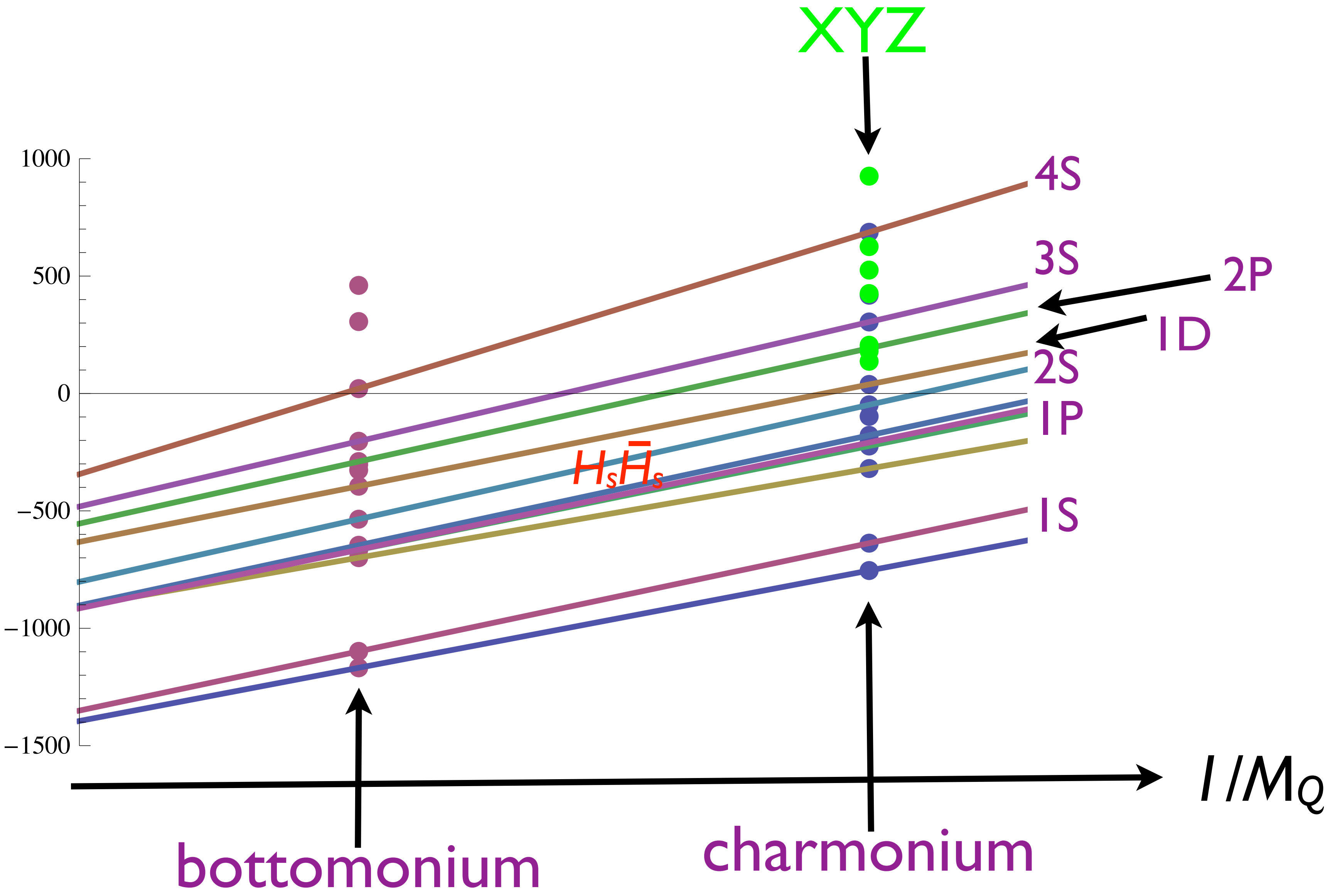
Scattering thresholds

for pairs of heavy-light mesons (relative to $H\bar{H}$)



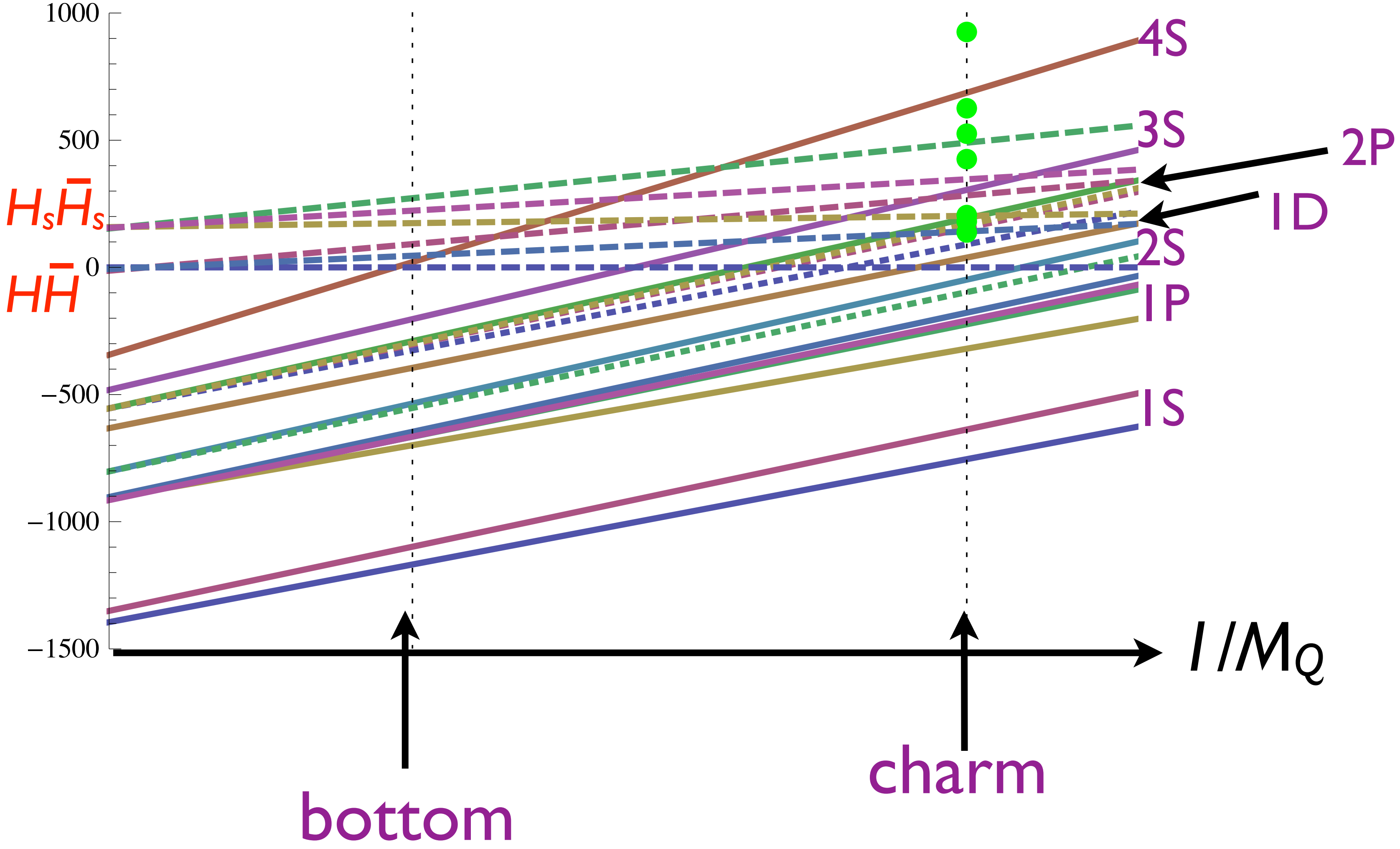
Quarkonium masses

relative to threshold for a heavy-light meson pair



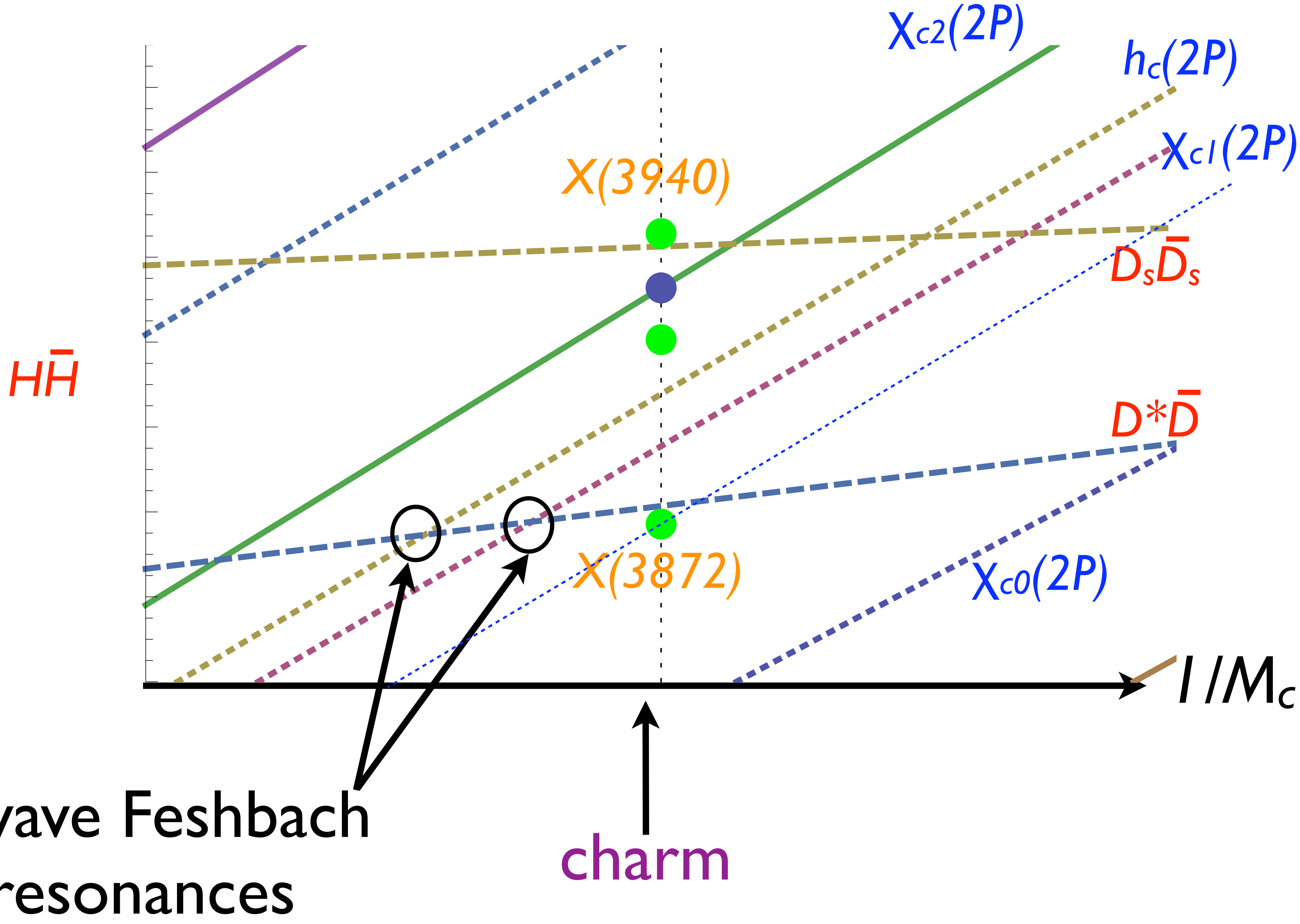
Quarkonium masses and Scattering thresholds

relative to threshold for a heavy-light meson pair



Quarkonium masses and Scattering thresholds

relative to threshold for a heavy-light meson pair



S-wave Feshbach resonances

charm

Lattice Gauge Theory

QCD calculations from 1st principles

QCD parameters: α_s

m_u, m_d, m_s (or m_π, m_K)

M_c, M_b (or M_D, M_B)

space-time lattice

spacing: a

volume: $L^3 x T$

calculate masses!

radiative transitions

decays into 2 nonrelativistic hadrons?

Lattice QCD

quantitative calculations for heavy hadrons require ...

- light quarks: dynamical *u, d, s*
  required for correct running of α_s
  even if there are no constituent light quarks
- heavy quarks: relativistic
  OR nonrelativistic with successive improvement terms
- several pion masses to extrapolate to m_π = 140 MeV
- several lattice spacings to extrapolate to $a \to 0$
- several lattice sizes to extrapolate to $L \to \infty$

Lattice QCD for $c\bar{c}$ Mesons

recent progress on $c\bar{c}$ mesons above $D\bar{D}$ threshold

exotic quantum numbers!
charmonium hybrids!
charm meson molecules?

- Regensburg Nov 2011
  Bali, Collins, and Ehmann

- Trinity/JLab/Old Dominion April 2012
  Liu, Moir, Peardon, Ryan, Thomas, Vilaseca,
  Dudek, Edwards, Joo, Richards

Regensburg group

- light quarks: dynamical *u*, *d* but no *s*

  incorrect running of α_s

- heavy quarks: relativistic

- 3 pion masses: 1010 MeV, 400 MeV, 280 MeV

- only 1 lattice spacing: no extrapolation to $a \to 0$

- only 1 lattice size: no extrapolation to $L \to \infty$

Regensburg group

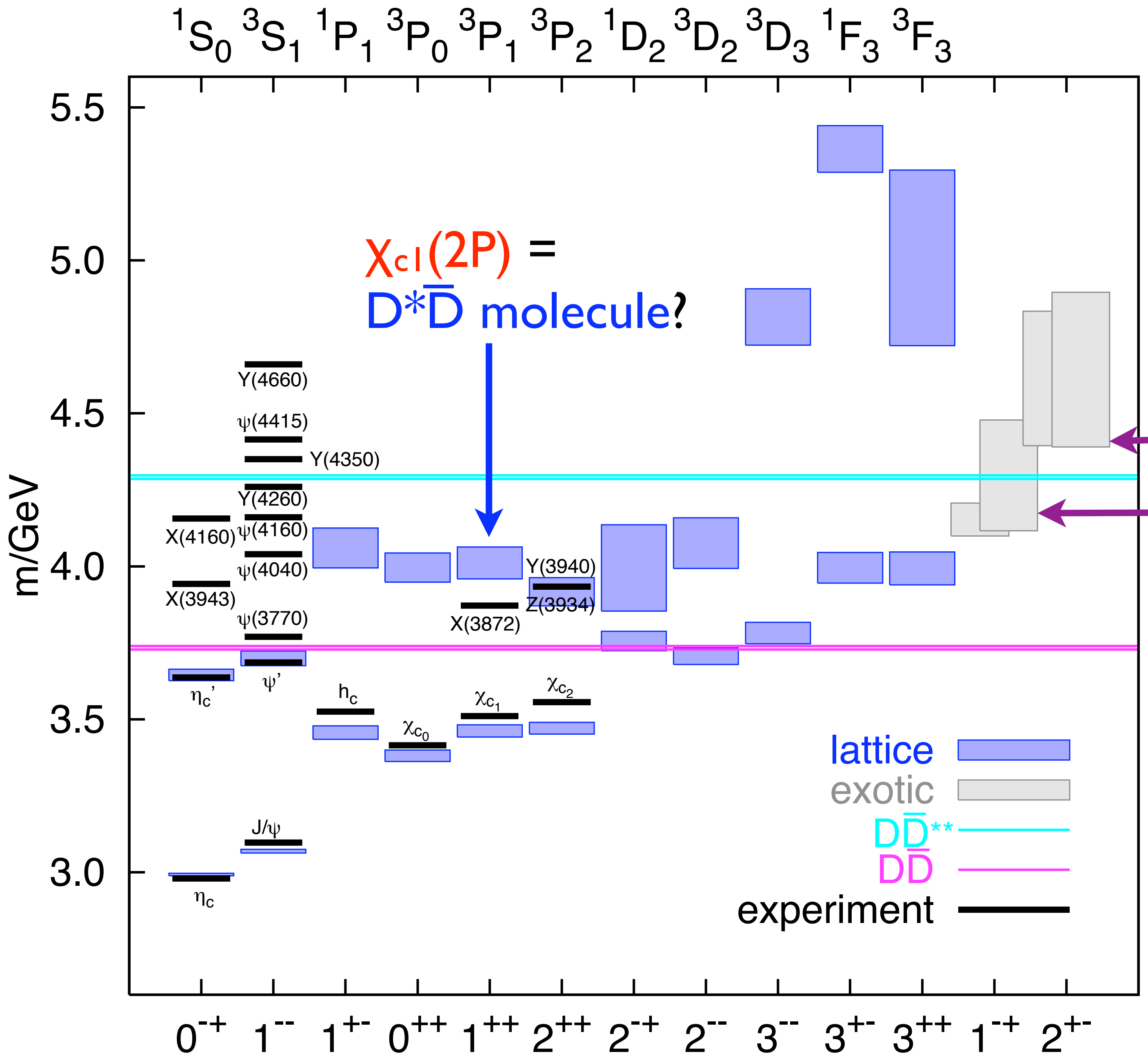
- ground state and 1st excited state
  for all J^{PC} with $J \leq 3$ except 0^{--}, 0^{-+}, 3^{-+} (exotic)

- lightest exotics: 1^{-+} 4150 MeV
  2^{+-} 4610 MeV

- molecules
  0^{-+}, 1^{--}: NO
  1^{++}: YES!?
  binding energy 88 MeV (if m_π = 280 MeV)

Lattice QCD for $c\bar{c}$ Mesons

Regensburg group



$\chi_{c1}(2P)$ =
$D^*\bar{D}$ molecule?

exotics

2^{+-}

1^{-+}

Lattice QCD for $c\bar{c}$ Mesons

Trinity/JLab/Old Dominion group April 2012

- light quarks: dynamical *u, d, s*

  correct running of α_s !

- heavy quarks: relativistic

- only 1 pion mass: $m_\pi = 400$ MeV

- only 1 lattice spacing: no extrapolation to $a \to 0$

- only 2 lattice sizes

Lattice QCD for cc Mesons

Trinity/JLab/Old Dominion group

- all J^{PC} with $J \leq 4$ except 0^{--}, 3^{-+}, 4^{+-} (exotic)
  ground state and up to 5 excited states!

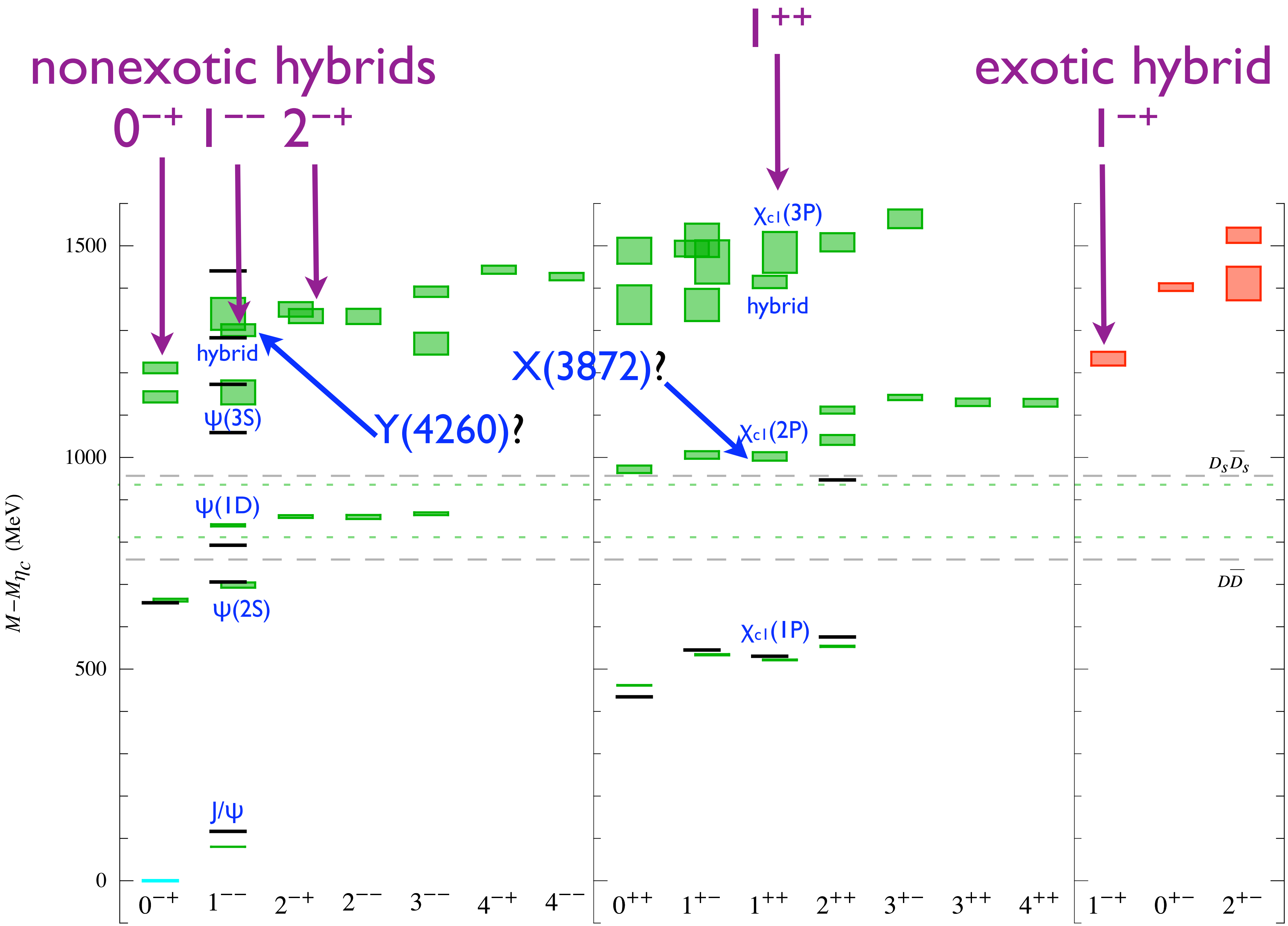
- exotics: 1^{-+}, 0^{+-}, 2^{+-}

- 6 complete charmonium multiplets:
  1S, 1P, 2S, 1D, 2P, 1F, 3S

- 2 complete charmonium hybrid multiplets:
  1^{--} $(0,1,2)^{-+}$ 4 states
  $(0,1,2)^{++}$ $(0,1,1,1,2,2,3)^{+-}$ 10 states

- 1^{--} charmonium hybrid candidate for *Y(4260)*

Lattice QCD for $c\bar{c}$ Mesons



Heavy quark limit

Born-Oppenheimer Approximation

Juge, Kuti, Morningstar 1999

$M_Q >> \Lambda_{\text{QCD}}$

Gluon fields respond quickly, minimizing their energy, in response to the slow motion of the heavy quarks

Step 1: Calculate heavy quark potential

Step 2: Solve Schroedinger equation

Heavy quark limit

Born-Oppenheimer Approximation

Step 1: Calculate heavy quark potential

$V(r)$ = ground-state energy of gluon field
in presence of static Q and Q
separated by distance r in z direction

gluon quantum numbers: $J_z^{(gluon)}$, CP_z

can use Lattice QCD

Juge, Kuti, Morningstar 1999

Step 2: Solve Schroedinger equation

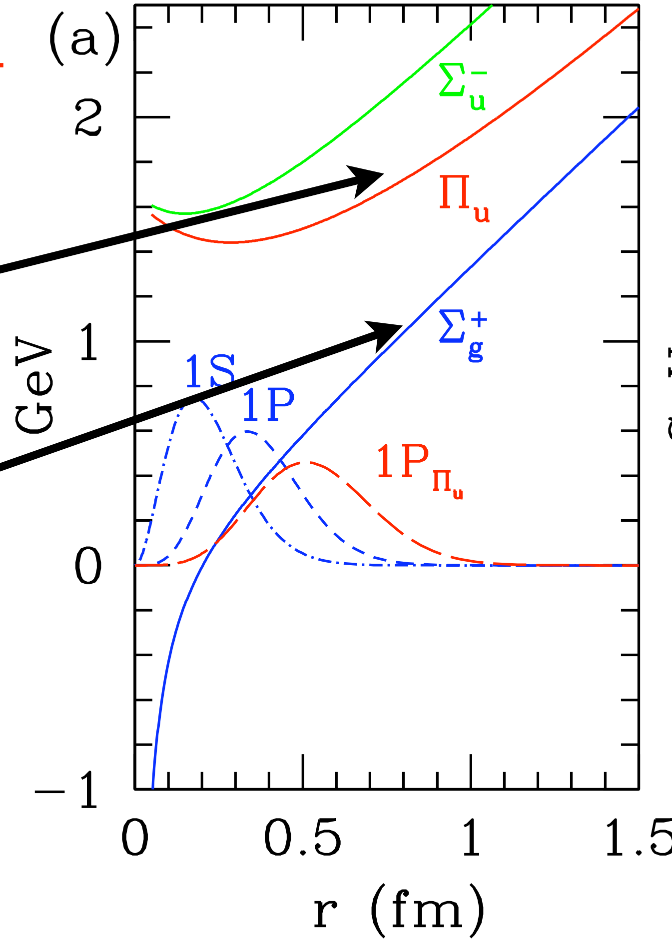
Heavy quark limit

Born-Oppenheimer Approximation

Step 1: Calculate heavy quark potential

$J_z^{(gluon)} = 1$:
quarkonium hybrid

$J_z^{(gluon)} = 0$:
quarkonium



Step 2: Solve Schroedinger equation

to find orbital (*L* = *S*,*P*,*D*,...)
and radial (*n*=1,2,3,...) states for each potential

Can Born-Oppenheimer Approximation be extended to Born-Oppenheimer Expansion?

Step 1a: Calculate adiabatic heavy quark potentials

gluon and light quark fields
in presence of static Q and $\bar{Q}$
separated by distance r in z direction

$V_n(r)$ = energy of n'th state
(ground state and excited states)

Step 1b: Calculate nonadiabatic transition potentials ??

$W_{nm}(r)$: transition of gluon, light quark fields
between energy levels n and m

Step 2: Solve Schroedinger equation

Heavy quark limit

Born-Oppenheimer Approximation

Step 1a: Calculate heavy quark potentials

many potentials with avoided crossings

Step 1b: Calculate transition potentials ??

Step 2: Solve Schroedinger equation

many-coupled-channel problem for each J^{PC}

Summary

Lattice gauge theory will soon be capable of definitive calculations of some properties of $c\bar{c}$ mesons above $D\bar{D}$ threshold:

masses!
radiative transitions
decays into 2 nonrelativistic hadrons?

$\Rightarrow$ strong constraints on phenomenological models

$\Rightarrow$ useful inputs for effective theories?

Will this be enough to understand XYZ states?

$c\bar{c}$ meson puzzle