Tidal Forces and the Roche Limit

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PHYS 305

Introduction

- Tidal Force: a differential forces that arises from the effects of gravity

- Cause tides and changes sea level

- Roche Limit: distance at which an orbiting satellite will disintegrate due to the tidal forces exerted on the satellite from the mass it's orbiting

Equations [1]

$$F_g = -\frac{Gm\mu}{r^2}$$

G = 6.67 x 10-11 m^3 kg^{-1} s^{-2}

$$F_t = \frac{2GMur}{d^3}$$

$$F_g = F_t$$

$$d = R\left(2\frac{\rho_M}{\rho_m}\right)^{\frac{1}{3}}$$

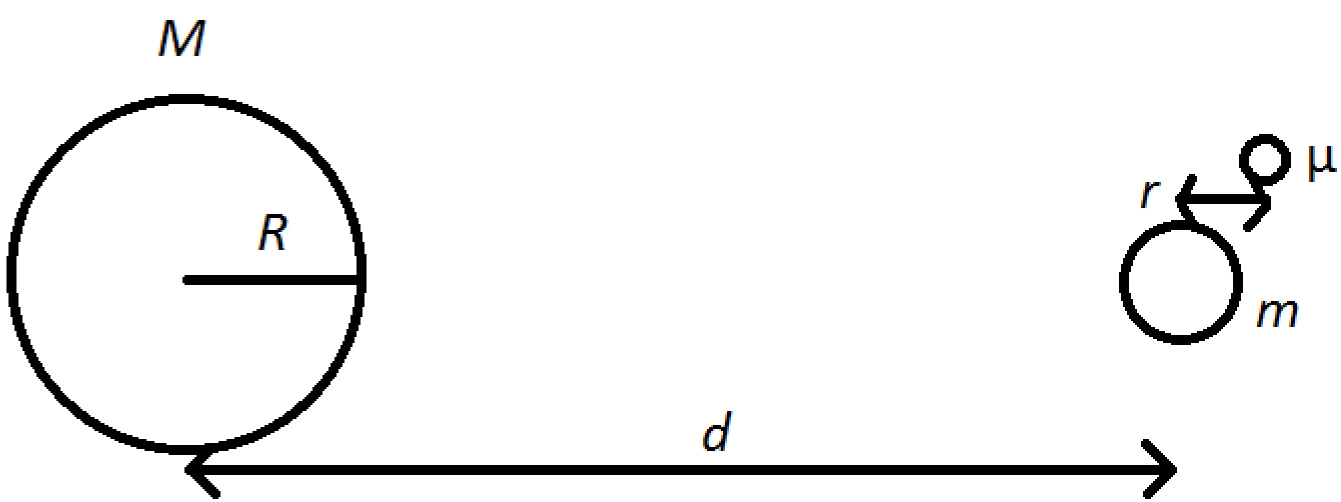


Fig 1: Forces and Roche limit

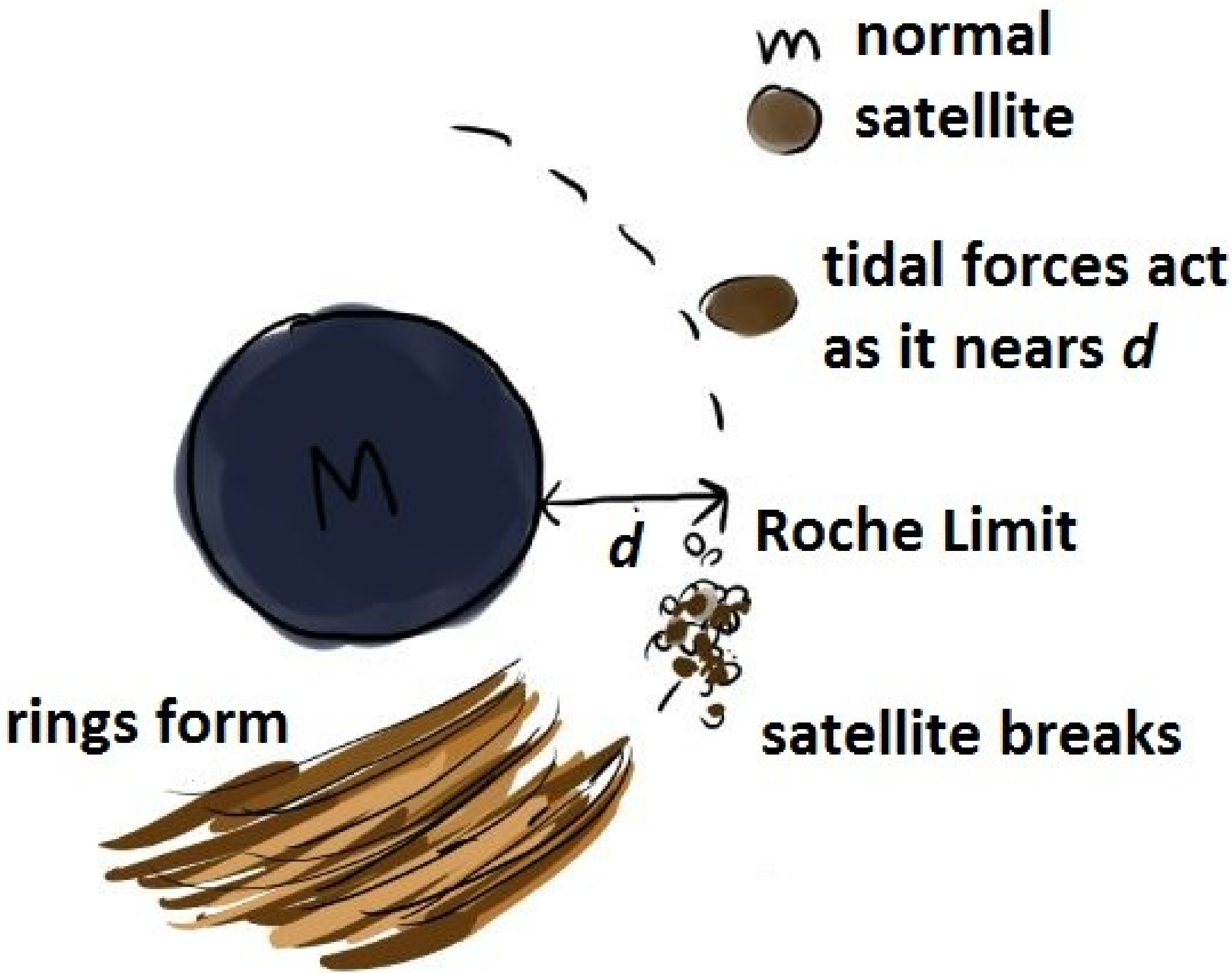
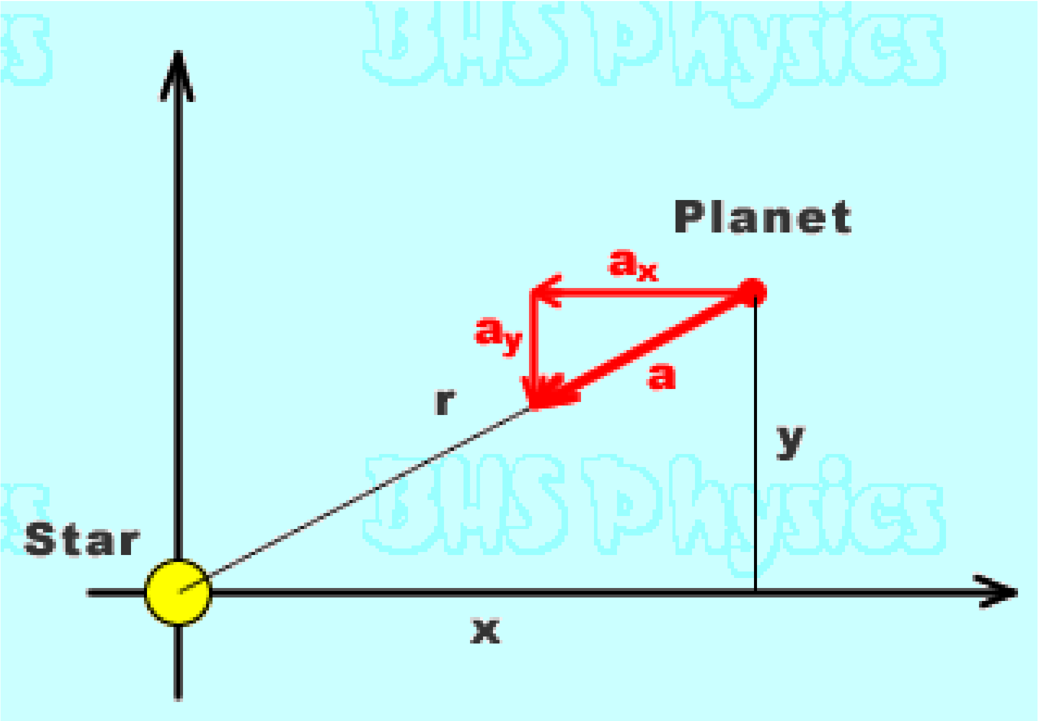


Fig. 2: Diagram describing the formation of rings due to Roche Limit

Equations – general orbit [2]

$$\frac{dv}{dt} = -\frac{GM}{r^2}$$

$$\frac{dv_x}{dt} = -\frac{GMx}{r^3}, \quad \frac{dv_y}{dt} = -\frac{GMy}{r^3}$$



$$r = \sqrt{x^2 + y^2}$$

Fig 3: Orbital diagram

```
G = 1.0;            //gravitational constant
M = 1.0;            // mass of main planet

dt = 0.1;
t0 = 0.0;

// set initial x,y components
x0 = 1.0;
y0 = 0.0;

// set initial velocity components
vx0 = 0.0;
vy0 = 1.8;

out << x0 << "\t" << y0 << "\t" << endl;

for (t = t0; t < 15; t += dt) {

      x = x0 + vx0*(dt/2.0);
      y = y0 + vy0*(dt/2.0);
      r = sqrt(pow(x,2) + pow(y,2));
      vx = vx0 - ((G*M*x)/pow(r,3))*dt;
      vy = vy0 - ((G*M*y)/pow(r,3))*dt;

      x0 = x;
      y0 = y;
      vx0 = vx;
      vy0 = vy;

      cout << x << "\t" << y << endl;
      out << x << "\t" << y << endl;
```

- Define constants
- Set variables (initial *x* and *y*, initial velocities
- Using Runge-Kutta second order method, calculate position and velocity
- Plot *x* and *y* values
- `for loop` repeats until orbit is plotted

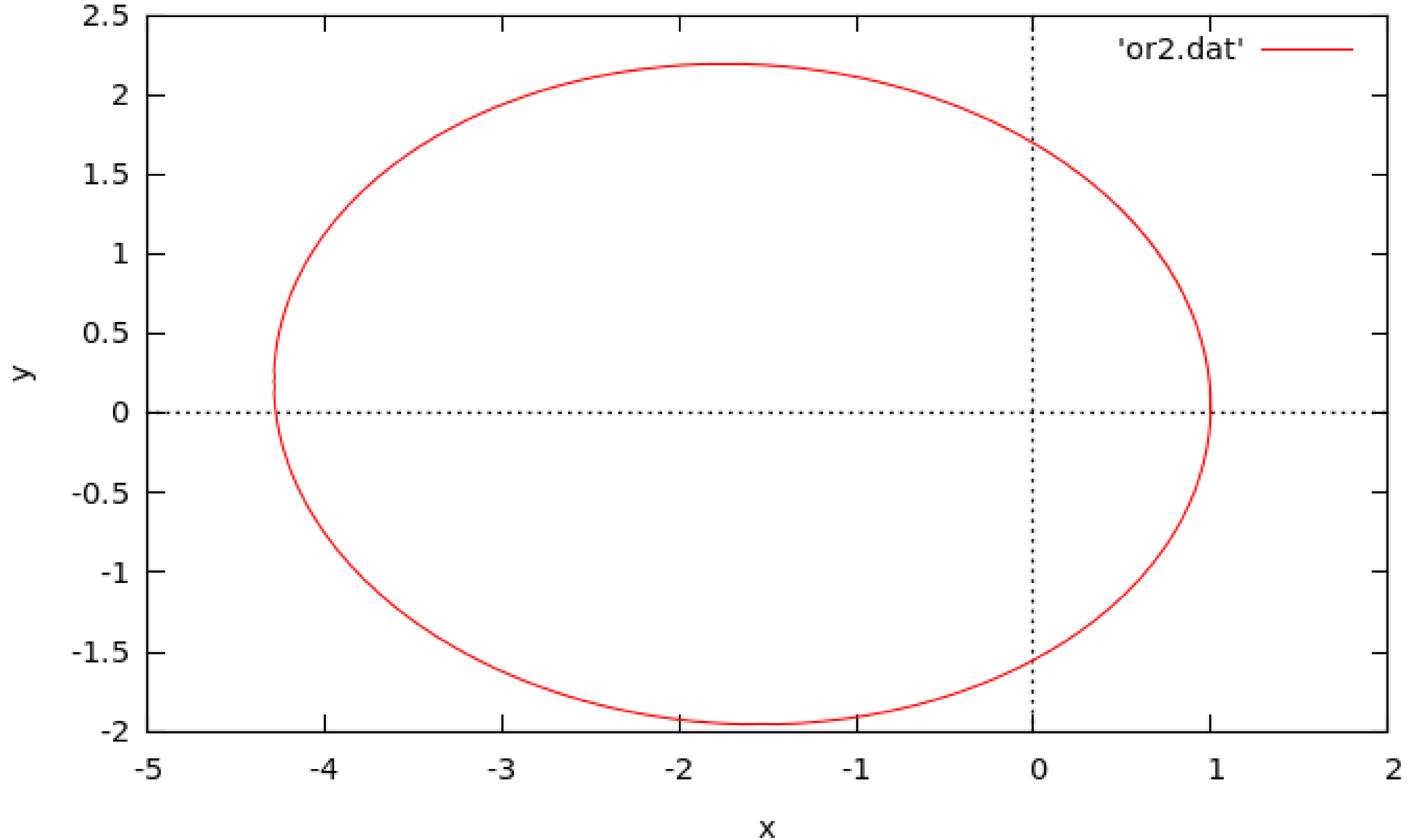


Fig. 4: Generic orbit of satellite starting at x = 1.0, y = 0.0, v_y = 1.8, and G, M = 1.0

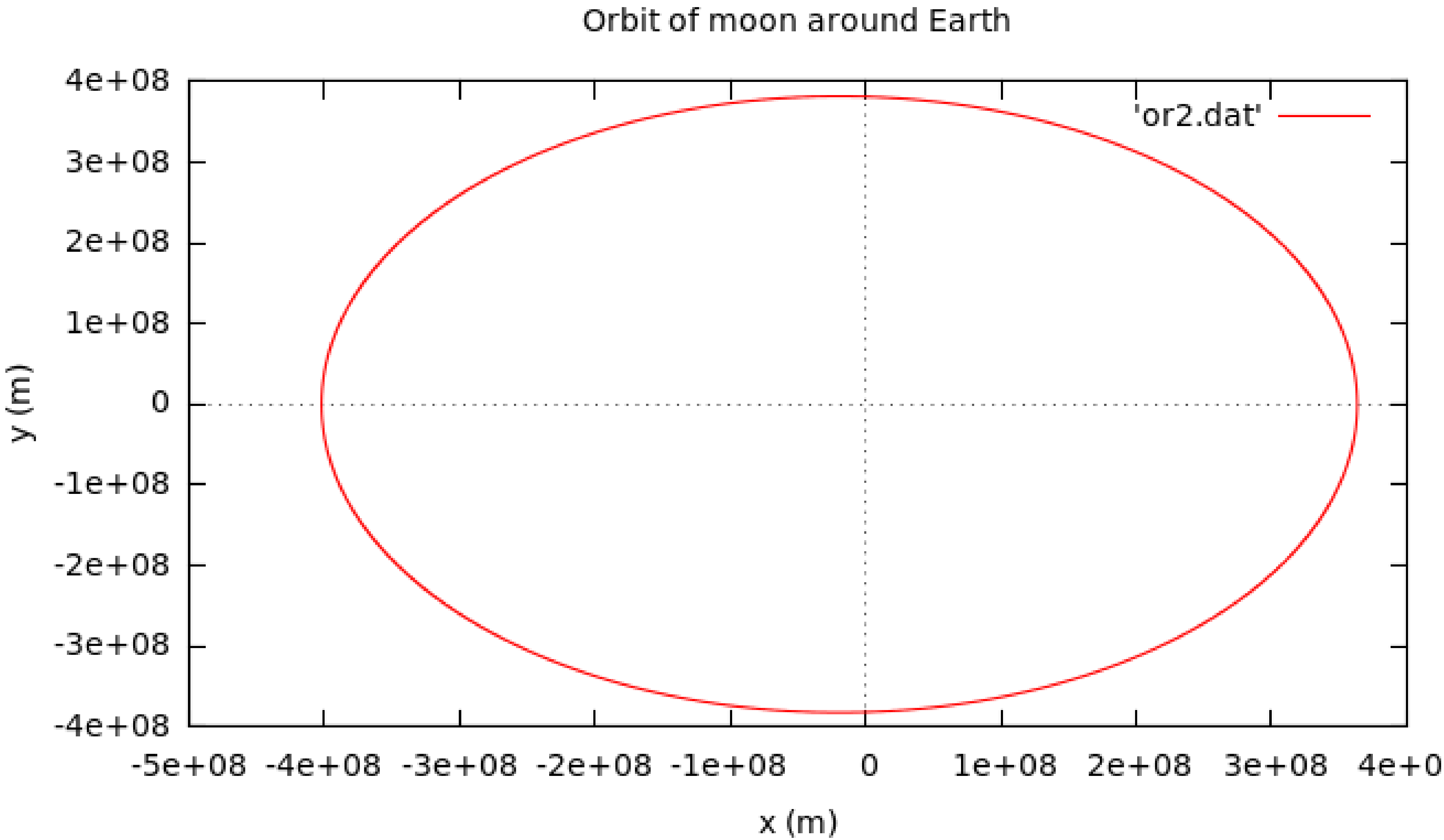


Fig 5:
x = 3.633 x 10^8 m vy = 1073.0 m/s, dt = 10000, t = 24,000
Orbital days: 27.8, actual = 27.3217 [3]

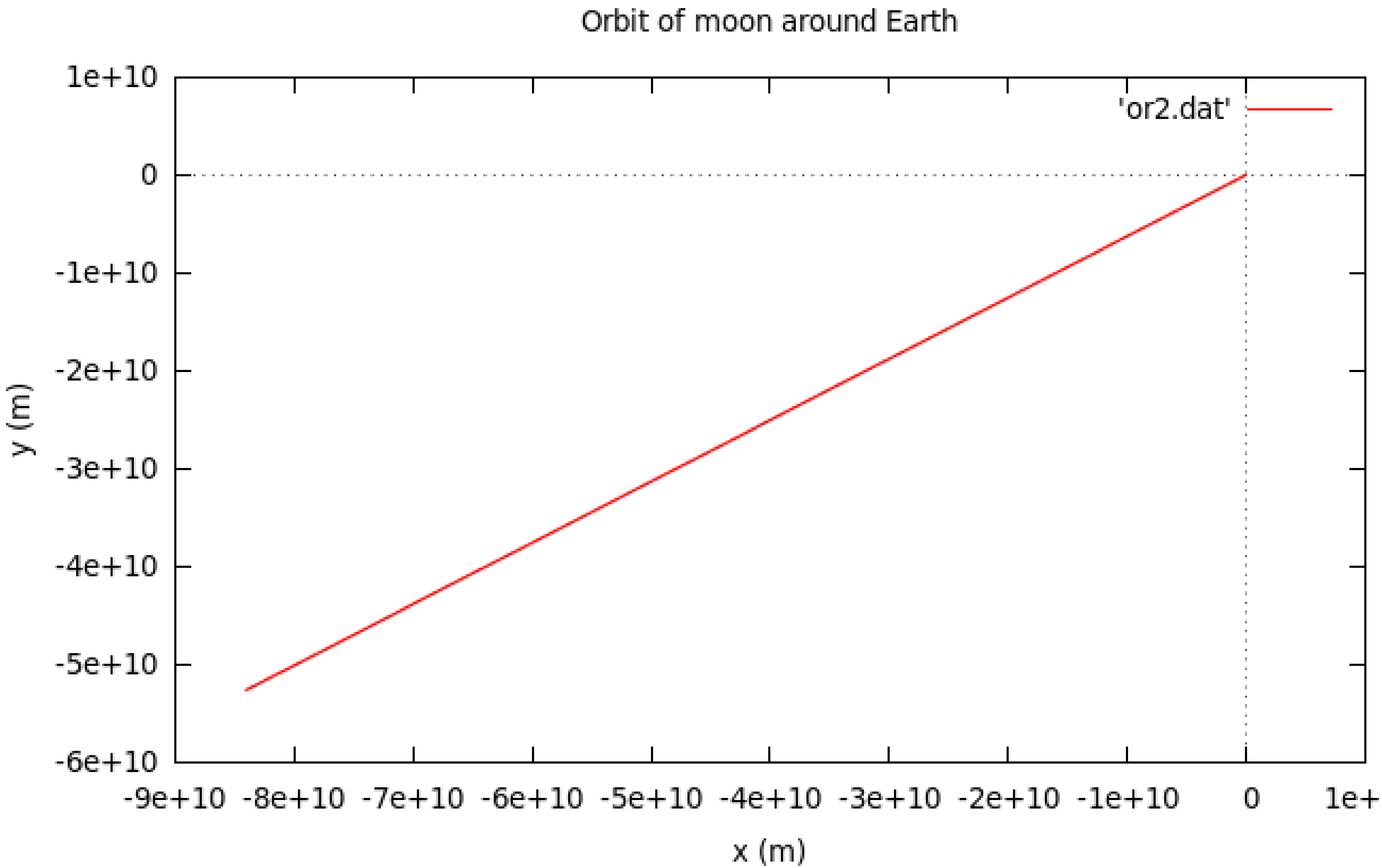


Fig 6: Roche limit of Earth/Moon = 9.497×10^6 m

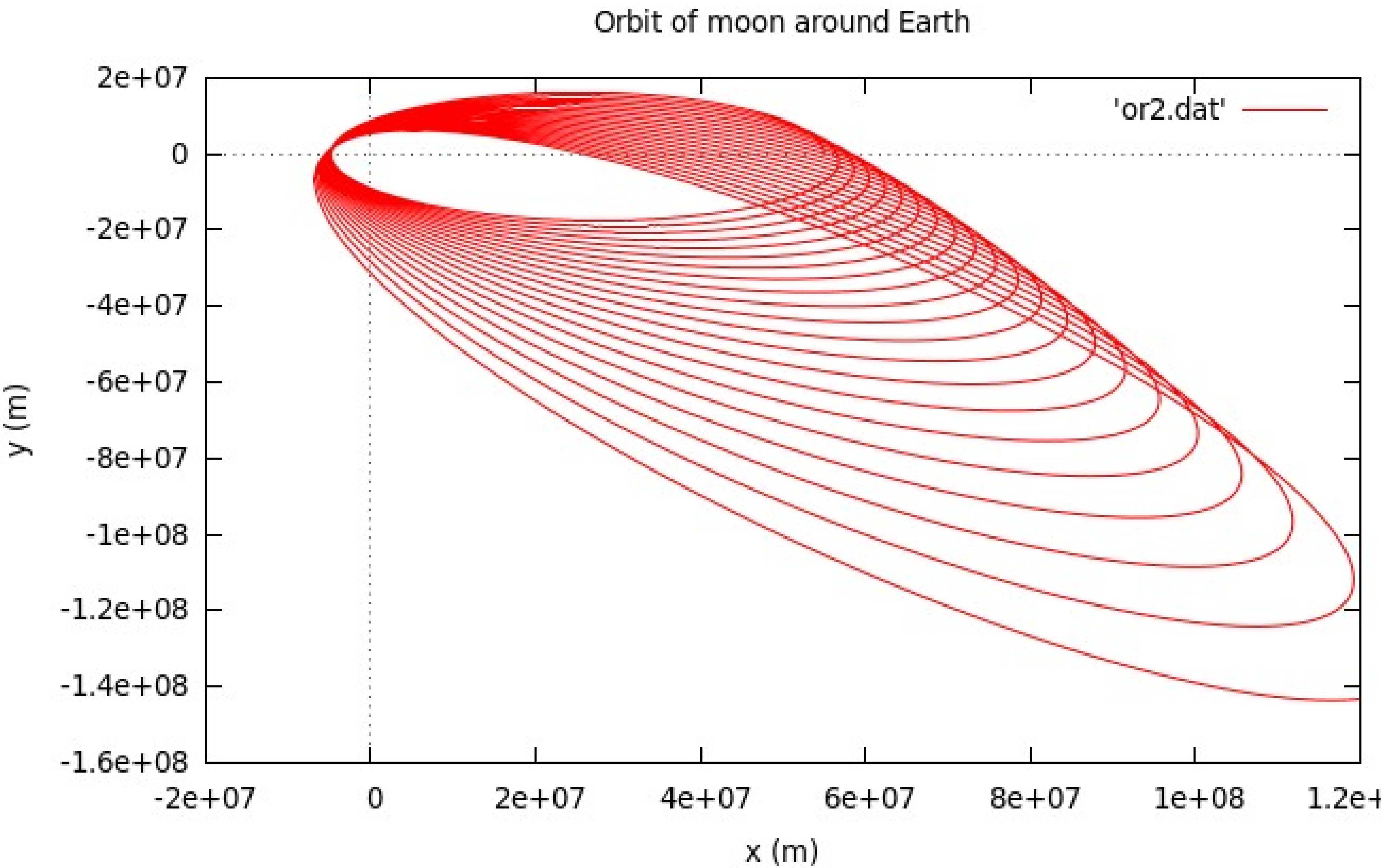


Fig 7: at x = 5.497×10^7 m

Project Expectations

- Finished C++ coding
- Finished plotting a general orbit
- Can use actual values and constants to plot orbit

Not done

- Compare satellite orbit with Roche Limit orbit

Sources

[1] http://en.wikipedia.org/wiki/Roche_limit

[2] http://www.batesville.k12.in.us/physics/phynet/mecl

[3] http://nssdc.gsfc.nasa.gov/planetary/factsheet/