

# An study of relativistic energy, momentum, and particle collisions

Natalie Nagata

ProjectReview

PHYS 305: Computational Physics

# Overview

- Background
- Objective
- Project Approach
- Results (thus far)
- Discussion
- Further work

# Background

- 1905 (“*Annus Mirabilis*”, the Miracle Year!)
- Albert Einstein—from patent office clerk to science rockstar!

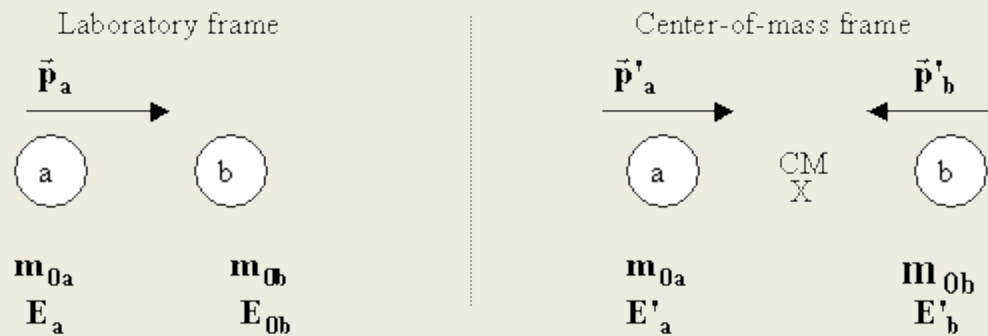


# Special Relativity

- Postulate 1
  - The laws of physics remain the same in all inertial frames
- Postulate 2
  - The speed of light is invariant in all inertial frames
- What does this mean???
  - Time dilation
  - Length contraction
  - Relativistic energies and momentum!

# Objective

- Show the relationship between relativistic momentum and energy (Lab frame vs CM frame)



**Fig. 1 Two-particle collision observed in lab and CM frames**

[http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/application\\_s\\_of\\_special\\_relativi.htm](http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/application_s_of_special_relativi.htm)

# Objective

Velocity of CM with respect to (WRT) to lab frame,

$$\mathbf{v}_{\text{cm}} = \frac{\mathbf{p}c^2}{\mathbf{E}} \quad (1)$$

Total momentum and energy in lab frame,

$$\mathbf{p} = \mathbf{p}_a + \mathbf{p}_b = \mathbf{p}_a \quad (2)$$

$$\mathbf{E} = \mathbf{E}_a + \mathbf{E}_b = \mathbf{E}_a + m_{0b}c^2 \quad (3)$$

# Objective

Using the energy-momentum relation,

$$\mathbf{E} = \mathbf{c} \cdot \sqrt{\mathbf{p}^2 + m_0^2 \mathbf{c}^2} \quad (4)$$

Eqn. (1) rewritten as function of total lab momentum and lab energy

$$\mathbf{v}_{\text{cm}} = \frac{\mathbf{p}_a \mathbf{c}^2}{\mathbf{E}_a + m_{0b} \mathbf{c}^2} \quad (5)$$

# Objective

Momentum WRT the CM frame

$$\mathbf{p}_a' = \frac{\mathbf{p}_a - \frac{\mathbf{v}_{cm}}{c^2} E_a}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad \mathbf{p}_b' = \frac{\mathbf{p}_b - \frac{\mathbf{v}_{cm}}{c^2} E_b}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad (6)$$

Energy WRT the CM frame

$$E_a' = \frac{E_a - \mathbf{p}_a \mathbf{v}_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad E_b' = \frac{E_b - \mathbf{p}_b \mathbf{v}_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad (7)$$



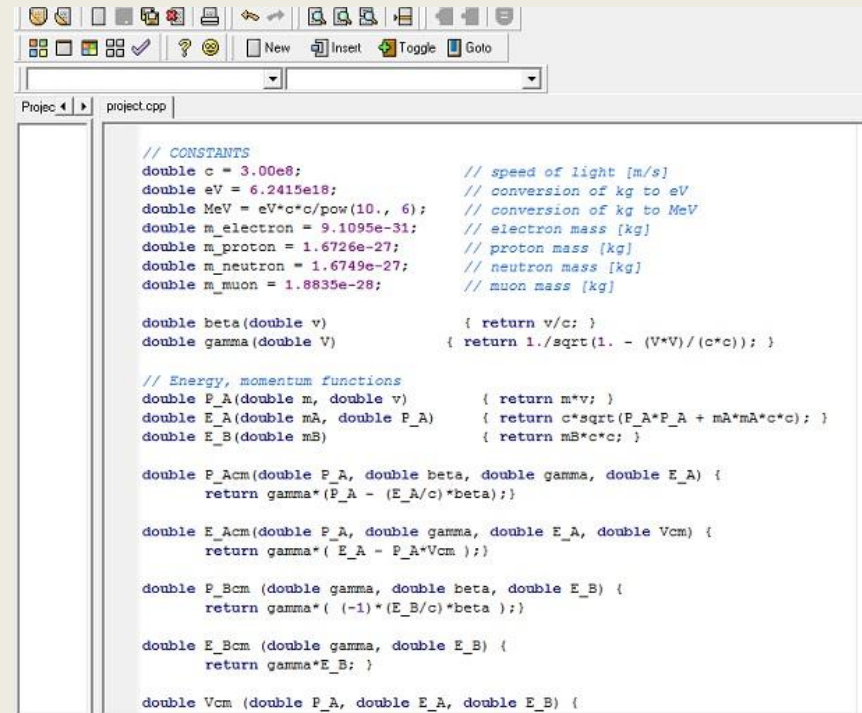
# Objective

- Graphically calculate and display the minimum threshold energies in some particle collisions
  - Necessary for the creation of certain particles

$$E_a^{\min} = \frac{E_T'^2 - (m_a^2 + m_b^2)c^4}{2m_b c^2}$$

# Project Approach

- Defining
  - Constants
  - Functions passing functions by value and/or reference (be careful of naming)



```
// CONSTANTS
double c = 3.00e8; // speed of light [m/s]
double eV = 6.2415e18; // conversion of kg to eV
double MeV = eV*c*c/pow(10., 6); // conversion of kg to MeV
double m_electron = 9.1095e-31; // electron mass [kg]
double m_proton = 1.6726e-27; // proton mass [kg]
double m_neutron = 1.6749e-27; // neutron mass [kg]
double m_muon = 1.8835e-28; // muon mass [kg]

double beta(double v) { return v/c; }
double gamma(double V) { return 1./sqrt(1. - (V*V)/(c*c)); }

// Energy, momentum functions
double P_A(double m, double v) { return m*v; }
double E_A(double mA, double P_A) { return c*sqrt(P_A*P_A + mA*mA*c*c); }
double E_B(double mB) { return mB*c*c; }

double P_Acm(double P_A, double beta, double gamma, double E_A) {
    return gamma*(P_A - (E_A/c)*beta);}

double E_Acm(double P_A, double gamma, double E_A, double Vcm) {
    return gamma*( E_A - P_A*Vcm );}

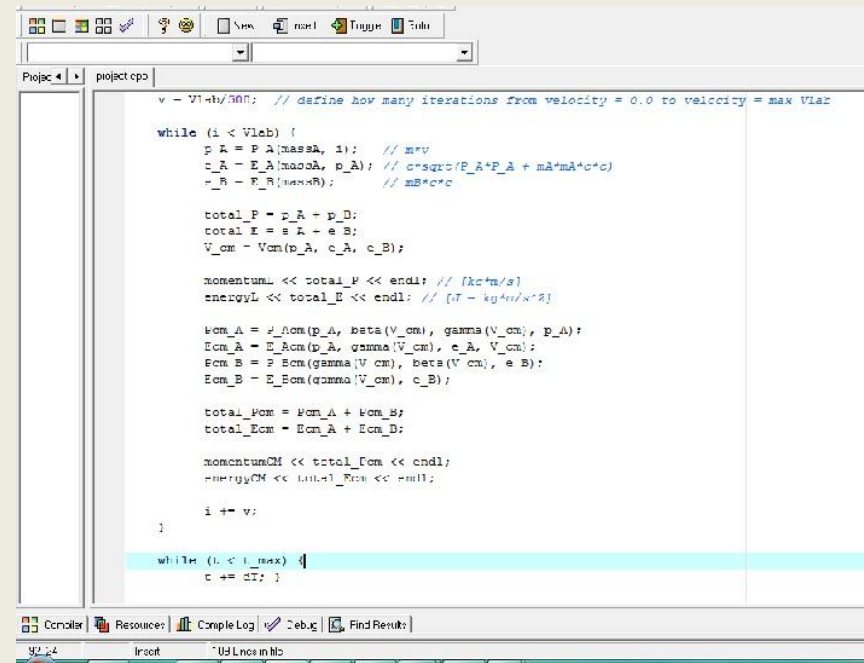
double P_Bcm (double gamma, double beta, double E_B) {
    return gamma*( (-1)*(E_B/c)*beta );}

double E_Bcm (double gamma, double E_B) {
    return gamma*E_B; }

double Vcm (double P_A, double E_A, double E_B) {
```

# Project Approach

- While-loop
  - Calls and defines energy and momentum functions
  - Calculates total momentum and energy over a varying velocity from 0 to nearly the speed of light



```
v = 14/100; // define how many iterations from velocity = 0.0 to velocity = max Vmax

while (i < Vmax) {
    p_A = P_A(massA, i); // m*v
    e_A = E_A(massA, p_A); // c*sqrt(P_A^2 + m^2*c^2)
    p_B = P_B(massB); // mB*c*c

    total_P = p_A + p_B;
    total_E = e_A + e_B;
    V_cm = Vcm(p_A, e_A, e_B);

    momentumL << total_P << endl; // [kg*m/s]
    energyL << total_E << endl; // [J = kg^2/m^2]

    fcm_A = f_cm(p_A, beta(V_cm), gamma(V_cm), p_A);
    fcm_B = f_cm(p_B, gamma(V_cm), e_A, V_cm);
    fcm_C = f_cm(gamma(V_cm), beta(V_cm), e_B);
    fcm_D = f_cm(gamma(V_cm), e_B);

    total_fcm = fcm_A + fcm_B;
    total_fcm = fcm_C + fcm_D;

    momentumCM << total_fcm << endl;
    energyCM << total_fcm << endl;

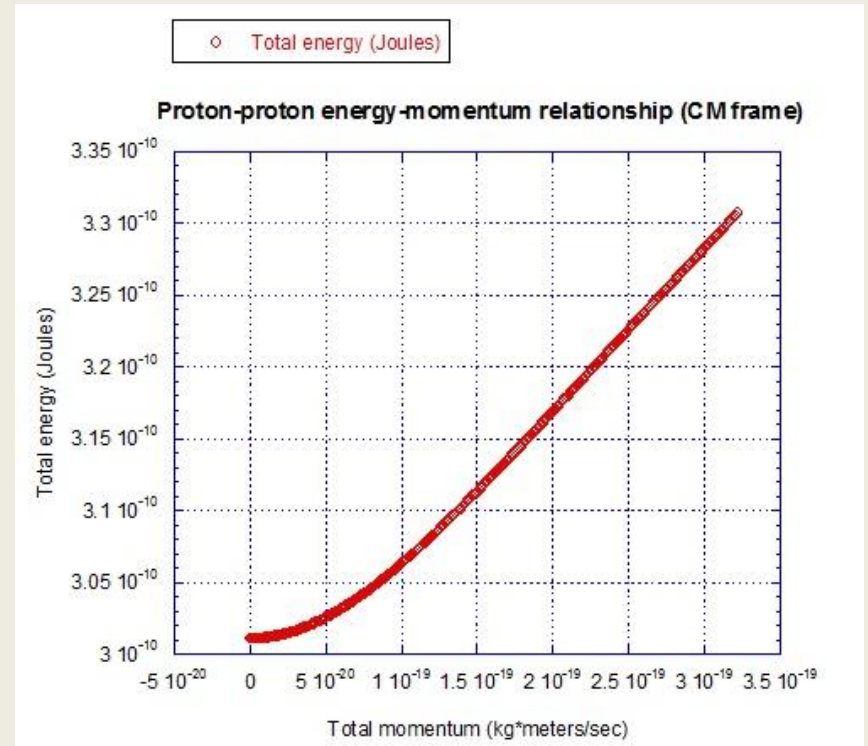
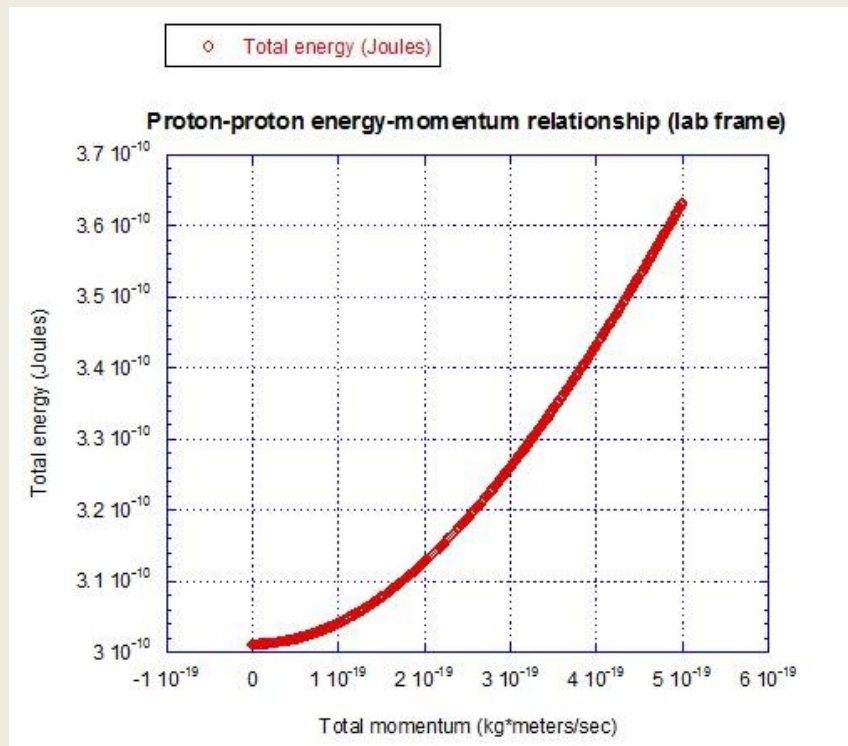
    i += v;
}

while (i < i_max) {
    c += dT; }
```

# Results (thus far)

Lab frame ( $0 < v/c < 1$ )

CM frame ( $0 < v/c < 1$ )



# Discussion

- As  $v \rightarrow c$ 
  - $E(\text{lab}) \gg E(\text{CM frame})$
- Energies of particle colliders using colliding beams (CM frame) instead of colliding with a stationary particle (lab frame)

# Discussion

- Some significant things to keep in mind:
  - Unit conversions (very tedious)
  - Many functions, many variables
    - If time permits, better organize better for clarity
    - Solution: stuctures? Arrays?

# Project Outlook

- Add asymptotic values of threshold energies needed to create new particles
- Unit conversions!!!
- Scenarios:



- Neutral pion mass = 135 MeV
- Proton rest energy = 938 MeV
- CM frame:  $v = \pm 0.36c$
- Lab frame (incoming proton):  $v = 0.64c$

# References

- Bitu, R. "Applications of Special Relativity on Particle Physics."  
[http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/applications\\_of\\_special\\_relativi.htm](http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/applications_of_special_relativi.htm)
- Fowler, M. "Transforming Energy into Mass: Particle Creation."  
[http://galileo.phys.virginia.edu/classes/252/particle\\_creation.html](http://galileo.phys.virginia.edu/classes/252/particle_creation.html)
- Taylor, John R. "Classical Mechanics". University Science Books. 2005