An study of relativistic energy, momentum, and particle colllisions

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ProjectReview

PHYS 305: Computational Physics

Overview

- Background
- Objective
- Project Approach
- Results (thus far)
- Discussion
- Further work

Background

- 1905 (“*Annus Mirabilis*”, the Miracle Year!)
- Albert Einstein—from patent office clerk to science rockstar!

Special Relativity

- Postulate 1
  - The laws of physics remain the same in all inertial frames

- Postulate 2
  - The speed of light is invariant in all inertial frames

- What does this mean???
  - Time dilation
  - Length contraction
  - Relativistic energies and momentum!

Objective

- Show the relationship between relativistic momentum and energy (Lab frame vs CM frame)

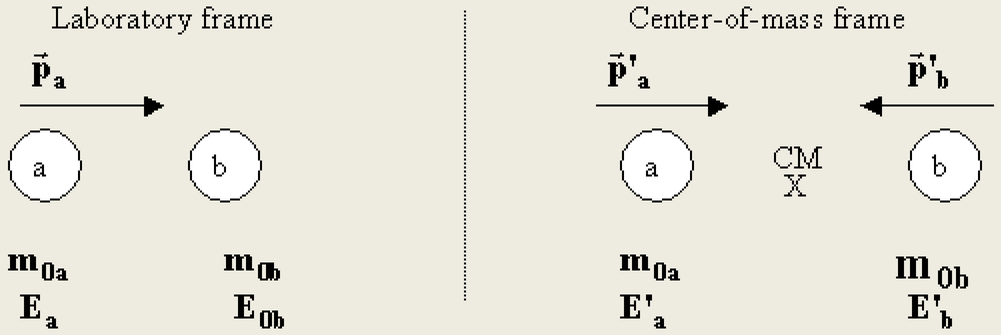


Fig. 1 Two-particle collision observed in lab and CM frames

http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/applications_of_special_relativi.htm

Objective

Velocity of CM with respect to (WRT) to lab frame,

$$v_{cm} = \frac{pc^2}{E} \qquad (1)$$

Total momentum and energy in lab frame,

$$p = p_a + p_b = p_a \qquad (2)$$

$$E = E_a + E_b = E_a + m_{0b}c^2 \qquad (3)$$

Objective

Using the energy-momentum relation,

$$E = c \cdot \sqrt{p^2 + m_0^2 c^2} \qquad (4)$$

Eqn. (1) rewritten as function of total lab momentum and lab energy

$$v_{cm} = \frac{p_a c^2}{E_a + m_{0b} c^2} \qquad (5)$$

Objective

Momentum WRT the CM frame

$$p_a' = \frac{p_a - \frac{v_{cm}}{c^2} E_a}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \qquad p_b' = \frac{p_b - \frac{v_{cm}}{c^2} E_b}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \qquad (6)$$

Energy WRT the CM frame

$$E_a' = \frac{E_a - p_a v_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \qquad E_b' = \frac{E_b - p_b v_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \qquad (7)$$

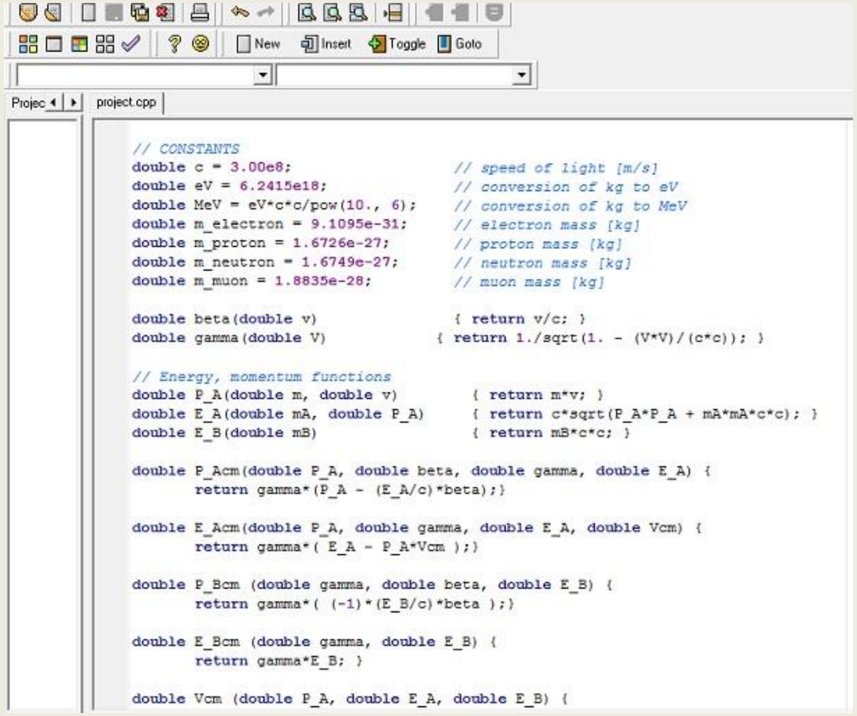
Objective

- Graphically calculate and display the minimum threshold energies in some particle collisions
    - Necessary for the creation of certain particles

$$E_a^{min} = \frac{E_T'^{\,2} - (m_a^{\,2} + m_b^{\,2})c^4}{2 m_b c^2}$$

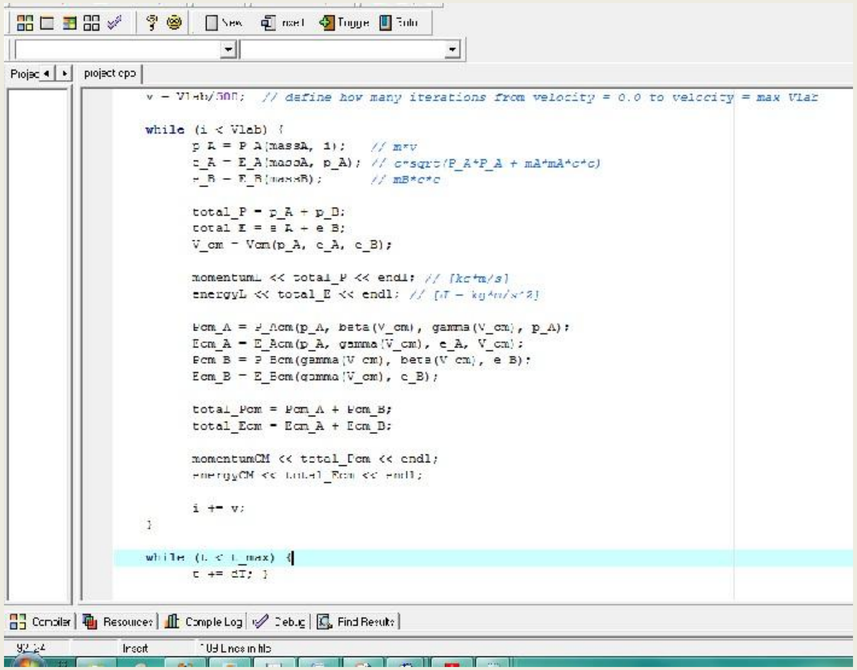
Project Approach

- Defining
  - Constants
  - Functions passing functions by value and/or reference (be careful of naming)



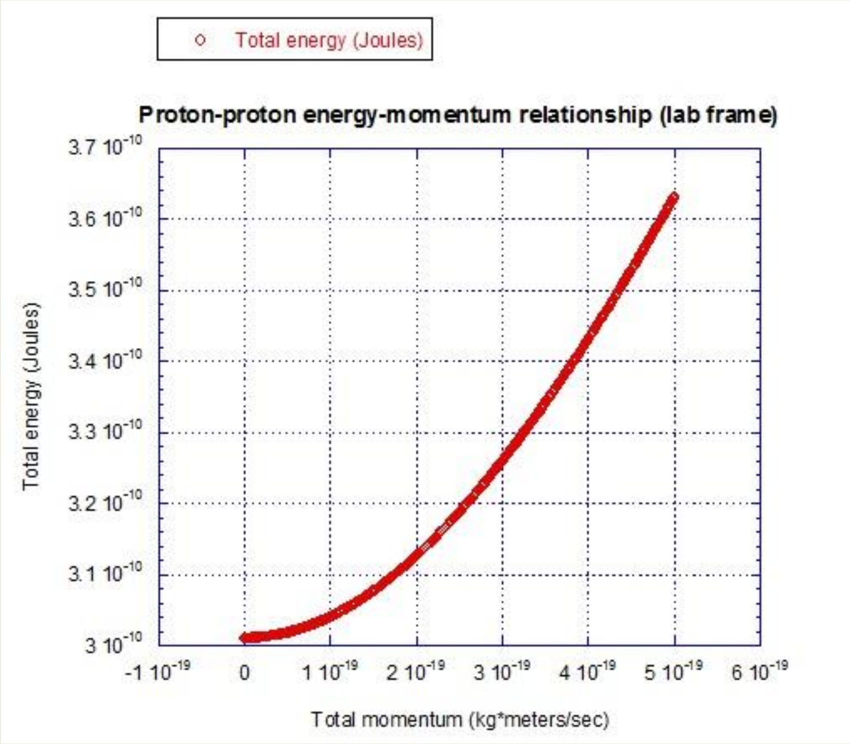
Project Approach

- While-loop
  - Calls and defines energy and momentum functions
  - Calculates total momentum and energy over a varying velocity from 0 to nearly the speed of light

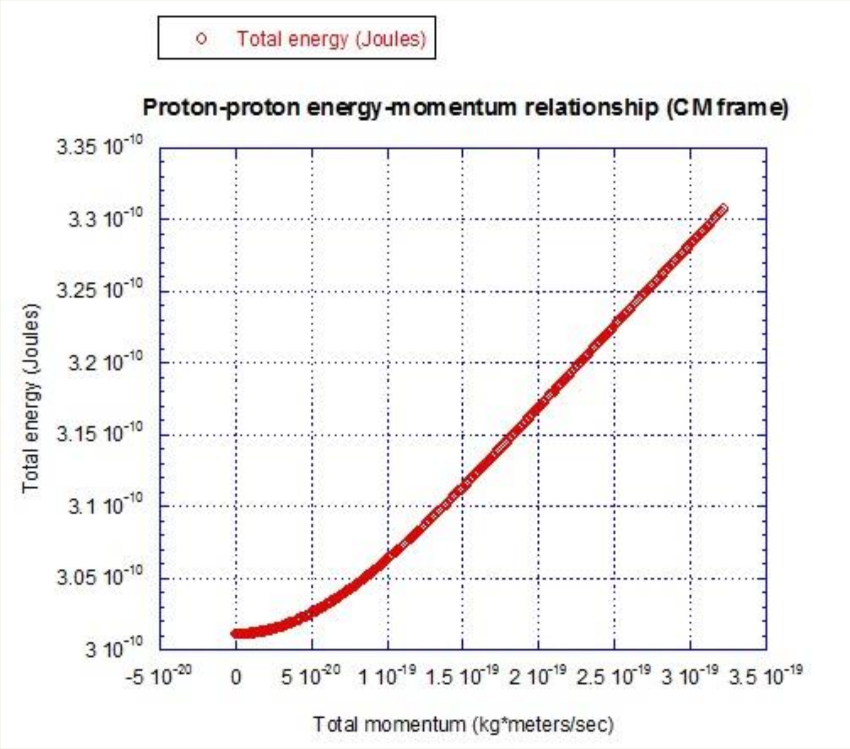


Results (thus far)

Lab frame (0 < v/c < 1)



CM frame (0 < v/c < 1)



Discussion

- As $v \rightarrow c$
  - E (lab) >> E (CM frame)

- Energies of particle colliders using colliding beams (CM frame) instead of colliding with a stationary particle (lab frame)

Discussion

- Some significant things to keep in mind:
  - Unit conversions (very tedious)
  - Many functions, many variables
    - If time permits, better organize better for clarity
    - Solution: stuctures? Arrays?

Project Outlook

- Add asymptotic values of threshold energies needed to create new particles
- Unit conversions!!!
- Scenarios:

$$p + p \rightarrow p + p + \pi^0$$

  - Neutral pion mass = 135 MeV
  - Proton rest energy = 938 MeV
  - CM frame: v = $\pm$0.36c
  - Lab frame (incoming proton): v = 0.64c

References

- Bitu, R. "Applications of Special Relativity on Particle Physics." http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/applications_of_special_relativi.htm
- Fowler, M. "Transforming Energy into Mass: Particle Creation." http://galileo.phys.virginia.edu/classes/252/particle_creation.html
- Taylor, John R. "Classical Mechanics". University Science Books. 2005