
The X Files
The Truth Is Out There

Effects of the $D\bar{D}\pi$ threshold on the $X(3872)$

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Discovery of $X(3872)$ in B decay

Belle collaboration, August 2003 hep-ex/0309032

$$B^{\pm} \longrightarrow X + K^{\pm}, \qquad X \longrightarrow J/\psi\, \pi^{+}\pi^{-}$$

confirmed by Babar hep-ex/0406022

Observation of $X(3872)$ in $p\bar{p}$ collisions

CDF collaboration, September 2003 hep-ex/0312021

$$p\bar{p} \longrightarrow X + \text{anything}, \qquad X \longrightarrow J/\psi\, \pi^{+}\pi^{-}$$

confirmed by D0 hep-ex/0405004

Mass and Width of the $X(3872)$

Mass — Belle, CDF, Babar, D0

$$M_X = 3871.9 \pm 0.5 \text{ MeV}$$

Width — Belle, hep-ex/0309032

$$\Gamma_X < 2.3 \text{ MeV} \qquad (90\% \text{ C.L.})$$

Mass is extremely close to the $D^{*0}\bar{D}^0$ threshold

$$M_X - (M_{D^{*0}} + M_{D^0}) = -0.31 \pm 0.64 \text{ MeV}$$

using recent precision determination of D^0 mass by CLEO

Quantum Numbers of the $X(3872)$

$X \longrightarrow J/\psi\,\gamma$ Belle, hep-ex/0505037

$\Longrightarrow C = +$

$X \longrightarrow J/\psi\,\pi^+\pi^-$ Belle, hep-ex/0505038

angular correlations $\Longrightarrow$ not $J^{PC} = 0^{++}$, 0^{-+}

$\pi\pi$ mass distribution $\Longrightarrow$ not 1^{-+}, 2^{-+}

1^{++} "strongly favored", but 2^{++} "not ruled out"

$X \longrightarrow D^0 \bar{D}^0 \pi^0$ Belle, preliminary

8 MeV above threshold $\Longrightarrow$ 2^{++} ruled out

$$J^{PC} = 1^{++}$$

Decays of the $X(3872)$

Measured branching ratios

$$\frac{\mathrm{Br}[X \to J/\psi\, \pi^+\pi^-]}{\mathrm{Br}[X \to D^0 \bar{D}^0\, \pi^0]} \approx 0.059 \pm 0.025 \text{ ??}$$

Belle, preliminary ??

$$\frac{\mathrm{Br}[X \to J/\psi\, \pi^+\pi^-\pi^0]}{\mathrm{Br}[X \to J/\psi\, \pi^+\pi^-]} = 1.0 \pm 0.4 \pm 0.3$$

Belle, hep-ph/0505038

$$\frac{\mathrm{Br}[X \to J/\psi\, \gamma]}{\mathrm{Br}[X \to J/\psi\, \pi^+\pi^-]} = 0.14 \pm 0.05$$

Belle, hep-ex/0505037

What is the $X(3872)$?

Belle: D-wave charmonium? $D^*\bar{D}$ molecule?

- charmonium

  Close and Paige (9-03); Pakvasa and Suzuki (9-03); Barnes and Godfrey (11-03); Eichten, Lane, and Quigg (1-04); Olsen (7-04); Meng, Gao, and Chao (6-05)

  motivated by J/ψ among decay products

- $D^*\bar{D}$ molecule

  Tornqvist (8-03, 2-04); Close and Paige (9-03); Pakvasa and Suzuki (9-03); Voloshin (9-03, 8-04, 9-05, 5-06); Wong (11-03); Braaten and Kusunoki (11-03, 2-04, 12-04, 6-05, 7-05, 9-06); Swanson (11-03, 6-04); Braaten, Kusunoki, and Nussinov (4-04); Kalashnikova (6-05); AlFiky, Gabbiani, and Petrov (6-05); ElHady (3-06)

  motivated by proximity to $D^*\bar{D}$ threshold

- threshold cusp from interactions with $D^*\bar{D}$

  Bugg (10-04)

  motivated by proximity to $D^*\bar{D}$ threshold

What is the $X(3872)$? (cont.)

- tetraquark: $(cq)_{3^*}\ (\bar{c}\bar{q})_3$

  Vijande, Fernandez, and Valcarce (7-04); Maiani, Piccinini, Polosa, and Riquer (12-04); Ishida, Ishida, and Maeda (9-05, 10-06); Ebert, Faustov, and Galkin (12-05); Karliner and Lipkin (1-06); Chiu and Hsieh (3-06)

- tetraquark: $(c\bar{c})_8\ (q\bar{q})_8$

  Hogassen, Richard, and Sorba (11-05)

- charmonium hybrid: $c\bar{c}g$

  Close and Paige (9-03); Li (10-04)

- glueball: gg

  Seth (11-04)

See also Ko (5-04); Rosner (8-04)

What is the $X(3872)$?

Experimental inputs

- $J^{PC} = 1^{++}$
  $\Longrightarrow$ S-wave coupling to $D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$

- Mass is extremely close to the $D^{*0}\bar{D}^0$ threshold

$$M_X - (M_{D^{*0}} + M_{D^0}) < -0.31 \pm 0.64 \text{ MeV}$$

Assumption: $M_X < M_{D^{*0}} + M_{D^0}$

Conclusion

$X(3872)$ is a weakly-bound charm meson molecule

$$X = \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

with universal properties that are insensitive to details of QCD

What is the X? (cont.)

Quantum Mechanics

2-body system with short-range interactions
and S-wave bound state sufficiently close to threshold
has universal properties
that depend only on the large scattering length a

"Universality of Few-Body Systems with Large Scattering Length"
Braaten and Hammer, Physics Reports (cond-mat/0410417)

- ^{4}He atoms: $a \approx +100$ Å
- spin-polarized ^{3}H atoms: $a_t \approx -82$ Å
- alkali atoms near Feshbach resonance: $-\infty < a(B) < \pm\infty$
- nucleons: $a_s \approx -23.8$ fm, $a_t \approx +5.4$ fm
- charm mesons $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$!!

What is the X? (cont.)

Long-distance interaction between low-energy D^{*0} and $\bar{D}^0$
are dominated by pion exchange

Natural scales from pion exchange

length: $1/m_\pi = 1.4$ fm
momentum: $m_\pi = 140$ MeV
energy: $m_\pi^2/(2M_{\mathrm{D^*D}}) = 10$ MeV

$\Longrightarrow$ large scattering length: $|a| \gg 1.4$ fm

$\Longrightarrow$ small binding energy: $E_X \ll 10$ MeV

Binding energy of $X(3872)$ Belle, Babar, CDF, D0,CLEO

$$E_X = 0.31 \pm 0.64 \text{ MeV}$$

What is the X? (cont.)

Since X decays,

there must be inelastic scattering channels for $D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$

$\Longrightarrow$ Scattering length a is complex

$$\frac{1}{a} = \gamma_{\rm re} + i\gamma_{\rm im}$$

Real and imaginary parts of $1/a$ determine

$$\text{binding energy}: \quad E_X = (\gamma_{\rm re}^2 - \gamma_{\rm im}^2)/(2M_{\rm D^*D})$$
$$\text{decay width}: \quad \Gamma_X = 2\gamma_{\rm re}\gamma_{\rm im}/M_{\rm D^*D}$$

Constraints on E_X and Γ_X

$$0 < E_X < 1.1 \text{ MeV}$$
$$0.07 \text{ MeV} < \Gamma_X < 2.3 \text{ MeV}$$

What is the X? (cont.)

Universal results

- elastic $D^{*0}\bar{D}^0$ cross section at low energy E

$$\sigma(E) = \frac{\pi}{\gamma_{\text{re}}^2 + (\gamma_{\text{im}} + \sqrt{2M_{\text{D}^*\text{D}}E})^2}$$

  huge cross section at threshold: $\sigma(E=0) > 60\ \text{fm}^2$!!

- shallow S-wave bound state: $X(3872)$

  wavefunction: $\psi(r) = \exp(-\gamma_{\text{re}}\, r)/r$

  $\Longrightarrow \langle r \rangle = 1/(2\gamma_{\text{re}})$
  huge mean separation: $\langle r \rangle_X > 2$ fm !!

What is the X? (cont.)

$X(3872)$ is a weakly-bound charm meson molecule

$$X = \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

with universal properties that depend on E_X, Γ_X
but are insensitive to other details of QCD

insensitive to mechanism for binding

- $\chi_{c1}(2P)$ near $D^{*0}\bar{D}^0$ threshold?
- 1^{++} tetraquark near $D^{*0}\bar{D}^0$ threshold?
- depth of pion-exchange potential between charm mesons?
- coupling of charmonium to charm meson pairs?

If couplings to $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$ are taken into account,
$X(3872)$ will be a weakly-bound charm meson molecule
regardless of the mechanism for binding

Decays of the $X(3872)$

Long-distance decays

$$X \longrightarrow D^0 \bar{D}^0 \pi^0$$
$$\longrightarrow D^0 \bar{D}^0 \gamma$$

dominated by $D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$ components of X
proceed through decays of constituent D^{*0} or $\bar{D}^{*0}$
universal, determined by E_X, Γ_X only

Short-distance decays

$$X \longrightarrow \text{charmonium} + \text{light hadrons}$$
$$\longrightarrow D^+ D^- \gamma$$
$$\longrightarrow \text{light hadrons}$$

dominated by short-distance ("core") components of X
factorization formula with universal long-distance factor

Long-distance Decays of $X(3872)$

$$X(3872) \longrightarrow D^0 \bar{D}^0\, \pi^0,\;\; D^0 \bar{D}^0\, \gamma$$

Voloshin (9-03); El-Hady (3-06)

dominated by $D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$ components
proceeds through decay of constituent: $D^{*0} \longrightarrow D^0\pi^0,\; D^0\gamma$
$\bar{D}^{*0} \longrightarrow \bar{D}^0\pi^0,\; \bar{D}^0\gamma$

- absolute predictions of partial widths

$$\Gamma[X \to D^0\bar{D}^0\pi^0] \;\approx\; \Gamma[D^{*0} \to D^0\pi^0]$$
$$\Gamma[X \to D^0\bar{D}^0\gamma] \;\approx\; \Gamma[D^{*0} \to D^0\gamma]$$

- lower bound on total width

$$\Gamma[X] \;>\; \Gamma[D^{*0}] \approx 70 \pm 15 \text{ keV}$$

- momentum distributions for $D^0\bar{D}^0\pi^0$, $D^0\bar{D}^0\gamma$
  determined primarily by E_X, Γ_X

Factorization

Braaten and Kusunoki (6-05), Braaten and Lu (6-06)

separate short distances $\lesssim 1/m_\pi,\ 1/\Lambda_{\text{QCD}},\ 1/m_c$
from long distances $\gtrsim |a|$

Factorization formulas for amplitudes

$$\mathcal{A} = \mathcal{A}_{\text{short}} \times \mathcal{A}_{\text{long}}$$

$\mathcal{A}_{\text{short}}$ has smooth limit as $a \longrightarrow \infty$
$\mathcal{A}_{\text{long}}$ is determined by a only

Amplitudes for $X(3872)$
$D^{*0}\bar{D}^0$ near threshold
$D^0\bar{D}^{*0}$ near threshold
same short-distance factor $\mathcal{A}_{\text{short}}$
long-distance factors $\mathcal{A}_{\text{long}}$ determined by E_X and Γ_X

Short-distance Decays of $X(3872)$

dominated by short-distance components of X

$$\begin{aligned} X &\longrightarrow \text{charmonium} + \text{light hadrons} \\ &\longrightarrow D^+ D^- \gamma \\ &\longrightarrow \text{light hadrons} \end{aligned}$$

Factorization formula

Braaten and Kusunoki (7-05); Braaten and Lu (6-06)

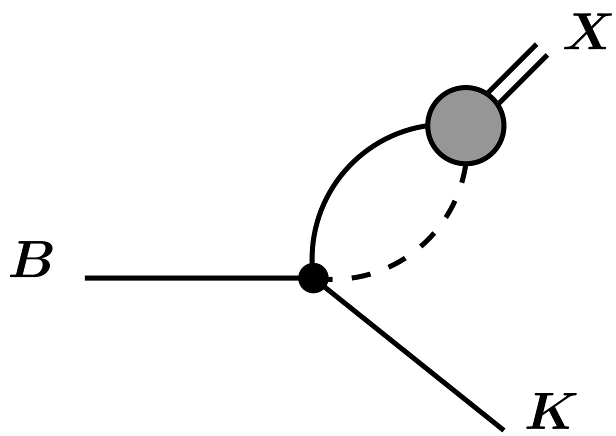
$$\Gamma[X \to H] = \Gamma_{\text{short}}[H] \times \left(E_X + \Gamma_X^2 / 16 E_X \right)^{1/2}$$

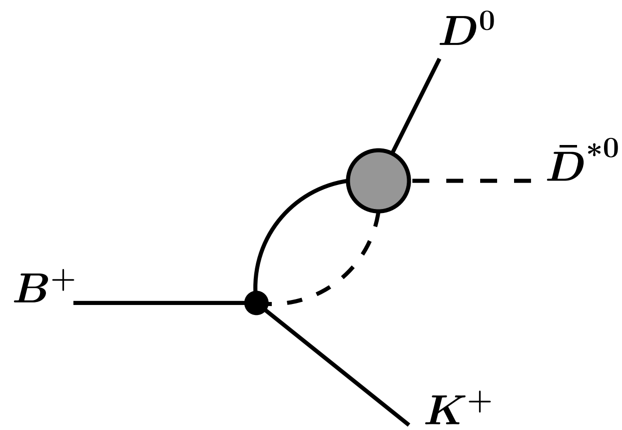
- long-distance factor cancels in ratio of decay rates

  $\Longrightarrow$ ratios are insensitive to E_X, Γ_X

Production of $X(3872)$, $D^{*0}\bar{D}^0$

Braaten, Kusunoki, and Nussinov (4-04), Braaten and Kusunoki (12-04)





Factorization formulas

$$\Gamma[B \to X + K] = \Gamma_{\text{short}} \times \frac{\pi}{M_{\text{D*D}}} \left[\gamma_{\text{re}}^2 + \gamma_{\text{im}}{}^2\right]^{1/2}$$

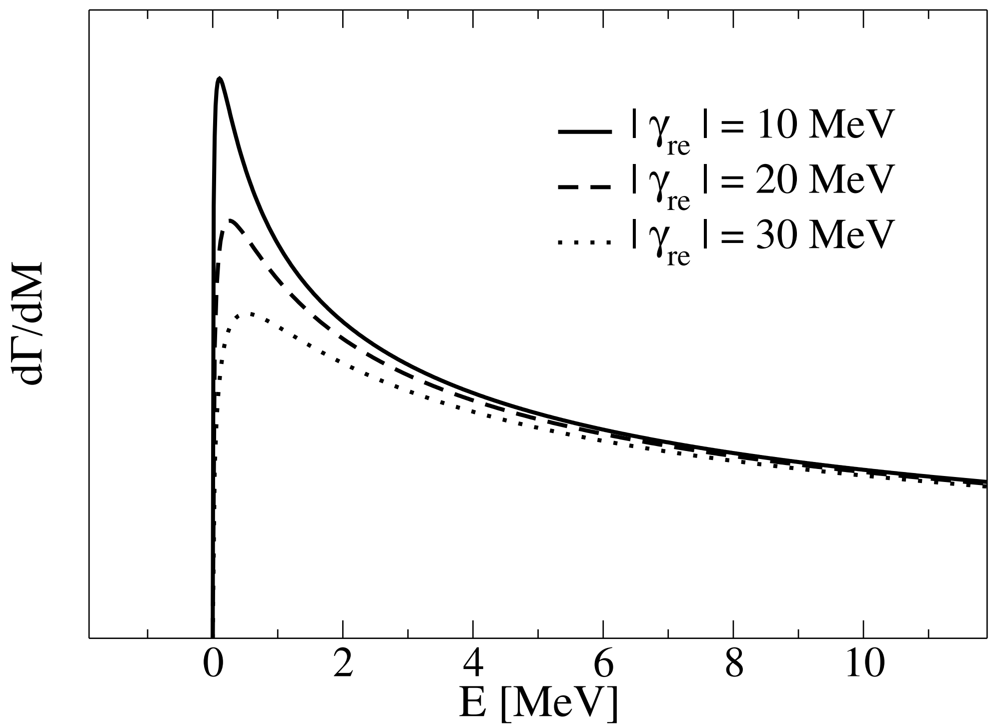
$$\frac{d\Gamma}{dM}[B \to D^{*0}\bar{D}^0 + K] = \Gamma_{\text{short}} \times \frac{\sqrt{2M_{\text{D*D}}E}}{\gamma_{\text{re}}^2 + (\gamma_{\text{im}} + \sqrt{2M_{\text{D*D}}E})^2}$$

$$M = (M_{D^0} + M_{D^{*0}}) + E$$

$B \to K + X$, $D^*\bar{D}$ (cont.)

Decay $B^+ \to K^+ + D^{*0}\bar{D}^0$

$$\frac{d\Gamma}{dM}[B \to K + D^{*0}\bar{D}^0] = \Gamma_{\text{short}} \times \frac{\sqrt{2M_{\text{D}^*\text{D}}E}}{\gamma_{\text{re}}^2 + (\gamma_{\text{im}} + \sqrt{2M_{\text{D}^*\text{D}}E})^2}$$



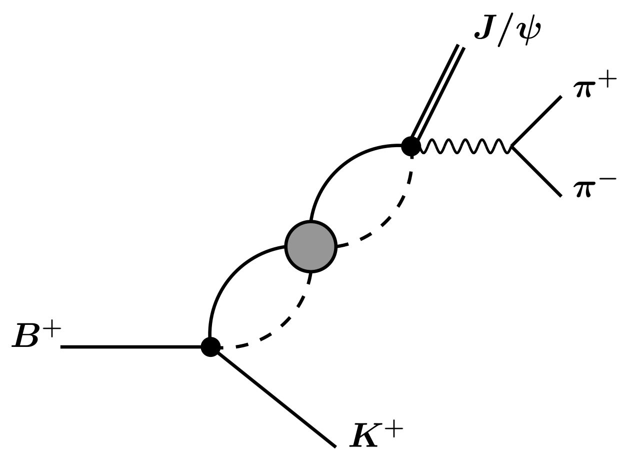
$D^{*0}\bar{D}^0$ invariant mass distribution peaks at $E = E_X + \Gamma_X^2/(16E_X)$

Line Shape of X

invariant mass distribution of decay products of X

short-distance decay channel: $X \longrightarrow H$

$$H = J/\psi\,\pi^+\pi^-,\ J/\psi\,\pi^+\pi^-\pi^0,\ J/\psi\,\gamma,\ \ldots$$



Factorization formula

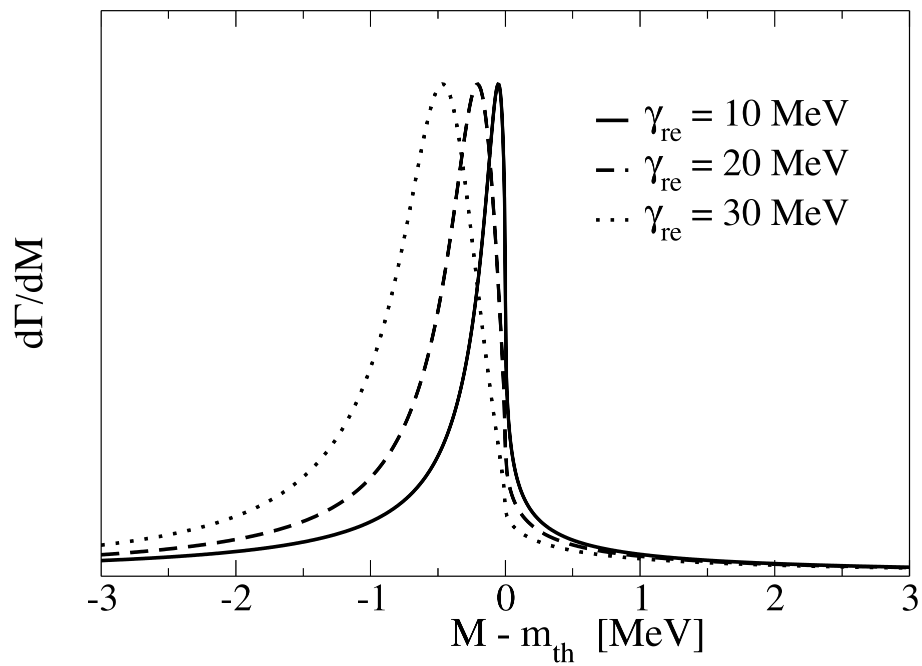
$$\frac{d\Gamma}{dM}[B \to K + H] = \Gamma_{\text{short}} \times \left| \frac{1}{\gamma_{\text{re}} + i\gamma_{\text{im}} - \sqrt{-2M_{\text{D*D}}E}} \right|^2$$

$$M = (M_{D^{*0}} + M_{D^0}) + E$$

Line Shape of X (cont.)

invariant mass of H: $M = (M_{D^{*0}} + M_{D^0}) + E$

$$\frac{d\Gamma}{dM}[B \to K + H] = \Gamma_{\text{short}} \times \left| \frac{1}{\gamma_{\text{re}} + i\gamma_{\text{im}} - \sqrt{-2M_{\text{D}^*\text{D}}E}} \right|^2$$



non-Breit-Wigner resonance near $E = -E_X$

Nearby Meson Thresholds

2-meson and 3-meson thresholds
that can be reached by $D^* \longleftrightarrow D\pi$

		order[†] in g
$D^{*+}D^-$, D^+D^{*-}	$+8.1$ MeV	g^2
$D^+D^-\pi^0$	$+2.5$ MeV	g^3
$D^+\bar{D}^0\pi^-$, $D^0\bar{D}^-\pi^+$	$+2.3$ MeV	g
$D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$	0	
$D^0\bar{D}^0\pi^0$	-7.1 MeV	g

† g = coupling constant for $D^* \leftrightarrow D\pi$

Nearby Meson Thresholds (cont.)

What is the effect of nearby $D^0\bar{D}^0\pi^0$ threshold?

Voloshin

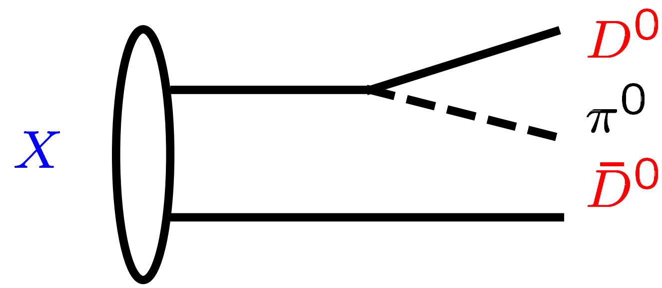
Phys. Lett. B **579**, 316 (2004)

"Interference and binding effects in decays of possible molecular component of $X(3872)$"

X decays into $D^0\bar{D}^0\pi^0$ through decay of constituent:

$$D^{*0} \longrightarrow D^0\pi^0 \quad \text{or} \quad \bar{D}^{*0} \longrightarrow \bar{D}^0\pi^0$$

Calculate momentum distribution of $D^0\bar{D}^0\pi^0$
using wavefunction for $D^{*0}\bar{D}^0$ / $D^0\bar{D}*0$
and treating decay as 1st order perturbation



Problem: effects of $D^0\bar{D}^0\pi^0$ become increasingly
nonperturbative as $M_{D^*} - M_D - m_\pi \longrightarrow 0$

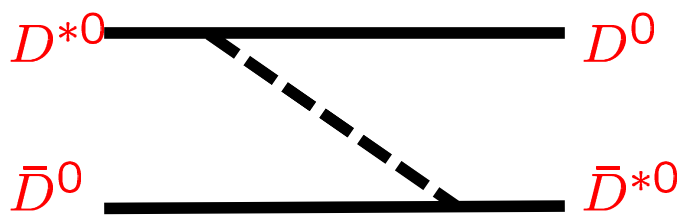
Nearby Meson Thresholds (cont.)

What is the effect of nearby $D^0\bar{D}^0\pi^0$ threshold?

Suzuki

Phys. Rev. D **72**, 114013 (2005)

"The $X(3872)$ boson: Molecule or charmonium"



exchanged π^0 can be on-shell, has momentum of order

$$\mu \equiv \left(2m_{\pi^0}(M_{D^{*0}} - M_{D^0} - m_{\pi^0})\right)^{1/2} \approx 44 \text{ MeV}$$

Static potential:

$$V(r) = -\text{constant} \times \delta^3(\mathbf{r}) + \text{terms suppressed by } \mu^2$$

"one-pion exchange produces practically no force
between D^* and $\bar{D}$, ... incapable of binding D^* and $\bar{D}$ "

Nearby Meson Thresholds

What is the effect of nearby $D\bar{D}\pi$ thresholds?

Meson Potential Models

2-body channels only, no 3-body channels

Tornquist (8-03, 2-04)

one-pion exchange

coupled channels: $D^*\bar{D}$, S-wave and P-wave

- predicted possible existence of $X(3872)$

Swanson (11-03, 6-04)

one-pion exchange + quark-exchange

5 coupled channels: $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$ S-wave and P-wave

$D^{*+}D^- + D^+D^{*-}$ S-wave

$J/\psi\ \rho$ S-wave; $J/\psi\ \omega$ S-wave

- correctly predicted Br[$X \to J/\psi\ \pi^+\pi^-\pi^0$] $\sim$ Br[$X \to J/\psi\ \pi^+\pi^-$]
- incorrectly predicted Br[$X \to J/\psi\ \pi^+\pi^-\pi^0$] $\gg$ Br[$X \to D^0\bar{D}^0\ \pi^0$]
- predicts weakly-bound 0^{++} $D^*\bar{D}^*$ molecule

Nearby Meson Thresholds

Question: Can effects of nearby $D\bar{D}\pi$ thresholds
be treated as perturbations?

Unnecessary complications

- spin-1 constituent: D^{*0} or $\bar{D}^{*0}$
- $C = + \implies X = (D^{*0}\bar{D}^0 + D^0\bar{D}^{*0})/\sqrt{2}$
- nearby 2-particle thresholds: $D^{*+}D^-$, D^+D^{*-}
- derivative coupling of pions: $(D^{*\mu})^\dagger\vec{\tau}\bar{D} \cdot \partial_\mu\vec{\pi}$

Question can be addressed in simpler model.

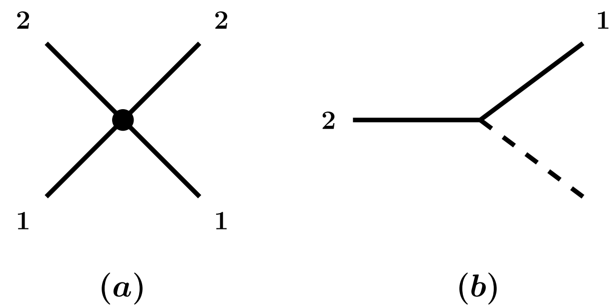
Scalar Meson Model

Scalar mesons: D_1, D_2, ϕ

Masses: $M_2 - (M_1 + m_\phi) \ll m_\phi \ll M_1 < M_2$

Channels near $D_1 D_2$ threshold: $D_1 D_2$, $D_1 D_1 \phi$

Interactions



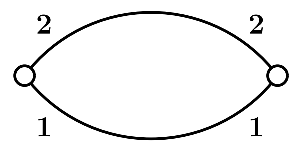
- $D_1 D_2$ coupling constant: λ_0
  must be treated nonperturbatively
  requires renormalization
  real, imaginary parts determined by E_X, Γ_X

- $D_2 \to D_1 \phi$ coupling constant: g
  determined by decay rate Γ_2 for $D_2 \to D_1 \phi$
  perturbative ??

Scalar Meson Model (cont.)

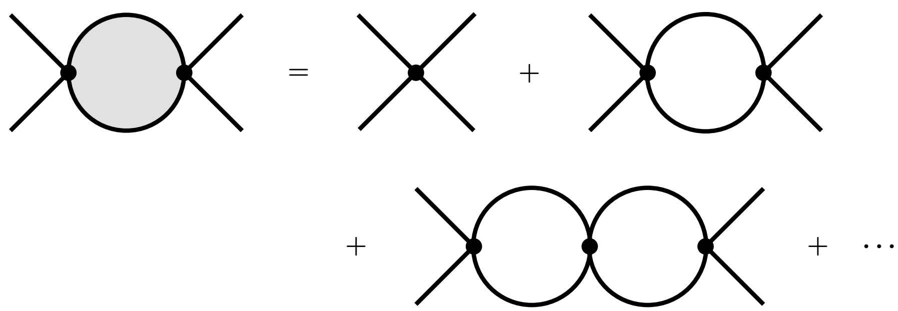
0th Order in g

amplitude for propagation of $D_1 D_2$ between contact interactions



$$L_0(E) = \infty + \kappa, \qquad \kappa = \sqrt{-2M_{12}(E - M_1 - M_2 + i\epsilon)}$$

amplitude for $D_1 D_2 \longrightarrow D_1 D_2$: sum to all orders in λ_0



$$\mathcal{A}_0(E) = \frac{1}{1/\lambda_0 - [\infty + \kappa]} = \frac{1}{1/\lambda - \kappa}$$

reproduces universal properties

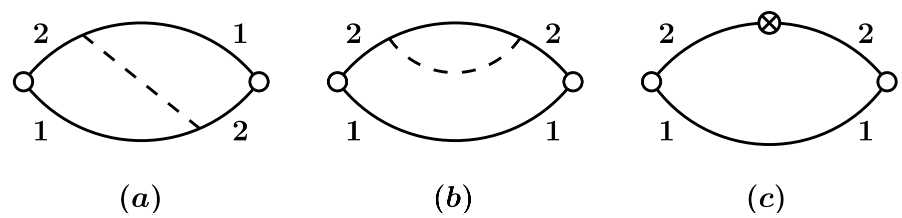
for particles with large scattering length

Scalar Meson Model (cont.)

2nd Order in g

corrections to amplitude

for propagation of $D_1 D_2$ between contact interactions



$$L_2(E) = \infty + g^2 F(\kappa), \qquad \kappa = \sqrt{-2M_{12}(E - M_1 - M_2 + i\epsilon)}$$

renormalization requires summing geometric series to all orders

$$\mathcal{A}_2(E) = \frac{1}{1/\lambda_0 - [\infty + \kappa] - [\infty + g^2 F(\kappa)]} = \frac{1}{1/\lambda - \kappa - g^2 F(\kappa)}$$

Infrared Problem:

$F(\kappa)$ diverges as $1/\kappa$ at the $D_1 D_2$ threshold ($\kappa \to 0$)

$\Longrightarrow$ nonperturbative as $\kappa \to 0$

Scalar Meson Model (cont.)

Infrared Problem at 2nd order in g

$$\mathcal{A}_2(E) = \frac{1}{1/\lambda - \kappa - g^2 F(\kappa)}$$

divergence in $F(\kappa)$ as $\kappa \to 0$

Resolution: divergence is proportional to width Γ_2 of D_2

$$g^2 F(\kappa) \longrightarrow -i\frac{M_{12}\Gamma_2}{2\kappa}$$

can be resummed into leading-order amplitude

by shifting mass: $M_2 \longrightarrow M_2 - i\Gamma_2/2$

$$\mathcal{A}_2(E) = \Big(1/\lambda - \sqrt{-2M_{12}(E - M_1 - M_2 + i\Gamma_2/2)} - \left[g^2 F(\kappa) + iM_{12}\Gamma_2/(2\kappa)\right]\Big)^{-1}$$

Resummed amplitude has same expansion to order g^2

smooth limit as $\kappa \to 0$

Summary

$X(3872)$ is weakly-bound charm meson molecule

$$X(3872) \;=\; \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

has universal properties that depend on E_X, Γ_X

but are insensitive to other details of QCD

Unique feature of $X(3872)$ is proximity to 3-meson thresholds:

$$D^0\bar{D}^0\pi^0, \quad D^+\bar{D}^0\pi^-, \quad D^0D^-\pi^+$$

Effect of 3-meson threshold can be calculated

using (resummation of) perturbation theory

Applications

- momentum distribution for $X \longrightarrow D^0\bar{D}^0\pi^0$
- line shape of X in $J/\psi\,\pi^+\pi^-$
- ...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