One-loop Corrections to *S* and *T* in Models with Extended Gauge Sectors

C.B. Jackson (w/ S. Dawson)
Brookhaven National Laboratory

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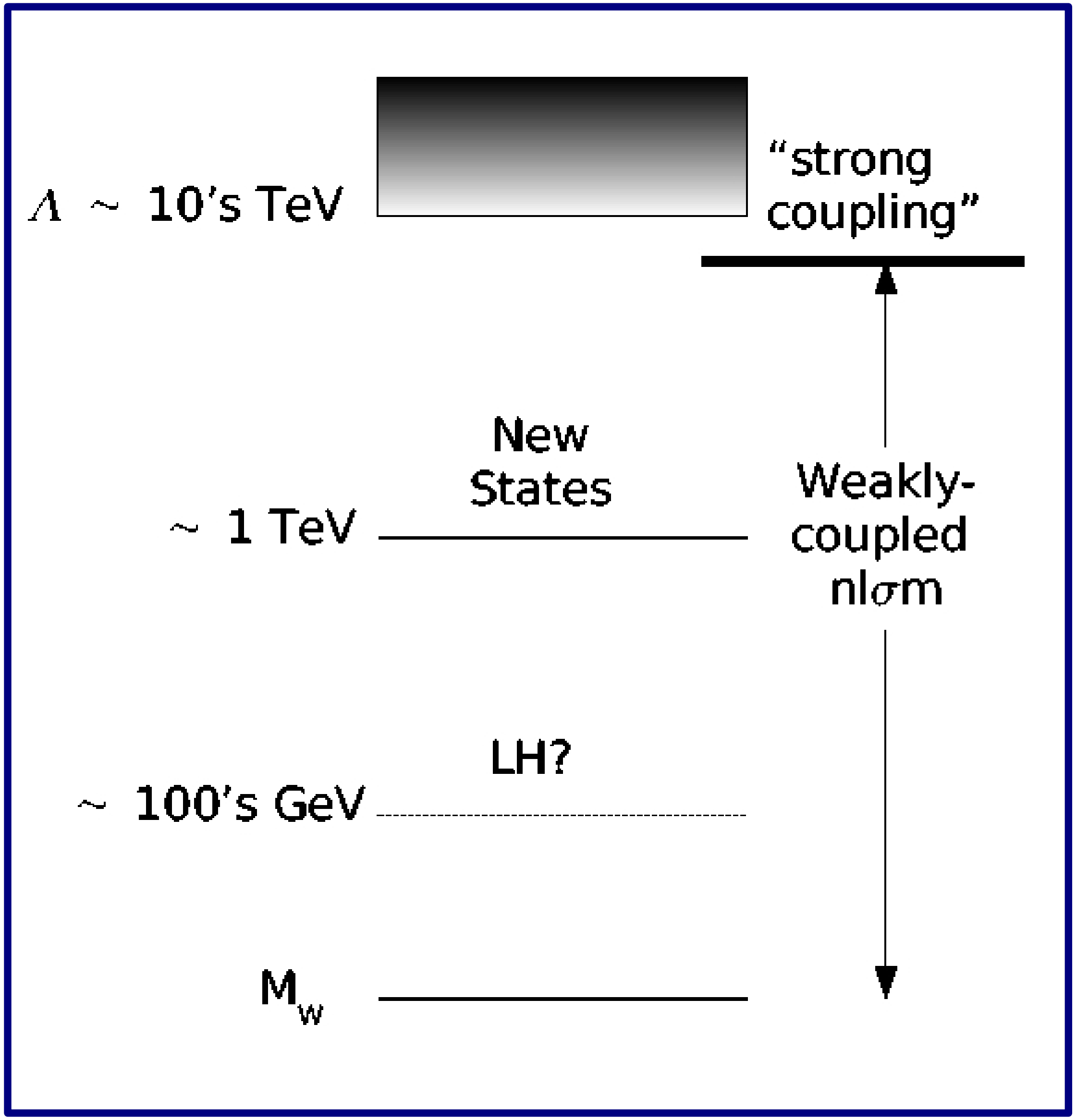
Outline

- Introduction to the model(s)
  - nlσm w/ extra *SU*(2)’s
  - More SU(2)’s = extra *W*’s and *Z*’s
- Loop corrections to self-energies, *S* and *T*
  - R_ξ gauge-dependence?
  - Unitary gauge $\rightarrow$ renormalizable?
- “Pinch” Technique...”one piece at a time”.
- Example: 3-site ~~Higgs~~ model
  - Chivukula et al., hep-ph/0607124(191)
  - M. Perelstein, JHEP 0410:010, 2004
  - Foadi et al., JHEP 0403:042, 2004

The Model(s)



- Gauge structure = $SU(2)^n$ x $U(1)$
- nlσm:

$$L_{gauge} = -\frac{1}{4g'^2} B_{\mu\nu}B^{\mu\nu} - \sum_{i=1}^{n} \frac{1}{2g_i^2} W_i^{\mu\nu} W_{i,\mu\nu}$$

$$L_{\Sigma} = \sum_{i=1}^{n} \frac{f_i^2}{4} Tr[D_\mu \Sigma_i (D_\mu \Sigma_i)^\dagger]$$

$$\Delta L = \sum_{j=1} c_j(\Lambda) O_j$$

- mass mixing = "towers" of W's/Z's

$$B = U_{00}A + U_{01}Z + Z' + \cdots$$

$$W_i^3 = U_{i0}A + U_{i1}Z + U_{i2}Z' + \cdots$$

$$W_i^{\pm} = V_{i1}W^{\pm} + V_{i2}W'^{\pm} + \cdots$$

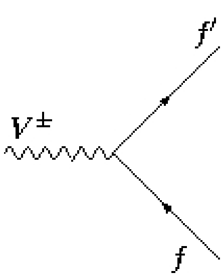
Coupling Fermions and Generic Feynman Rules

- Assume fermions only couple to $SU_1(2)$ and $U(1)$
- However, mass-mixing results in couplings of fermions to "new" triplets:

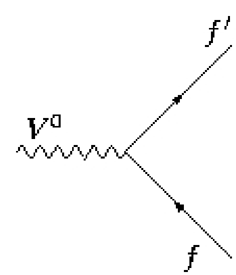
$$L_{CC} = -\frac{g_1}{2\sqrt{2}} \bar{\psi} \gamma^\mu (1-\gamma_5)\psi \; W^{\pm}_{1,\mu}$$

$$= -\frac{g_1}{2\sqrt{2}} \bar{\psi} \gamma^\mu (1-\gamma_5)\psi \; (V_{11} W^{\pm} + V_{12} W'^{\pm} + \cdots)$$

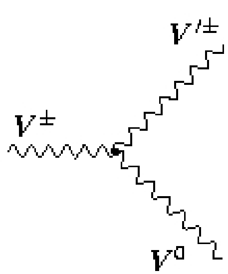
- Generic Feynman rules:



$$= g_{V^\pm f f} \; \gamma^\mu (1-\gamma_5)$$



$$= g_{V^0 f f} (Y_L P_L + Y_R P_R)$$



$$= g_{V^\pm V'^\pm V^0} \; V^{\mu\nu\lambda}$$

Generic couplings = functions of g_i's, U_{ij}'s and V_{ij}'s

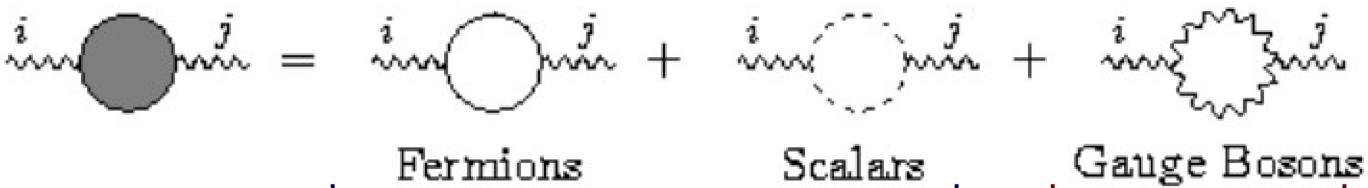
Gauge-dependence of *S* and *T*?

- *S* and *T* written in terms of Π_{ij}'s:

$$\frac{\alpha S}{4 s_w^2 c_w^2} = \bar{\Pi}'_{ZZ}(0) - \bar{\Pi}'_{AA}(0) - \frac{c_w^2 - s_w^2}{s_w c_w} \bar{\Pi}'_{ZA}(0)$$

$$\alpha T = \frac{\bar{\Pi}_{WW}(0)}{M_W^2} - \frac{\bar{\Pi}_{ZZ}(0)}{M_Z^2}$$

- Contributions to Π_{ij}'s:



R_ξ Gauge-independent ???

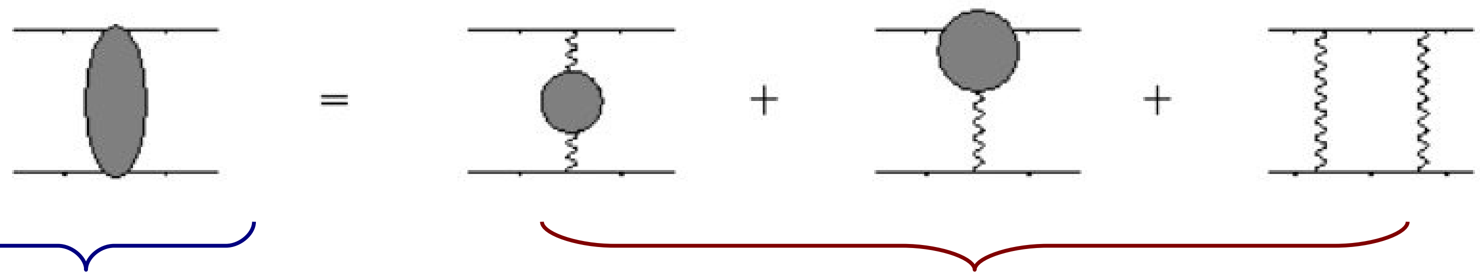
- Gauge-boson propagator:

$$D_i^{\mu\nu}(q) = \frac{-i}{(q^2 - M_i^2)} \left\{ g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2 - \xi M_i^2} \right\}$$

Π_{ij}'s are gauge-dependent!
(e.g., see Degrassi & Sirlin, NPB383(1992),73)

Non-renormalizable, too?

- Unitary gauge ($\xi \to \infty$): non-physical states decouple...fewer diagrams to calculate!
- As $q \to \infty$, $D_i^{\mu\nu} \sim 1$...disaster for loop-diagrams!
- Self-energies develop q^4 and q^6 (non-renormalizable) terms.
- Gauge-invariant & renormalizable *S/T* doomed?
- Consider 4-fermion scattering:

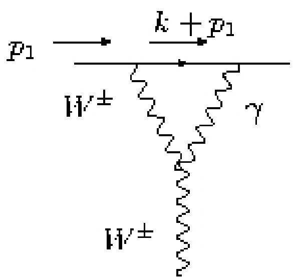


Gauge-invariant
and Renormalizable

Individually, gauge-dep.
and non-renormalizable

The Pinch Technique

- PT: isolate propagator-like, or pinch, contributions in vertex/box diagrams...e.g.,

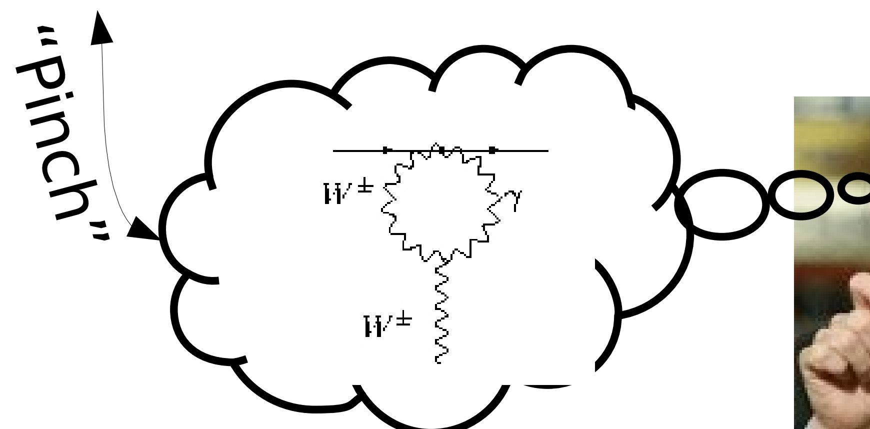


$$\sim \bar{u}(p_2)\left\{\cdots(\not{k}+\not{p}_1)\not{k}\cdots\right\}u(p_1)\frac{1}{(k^2-M_W^2)(k+p_1)^2(k-q)^2}$$

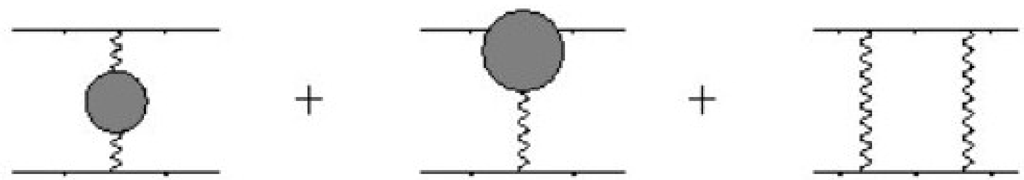
$$= \bar{u}(p_2)\left\{\cdots(\not{k}+\not{p}_1)((\not{k}+\not{p}_1)-\not{p}_1)\cdots\right\}u(p_1)\frac{1}{(k^2-M_W^2)(k+p_1)^2(k-q)^2}$$

$$= \bar{u}(p_2)\left\{\cdots(k+p_1)^2\cdots\right\}u(p_1)\frac{1}{(k^2-M_W^2)(k+p_1)^2(k-q)^2} + \text{“Non-pinch”}$$

Pinch terms carry the exact ξ dependence and q^4/q^6 dependence needed to cancel “bad” terms in two-point functions.



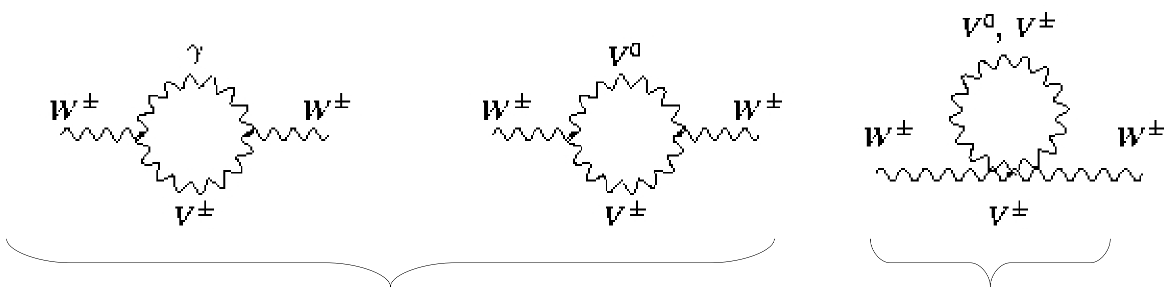
PT Self-Energies

- PT self-energy: 
- Consider W self-energy:

$$= \frac{g^2_{Wff}}{(q^2-M^2_W)^2}\left\{\bar{\Pi}_{WW} + \frac{(q^2-M^2_W)}{g_{Wff}}\{V\} + \frac{(q^2-M^2_W)^2}{g^2_{Wff}}\{B\}\right\} \equiv \frac{g^2_{Wff}}{(q^2-M^2_W)^2}\,\bar{\Pi}^{PT}_{WW}$$

"Pinch" terms

- Two-pt. Function contains 3 types of diagrams:



q^4 and q^6 pieces

q^2 independent

"♬ One Piece at a Time ♬"

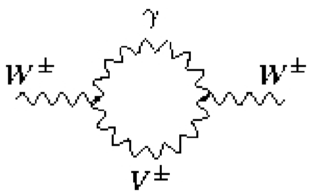
(Johnny Cash)

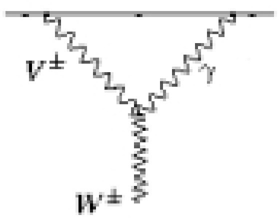
- In particular, the poles:

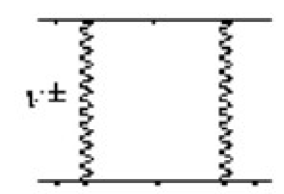
$$S_{\gamma V^\pm} = \frac{1}{16\pi^2\epsilon}\left[-\frac{5}{6}\,\frac{q^4}{M^2_{V^\pm}} + \cdots\right]$$

$$S_{V^\pm V^0}(M_{V^\pm}, M_{V^0}) = \frac{1}{16\pi^2\epsilon}\left\{-\frac{7}{12}q^4\left(\frac{1}{M^2_{V^\pm}} + \frac{1}{M^2_{V^0}}\right) - \frac{q^6}{12\mathrm{M}^2_{V^\pm}M^2_{V^0}} + \cdots\right\}$$

- Organize in a "diagram-by-diagram" manner:

$S^{PT}_{\gamma V^\pm} =$ 

$+ \frac{(q^2 - M^2_W)}{g_{Wff}} \Big\{$ 

$\Big\} + \frac{(q^2 - M^2_W)^2}{g^2_{Wff}} \Big\{$ 

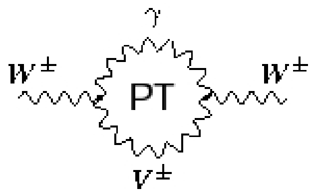
$\Big\}$

$$S^{PT}_{\gamma V^\pm} \equiv S_{\gamma V^\pm} + (q^2 - M^2_W)\left[\frac{g_{V^\pm ff}\cdot g_{\gamma ff}}{g_{Wff}\cdot g_{W\gamma V^\pm}}\right]\left\{V_{\gamma V^\pm}\right\} + (q^2 - M^2_W)^2\left[\frac{g_{V^\pm ff}\cdot g_{\gamma ff}}{g_{Wff}\cdot g_{W\gamma V^\pm}}\right]^2\left\{B_{\gamma V^\pm}\right\}$$

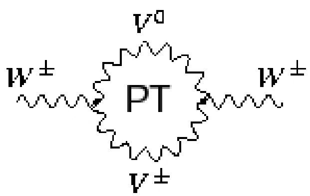
$$\left\{V_{\gamma V^\pm}\right\} = -\left(\frac{7}{2} + \frac{11}{6}\,\frac{q^2}{M_{V^\pm}}\right) \qquad \left\{B_{\gamma V^\pm}\right\} = \frac{1}{M^2_{V^\pm}}$$

Bloody Detail

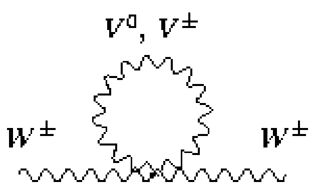
- PT "diagrams":



$$S^{PT}_{\gamma V^{\pm}}(M_{V^{\pm}}) = \left(\frac{15}{4}M^2_{V^{\pm}} - \frac{7}{2}M^2_W - \frac{M^4_W}{M^2_{V^{\pm}}}\right) + q^2\left(\frac{43}{6} + \frac{M^2_W}{6M^2_{V^{\pm}}}\right)$$



$$S^{PT}_{V^{\pm}V^0}(M_{V^{\pm}}, M_{V^0}) = 3\mathrm{M}^2_{V^{\pm}} + \frac{3}{2}M^2_W\left(\frac{M^2_{V^{\pm}}}{M^2_{V^0}} - 1\right) - \frac{3}{4}\left(\frac{M^4_W}{M^2_{V^{\pm}}} + \frac{M^4_{V^{\pm}}}{M^2_{V^0}}\right)$$
$$+ q^2\left[\frac{43}{12} + \frac{M^2_W}{6M^2_{V^{\pm}}} - \frac{1}{12}\left(\frac{M^4_W}{2M^2_{V^{\pm}}M^2_{V^0}} + \frac{M^2_{V^{\pm}}}{M^2_{V^0}}\right)\right] + \left(M_{V^{\pm}} \leftrightarrow M_{V^0}\right)$$



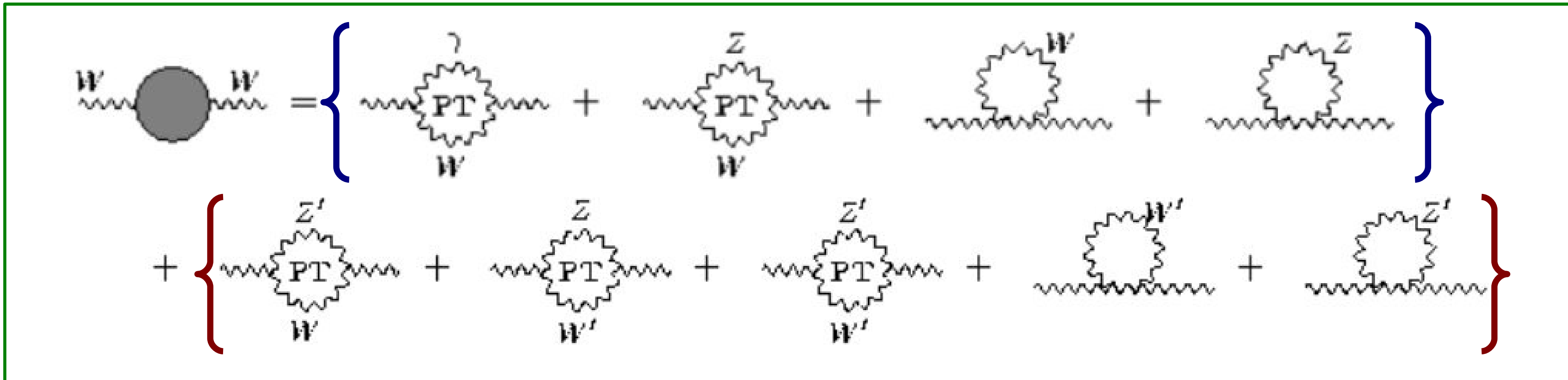
$$S_V(M_V) = -\frac{9}{4}M^2_V$$

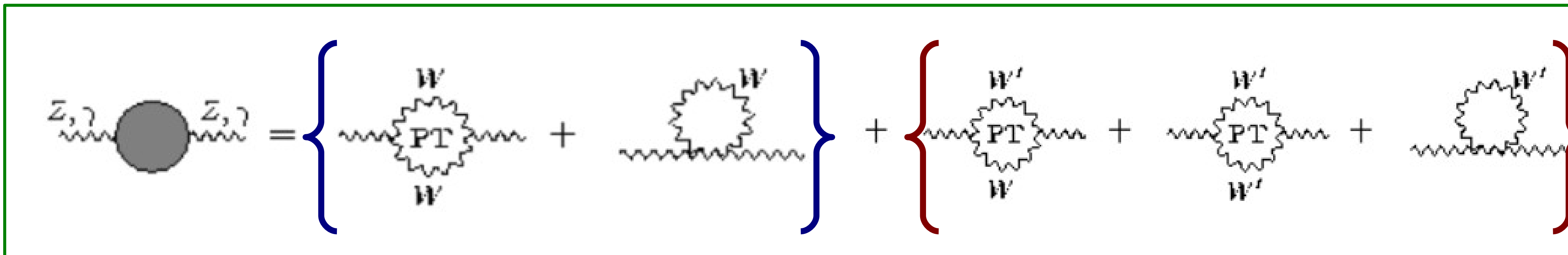
Neutral SE's contain these same structures with

$$(M_{V^{\pm}}, M_{V^0}) \rightarrow (M_{V_1^{\pm}}, M_{V_2^{\pm}})$$

Building the Self-Energies

- With the PT "diagrams", we can construct gauge-invariant* and renormalizable self-energies:
- For *n*=2 (*SU(2) x SU(2) x U(1)*):





(Blue = SM triplet + photon)   (Red = "new" triplet)

Results for a 3-site ~~Higgs~~ Model...again

- Apply our results to the deconstructed 3-site ~~Higgs~~ model (with *"localized"* fermions).
- Large mass limit: $M_W^2 \ll M_{W'}^2 \ll \Lambda^2 \quad (M_{W'} \simeq M_{Z'})$
- Leading chiral-logs: $$(4\pi)^{\epsilon}\frac{\Gamma(1+\epsilon)}{\epsilon} \to \log\left(\frac{\Lambda^2}{M_i^2}\right)$$
- Notation: $\bar{\Pi}_{ij}^{\mu\nu}(q) = g^{\mu\nu}\left(\bar{\Pi}_{ij}^{S} + q^2\,\bar{\Pi}_{ij}^{T}\right)$

$$\bar{\Pi}_{WW}^{S} = \frac{e^2}{16\pi^2 s_w^2}\left[-\frac{3}{4}(M_W^2 + M_Z^2)\log\frac{\Lambda^2}{M_W^2} + O\left(\frac{M_W^2}{M_{W'}^2}\right)\log\frac{\Lambda^2}{M_{W'}^2}\right]$$

$$\bar{\Pi}_{WW}^{T} = \frac{e^2}{16\pi^2 s_w^2}\left[\frac{29}{4}\log\frac{\Lambda^2}{M_W^2} + \frac{7}{4}\log\frac{\Lambda^2}{M_{W'}^2}\right]$$

$$\vdots$$

$$\vdots$$

S and T in the 3-site ~~Higgs~~ Model

- The S parameter: $$\frac{\alpha S_{1-loop}}{4 s_w^2 c_w^2} = \bar{\Pi}^T_{ZZ}(M_Z^2) - \bar{\Pi}^T_{\gamma\gamma}(M_Z^2) - \frac{c_w^2 - s_w^2}{s_w c_w}\bar{\Pi}^T_{Z\gamma}(M_Z^2)$$

- In the large mass limit, we find:

$$\alpha S_{1-loop} = \frac{\alpha}{12\pi}\log\frac{M_{W'}^2}{M_W^2} - \frac{5\alpha}{3\pi}\log\frac{\Lambda^2}{M_{W'}^2}$$

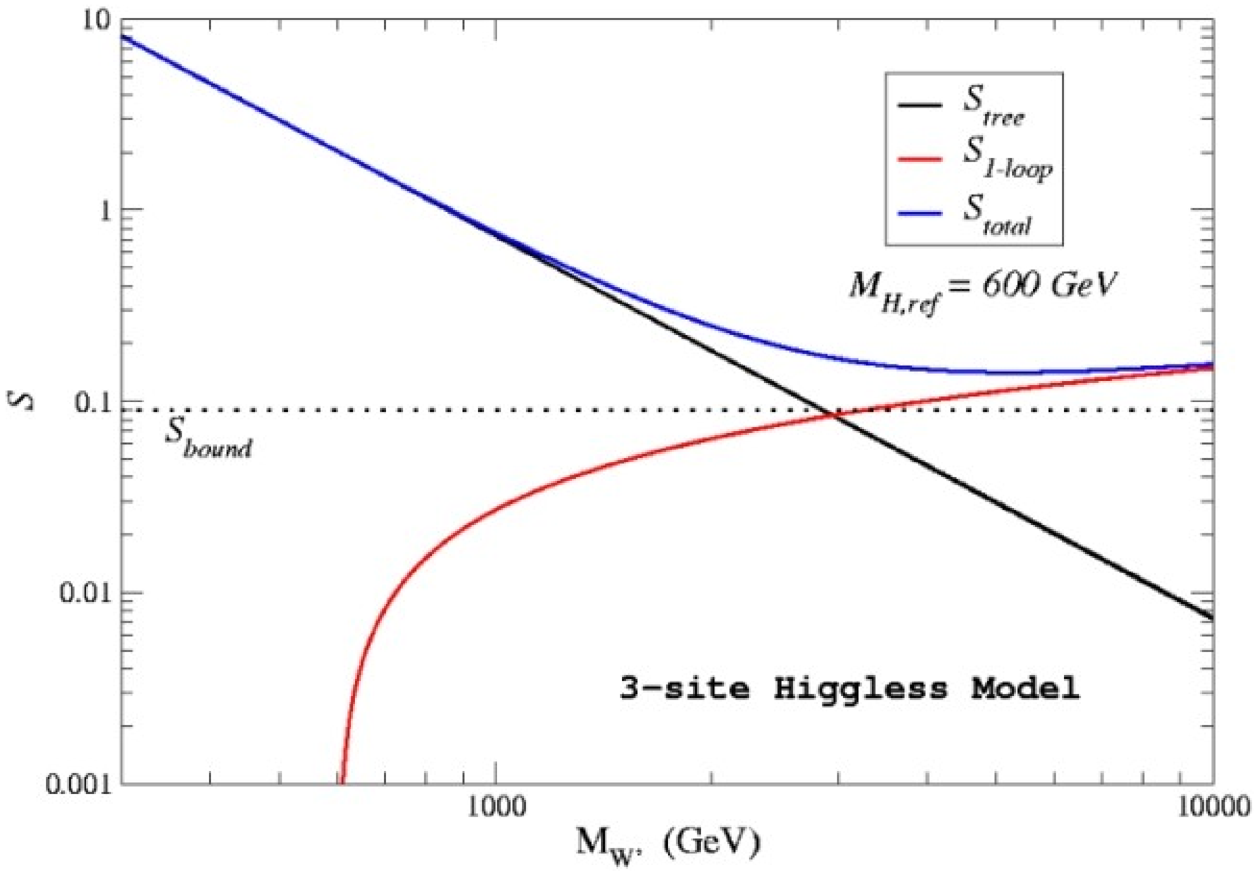
(Note: (5/3) versus (17/24) for "delocalized" fermions)

- For the T parameter: $$\alpha T_{1-loop} = \frac{\bar{\Pi}^S_{WW}}{M_W^2} - \frac{\bar{\Pi}^S_{ZZ}}{M_Z^2}$$

$$\alpha T_{1-loop} = -\frac{3\alpha}{16\pi c_w^2}\log\frac{\Lambda^2}{M_W^2} + O\left(\frac{M_W^2}{M_{W'}^2}\right)\log\frac{\Lambda^2}{M_{W'}^2}$$

(Note: "new" triplet decouples from T at one-loop level)

Chiral Log Contributions to *S*



- Subtract Higgs piece:

$$\alpha S_{Higgs} = \frac{\alpha}{12\pi} \log\left(\frac{M_{H,ref}^2}{M_W^2}\right)$$

- Tree-level contribution:

$$\alpha S_{tree} = \frac{4 s_w^2 M_W^2}{M_{W'}^2}$$

- Bound on *S* for TC-like models:

$$S_{bound} \leq 0.09 \,,\ (M_{H,ref} = 600\ \ GeV)$$

Summary

- Gauge boson contributions to EW corrections contain non-trivial R_ξ gauge-dependence.
- Also, in the Unitary gauge, two-point functions are non-renormalizable.
- Pinch Technique: combine propagator-like terms from Vertex/Box corrections with traditional SE’s.
- PT self-energies and, thus, *S* and *T* are gauge-invariant and renormalizable quantities.
- 3-site ~~Higgs~~ Model (with localized fermions): sizeable one-loop corrections to *S*, while contributions to *T* decouple.

(Special Thanks to Sekhar and Shinya)

The Rest of the SE's

$$\bar{\Pi}^{S}_{ZZ} = \frac{e^2}{16\pi^2 s_w^2 c_w^2}\left[-\frac{3}{2}M_W^2 \log\left(\frac{\Lambda^2}{M_W^2}\right) + O\left(\frac{M_Z^2}{M_{W'}^2}\right)\log\left(\frac{\Lambda^2}{M_{W'}^2}\right)\right]$$

$$\bar{\Pi}^{T}_{ZZ} = \frac{e^2}{16\pi^2 s_w^2 c_w^2}\left[\left(7c_w^4 + \frac{1}{3}c_w^2 - \frac{1}{12}\right)\log\left(\frac{\Lambda^2}{M_W^2}\right) + \left(7c_w^4 - \frac{7}{2}c_w^2\right)\log\left(\frac{\Lambda^2}{M_{W'}^2}\right)\right]$$

$$\bar{\Pi}^{T}_{\gamma\gamma} = \frac{e^2}{16\pi^2}\left[7\log\left(\frac{\Lambda^2}{M_W^2}\right) + 7\log\left(\frac{\Lambda^2}{M_{W'}^2}\right)\right]$$

$$\bar{\Pi}^{T}_{Z\gamma} = \frac{e^2}{16\pi^2 s_w c_w}\left[\left(7c_w^2 + \frac{1}{6}\right)\log\left(\frac{\Lambda^2}{M_W^2}\right) + \left(7c_w^2 + O\left(\frac{M_W^2}{M_{W'}^2}\right)\right)\log\left(\frac{\Lambda^2}{M_{W'}^2}\right)\right]$$

3-site ~~Higgs~~ Model

- nlσm based on $SU(2)^3 \rightarrow SU(2)$ (Custodial)
- Gauged sub-group: $SU(2)$ x $SU(2)$ x $U(1)$
- Symmetry breaking achieved by 2 Σ fields:

$$L = -\frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - \sum_{i=1}^{n} \frac{1}{2g_i^2} W_i^{\mu\nu} W_{i,\mu\nu} + f_0^2 Tr|D_\mu \Sigma_0|^2 + f_1^2 Tr|D_\mu \Sigma_1|^2$$

$$D_\mu \Sigma_0 = \partial_\mu \Sigma_0 + i W_{1,\mu}^a \tau^a \Sigma_0 - i B_\mu \Sigma_0 \tau^3 \qquad D_\mu \Sigma_0 = \partial_\mu \Sigma_0 + i W_{2,\mu}^a \tau^a \Sigma_1 - i \Sigma_1 W_{1,\mu}^a \tau^a$$

- Σ's acquire vev's: $\langle \Sigma_0 \rangle = \langle \Sigma_1 \rangle = diag(1\ ,\ 1)$
- $M_i(g_0, g_1, g', f_0, f_1) \rightarrow g_0(M_i\text{'s})$, e.g.
- Also, must add higher-derivative terms (parameterize strongly-coupled physics):

$$\Delta L = c_1 B^{\mu\nu} Tr(W_{1,\mu\nu} \Sigma_0 \tau^3 \Sigma_0^\dagger) + c_2 Tr(W_{1,\mu\nu} \Sigma_1^\dagger W_2^{\mu\nu} \Sigma_1)$$