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for CDF and $D_0$ Collaborations

Combined Measurement of Top Quark Mass from Tevatron

November 01, 2006
DPF+JPS Meeting
Honolulu, Hawaii
1. The outline

1. Importance of precise $m_t$ value
2. Measuring $m_t$ at the Tevatron
3. Single most precise $m_t$ measurement to date
4. Combining 11 best $m_t$ measurements
5. Results, conclusions and outlook
2. $m_t$ is important detail of Standard Model and beyond

Higgs mass range is extremely sensitive to the $m_t$ precision

$$m_W \approx a + b \cdot \left( \frac{m_t}{172 \text{ GeV}} \right)^2 - 1 - c \cdot \ln \left( \frac{m_H}{100 \text{ GeV}} \right)$$

and so are SUSY models
3. Direct $m_t$ measurements come only from Tevatron

"dilepton" 5%
"alljets" 46%
"lepton+jets" 29%
"lepton" = e, mu

Run I 1992−1996 [100pb⁻¹/exp]
Run II 2001−... [1000pb⁻¹/exp ...so far]
4. Jet combinatorics and missing $\nu$'s complicate reco

- **dilepton** combinations: 2
- **lepton+jets** combinations: 12
- **alljets** combinations: 360

- **after b-jet ID:** 2
- **after b-jet ID:** 6
- **after b-jet ID:** 90

- **missing $\nu$'s:** 2
- **missing $\nu$'s:** 1
- **missing $\nu$'s:** none

★ many possible combinations when assigning jets to partons, but only one is correct

★ deduce escaping $\nu$'s from missing $E_T$ [ MET ]

★ will focus on lepton+jets channel in this talk

★ b-quark ID helps reduce combinatorics and backgrounds...
5. **Lifetime of b-quark helps its identification**

- can reconstruct secondary decay vertex from b-quark
- significantly reduces combinatorial and other backgrounds
- improves overall $m_t$ sensitivity
6. Measuring $m_t$ at Tevatron is possible, but not easy

1. not just a calculation of trijet invariant mass
2. quarks turn into hadronic jets through complicated process
3. additional jets from gluon radiation
4. thus must have excellent energy corrections and good modeling of gluon radiation

$m_t = 175$ GeV simulation

lepton+jets
4 jets...

$N_{jets}$
7. Jet energy scale corrections and calibration

1. In order to obtain parton energies, multiple corrections applied to jet energies to account for instrumental, physical and reconstruction effects.

2. In situ $W \rightarrow jj$ calibration adds crucial correction and allows precision measurement of $m_t$. JES uncertainty now scales with statistics.
8. $m_t$ measured in all channels using many methods

**Template.** For each event choose best reconstructed $m_t$ using over-contrained kinematic fit. Obtain probability using MC templates for different hypothetical masses. Get $m_t$ using maximum likelihood fit


**$L_{xy}$.** Recently proposed by Chris Hill et al in CDF. Lorentz boost given to b-quark in top decay is proportional to $m_t$. Measuring transverse decay length $L_{xy}$ of b-hadrons from top decay gives $m_t$

**Dilepton** assumes $\eta(\nu)$, $\phi(\nu)$, $p_z(t\bar{t})$ to deal with 2 missing $\nu'$s [talks by T. Maki, B. Jayatilaka and J. Temple]. **Alljets** employs neural network to reduce huge background [F. Margaroli’s talk]
9. Recap

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10. **Comes from CDF matrix element technique analysis**

- Maximize kinematic and dynamic information
- Calculate a probability per event to be signal or background as a function of the top mass
- Signal probability for a set of measured jets and lepton (x)

\[
P(x;M_{\text{top}},JES) = \frac{1}{\sigma} \int dq_1 dq_2 f(q_1) f(q_2) \, d\sigma(y;M_{\text{top}}) \, W(x,y,JES)
\]

- **Differential cross section:** LO ME (qq->tt) only
- **Transfer function:** probability to measure x when parton-level y was produced

- JES is a free parameter, constrained in situ by mass of the W

- Background probability is similar, but no dependence on \(M_{\text{top}}\)

\[
L(f_{\text{top}},M_{\text{top}},JES) \propto \prod_{i} \left( f_{\text{top}} \, P_{\text{top},i}(M_{\text{top}},JES) + (1 - f_{\text{top}}) \, P_{\text{bkgd},i}(JES) \right)
\]
11. Transfer function

- Detector resolution modeled with “transfer functions” using Monte Carlo
- HERWIG/PYTHIA generation and parton showering
- GEANT detector simulation
- Probability of jet with energy $E_{\text{jet}}$ originating from a parton with energy $E_{\text{parton}}$

\[
W_{\text{jet}}(\delta, E_{\text{parton}}) = \frac{1}{\sqrt{2\pi p_2 + p_3 p_5}} \left[ \exp\left[\frac{-(\delta - p_1)^2}{2p_2}\right] + p_3 \exp\left[\frac{-(\delta - p_4)^2}{2p_5}\right]\right]
\]

\[
\delta \equiv E_{\text{parton}} - E_{\text{jet}}, p_i = a_i + b_i E_{\text{parton}}
\]
12. **CDF lepton+jets signal selection and acceptance**

- **lepton**: $E_T > 20$ GeV (electron), $p_T > 20$ GeV/c ($\mu$)
- **4 jets**: $E_T > 15$ GeV, $|\eta| < 2.0$
- **missing $E_T$**: $\not{E}_T > 20$ GeV
- **$b$-tag**: $\geq 1$ from secondary vertex
- **non-$W$ veto**: $\Delta \phi < 0.5$ or $\Delta \phi > 2.5$ ($\not{E}_T < 30$ GeV)

[See E. Thomson’s talk for selection details]

1. Relative to $m_t = 175$ GeV and JES=1, acceptance varies by about $\pm 50\%$ over relevant range
2. Important to account for this when forming likelihood function
3. Used Pythia MC samples at 5 GeV intervals from 130-230 and interpolated between intermediate points
13. Backgounds are well understood and simulated

- Fractions were scaled from 695 \( pb^{-1} \) to 940 \( pb^{-1} \)
- Important for checking the method, but not used in \( m_t \) measurement
14. **Data is well described by signal and background MC**

CDF Run II Preliminary (940 pb⁻¹)

Monte Carlo
- mean: 181.9 GeV/c²
- RMS: 59.4 GeV/c²

Data
- mean: 182.0 GeV/c²
- RMS: 58.9 GeV/c²
15. Pseudo-experiments prove that ME method works well

- Pseudo-experiments were generated using signal and background MC mixed appropriately. This study proves that:

1. estimator has no bias
2. method is consistent and robust
3. statistical uncertainty has Gaussian nature

⇒ can apply to data
16. Maximum likelihood fit result

1. \[ m_t = 170.9 \pm 2.1(\text{stat} + \text{JES}) \text{ GeV}^2/c^2 \]
2. \[ m_t = 170.9 \pm 1.6(\text{stat}) \text{ GeV}^2/c^2 \]
3. \[ \text{JES} = 0.99 \pm 0.02(\text{stat}) \]
17. Complete $m_t$ result and its systematic uncertainties

$$[170.9 \pm 2.1(\text{stat} + JES) \pm 1.4(\text{syst})] \text{ GeV/c}^2$$
$$[170.9 \pm 1.6(\text{stat}) \pm 1.9(JES + syst)] \text{ GeV/c}^2$$
$$[170.9 \pm 2.5 \text{ GeV/c}^2] \text{ GeV/c}^2$$

⇒ Best single measurement in the world!

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>CDF Magnitude (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b-JES</td>
<td>0.6</td>
</tr>
<tr>
<td>Signal (Initial and final state radiation, parton distribution functions)</td>
<td>1.1</td>
</tr>
<tr>
<td>Background (composition and shape)</td>
<td>0.2</td>
</tr>
<tr>
<td>Fit (Method, Monte Carlo statistics)</td>
<td>0.4</td>
</tr>
<tr>
<td>Monte Carlo (Modeling of ttbar)</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.4</strong></td>
</tr>
</tbody>
</table>
18. I showed the best, but one of many...
19. **Example of original new method: \( L_{xy} \)**

- Uses the average transverse decay length, \( L_{xy} \) of the b-hadrons
- B hadron decay length \( \propto \) b-jet boost \( \propto M_{\text{top}} \) (\( \geq 3 \) jets)

**PRD 71, 054029 by C. Hill et al.**

**CDF Run 2 Preliminary - 695 pb\(^{-1}\)**

\[
M_{\text{top}} = 183.9^{+15.7}_{-13.9} \text{ (stat)} \pm 0.3 \text{ (JES)} \pm 5.6 \text{ (syst)} \text{ GeV}/c^2
\]

Statistics limited, but can make big contributions at Run IIb, LHC

**Insensitive to JES, but need \( L_{xy} \) simulation**
20. Recap

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5. Results, conclusions and outlook
21. Combine measurements to reduce $\sigma m_t$

<table>
<thead>
<tr>
<th>Measurement</th>
<th>CDF-I</th>
<th>DØ-I</th>
<th>CDF-II</th>
<th>DØ-II</th>
<th>CDF-I</th>
<th>DØ-I</th>
<th>CDF-II</th>
<th>DØ-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>di-l</td>
<td>167.4±11.4</td>
<td>168.4±12.8</td>
<td>164.5±5.6</td>
<td>178.1±8.3</td>
<td>176.1±7.3</td>
<td>180.1±5.3</td>
<td>170.9±2.5</td>
<td>170.3±4.5</td>
</tr>
<tr>
<td>l+j</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>186.0±11.5</td>
<td>174.0±5.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l+j*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lxy*</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

$M_{top}$ [GeV/c²]

[see arXiv:hep-ex/0608032 for details]
22. **Combination method**

- **Best Linear Unbiased Estimator (BLUE)**

- Returns a weighted average, including breakdown of uncertainties by input category

- Results cross-checked with a MINUIT $\chi^2$ – minimization

- Was used for final (CDF+D0) Run-I average and all world averages since then
23. Must decide on error categories

★ Jet energy scale:
  • aJES: $D0$ Run II e/h calibration
  • bJES: JES uncertainties specific to b-jets
  • cJES: fragmentation and OOC showering
  • dJES: correlated within experiment but not between RunI&II
  • iJES: in-situ calibration from Wjj
  • rJES: remaining JES

★ Signal: signal modeling (ISR, FSR, PDF)
★ Bgd: background normalization and shape
★ UN/MI: $D0$ Run I uranium noise and MI
★ Fit: fit method, finite MC sample size
★ MC: Pythia vs Herwig (vs ISAJET)
★ Statistical: limited data size
24. **All uncertainties were carefully categorized**

<table>
<thead>
<tr>
<th>Run-I published</th>
<th>Run-II preliminary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CDF</strong></td>
<td><strong>DØ</strong></td>
</tr>
<tr>
<td>all-j</td>
<td>l+ j</td>
</tr>
<tr>
<td><strong>Lumi (pb⁻¹)</strong></td>
<td>110</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>186.0</td>
</tr>
<tr>
<td>iJES</td>
<td>0.0</td>
</tr>
<tr>
<td>aJES</td>
<td>0.0</td>
</tr>
<tr>
<td>bJES</td>
<td>0.6</td>
</tr>
<tr>
<td>cJES</td>
<td>3.0</td>
</tr>
<tr>
<td>dJES</td>
<td>0.3</td>
</tr>
<tr>
<td>rJES</td>
<td>4.0</td>
</tr>
<tr>
<td>Signal</td>
<td>1.8</td>
</tr>
<tr>
<td>BG</td>
<td>1.7</td>
</tr>
<tr>
<td>Fit</td>
<td>0.6</td>
</tr>
<tr>
<td>MC</td>
<td>0.8</td>
</tr>
<tr>
<td>UN/MI</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Syst.</strong></td>
<td>5.7</td>
</tr>
<tr>
<td><strong>Stat.</strong></td>
<td>10.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>11.5</td>
</tr>
<tr>
<td><strong>Syst.</strong></td>
<td>4.5</td>
</tr>
</tbody>
</table>
25. **Must decide on correlations**

- **Uncorrelated**: Stat, Fit, iJES

- **Correlated across all inputs**
  - in same run: dJES
  - in same channel: Bgd
  - everywhere: Signal, bJES, cJES, rJES, MC

- **Correlation taken to be 0 or 100%**
  - Variations considered as part of cross-checks
26. **Combination result**

**Mass of the Top Quark (*Preliminary)**

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$M_{\text{top}}$ [GeV/c$^2$]</th>
</tr>
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<tbody>
<tr>
<td>CDF-I di-l</td>
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<td>164.5 ± 5.6</td>
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<tr>
<td>DØ-II di-l*</td>
<td>178.1 ± 8.3</td>
</tr>
<tr>
<td>CDF-I l+j</td>
<td>176.1 ± 7.3</td>
</tr>
<tr>
<td>DØ-I l+j</td>
<td>180.1 ± 5.3</td>
</tr>
<tr>
<td>CDF-II l+j*</td>
<td>170.9 ± 2.5</td>
</tr>
<tr>
<td>DØ-II l+j*</td>
<td>170.3 ± 4.5</td>
</tr>
<tr>
<td>CDF-I all-j</td>
<td>186.0 ± 11.5</td>
</tr>
<tr>
<td>CDF-II all-j*</td>
<td>174.0 ± 5.2</td>
</tr>
<tr>
<td>CDF-II lxy*</td>
<td>183.9 ± 15.8</td>
</tr>
<tr>
<td>Tevatron Run-I/II*</td>
<td>171.4 ± 2.1</td>
</tr>
</tbody>
</table>

$\chi^2 / \text{dof} = 10.6 / 10 \Rightarrow (39\%)$

1. Measurements are consistant
2. CDF RunII uses $700 - 1000 \, pb^{-1}$
3. D0 Run II uses $380 \, pb^{-1}$
27. Cross Checks

- Repeated combination with these variations
  - Used each extreme of asymmetric stat uncertainty
  - Varied $\rho$(LJT-Lxy) by +/- 5% for their stat errors
  - Varied correlations among all inputs by 10% for bJES, cJES, rJES, Signal, MC, and Bgd simultaneously
  - Varied treatment of Run I uncertainties

- Central value and total uncertainty both affected by <100 MeV/c$^2$ level in all cases
28. Results are consistent among the channels

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (GeV/c²)</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{t}^{\text{all-j}}$</td>
<td>173.4 ± 4.3</td>
<td>1.00</td>
</tr>
<tr>
<td>$M_{t}^{l+j}$</td>
<td>171.3 ± 2.2</td>
<td>0.29 1.00</td>
</tr>
<tr>
<td>$M_{t}^{\text{di-l}}$</td>
<td>167.0 ± 4.3</td>
<td>0.46 0.37 1.00</td>
</tr>
</tbody>
</table>

Comparison of $M_{\text{top}}$ in Different Final States
(Tevatron Preliminary, July 2006)

- Dilepton - All-Jets
  - $6.4 \pm 4.4$
  - $\chi^2 = 2.0/1 (16\%)$

- Lepton+Jets - All-Jets
  - $2.1 \pm 4.3$
  - $\chi^2 = 0.2/1 (65\%)$

- Lepton+Jets - Dilepton
  - $4.3 \pm 4.0$
  - $\chi^2 = 1.2/1 (27\%)$
29. \( m_t = 171.4 \pm 2.1 \, GeV^2/c^2 \): lighter Higgs and SUSY

\[
m_H = 85^{+39}_{-28} \, GeV/c^2 \ (68\% \, CL) \\
< 166 \, GeV/c^2 \ (one \ sided \ 95\% \, CL) \\
< 199 \, GeV/c^2 \ (95\% \, CL, \ if \ LEP2 \ limit \ of \ 114 \, GeV \ included) \quad (1)
\]
30. **Bright prospects for** $m_t$ 

1. New combined result exceeds Run II TDR goal already!

2. Adding JES to alljets may result in sensitivity comparable to leptop+jets

3. Tevatron performance and combination of results from CDF and $D_0$ will reduce uncertainty even further

4. Tevatron Run II result is expected to achieve uncertainty of about 1 GeV
31. ... but improvements are required

The Fit (all quantities in GeV/c²)

- **JES**: 1.4
  - **iJES**: 0.9
  - **aJES**: 0.2
  - **bJES**: 0.6
  - **cJES**: 0.5
  - **dJES**: 0.7
  - **rJES**: 0.3

- **Signal**: 0.9
- **Bgd**: 0.3
- **Fit**: 0.3
- **MC**: 0.3
- **UN/MI**: 0.1

- **Total Systematic**: 1.7
- **Statistical**: 1.2

- **Total**: 2.1

1. Will have to improve understanding of systematic uncertainties
2. Similar to $m_W$ measurement at LEP 2
3. Real precision measurement at hadron collider!
32. How large are these QCD effects?

- Initial State radiation
- Causes confusion in top mass reconstruction
- Underlying Event
- Changes color connections, hence fragmentation
- Tune A showed that color re-connections are important

Stephen Mrenna (CD) The Top Mass in Pythia

DPF2006, Alexander Golossanov [Fermilab]
33. Summary and outlook:

New world average combines Tevatron Run I and best recent Run II measurements with up to 1 fb$^{-1}$ of data [$\sigma_{mt} / m_t \approx 1.2\%$!]

$$m_t = 171.4 \pm 2.1 \text{ GeV}^2/c^2$$

New single most accurate measurement from CDF dominates the world average

$$m_t = 170.9 \pm 2.5 \text{ GeV}^2/c^2$$

Expect to approach $\sigma_{mt} \sim 1 \text{ GeV}$ in combined precision if Tevatron provides more data and physicists improve their understanding of systematics: challenging even at LHC!

New $m_t$ and $m_W$ values hint lighter Standard Model Higgs... and lighter Higgs is easier to find at Tevatron!

$$\Rightarrow m_{H}^{SM} = 85^{+39}_{-28} \text{ GeV}/c^2$$