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Exclusive Drell-Yan Production at NNLO

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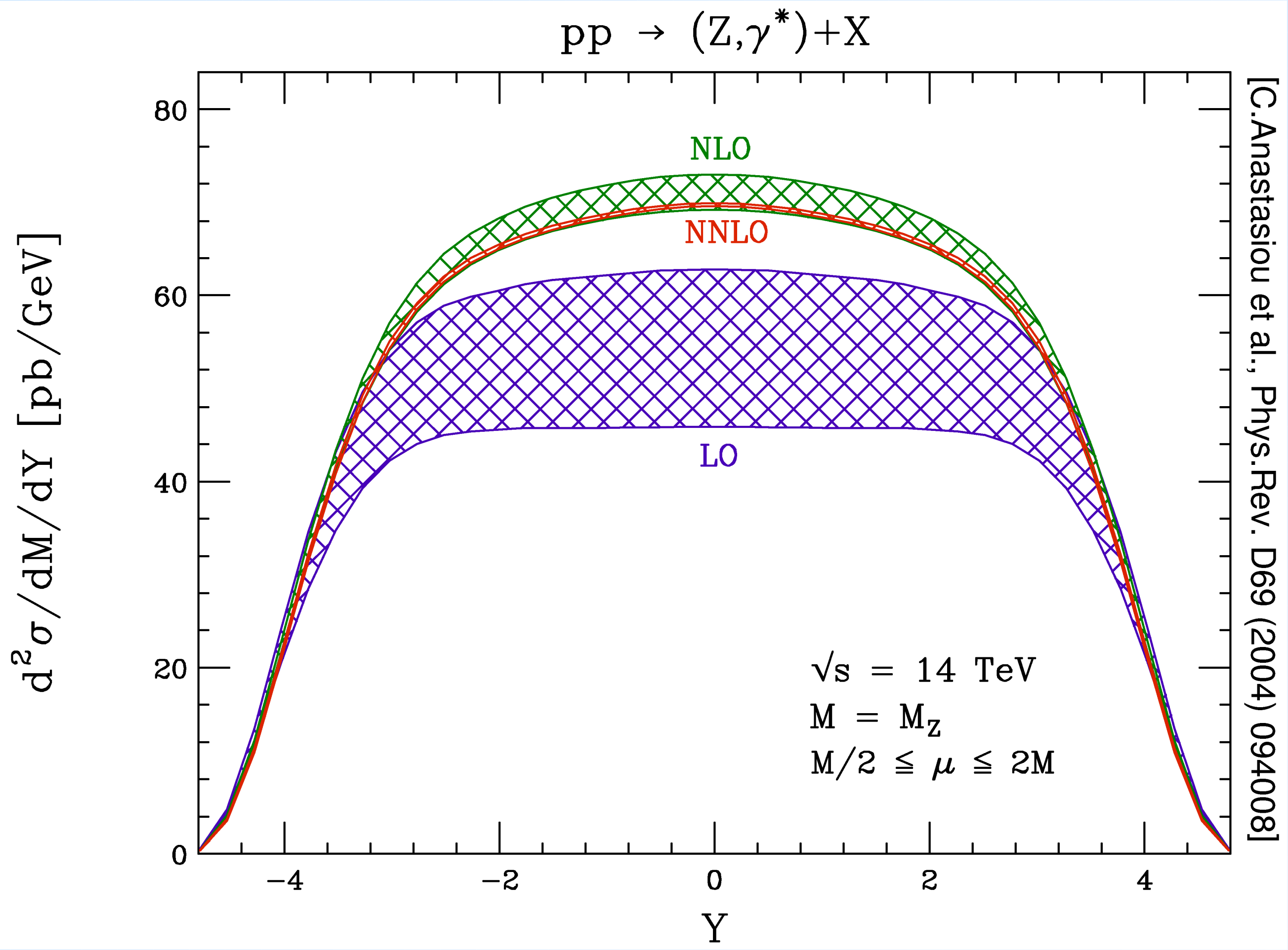
Why keep flogging Drell-Yan?

- Inclusive NNLO known for 15 years!

  [Hamberg *et al.* (1991);Harlander & W.K. (2002)]

- Rapidity Distributions at NNLO are now known.



Why keep flogging Drell-Yan?

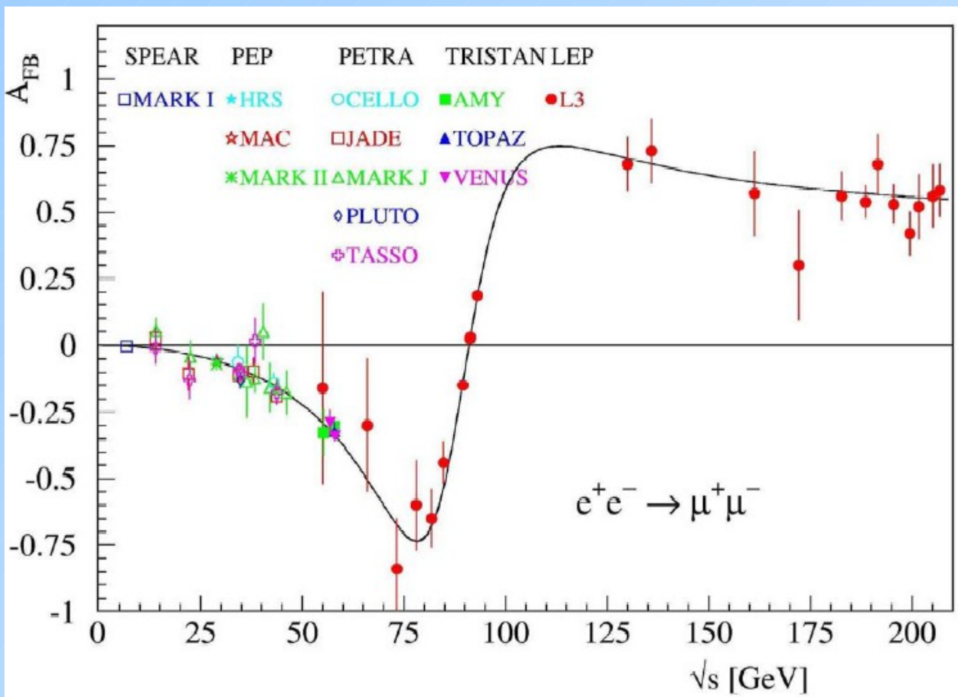
- Until now NNLO calculations compute massive vector boson production, not di-lepton production
  - Melnikov and Petriello have taken the first step by computing $W^{\pm} \rightarrow \ell^{\pm} \nu$ production using sector decomposition.
- We need a fully exclusive calculation of Drell-Yan.
  - Particle detectors detect leptons. There are measurements to exploit with a fully exclusive calculation of Drell-Yan.
- We need experience with parton-level Monte Carlo calculations at NNLO.
  - Drell-Yan provides the simplest and most useful hadronic production process to study.

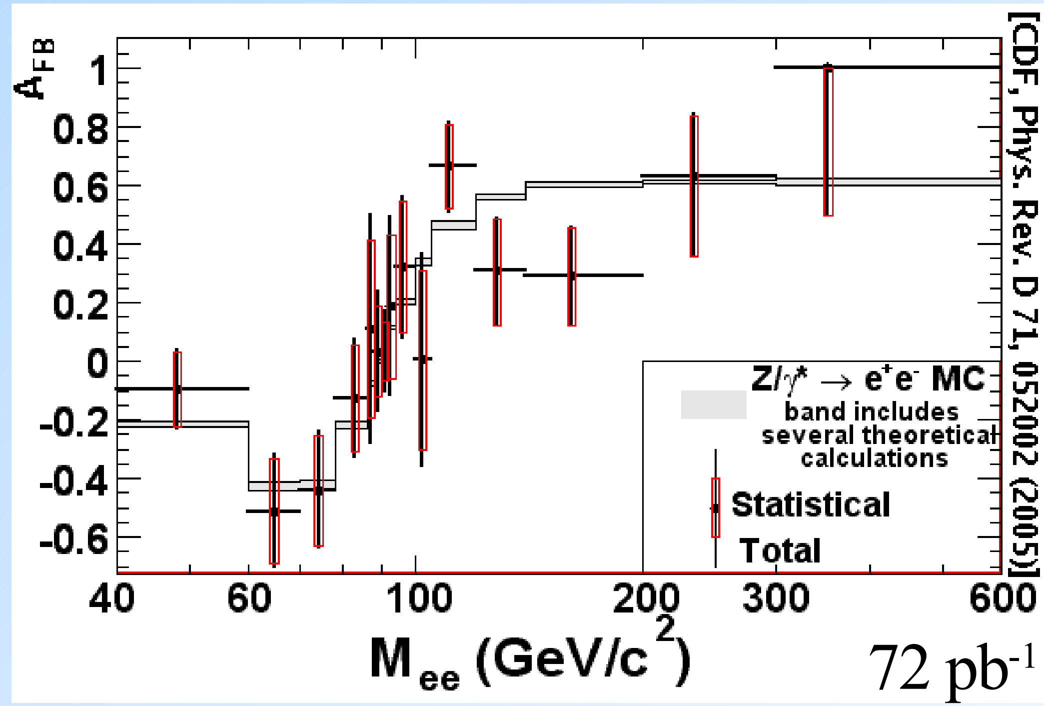
Applications of Exclusive Drell-Yan

Fully exclusive Drell-Yan at NNLO opens the possibility of Precision Electroweak Physics at hadron colliders. Measurements include:

- Neutral Current Forward-Backward Asymmetry
- W Mass Measurement
- LHC Luminosity Monitor
- PDF extraction

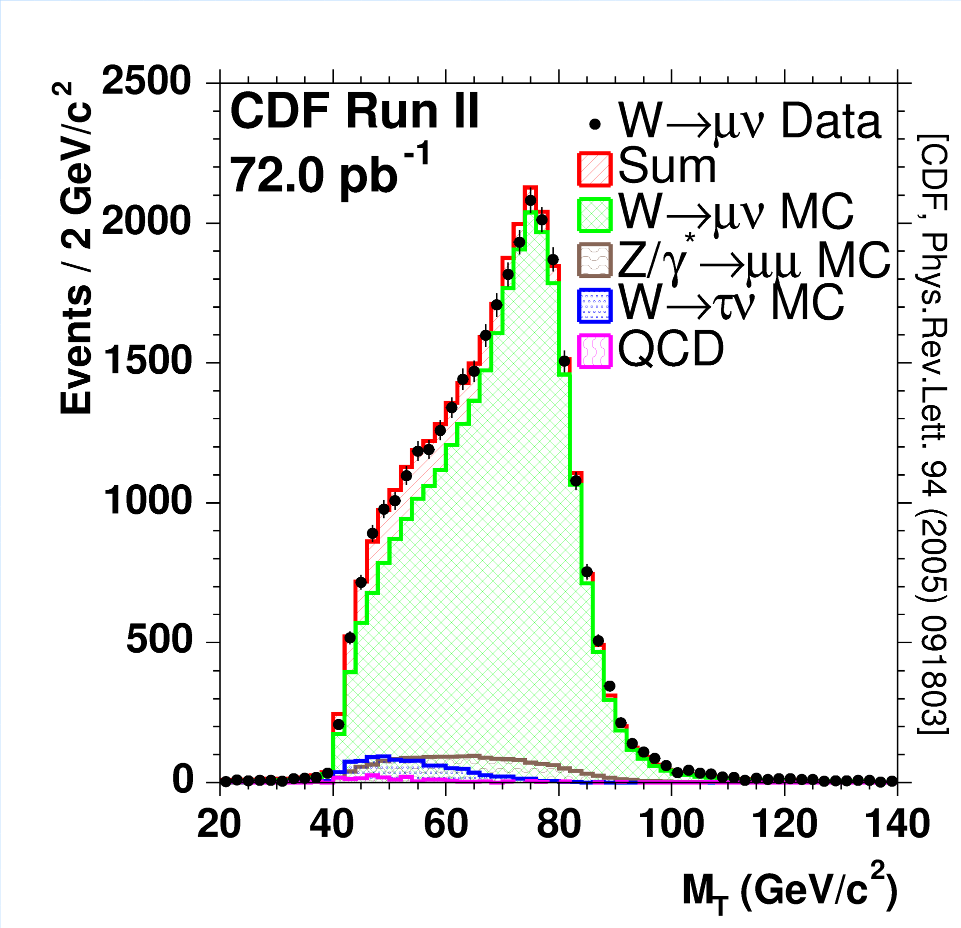
Neutral Current A_{FB}





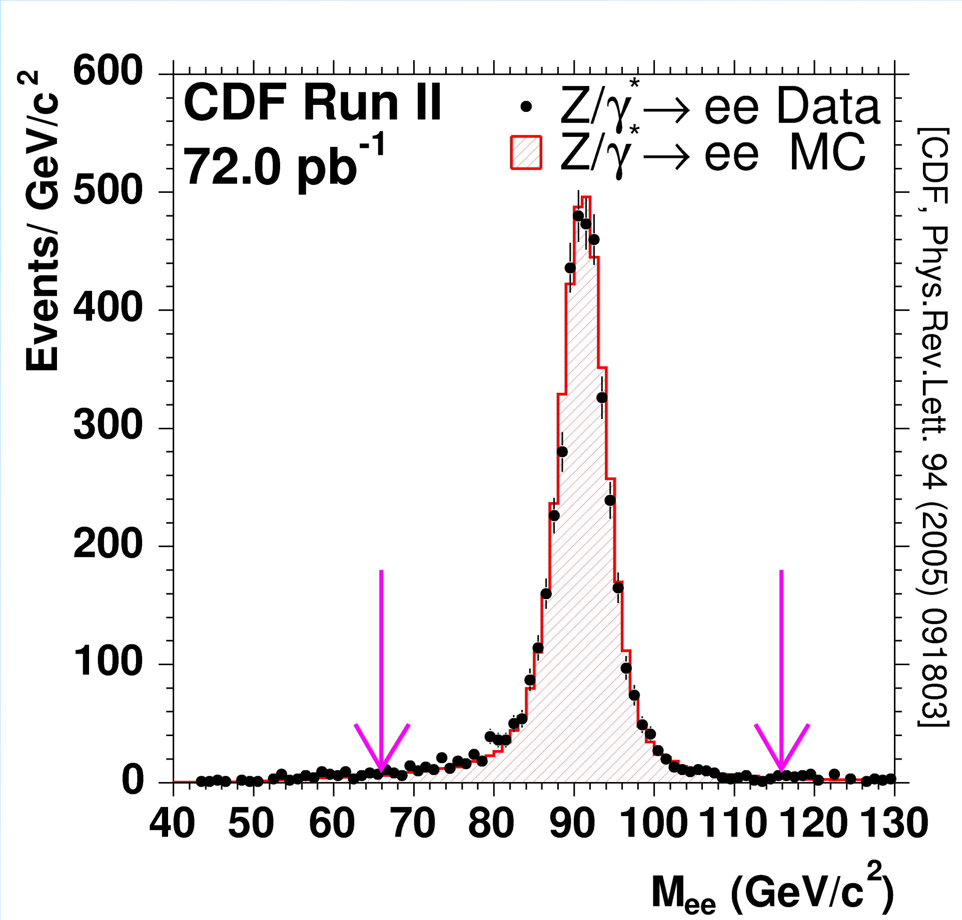
The forward backward asymmetry is well measured in e^+e^- where data are plentiful. Elsewhere there is room for improvement. The Tevatron needs ~10 fb^{-1} to compete with L3.

$W^{\pm}$ Mass Measurement



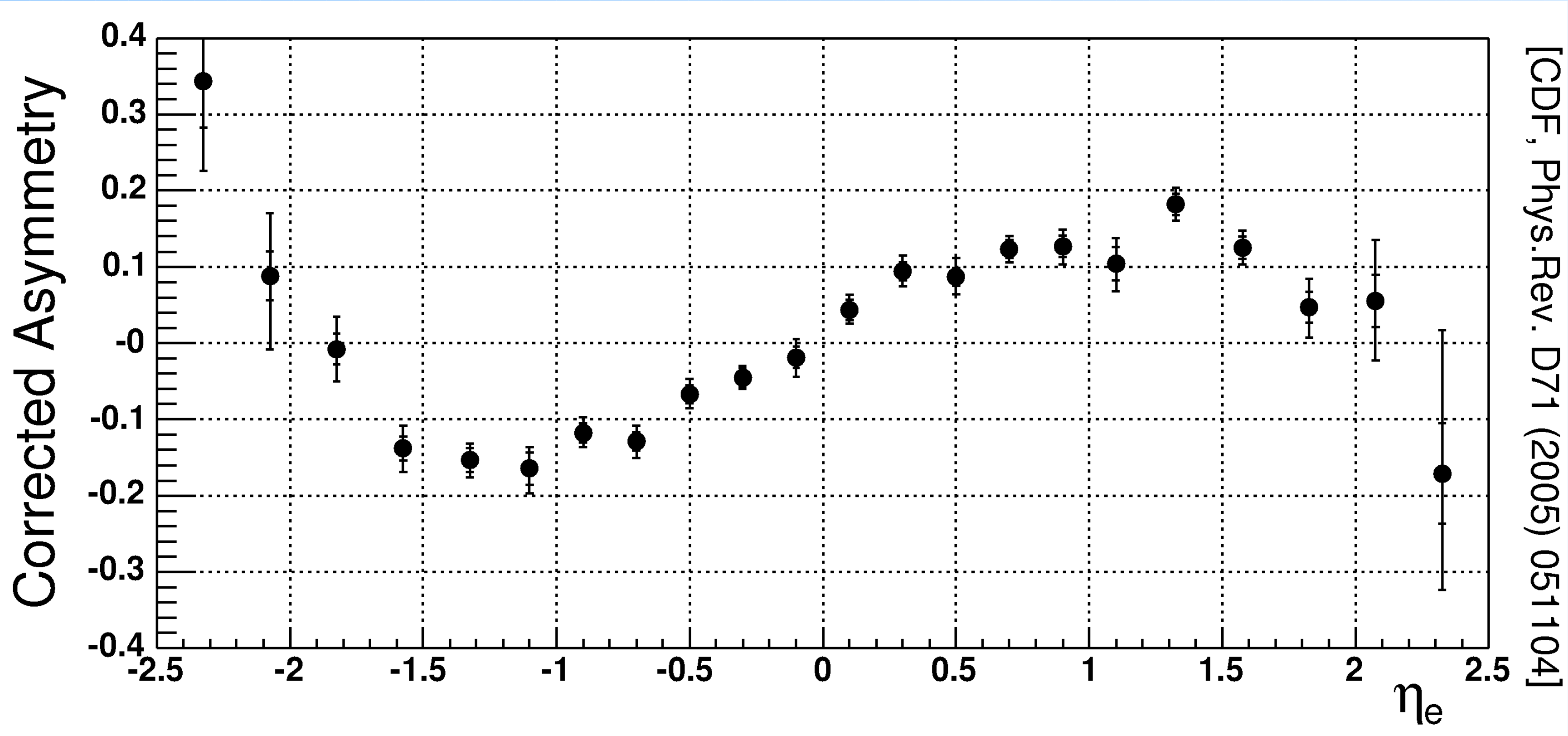
$W^{\pm}$ mass is determined from the transverse mass and other lepton distributions. We need to know where the leptons go.

Z/γ^* Mass and Rate Measurements



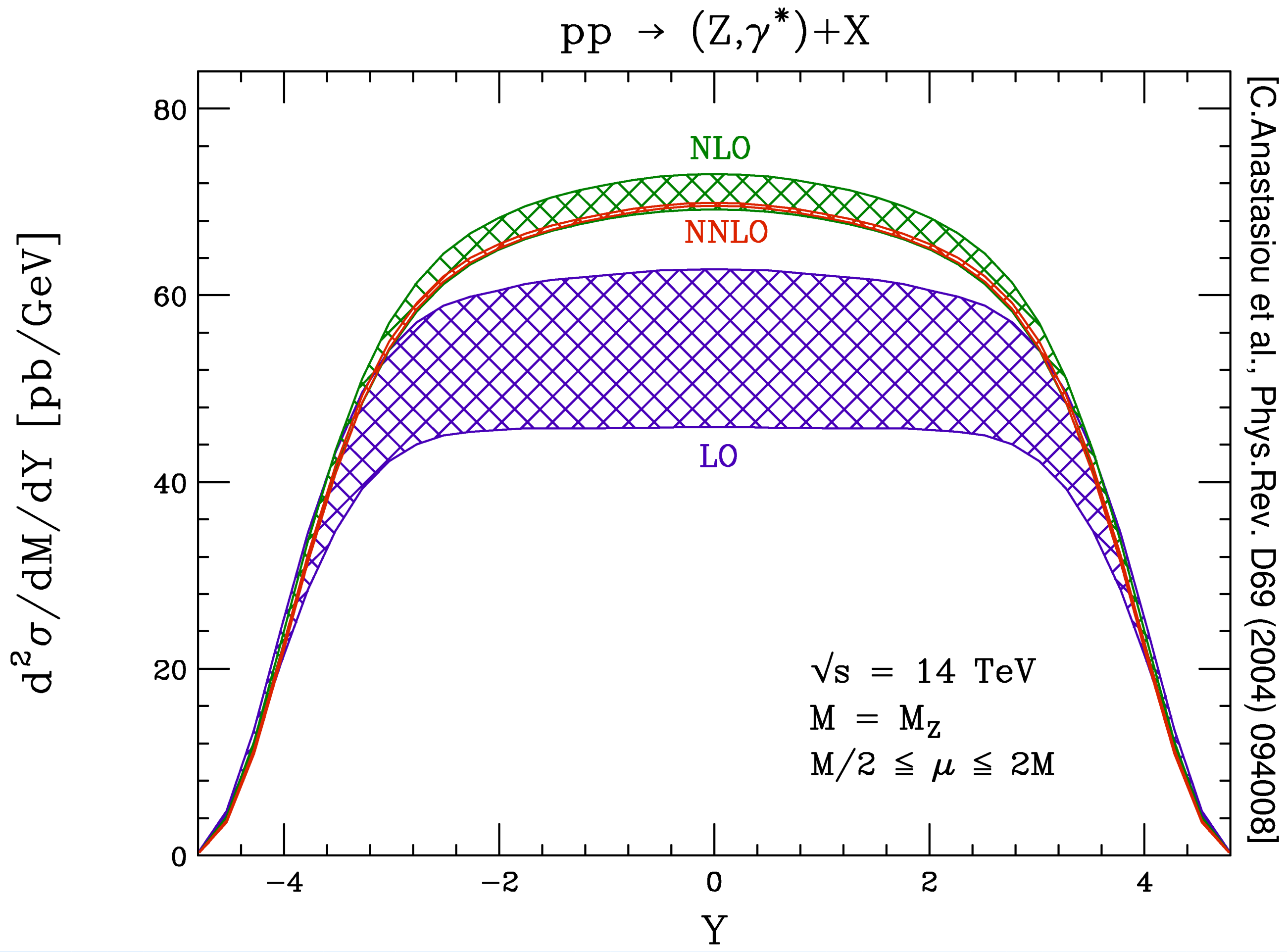
Di-lepton mass distribution will be used to calibrate detectors and with the rate can be used as a luminosity monitor.

PDF Extraction



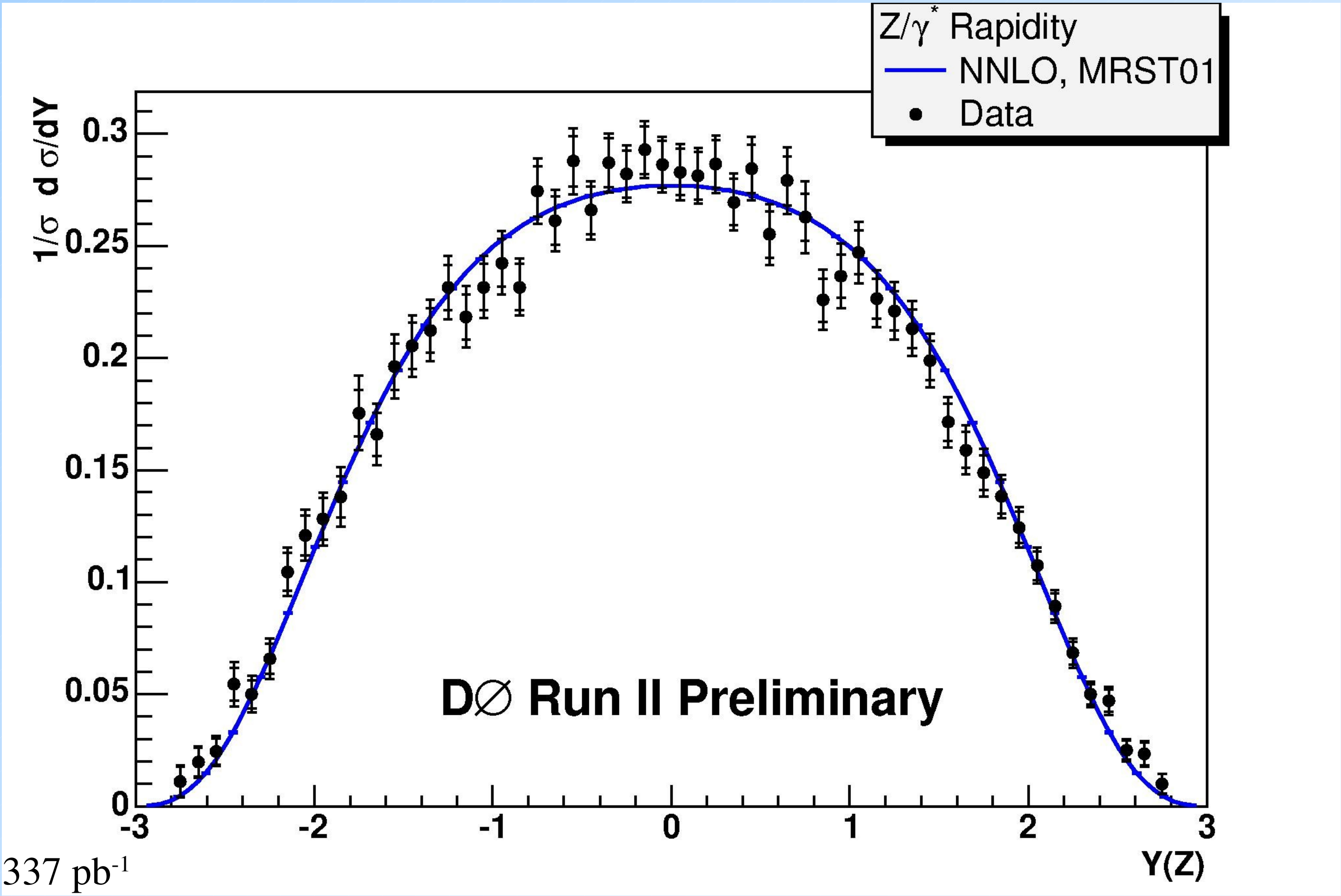
The W boson charge asymmetry is an important input to fitting parton distribution functions.

PDF Extraction



Precision measurements of Neutral Current Drell-Yan can also be used for PDF studies.

PDF Extraction



With a fully exclusive calculation, systematics would be reduced because acceptance could be computed.

Objective: Fully Exclusive Drell-Yan

Large efforts are underway to construct general-purpose subtraction schemes for NNLO calculations. Most focus on $e^+e^- \to$ jets.

- I am interested in hadronic production processes and will sacrifice generality for faster results.

I will describe a subtraction scheme that leverages the techniques of inclusive calculations to define a suitable counter-term.

- It is not clear how this method could be applied to processes for which the inclusive cross section cannot be computed.

NLO Calculations

Next-to-Leading Order calculations consist of two contributions:

Virtual Corrections to one loop.

Single Real Emission Corrections at tree-level.

$$\sigma^{NLO} = \int_{n+1} d\sigma_{n+1}^{(0)} + \int_{n} d\sigma_{n}^{(1)}$$

Both terms are infrared singular.

Subtraction at NLO

A subtraction scheme adds (and subtracts back out) a local counter-term to both Virtual and Real Correction terms, canceling the infrared singularities.

$$\sigma^{NLO} = \int_{n+1} \left(d\sigma^{(0)}_{n+1} - \quad d\alpha^{(0)}_{n+1} \right) + \int_{n} \quad d\sigma^{(1)}_{n} \; + \int_{n+1} d\alpha^{(0)}_{n+1}$$

Both terms are now infrared finite.

Subtraction at NLO

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Both terms are now infrared finite.

NNLO Calculations

At Next-to-Next-to-Leading Order, there are three contributions:

- Double Real Emission Corrections at tree-level.
- Single Real Emission Corrections to one loop.
- Virtual Corrections to two loops.

$$\sigma^{NNLO} = \int_{n+2} d\sigma_{n+2}^{(0)} + \int_{n+1} d\sigma_{n+1}^{(1)} + \int_{n} d\sigma_{n}^{(2)}$$

All three terms are infrared singular.

Subtraction at NNLO

A normal NLO subtraction scheme will take care of the singly-infrared singular regions.

$$\sigma^{NNLO} = \int_{n+2} \left(d\sigma_{n+2}^{(0)} - d\alpha_{n+2}^{(0)} \right) + \int_{n+2} d\alpha_{n+2}^{(0)} + \int_{n+1} d\sigma_{n+1}^{(1)} + \int_{n} d\sigma_{n}^{(2)}$$

We still must deal with doubly-infrared regions of both $d\sigma$ and $d\alpha$.

Subtraction at NNLO

We need counter-terms to tree-level double real emission ($d\gamma^{(0)}$), one-loop single real emission ($d\alpha^{(1)}$) and a counter-term to single real emission from the NLO counter-term ($d\beta^{(0)}$)!

$$\begin{aligned}\sigma^{NNLO} = & \int_{n+2} \left(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} + d\beta^{(0)}_{n+2} - d\gamma^{(0)}_{n+2} \right) \\ & + \int_{n+1} \left(d\sigma^{(1)}_{n+1} - d\alpha^{(1)}_{n+1} \right) + \int_{n+2} \left(d\alpha^{(0)}_{n+2} - d\beta^{(0)}_{n+2} \right) \\ & + \int_{n} d\sigma^{(2)}_{n} + \int_{n+1} d\alpha^{(1)}_{n+1} + \int_{n+2} d\gamma^{(0)}_{n+2}\end{aligned}$$

Subtraction at NNLO

We need counter-terms to tree-level double real emission, one-loop single real emission and a counter-term to single real emission from the counter-term!

$$
\begin{aligned}
\sigma^{NNLO} = & \int_{n+2} \left(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} + d\beta^{(0)}_{n+2} - d\gamma^{(0)}_{n+2} \right) \\
& + \int_{n+1} \left(d\sigma^{(1)}_{n+1} - d\alpha^{(1)}_{n+1} \right) + \int_{n+2} \left(d\alpha^{(0)}_{n+2} - d\beta^{(0)}_{n+2} \right) \\
& + \int_{n} d\sigma^{(2)}_{n} + \int_{n+1} d\alpha^{(1)}_{n+1} + \int_{n+2} d\gamma^{(0)}_{n+2}
\end{aligned}
$$

NNLO Counter-terms

It would seem that I need three new counter-terms for double real emission. In general, this would be so. But I am not interested in generality, so a very simple choice is available to me:

Because I can do the total integrals analytically, I can use the matrix elements themselves as the counter-terms and map them to the appropriate doubly-singular point in phase space.

$$\begin{aligned} d\alpha_{n+1}^{(1)} &= d\sigma_{n+1}^{(1)} \ , \\ d\gamma_{n+2}^{(0)} &= d\sigma_{n+2}^{(0)} \ , \\ d\beta_{n+2}^{(0)} &= d\alpha_{n+2}^{(0)} \ . \end{aligned}$$

Setting Up the calculation

The cross section is the scalar product of hadronic and leptonic tensors, $\sigma = H^{\mu\nu} L_{\mu\nu}$. If I average over the lepton observables, $\int L_{\mu\nu} \propto g_{\mu\nu}$. Inclusive calculations need only the trace of the hadronic tensor, $H^{\mu}{}_{\mu}$.

Computing the exclusive Drell-Yan cross section means computing the full tensor $H^{\mu\nu}$. Since this is a covariant object, I must choose a reference frame for the calculation. Furthermore, this frame and the parameterization of phase space must allow me to capture production information.

Parameterizing Phase Space

In calculating the inclusive cross section, van Neerven *et al.* chose different parameterizations of phase space to simplify the calculation of each term. Of particular concern is the presence of angular variables in massive propagators.

Since I want to integrate out the radiation and have ready access to production information, I do not have the luxury of choosing the most convenient parameterization for each term.

Choosing a Parameterization

In particular, ready access to production information demands that the initial state massive propagators be free of angular terms.

> In this parameterization, for a given $\hat{s}$ and M_V, a single invariant of the gauge boson and an initial state parton, say s_{1V}, will determine the rapidity of the gauge boson at the doubly-singular point.

So, if one maps all configurations with a particular value of s_{1V} to the corresponding doubly-singular configuration, one obtains a suitable subtraction counter-term: $\frac{dH^{\mu\nu}}{dM_V^2\, ds_{1V}}$.

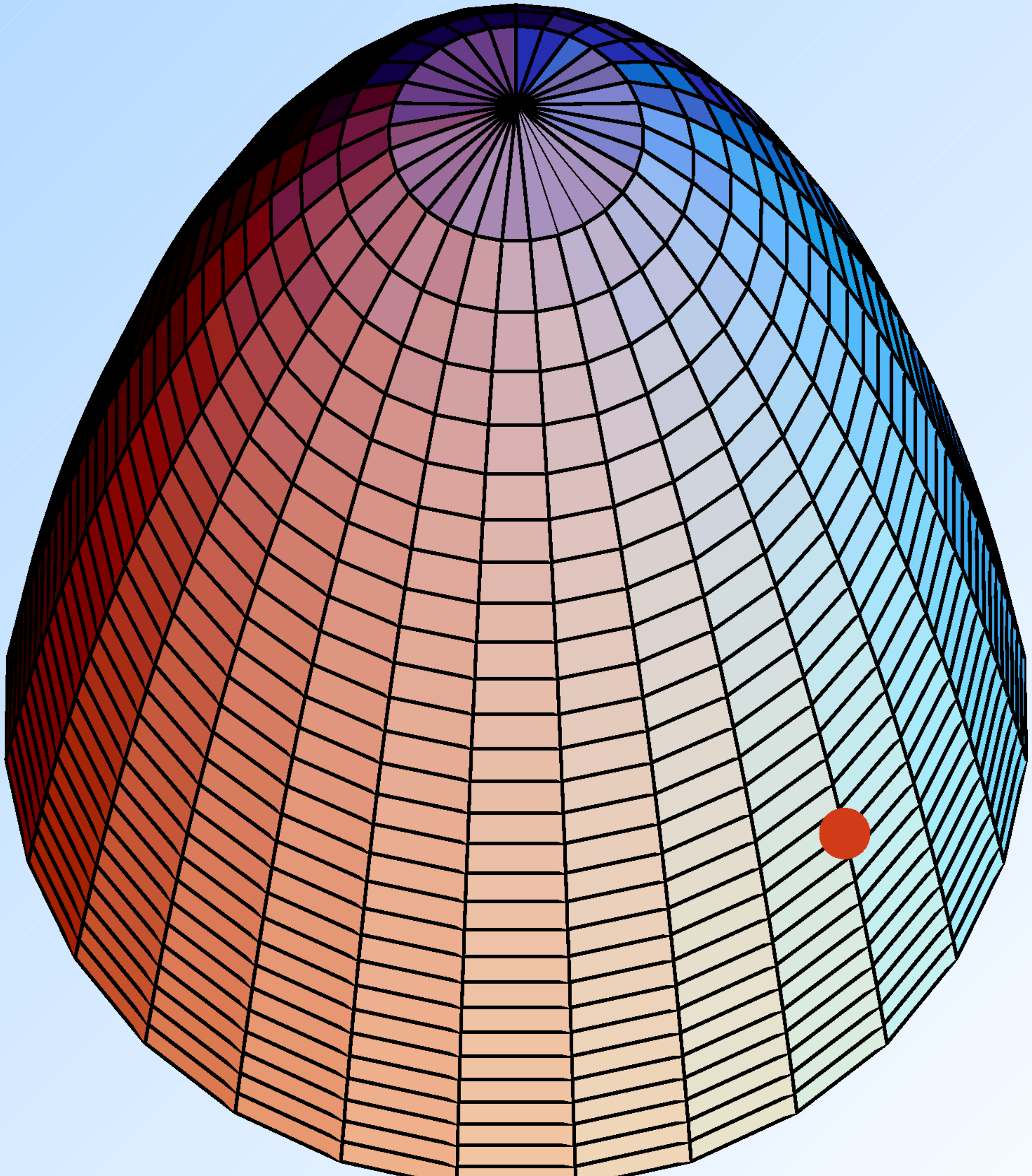
Performing the Calculation

I perform the calculation in two steps. In the first step, I integrate out the angular dependence of the radiation in the radiation rest frame.

I then transform to the gauge boson rest frame and orient it as a Gottfried-Jackson frame, where incoming parton momentum p_1 defines the z axis, the lepton momenta define the x-z plane and s_{1V} defines the rapidity of the subtraction point.

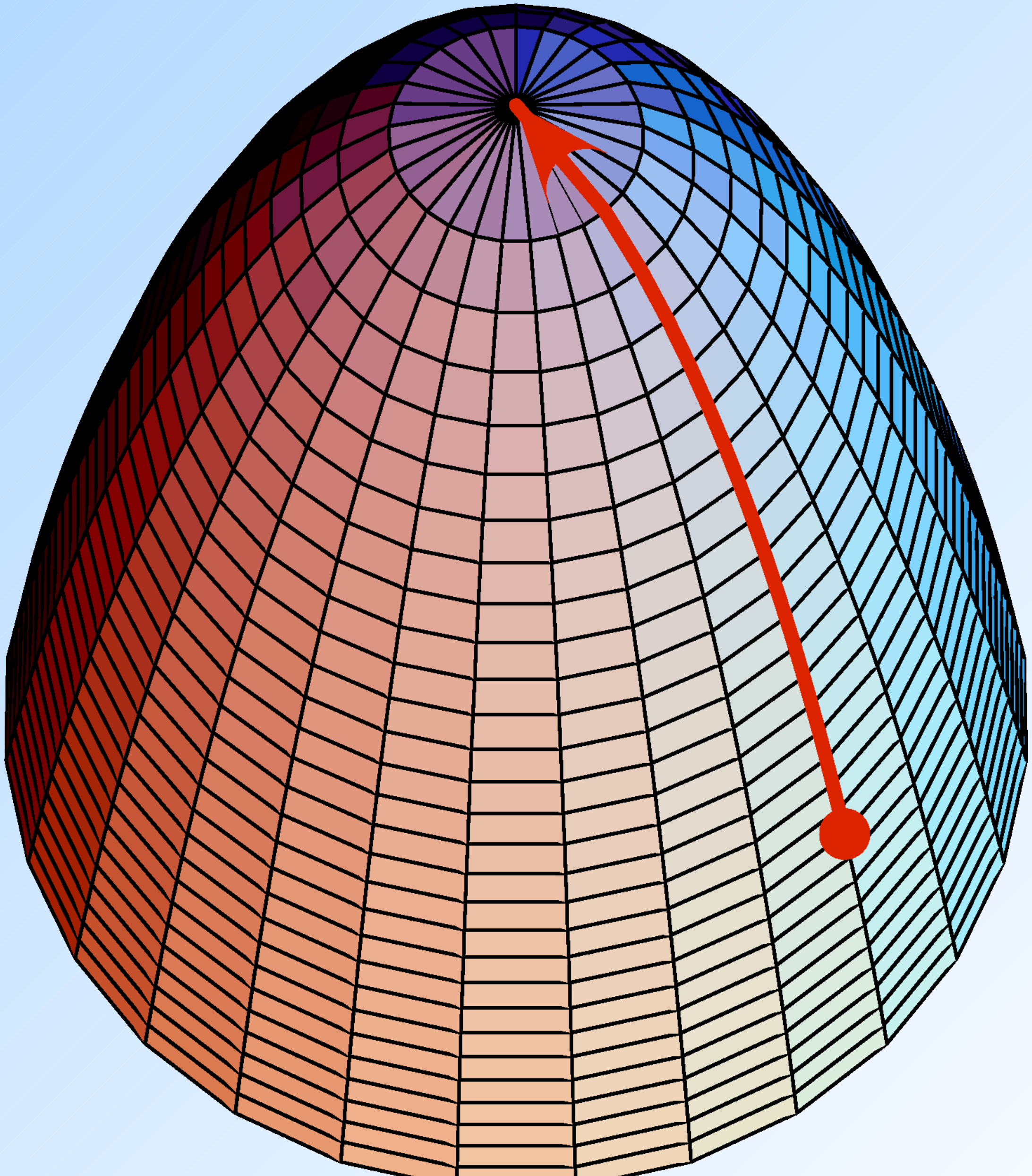
Mapping to the Doubly-Singular Point

This surface describes the possible values of $\vec{p}_2$ in the Gottfried-Jackson frame for given values of $\hat{s}$, M_V, s_{1V} .



Mapping to the Doubly-Singular Point

All points on the surface map to the same doubly-singular point.



Axial Terms

Capturing the full structure of the hadronic tensor means computing axial-vector terms, σ^{AV}, which vanished in taking the trace $H^{\mu}{}_{\mu}$ in inclusive results.

The only new two-loop calculation needed are anomaly terms, which are now known. Consistency between σ^{VV} and σ^{AA} and the universal structure of infrared singularities fully determines σ^{AV}.

The axial components of one-loop single emission can be computed by standard methods and verified by again checking the consistency of σ^{VV} and σ^{AA}.

Subtraction at NNLO

Thus, an NNLO cross section takes the form:

$$\sigma^{NNLO} = \int_{n+2} (d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} + d\bar{\alpha}^{(0)}_{n+2} - d\bar{\sigma}^{(0)}_{n+2}) \otimes \mathcal{S}$$
$$+ \left(\int_{n+1} d\sigma^{(1)}_{n+1} + \int_{n+2} (d\alpha^{(0)}_{n+2} - d\bar{\alpha}^{(0)}_{n+2}) - \int_{n+1} d\bar{\sigma}^{(1)}_{n+1} \right) \otimes \mathcal{S}$$
$$+ \left(\int_{n} d\sigma^{(2)}_{n} + \int_{n+1} d\bar{\sigma}^{(1)}_{n+1} + \int_{n+2} d\bar{\sigma}^{(0)}_{n+2} \right) \otimes \mathcal{S}$$

$\mathcal{S}$ factors represent measurement functions, including lepton cuts, isolation cuts and even jet vetoes.

Barred terms indicate counter-terms that take the same values as unbarred terms but are mapped to the subtraction point. Terms on the bottom line have been heavily shaken to permit analytic integration.

Cuts and Vetoes

$$
\begin{aligned}
\sigma^{NNLO} = & \int_{n+2} \left(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} + d\bar{\alpha}^{(0)}_{n+2} - d\bar{\sigma}^{(0)}_{n+2} \right) \otimes \mathcal{S} \\
& + \left(\int_{n+1} d\sigma^{(1)}_{n+1} + \int_{n+2} \left(d\alpha^{(0)}_{n+2} - d\bar{\alpha}^{(0)}_{n+2} \right) - \int_{n+1} d\bar{\sigma}^{(1)}_{n+1} \right) \otimes \mathcal{S} \\
& + \left(\int_{n} d\sigma^{(2)}_{n} + \int_{n+1} d\bar{\sigma}^{(1)}_{n+1} + \int_{n+2} d\bar{\sigma}^{(0)}_{n+2} \right) \otimes \mathcal{S}
\end{aligned}
$$

Lepton cuts can be applied to all terms in the formula, but hadronic cuts (isolation cuts, jet vetoes) can only be applied to un-barred terms.

To avoid a failure of the cancellation near the subtraction point, one must avoid vetoing invisible jets and isolating undetected leptons.

Subtraction at NNLO Summary

You put the double-real in

You take the double-real out

You put the double-real in and you shake it all about

Apply the cuts and vetoes and get the answer out.

$$\sigma^{NNLO} = \int_{n+2} \left(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} + d\overline{\alpha}^{(0)}_{n+2} - d\overline{\sigma}^{(0)}_{n+2} \right) \otimes \mathcal{S}$$
$$+ \left(\int_{n+1} d\sigma^{(1)}_{n+1} + \int_{n+2} \left(d\alpha^{(0)}_{n+2} - d\overline{\alpha}^{(0)}_{n+2} \right) - \int_{n+1} d\overline{\sigma}^{(1)}_{n+1} \right) \otimes \mathcal{S}$$
$$+ \left(\int_{n} d\sigma^{(2)}_{n} + \int_{n+1} d\overline{\sigma}^{(1)}_{n+1} + \int_{n+2} d\overline{\sigma}^{(0)}_{n+2} \right) \otimes \mathcal{S}$$

Status Report and Conclusions

- All analytic calculations are complete.
  - All poles in ϵ explicitly canceled.
  - All integrals verified using threshold expansion technique.
  - Total cross section agrees with known result.
- Numerical code is written.
  - First results will come very soon.
- This approach leverages fully-inclusive calculations to obtain a counter-term for computing exclusive Drell-Yan production at NNLO using subtraction.
  - Allows Flexible cuts and vetoes.
  - Addresses important physics needs at LHC and Tevatron.