Elliptic Inflation

Generating curvature perturbations at the end of inflation

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References

- T. Matsuda,
  "*Elliptic Inflation:Generating the curvature perturbation without slow-roll*",
  JCAP 0609:003,2006 [hep-ph/0606137]

- T. Matsuda,
  "*Brane inflation without slow-roll*", [astro-ph/0610402]

- T. Matsuda,
  "*Generating the curvature perturbation with instant preheating*",
  hep-th/0610232

I. Motivation ~ Why we need "alternatives"?

⋆ Slow-Roll Inflation

~ The density perturbations are produced by the "light inflaton"

→ Loop corrections △ / "η"-problem △

⋆ Alternatives ~ "light field" $\neq$ inflaton

→ "slow-roll" conditions are not critical ◯

− Mechanism is : "Conversion"

Isocurvature perturbation → Curvature perturbation

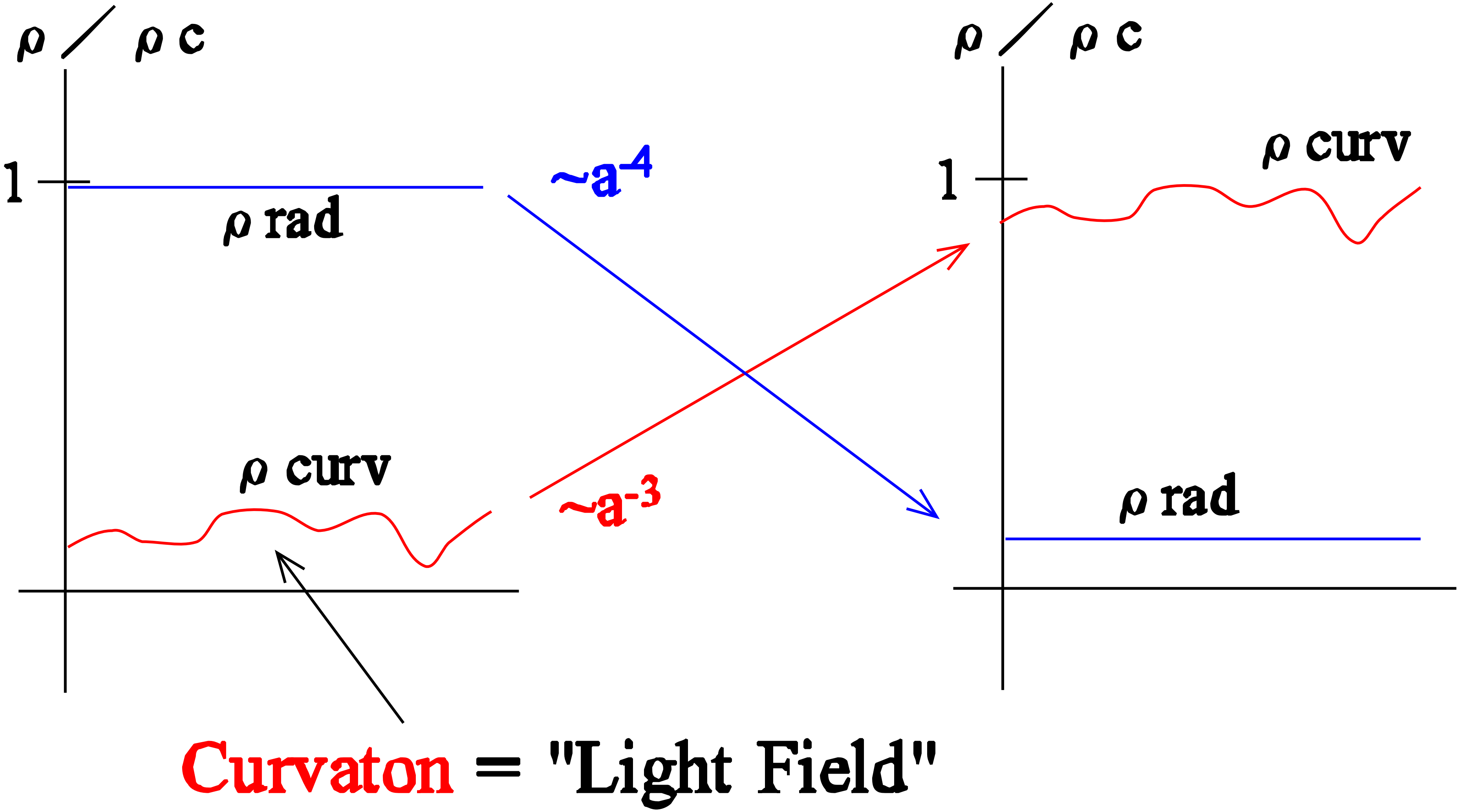
Ex.1 **Curvaton**

~ Late-time domination → decay + reheating

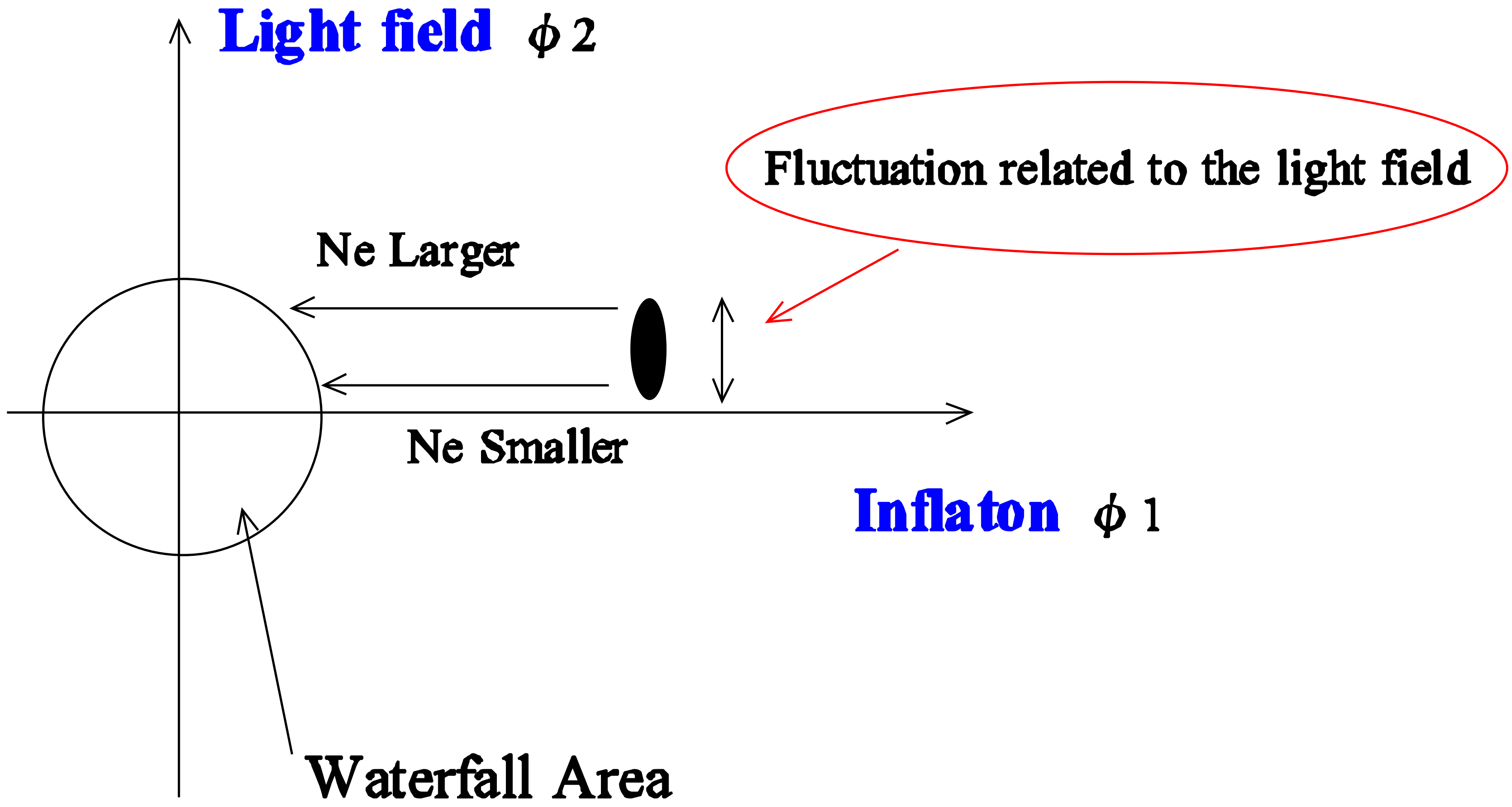
Ex.2 **Generating the curvature perturbation at the end of inflaton**

~ δN_e at the end of inflation ("Waterfall" or "Preheating")

Model A : **Curvatons** (Late-time dominance → decay to reheat)

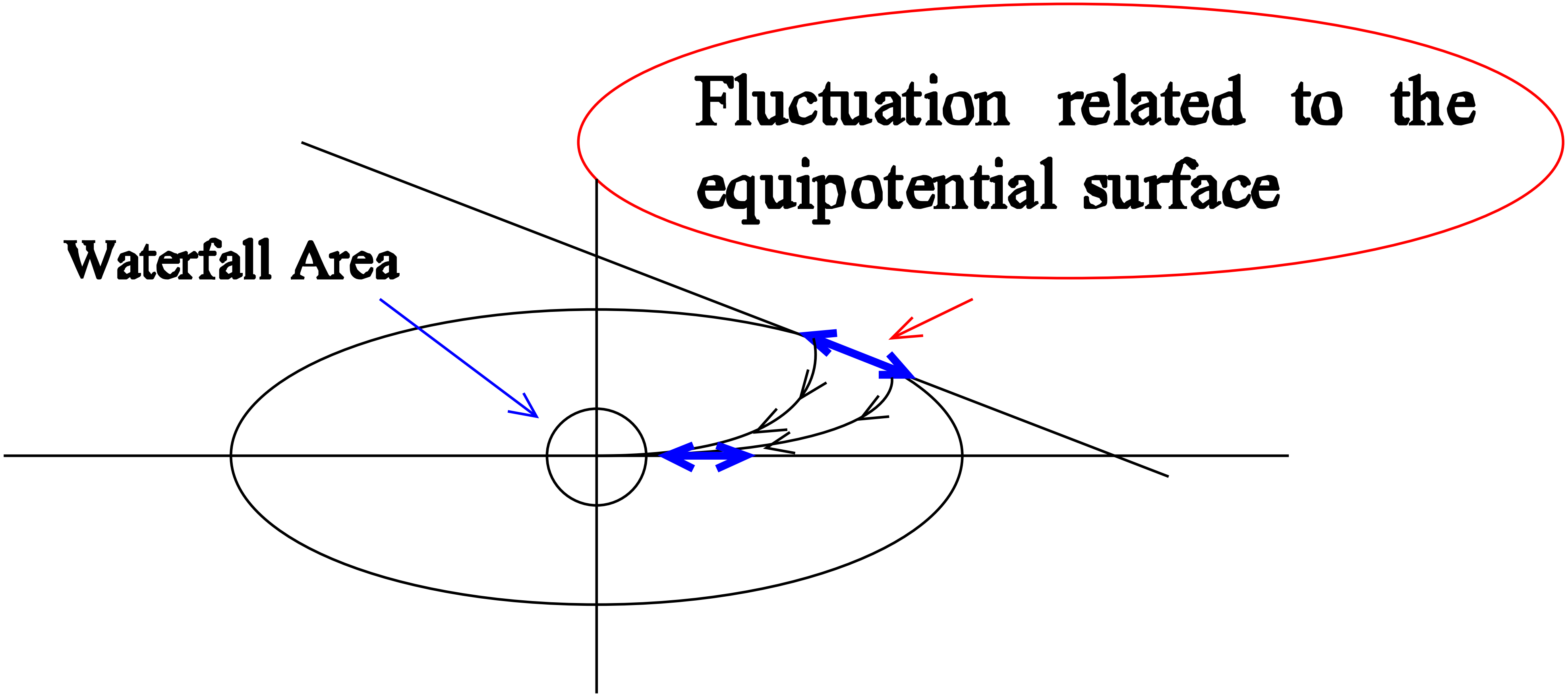


Model B-1 : $\underline{\delta N_e}$ from **hybrid potential** with $m_2 \simeq 0$ (Lyth '05)



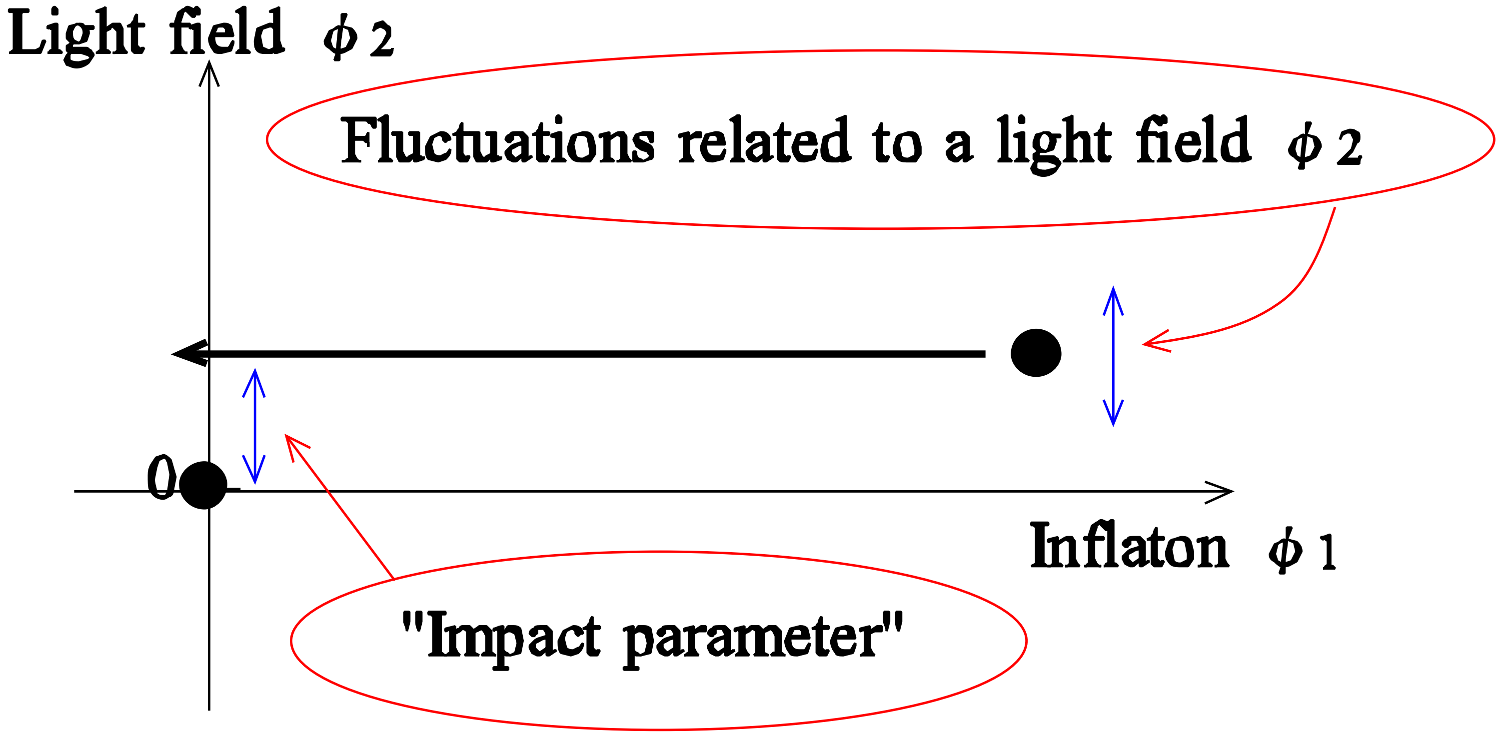
Fluctuation related to a light field $\delta\phi_2$ is converted into the fluctuation related to the number of e-foldings δN_e(=curvature perturbation)

Model B-2 : $\underline{\delta N_e}$ from **hybrid potential** with $m_1 \sim m_2$ (Matsuda '06)



Fluctuation along the equipotential surface is converted into δN_e
(generating the curvature perturbation at the end of fast-roll inflation)

Model C :$\underline{\delta N_e}$ from **Instant preheating** with $m_1 \gg m_2$ (Matsuda '06)

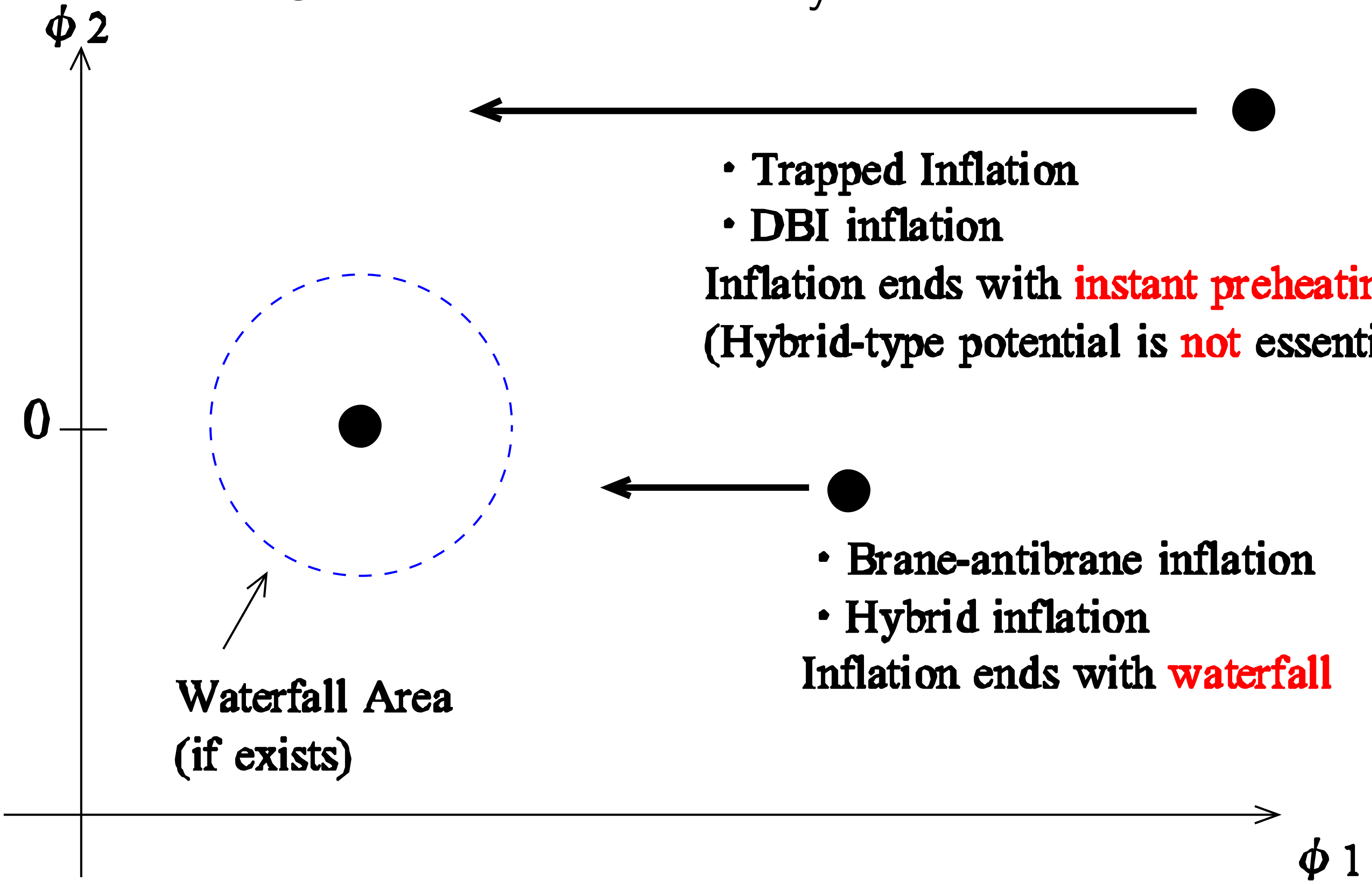


$\delta\phi_2$ is converted into the fluctuation of the preheat field $\delta n_\chi/n_\chi$

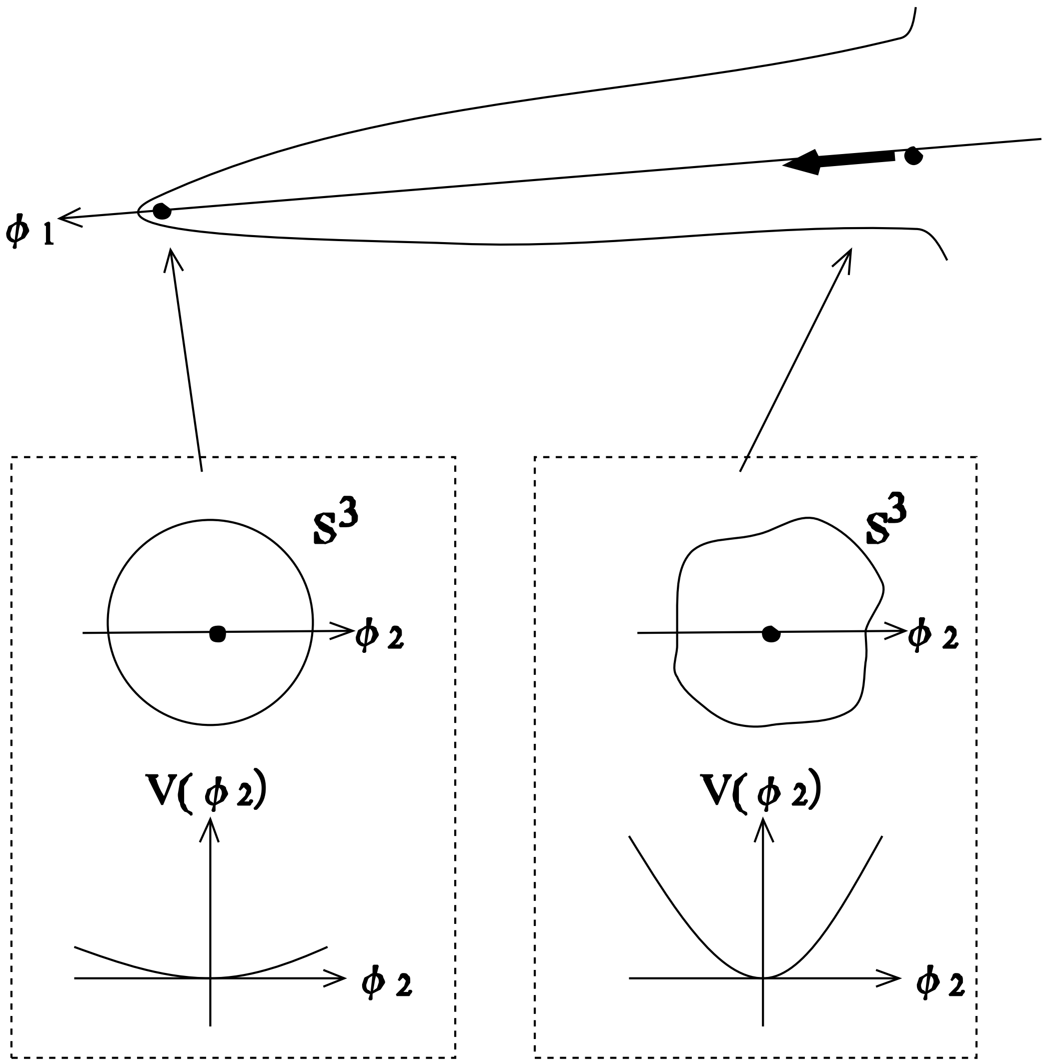
i.e. $\delta\phi_2 \longrightarrow \frac{\delta n_\chi}{n_\chi} \longrightarrow \frac{\delta\rho_\chi}{\rho_\chi} \longrightarrow \frac{\delta T}{T}$

Due to $$n_\chi = \frac{(g|\dot{\phi}_1|)^{3/2}}{8\pi^3}\exp\left[-\frac{\pi g|\phi_2|^2}{|\dot{\phi}_1|}\right]$$

Model B-1 and C-1 in brane inflationary models



* Model B-1=Hybrid inflation / Molde C-2=Instant preheating



These mechanisms are very important because the isometries at the tip play the required role of the light field ϕ_2.

Mechanisms for generating the curvature perturbation at the end of inflation

	Inflation ends with waterfall	Inflation ends with preheating
$m_1 \gg m_2$	D. H. Lyth	T. Matsuda
$m_1 \sim m_2$	T. Matsuda	W. Kolb, A. Riotto and A. Vallinotto

D. H. Lyth, *Generating the curvature perturbation at the end of inflation, JCAP 0511:006,2005* [astro-ph/0510443].

W. Kolb, A. Riotto and A. Vallinotto, *Curvature perturbations from broken symmetries, Phys.Rev.D71:043513,2005* [astro-ph/0410546];

Specific Examples

1. Generating δN_e at the end of inflation (Lyth '05)

Using "hybrid-type potential"

$$V(\varphi, \sigma) = \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\varphi^2\sigma^2 + \frac{1}{4}\left(\sigma^2 - M^2\right), \tag{1}$$

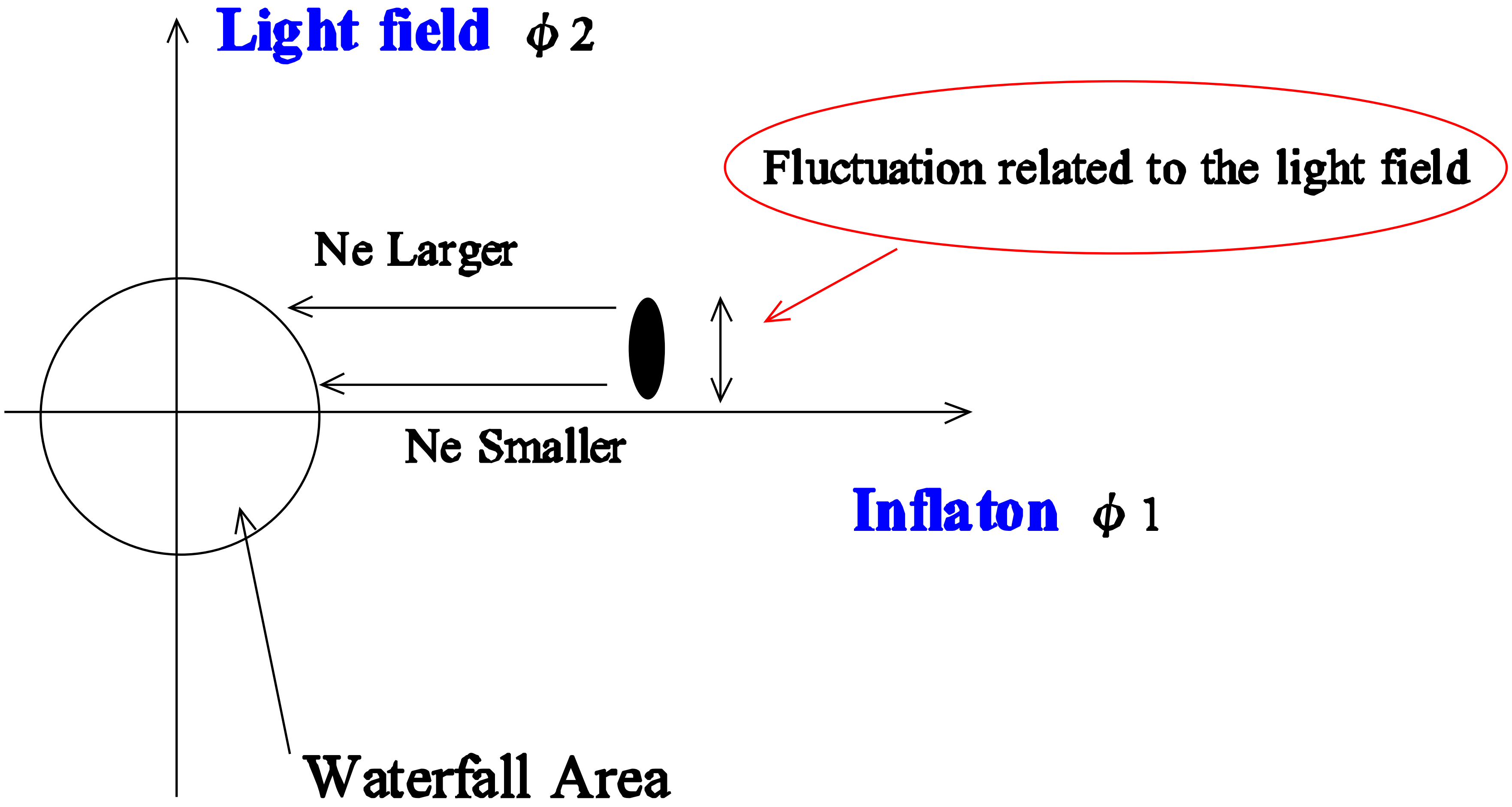
$\varphi = \phi_1 + i\phi_2$; ($\phi_1 =$Inflaton, $\phi_2 =$Light field, $\sigma =$Waterfall field).

The number of e-foldings elapsed during fast-roll inflation is

$$N_F \simeq \frac{1}{F}\ln\left(\frac{\phi_{1,ini}}{\phi_{1,end}}\right), \;\; \phi_{1,end} \equiv \sqrt{M^2 - \phi_2^2} \tag{2}$$

where F is given by

$$F \equiv \frac{3}{2}\left(1 - \frac{\sqrt{9-4\alpha^2}}{3}\right), \alpha \equiv H_I/m \tag{3}$$



N_e is larger in the outside, and smaller in the inside. The mean value of the light field ϕ_2 is a constant during inflation. The gaussian fluctuation $\phi_2 \simeq H_I/2\pi$ is induced during inflation.

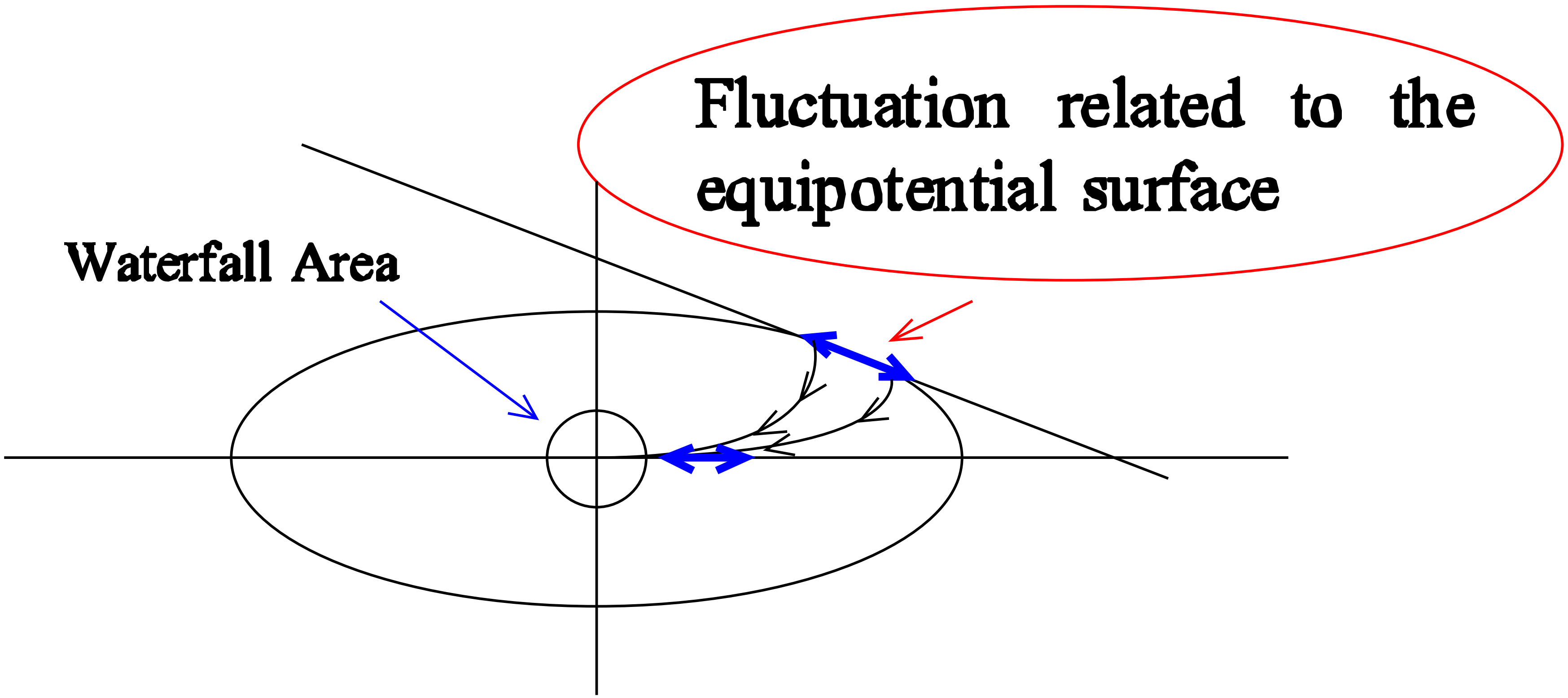
Due to the fluctuation related to the Light field $\phi_2 \simeq H_I/2\pi$, **the curvature perturbation** $\zeta = \delta N_e$ is generated at the end of inflation.

$$\zeta = \frac{\partial N}{\partial \phi_1}\frac{\partial \phi_1}{\partial \phi_2}\delta\phi_2 + \frac{1}{2}\left\{2\frac{\partial^2 N}{\partial \phi_1^2}\left(\frac{\partial \phi_1}{\partial \phi_2}\right)^2 + \frac{\partial N}{\partial \phi_1}\frac{\partial^2 \phi_1}{\partial \phi_2^2}\right\}(\delta\phi_2)^2$$
$$= \frac{\gamma H_I}{2\pi F_1(1-\gamma^2)M} + \frac{(2\gamma^2+1)H_I^2}{8\pi^2 F_1(1-\gamma^2)^2 M^2}. \quad (4)$$

$\delta\phi_2$ appears in the above equation because the end-point ($\phi_{1,end}$) of the inflaton field depends on ϕ_2

$$N_F \simeq \frac{1}{F}\ln\left(\frac{\phi_{1,ini}}{\phi_{1,end}}\right), \ \phi_{1,end} \equiv \sqrt{M^2 - \phi_2^2} \quad (5)$$

Even if both fields are **heavy** ($m_2 > m_1 > H_I$) and fast-roll occurs in both ϕ_1 and ϕ_2, fluctuations generated along the equipotential surface is **always** present and they may be converted into δN_e at the end of inflation.



Fluctuations on the elliptic surface $\rightarrow \delta N$
(These effects could be Dominant / or Subdominant)

Naive integration is incorrect in [Fast-roll + Heavy field (osc)]

[×] Integrating out $\phi_2 \rightarrow$ Single field inflation($m_1 > H_I$) + $\delta\phi_1 \simeq 0$

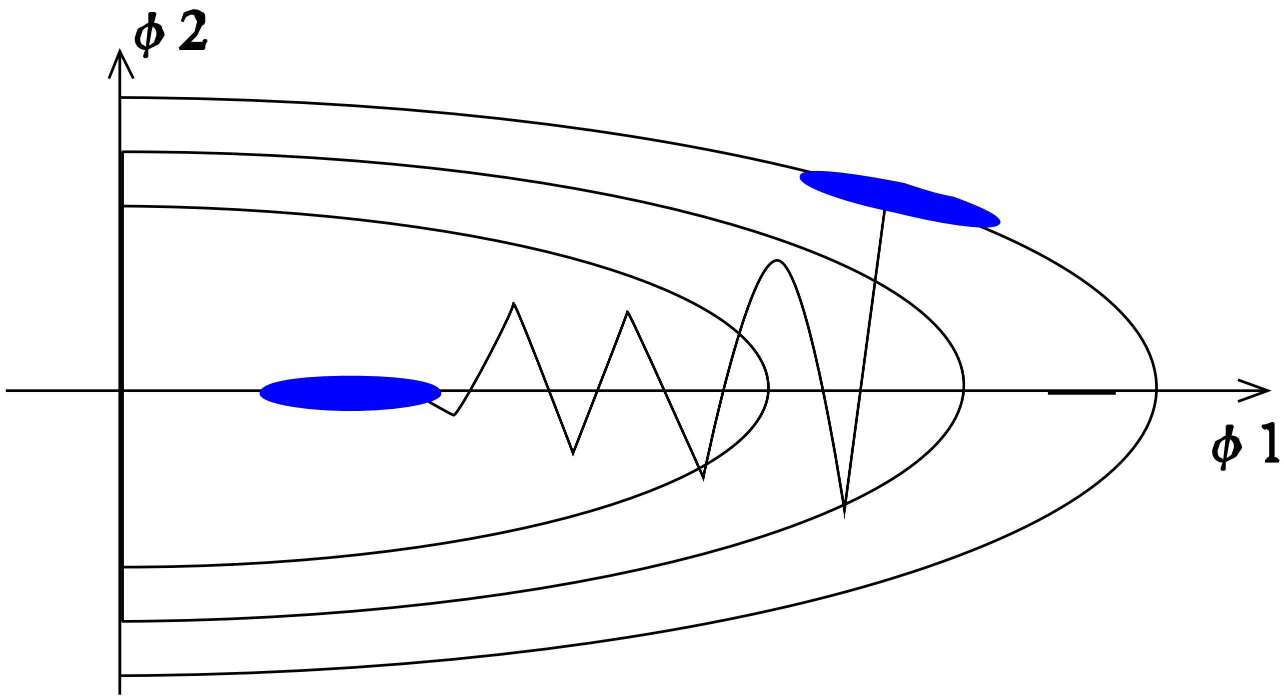
[◯] Fluctuation on the equipotential surface

+Eq.of mo. for $\phi_{1,2} \rightarrow$ Single field inflation($m_1 > H_I$) + $\delta\phi_1 \neq 0$

"heavy field" appears **always** in brane inflationary models!

Fast-roll ϕ_1 →This effect is **dominant** and **observable**.

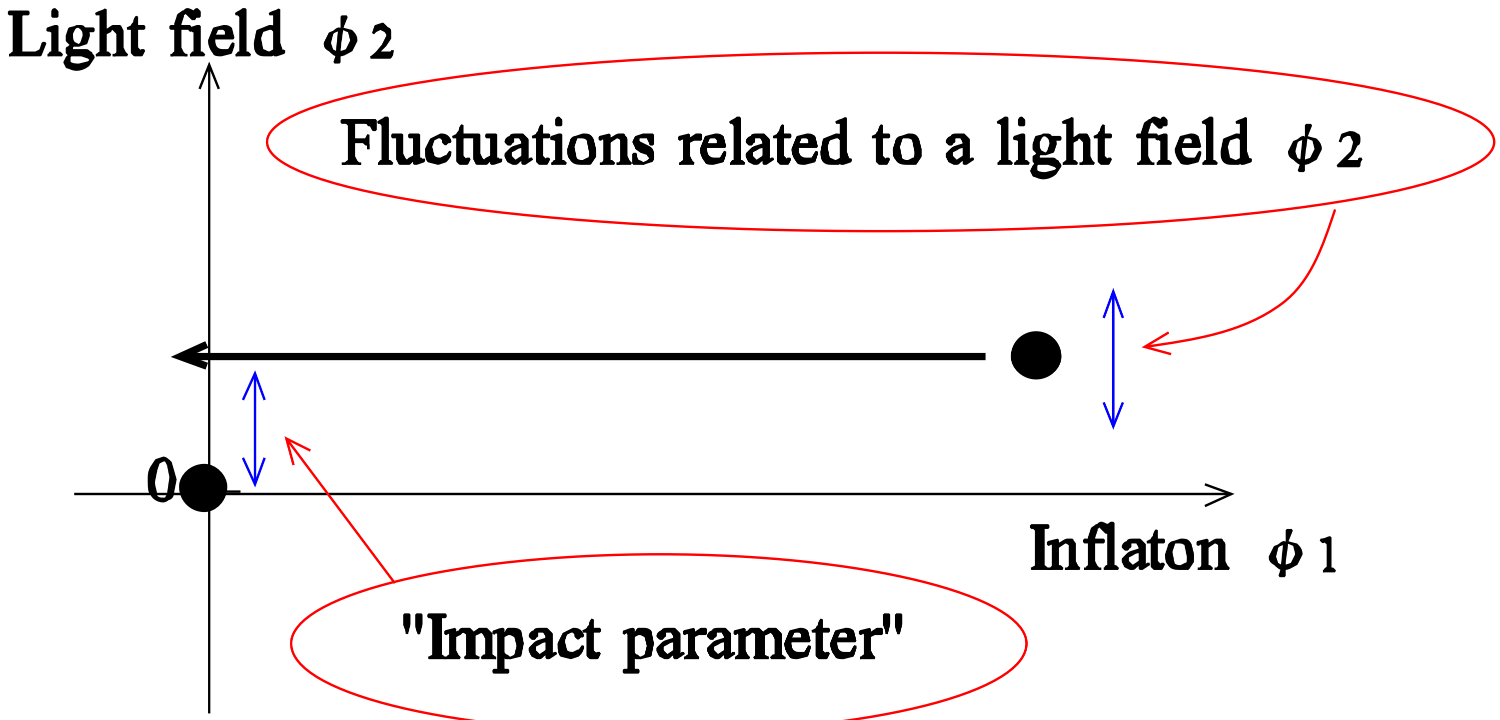
*If slow-roll→This effect is **subdominant** / could be **observable**.



2. Generating δN_e at the end of inflation (Matsuda '06)

Using "instant preheating"

$$n_\chi = \frac{(g|\dot{\phi}_1|)^{3/2}}{8\pi^3} \exp\left[-\frac{\pi g|\phi_2|^2}{|\dot{\phi}_1|}\right] \tag{6}$$



Due to the fluctuations related to the Light field $\delta\phi_2 \simeq \frac{H_I}{2\pi}$,

$$\frac{\delta n_\chi}{n_\chi} = \frac{2\pi g|\phi_2(t_*)|^2}{|\dot{\phi}_1(t_*)|}\frac{|\delta\phi_2(t_*)|}{|\phi_2(t_*)|} \rightarrow \frac{\delta\rho}{\rho} \rightarrow \zeta \tag{7}$$

Ex.1 Trapped inflation

The fluctuation related to N_e elapsed during "trapped inflation" is

$$\zeta \sim \delta N \sim \frac{\phi_2}{v}\delta\phi_2. \tag{8}$$

Ex.2 DBI inflation

$$\zeta \sim \frac{\delta n_\chi}{n_\chi} \sim -\frac{b\delta b}{\eta}, \tag{9}$$

where $b=$ impact parameter, $\eta \equiv$arctanh v,
$\eta \gg 1$ for ultra-relativistic motion $v \rightarrow 1$

Summary

The results I have shown you today leads to a new paradigm of **"generating curvature perturbations at the end of inflation"**

	Inflation ends with waterfall	Inflation ends with preheating
$m_1 \gg m_2$	D. H. Lyth	T. Matsuda
$m_1 \sim m_2$	T. Matsuda	W. Kolb, A. Riotto and A. Vallinotto

which is

(1) very natural in the brane Universe
(*The correction is significant in KS-throat) and

(2) may solve the η-problem in SUGRA.

Even if these mechanisms do not generate the dominant part of the curvature perturbations, there could be an observable signature of the curvature perturbations related to
(1) the "heavy" scalar fields other than the inflaton, rolling fast (or oscillate) during inflation, or
(2) the "light field" that appears in KS-throat inflation.

The study of the observable evidences is still in progress.