Two Small Numbers: Baryon Number and the Cosmological Constant, Are they interrelated?

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Azusa Minamizaki,
with Prof.Akio Sugamoto

Ochanomizu University

1.Plan of my talk

1. Motivation
   - . Two numbers
   - . Earlier models
     - -Susskind,Dimopoulos (non-zero baryon number density)
     - -Yoshimura (small cosmological constant)
2. Our toy-model
3. Summary

1.Motivation

Two Numbers

The cosmological constant , Λ

The constant term added to Einstein equation.

corresponds to

the energy density of the universe

$$\langle \text{the energy density of } \phi \rangle \equiv \Lambda$$

ϕ : some cosmic field

From the observation of WMAP (2003)

Non-zero Λ is consistent with our Universe

The cosmological constant

$$\Lambda \sim O(meV)^4 = 10^{-48}(GeV)^4$$

Baryon-to-photon ratio

$$\eta \equiv \frac{n_B}{n_\gamma} = (6.14 \pm 0.25) \times 10^{-10}$$

Baryon number density

$$n_B = n_\gamma \times \frac{n_B}{n_\gamma} = 10^{-48}(GeV)^3$$

Early models

◊ **L.Susskind, S.Dimpoulos**

Phys.Rev. ***D18****, 1978*

. a non-zero baryon number
. the time dependence of the phase
of a scalar field

◊ **M.Yoshimura** *Phys.Lett.* ***B608****, 2005*

. a small cosmological constant
. the competition of the restoring force
and the analogue of the centrifugal force

In our model,

Baryon number (Susskind & Dimpoulos)
provides the centrifugal force (Yoshimura)

Baryon number generation
is responsible for
the small positive cosmological constant.

Sakharov's conditions for baryogenesis

1. B violation
2. C, CP violation
3. Loss of thermal equilibrium

(1 and 2: built into the model)

(3: → the expanding universe)

All three conditions must be satisfied to generate baryon number.

Susskind, Dimopoulos

Phys.Rev. ***D18***, *1978*

. a complex scalar field $\phi(\chi)$ carring the baryon number

. potential :

α : a complex constant

$$V(\phi) = \lambda\left(\phi\phi^*\right)^n \underline{\left(\phi + \phi^*\right)\left(\alpha\phi^3 + \alpha^*\phi^{*3}\right)}$$

B, C, CP violating

. metric :

$$(ds)^2 = (dt)^2 - R(t)^2(dx)^2$$

. action :

$$S = \int d^4\chi\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - V(\phi)\right]$$

Susskind, Dimopoulos

- $t \longrightarrow \tau = \sqrt{2t}$ conformal time
- $\phi = e^{i\theta(t)}$ ϕ : spatially const.

⬇

1. Lagrangian for the phase θ

$$\mathcal{L} = \left(\frac{d\theta}{d\tau}\right)^2 - \tau^{2n} V(\theta)$$

looks like the Lagrangian of a pendulum

Susskind, Dimopoulos

2. "baryon number" density

The time-component of the baryon current

$$n_B = R(t)^3 \left\{ i\phi\partial_0\phi^* + c.c. \right\}$$

$$= 2\frac{d\theta}{d\tau}$$

$\phi = e^{i\theta(t)}$

n_B is given by the angular velocity of the pendulum.

$\frac{d\theta}{d\tau}$ must be kept

- non-zero
- constant

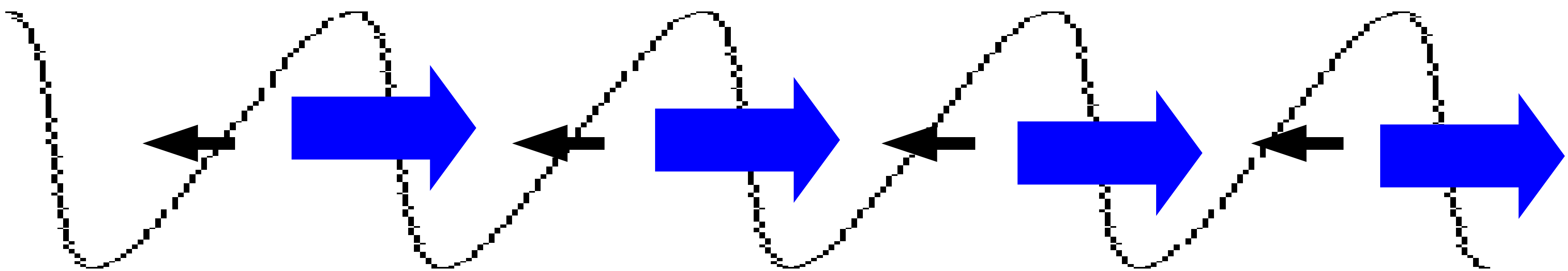
Susskind, Dimopoulos

the angular velocity is kept non-zero

$$V(\phi) = \lambda \left(\phi\phi^*\right)^n \left(\phi + \phi^*\right)\left(\alpha\phi^3 + \alpha^*\phi^{*3}\right)$$

$$V(\theta) = 4\lambda \cos\theta \cos(3\theta + \beta)$$

β: a constant



the potential is not left-right symmetric

the value of θ is accelerated preferentially toward one direction.

Susskind, Dimopoulos

Constant angular velocity

Introduce a damping term

$$\frac{d^2\theta}{d\tau^2} + \tau^{-2n}\frac{dV}{d\theta} + \frac{\lambda^2}{\tau^{4n}}\frac{d\theta}{d\tau} = 0$$

They showed that
the model can generate baryon number

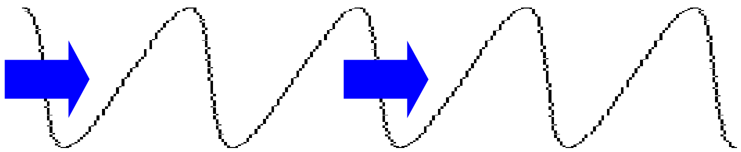
Susskind, Dimopoulos

the Susskind-Dimopoulos model

Baryon number density is
given by the angular velocity of θ

$$n_B = 2\frac{d\theta}{d\tau}$$

In order to generate
a non-zero and constant angular velocity



not left-right symmetric

the damping term

Yoshimura

Phys.Lett. ***B608***, *2005*

$\langle$the energy density of $\underline{\phi_i}$ $\rangle \equiv \Lambda \qquad i = 1,2$

Brans and Dicke type Lagrangian

$$\mathcal{L} = \sqrt{-g}\left[-f(\phi_i)R + \frac{1}{2}(\partial\phi_i)^2 - V\right]$$

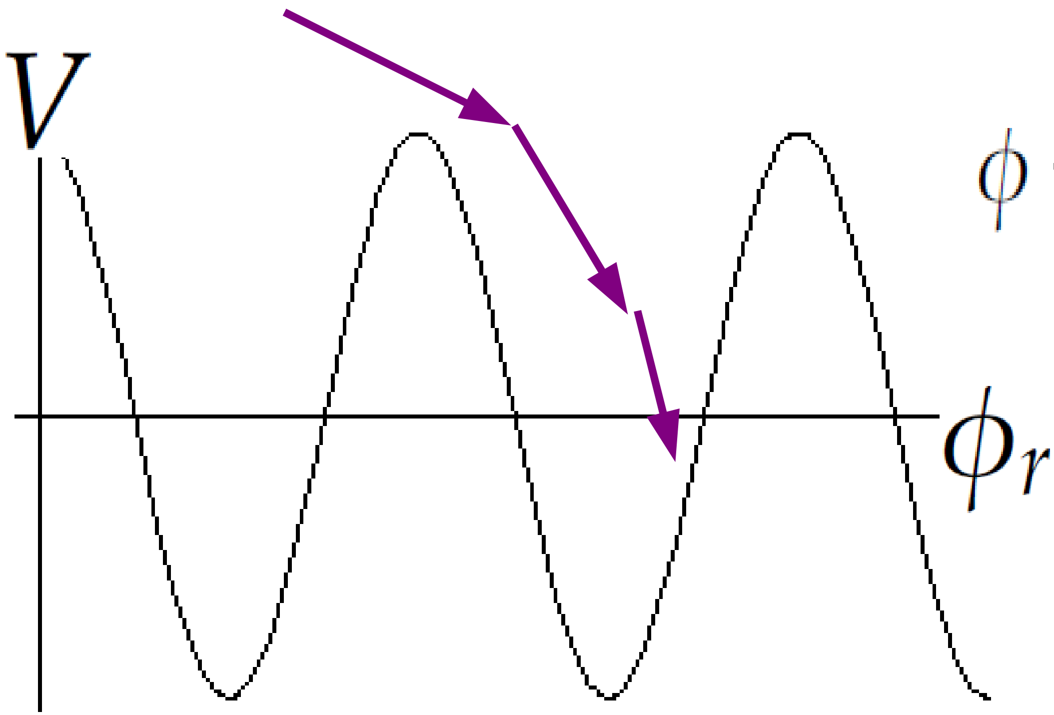
Brans and Dicke theory

$$f(\phi_i) = \epsilon{\phi_i}^2, < f >= \frac{1}{16\pi G_N}$$

Yoshimura's potential

$$V(\phi) = V_0 \cos \frac{\phi_r}{M}$$

$$\begin{cases} \phi_r = \sqrt{\phi_i^2} \\ V_0 \simeq O[M]^4 \\ M \simeq 1TeV \end{cases}$$



ϕ falls and settles into one of the minima

$\rightarrow \phi_r = const$

$\leftrightarrow G_N = const$

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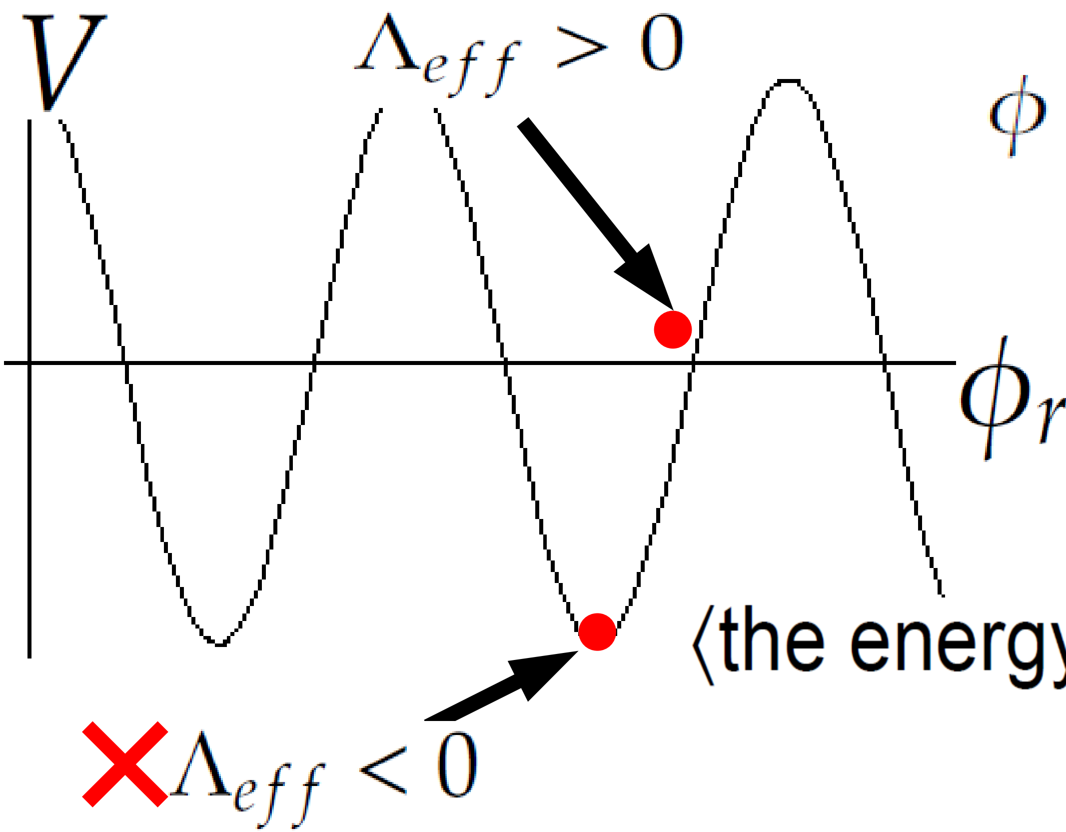


ϕ falls and settles into one of the minima

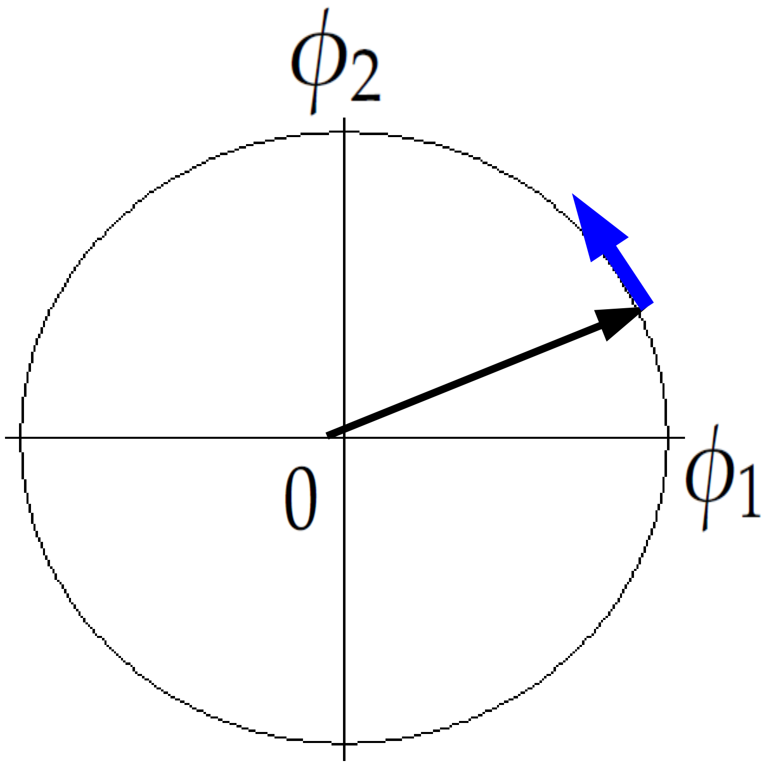
$\rightarrow \phi_r = const$

$\langle$the energy density of $\phi_i \rangle \equiv \Lambda$

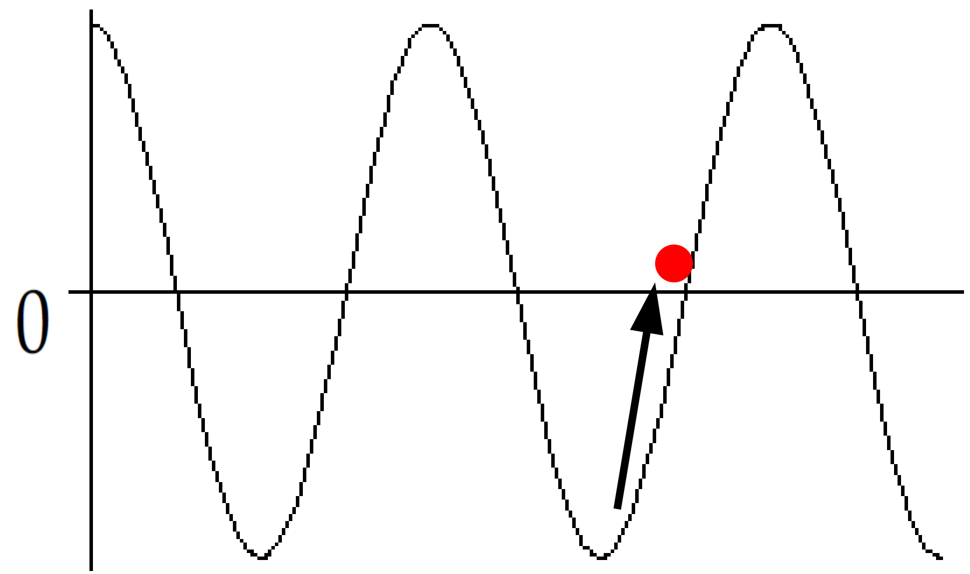
$\sim O(meV)^4$

Yoshimura

Rotationally symmetric in the 2-dimensional complex ϕ space



"angular momentum" which provides a "centrifugal force"



generate a small, positive cosmological constant

the Yoshimura model

$\langle$the energy density of ϕ_i $\rangle \equiv \Lambda$
the cosmological constant.

determined by a competition
between the potential and
the centrifugal barrier.

"angular momentum"

the Yoshimura model

$\langle$the energy density of ϕ_i $\rangle \equiv \Lambda$
the cosmological constant.

determined by a competition between the potential and the centrifugal barrier.

"angular momentum"

$$n_B = 2\frac{d\theta}{d\tau}$$

2.Our Toy-model

The scenario of baryon number generation

baryon number conservation

$$\psi_i \longrightarrow e^{ib_i\alpha}\psi, \quad \bar{\psi}_i \longrightarrow e^{-ib_i\alpha}\bar{\psi}_i$$

conserved baryon-number current

$$\boxed{j_B{}^{\mu} = \sum_i b_i \bar{\psi}_i \gamma^{\mu} \psi_i}$$

where b_i is baryon number of ψ_i

Introduce a complex scalar field ϕ with baryon number q

$$\phi(x) = \phi_r e^{i\theta(x)}$$

"baryon number" conservation

$$\phi \longrightarrow e^{ib_i q}\phi, \quad \phi^* \longrightarrow e^{-ib_i q}\phi^*$$

"baryon current"

$$j_\phi{}^\mu \equiv -2q\phi^* \partial_\mu \phi + h.c. \equiv q\phi_r{}^2 \partial_\mu \theta$$

analogues to the "baryon current" of Susskind-Dimopoulos model

We couple the fermion and scalar sector

$$S = \int d^4\chi \sqrt{-g}\left[g_{\mu\nu}\partial^\mu\phi^*\partial^\nu\phi - V(\phi) + q g_{\mu\nu}\partial^\mu\theta j_B{}^\nu\right]$$

- $q\partial_\mu\theta j_B{}^\mu$: A.G.Cohen, D.B.Kaplan Phys. Lett. B199 (1987)

Familiar interaction in PCAC.
(in 1960's)

Sum of these two currents is conserved

$$\partial_\mu \left(j_B{}^\mu + j_\phi{}^\mu \right) = 0$$

fermimonic — bosonic

$n_B \equiv j_B{}^0$ — $q\phi_r{}^2 \partial_\mu \theta$

$$\Longrightarrow \quad \boxed{n_B + q\phi_r{}^2 \dot{\theta} = 0}$$

The relation of baryon number denity and “angular velocity”

ϕ : spatially const.

The action for our model

$$S = \int d^4\chi \sqrt{-g}\left[g_{\mu\nu}\partial^\mu\phi^*\partial^\nu\phi - V\left(\phi\right) + q g_{\mu\nu}\partial^\mu\theta {j_B}^\nu\right]$$

Robertson-Walker metric

$$(ds)^2 = (dt)^2 - \left(\frac{a(t)}{a_0}\right)^2 (d\vec{x})^2$$

$\phi(t) = \phi_r e^{i\theta(t)}$: spatially const.

$$S = \int d^4\chi \left(\frac{a(t)}{a_0}\right)^3 \left[{\dot{\phi}_r}^2 + \phi_r^2\dot{\theta}^2 - V\left(\phi_r, \theta\right) + q\dot{\theta} n_B\right]$$

The potential for our model

B, C, CP violating

$$V = V_0 \left(1 + \cos\frac{\phi_r}{M}\right) \frac{1}{{\phi_r}^6} \left(\phi\phi^*\right)\left(\phi + \phi^*\right)\left(\alpha\phi^3 + \alpha^*\phi^{*3}\right)$$

Yoshimura

Susskind-Dimopoulos

α: a complex constant

$$\phi(t) = \phi_r e^{i\theta(t)}$$

$$V\left(\phi_r, \theta\right) = V_0\left(1 + \cos\frac{\phi_r}{M}\right) 2\cos\theta \cdot 2\cos\left(3\theta + \beta\right)$$

β : a constant

ϕ_r direction

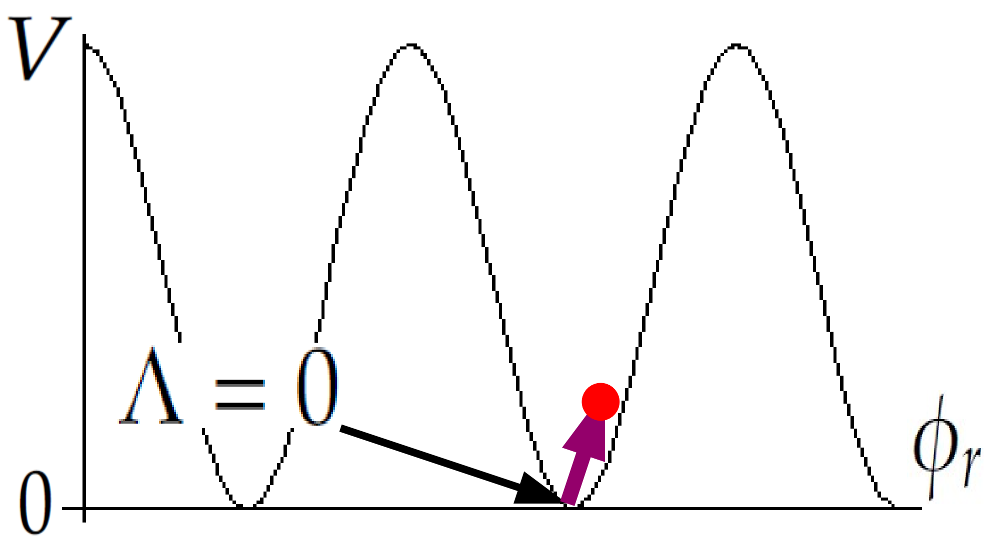
$$V\left(\phi_r,\theta\right)=V_0\left(1+\cos\frac{\phi_r}{M}\right)2\cos\theta\cdot 2\cos\left(3\theta+\beta\right)$$

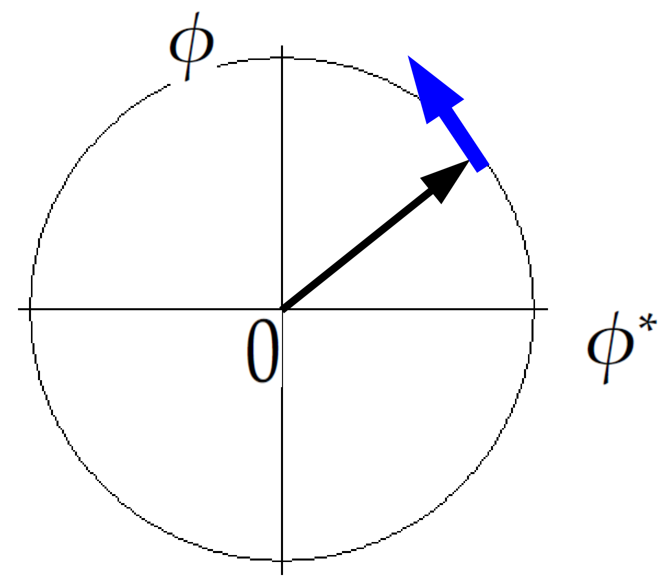
$V_0 \simeq O[M]^4$

$M \simeq 1TeV$

potential minimum=0

We only need
a small “angular momentum”

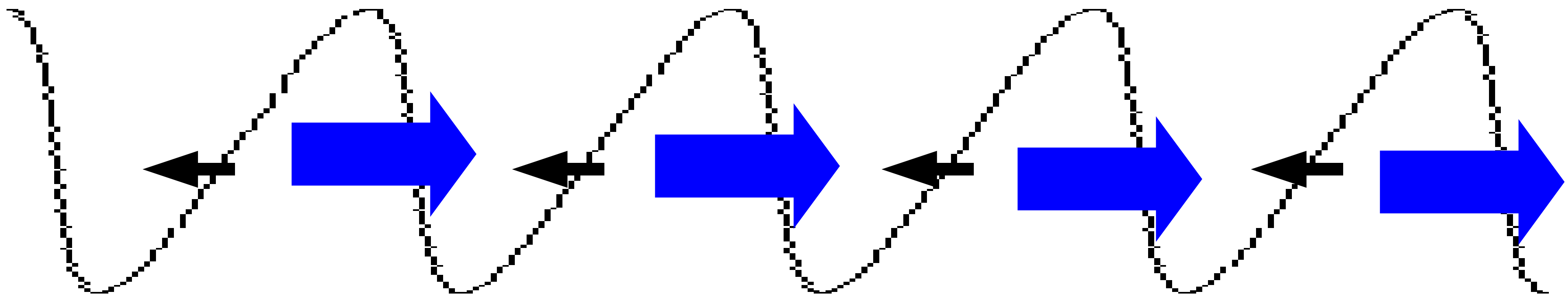




θ direction

$$V(\phi_r, \theta) = V_0 \left(1 + \cos\frac{\phi_r}{M}\right) 2\cos\theta \cdot 2\cos(3\theta + \beta)$$

identical to the Susskind-Dimopoulos model



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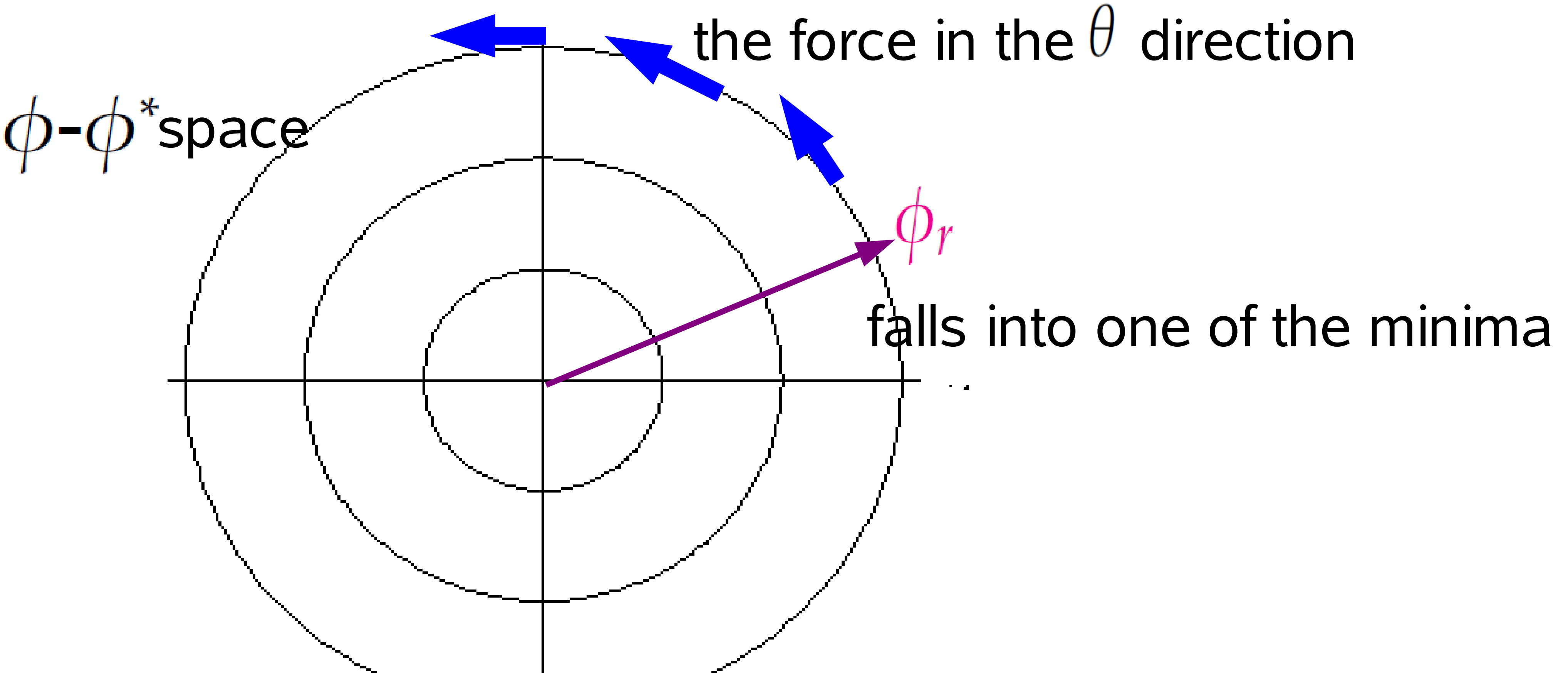
The potential for our model

$$V\left(\phi_r,\theta\right)=V_0\left(1+\cos\frac{\phi_r}{M}\right)2\cos\theta\cdot 2\cos\left(3\theta+\beta\right)$$



looks like a CD

n_B and $\dot{\theta}$

the equation of motion of θ

$$\ddot{\theta} + 3H\dot{\theta} + 2\frac{\dot{\phi}_r}{\phi_r}\dot{\theta} + \frac{1}{\phi_r{}^2}\frac{\partial V}{\partial \theta} = \frac{q}{\phi_r{}^2}(\dot{n}_B + 3Hn_B)$$

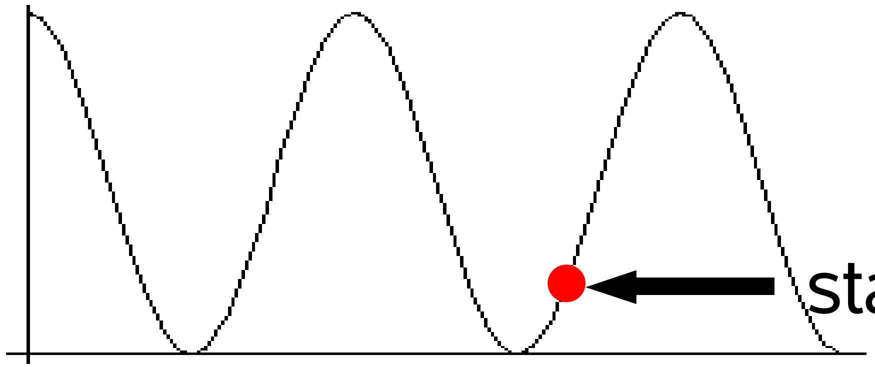
after enough time

average this equation over one period of θ

$$< \dot{\theta} >= -\frac{q}{\phi_r{}^2} < n_B >$$

“angular velocity” baryon number density

Small cosmological constant



centrifugal barrier + potential

$$\mathcal{H} = \left(\frac{a(t)}{a_0}\right)^3 \left\{\frac{1}{2}{\phi_r}^2\dot{\theta}^2 + \frac{1}{2}\dot{\phi_r}^2 + V(\phi_r, \theta)\right\} = const$$

$$\left(\frac{a(t)}{a_0}\right)^3 \left\{\frac{1}{2}\frac{q^2}{<\phi_r>^2} <n_B>^2\right\} = \Lambda_{eff}$$

Estimation

$$\left(\frac{a(t)}{a_0}\right)^3 \left\{\frac{1}{2}\frac{q^2}{<\phi_r>^2} <n_B>^2\right\} = \Lambda_{eff}$$

- the present time $\frac{a(t)}{a_0} = 1$
- $\Lambda_{eff} = (1meV)^4$
- $<n_B> = 10^{-48}(GeV)^3$

$$<\phi_r> \sim 1TeV$$

$$\rightsquigarrow q \sim 10^{-3}$$

3.Summary

- We constructed a model by combining the models of Susskind, Dimopoulos and Yoshimura

- In order to generate small and positive cosmological constant

  ↑

  The model needs "angular momentum"

  ↓

  It also generates baryon number.

- $$\left(\frac{a(t)}{a_0}\right)^3 \left\{\frac{1}{2}\frac{q^2}{<\phi_r>^2}<n_B>^2\right\} = \Lambda_{eff}$$