The decay constants $f_B$ and $f_{D^+}$ from three-flavor lattice QCD

Flavor Physics from Lattice QCD

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Unitarity Triangle

Over constrain $(\bar{\rho}, \bar{\eta})$

\[
\begin{align*}
|V_{ud}V_{ub}^*| & \quad |V_{td}V_{tb}^*| \\
|V_{cd}V_{cb}^*| & \quad |V_{cd}V_{cb}^*|
\end{align*}
\]

(0,0) \quad (1,0)

QCD form factors have been a leading source of uncertainty in many important cases.

Precision Lattice QCD is required
\[ N_f = 2 + 1 \] dynamical quarks

Neglecting vacuum polarization \((n_f = 0\), quenched QCD\) leads to 10-20% uncertainties

The MILC collaboration has made publicly available sets of gluon configurations having three flavors dynamical quarks (google: gauge connection)

- quenching no longer dominant systematic!
- one flavor \(m_{\ell} \approx m_s\), two flavors \(m_s/10 \leq m_{\ell} \leq m_s\)
- numerically less expensive than other methods
- lighter quarks reduce “chiral” extrapolation systematics
- improved! gluon \(O(\alpha_s^2 a^2)\), quarks \(O(\alpha_a a^2)\)
testing three flavor QCD

Do “gold plated” quantities match experiment?

Gold plated in lattice QCD:

- stable particles not near threshold
- decays having at most one stable initial and final state meson

check lights, baryons, heavy-lights and -onia...


Next slide from A. Kronfeld LAT2003
Unquenched QCD

Davies et al., hep-lat/0304004

Heavy Quarks

LQCD/Exp’t ($n_f = 0$) vs LQCD/Exp’t ($n_f = 3$)

- $f_\pi$
- $f_K$
- $3M_{\Xi} - M_N$
- $2M_{B_s} - M_{\Upsilon}$
- $\psi(1P - 1S)$
- $\Upsilon(1D - 1S)$
- $\Upsilon(2P - 1S)$
- $\Upsilon(3S - 1S)$
- $\Upsilon(1P - 1S)$
B_c mass prediction

“In an unprecedented feat of computation, particle theorists made the most precise prediction yet of the mass of the ‘charm-bottom’ particle. Days later, experimentalists dramatically confirmed that prediction.” I. Shipsey, Nature 436 (2005)

AIP Physics News Update: *Most Precise Mass Calculation For Lattice QCD* among The Top Physics Stories for 2005

A Precision test of HQ effective theories on the lattice. Discretization effects for HQ’s are under control.

**“Gold” Modes for CKM Matrix**

Leptonic and semileptonic decays plus mixing

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}|
\pi \to \ell \bar{\nu} & K \to \ell \bar{\nu} & B \to \pi \ell \bar{\nu}
|V_{cd}| & |V_{cs}| & |V_{cb}|
D \to \ell \bar{\nu} & D_s \to \ell \bar{\nu} & B \to D^* \ell \bar{\nu}
D \to \pi \ell \bar{\nu} & D \to K \ell \bar{\nu} & B \to D \ell \bar{\nu}
|V_{td}| & |V_{ts}| & |V_{tb}| \approx 1
B-\bar{B} \text{ mixing:} & B_s-\bar{B}_s \text{ mixing:} & \\
\hat{B}_{B_d} \text{ and } f_B & \hat{B}_{B_s} \text{ and } f_{B_s} & \\
K-\bar{K} \text{ mixing:} & |\epsilon_K| \sim B_K \bar{\eta}(1 - \rho) & \\
\end{pmatrix}
\]
Decay constant $f_{D^+}$ predicted

“It became clear that both groups [CLEO-c and Fermilab Lattice + MILC Collaborations] could have substantial results just in time for the Lepton-Photon Symposium in Uppsala at the end of June. Since both communities felt that it was very important for the LQCD result to be a real prediction, they agreed to embargo both of their results until the conference... The two results agree well within the errors of about 8% for each.” D. Cassel, CERN Courier 45, 6 (2005)

$D$ decays constants are an important test of the lattice techniques needed for $f_B$.

Simulated masses down to $m_q = m_s/10 + \chi_{PT}$.

NLO Staggered $\chi$PT

Squared pseudoscalar meson masses are split

$$M_{ab,\xi}^2 = (m_a + m_b)\mu + a^2\Delta\xi.$$  

The (sixteen) mesons are labeled by their taste representation $\xi = P, A, T, V, I$. $\Delta P = 0$.

NLO $\chi$PT for $\phi_{H_q} \equiv f_{H_q} \sqrt{m_{H_q}}$:

$$\phi_{H_q} = \Phi_H [1 + \Delta f_H (m_q, m_l, m_h) + p_H (m_q, m_l, m_h)]$$  

At finite $a$, taste breaking effects arise in the logarithmic terms $\Delta f_H$ and the analytic terms $p_H$. Effects parameterized by $a^2\Delta\xi$ and additional LECs $a^2\delta'_V$ and $a^2\delta'_A$.

NLO Staggered $\chi$PT

$\phi_{\Omega q}(m_q, m_l, m_h)$

slices (red: $a \neq 0$)

- finite $a$ (taste) effects dilute logarithmic behavior
- QCD “chiral log” recovered when $a \to 0$
- in continuum limit, same LECs as QCD
- $f_{D+}$ and $f_{Ds}$ in limits $m_q, m_l, m_h \to$ physical masses
Simulations

Decay constants are computed for many combinations of $(m_q, m_l)$. The “partially quenched” values correspond to $m_q \neq m_l$.

At each lattice spacing, entire set of results are fit using NLO SχPT.

![Graphs showing decay constants for different lattice spacings](image)
$D$ meson $\chi$ extrapolations

- $a = 0.09$ fm (red) and $a = 0.12$ fm (blue)
- only subset of fitted pts along $m_q = m_l$ visible
- square symbols correspond to $f_{D^+}$ and $f_{D_s}$
$D$ meson decay constants

$f_{D^+}$ is an important check of Staggered $\chi$-PT.

\begin{align*}
f_{D^+} &= 201 \pm 3 \pm 17 \text{ MeV} \\
f_{D_s} &= 249 \pm 3 \pm 16 \text{ MeV} \\
f_{D_s}/f_{D^+} &= 1.24 \pm 0.01 \pm 0.07 \quad \text{hep-lat/0506030}
\end{align*}

bulk of common uncertainties cancel in ratio
HPQCD $f_{B^+}$ and $f_{B_s}$

HPQCD uses the same MILC lattices

NRQCD used to simulate the bottom quark.

FPCP’06: Belle $f_{B^+}$


$$\frac{f_{B_s}}{f_{B^+}} = 1.20 \pm 0.03 \pm 0.01$$

Ratio input for $\Delta M_{B_s}/\Delta M_{B_d}$ constraint
CKM constraints and $f_{B^+}$

Below: constraints from $\Delta M_d$ and Belle $B \to \tau \nu$

Left: with HPQCD $f_B$ and JLQCD $\hat{B}_{Bd} (N_f = 2)$

LATTICE 05 $f_{B_d}^2 B_{Bd}$

EPS05 inputs
Fermilab-MILC B meson results

Preliminary result only at lattice spacing $a = 0.09\,\text{fm}$.
Calculations underway at $a = 0.12$ and $0.15\,\text{fm}$.

$$f_{B_s}/f_{B^+} = 1.27 \pm 0.02 \pm 0.06$$
Decay constant ratios

Preliminary ratios of decay constants at a lattice spacing $a = 0.09$ fm.

\[
\begin{align*}
    f_{D_s}/f_{D^+} &= 1.21 \pm 0.01 \pm 0.04 \\
    f_{B_s}/f_{D_s} &= 0.99 \pm 0.02 \pm 0.06 \\
    f_{B^+}/f_{D^+} &= 0.95 \pm 0.03 \pm 0.06 \\
    R = (f_{B_s}/f_{B^+})/(f_{D_s}/f_{D^+}) &= 1.04 \pm 0.01 \pm 0.02
\end{align*}
\]

$R - 1$ is a measure of both $SU(3)$ and HQ flavor symm. breaking. Result above indicates contributions from analytic terms are larger than just the $\chi$-log contributions, which were estimated to be $R - 1 = -3.3\%$, [B. Grinstein, hep-ph/9308226].
More CKM physics

Lattice QCD is capable of providing form factors needed in CKM studies.


- $B \rightarrow D^* \ell \nu$: eliminate quenching error and reduce $\chi$-extrap. uncertainty in $h_{A_1}(1)$

- $B \rightarrow \pi \ell \nu$: HQS and unitarity constraints applied to lattice results

- HQET matrix elements $\bar{\Lambda}$ and $\lambda_1$: appear in HQET expansion for inclusive $B$ decay rates.

- $B - \bar{B}$ matrix elements from MILC lattices

- $B_K$: Mixed staggered (sea) domain wall (valence) action simplifies $\chi$-P.Th