The decay constants f_B and f_{D^+} from three-flavor lattice QCD

Flavor Physics from Lattice QCD

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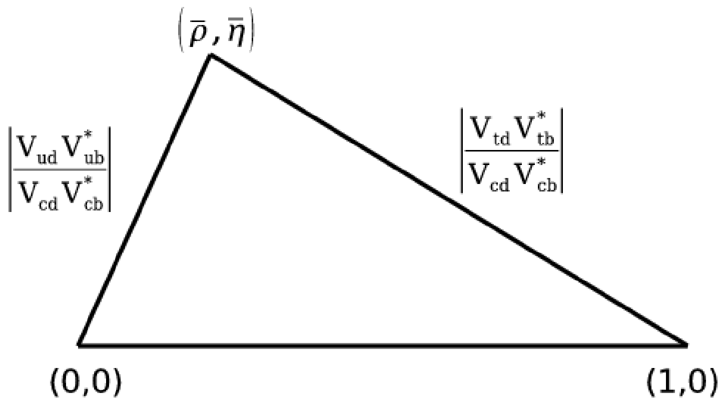
Unitarity Triangle

Over constrain $(\bar{\rho}, \bar{\eta})$



QCD form factors have been a leading source of uncertainty in many important cases.

Precision Lattice QCD is required

— $\mathrm{N_f} = 2 + 1$ dynamical quarks —

Neglecting vaccuum polarization ($\mathrm{n_f} = 0$, quenched QCD) leads to 10-20% uncertainties

The MILC collaboration has made publicly available sets of gluon configurations having three flavors dynamical quarks (google: gauge connection)

- **quenching no longer dominant systematic!**
- one flavor $m_h \approx m_s$, two flavors $m_s/10 \leq m_l \leq m_s$
- numerically less expensive than other methods
- **lighter quarks reduce "chiral" extrapolation systematics**
- **improved!** gluon $\mathcal{O}\left(\alpha_s^2 a^2\right)$, quarks $\mathcal{O}\left(\alpha_s a^2\right)$

testing three flavor QCD

Do "gold plated" quantities match experiment?

Gold plated in lattice QCD:

- stable particles not near threshold
- decays having at most on stable initial and final state meson

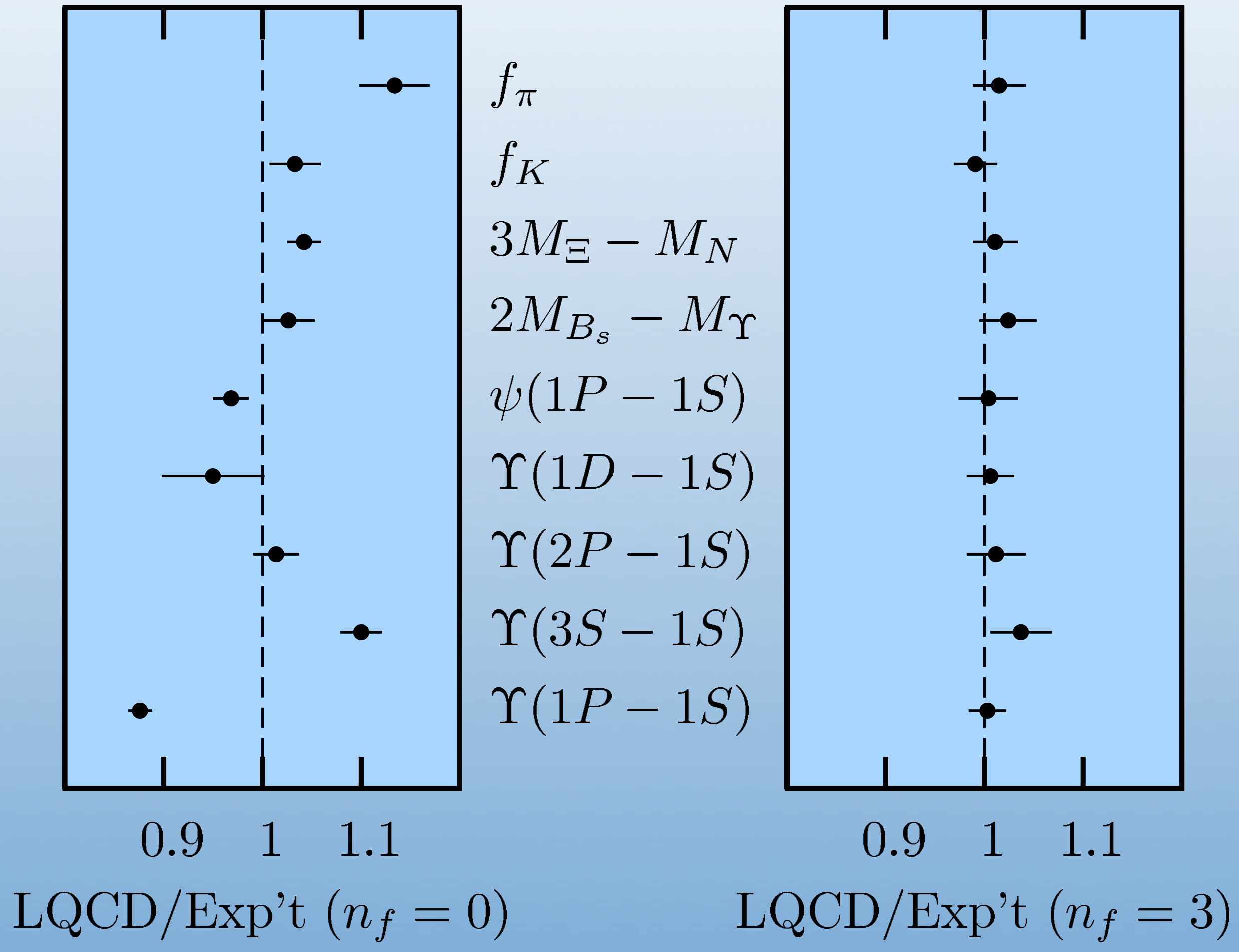
check lights, baryons, heavy-lights and -onia. . .

Davies *et al.*, Phys. Rev. Lett. **92**, 022001 (2004)

Next slide from A. Kronfeld LAT2003

Unquenched QCD

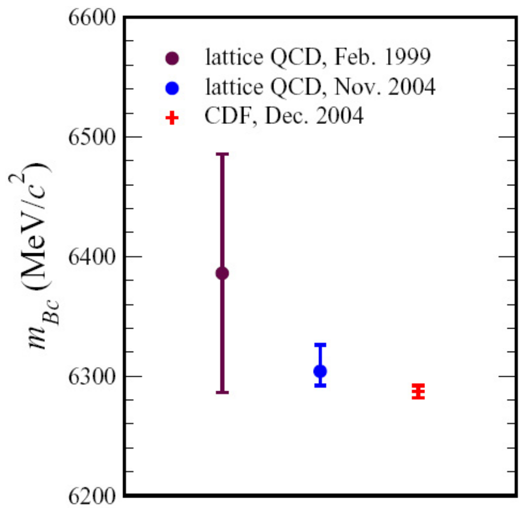
Davies et al., hep-lat/0304004



B_c mass prediction

"In an unprecedented feat of computation, particle theorists made the most precise prediction yet of the mass of the 'charm-bottom' particle. Days later, experimentalists dramatically confirmed that prediction." I. Shipsey, Nature **436** (2005)

AIP Physics News Update: *Most Precise Mass Calculation For Lattice QCD* among **The Top Physics Stories for 2005**



I. Allison, *et al.*, Phys. Rev. Lett. **94** (2005)

A Precision test of HQ effective theories on the lattice. Discretization effects for HQ's are under control.

"Gold" Modes for CKM Matrix

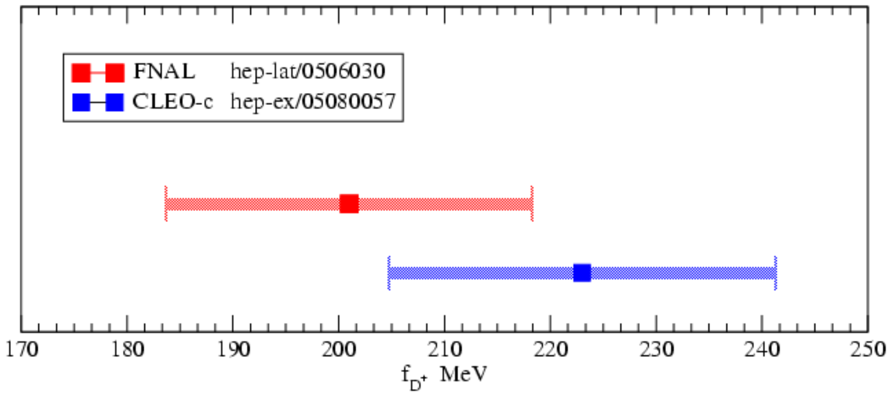
leptonic and semileptonic decays plus mixing

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ \pi \to \ell\bar{\nu} & K \to \ell\bar{\nu} & B \to \pi\ell\bar{\nu} \\ & K \to \pi\ell\bar{\nu} & B \to \ell\bar{\nu} \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ D \to \ell\bar{\nu} & D_s \to \ell\bar{\nu} & B \to D^*\ell\bar{\nu} \\ D \to \pi\ell\bar{\nu} & D \to K\ell\bar{\nu} & B \to D\ell\bar{\nu} \\ |V_{td}| & |V_{ts}| & |V_{tb}| \approx 1 \\ B\text{-}\bar{B} \text{ mixing:} & B_s\text{-}\bar{B}_s \text{ mixing:} & \\ \hat{B}_{B_d} \text{ and } f_B & \hat{B}_{B_s} \text{ and } f_{B_s} & \end{pmatrix}$$

K-$\bar{K}$ mixing: $|\epsilon_K| \sim B_K \bar{\eta}(1 - \bar{\rho})$

— Decay constant f_{D^+} predicted —

"It became clear that both groups [CLEO-c and Fermilab Lattice + MILC Collaborations] could have substantial results just in time for the Lepton-Photon Symposium in Uppsala at the end of June. Since both communities felt that it was very important for the LQCD result to be a real prediction, they agreed to embargo both of their results until the conference. . . The two results agree well within the errors of about 8% for each." D. Cassel, CERN Courier **45**, 6 (2005)



D decays constants are an important test of the lattice techniques needed for f_B.

Simulated masses down to $m_q = m_s/10 + \chi$PT.

Aubin *et al.*, Phys. Rev. Lett. **95** (2005) 122002

NLO Staggered χPT

Squared pseudoscalar meson masses are split

$$M^2_{ab,\xi} = (m_a + m_b)\mu + a^2\Delta_\xi \ .$$

The (sixteen) mesons are labeled by their taste representation $\xi = P, A, T, V, I$. $\Delta_P = 0$.

NLO χPT for $\phi_{H_q} \equiv f_{H_q}\sqrt{m_{H_q}}$:

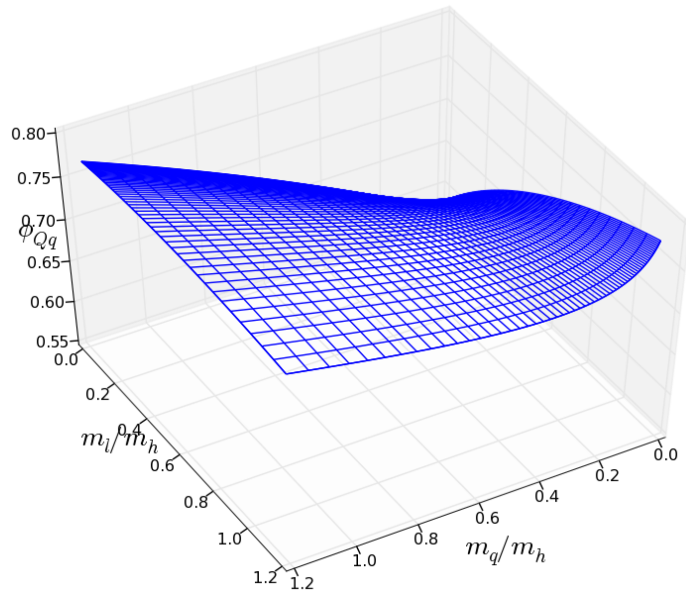
$$\phi_{H_q} = \Phi_H \left[1 + \Delta f_H(m_q, m_l, m_h) + p_H(m_q, m_l, m_h)\right]$$

At finite a, taste breaking effects arise in the logarithmic terms Δf_H and the analytic terms p_H. Effects parameterized by $a^2\Delta_\xi$ and additional LECs $a^2\delta'_V$ and $a^2\delta'_A$.

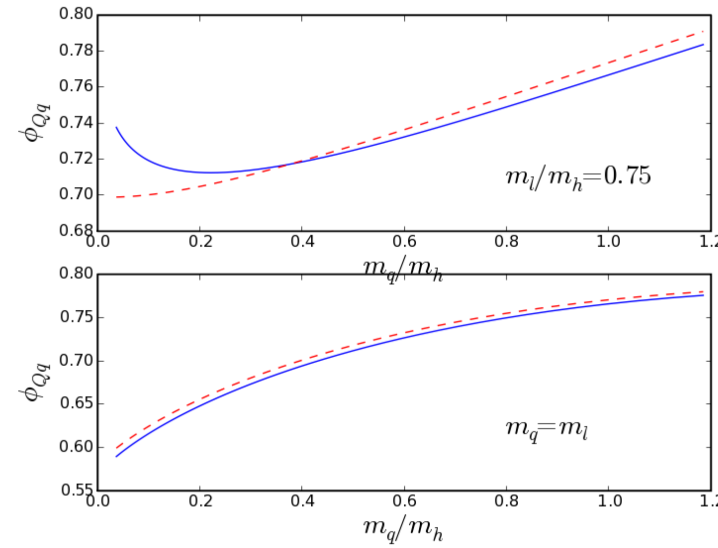
Aubin *et al.*, Phys. Rev. D. **70** (2004) 094505

NLO Staggered χPT

$\phi_{Qq}(m_q, m_l, m_h)$



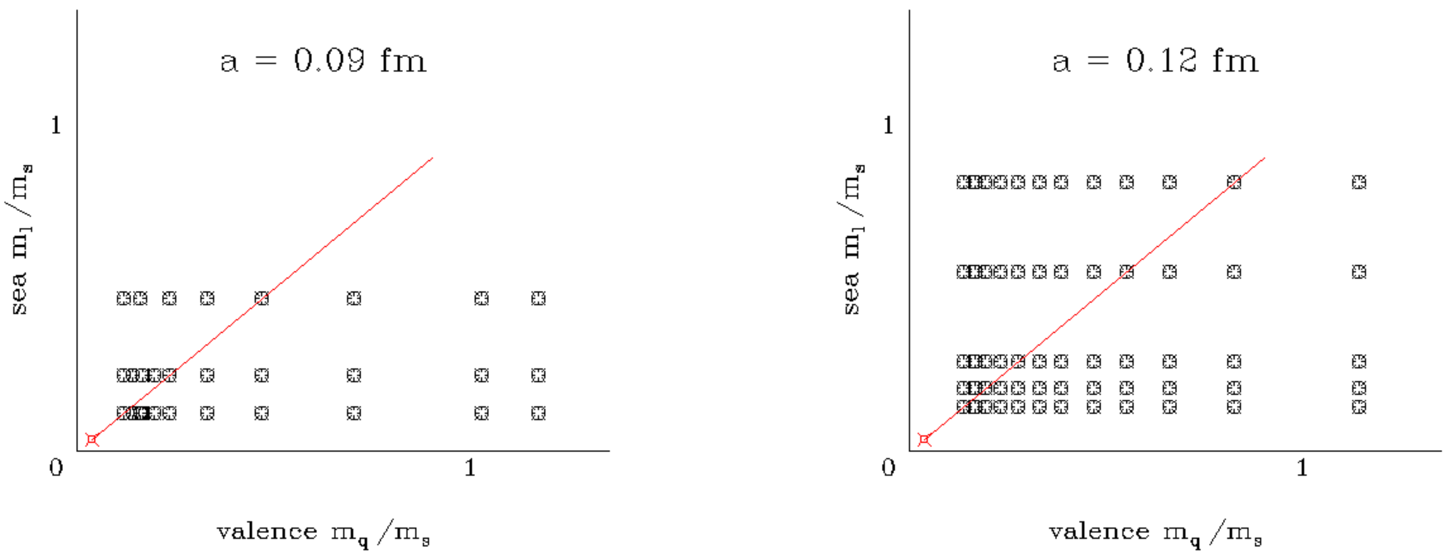
slices (red: $a \neq 0$)



- finite a (taste) effects dilute logarithmic behavior
- QCD "chiral log" recovered when $a \to 0$
- in continuum limit, same LECs as QCD
- f_{D^+} and f_{D_s} in limits $m_q, m_l, m_h \to$ physical masses

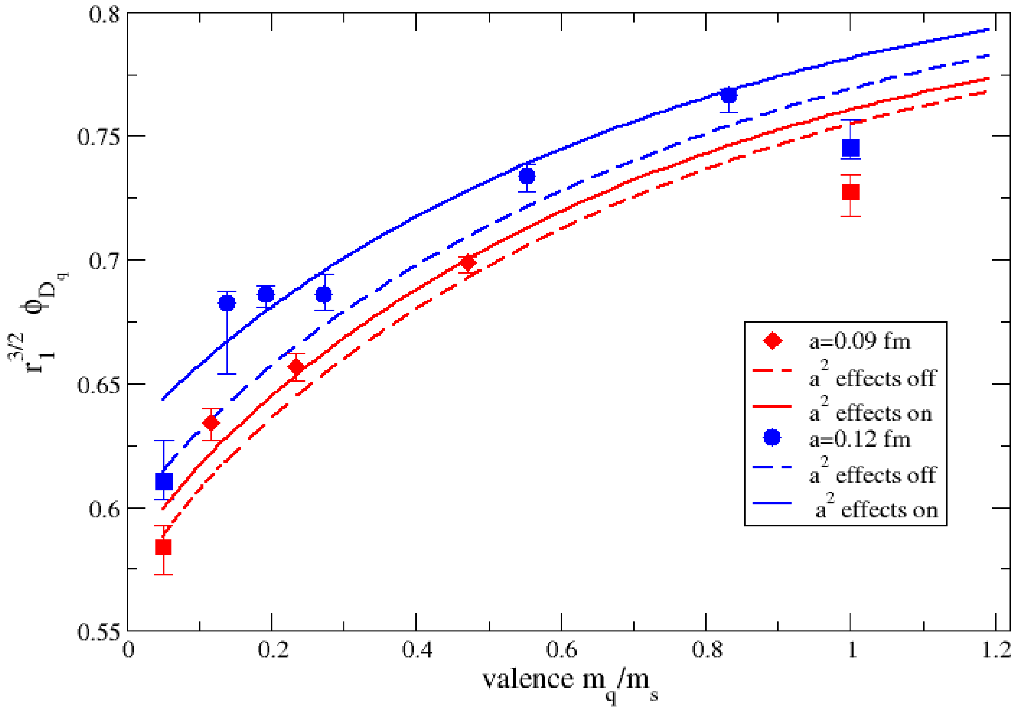
Simulations

Decay constants are computed for many combinations of (m_q, m_l). The "partially quenched" values correspond to $m_q \neq m_l$.



At each lattice spacing, entire set of results are fit using NLO SχPT.

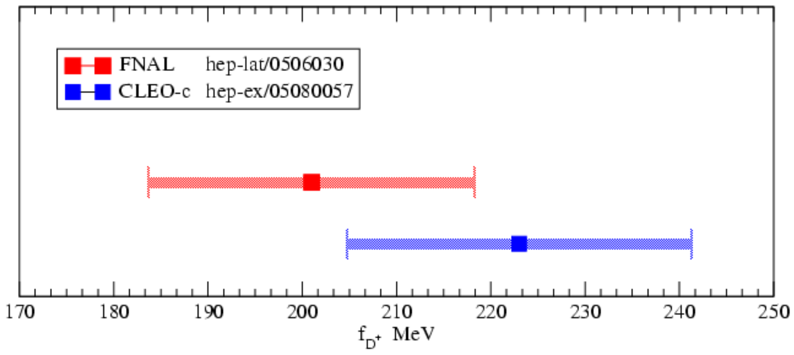
D meson χ extrapolations

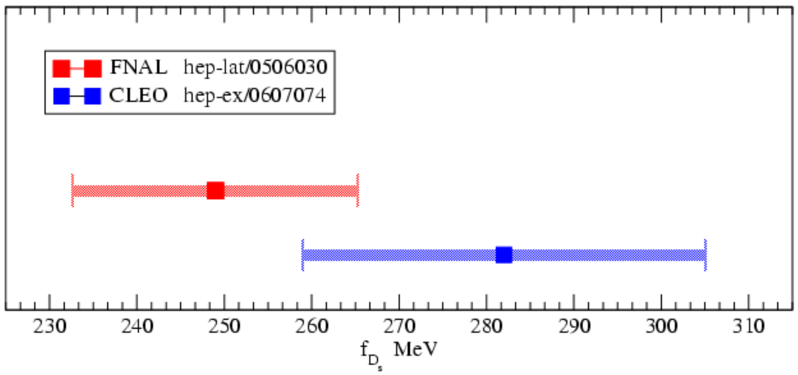


- $a = 0.09\,\mathrm{fm}$ (red) and $a = 0.12\,\mathrm{fm}$ (blue)
- only subset of fitted pts along $m_q = m_l$ visible
- square symbols correspond to f_{D+} and f_{D_s}

D meson decay constants

f_{D^+} is an important check of Staggered χ-PTh.





$$f_{D^+} = 201 \pm 3 \pm 17 \text{ MeV}$$

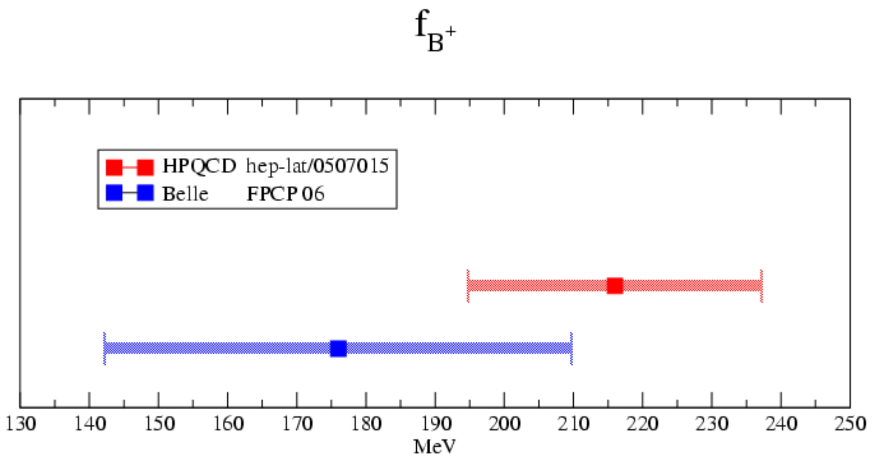
$$f_{D_s} = 249 \pm 3 \pm 16 \text{ MeV}$$

$$f_{D_s}/f_{D^+} = 1.24 \pm 0.01 \pm 0.07 \qquad \text{hep-lat/0506030}$$

bulk of common uncertainties cancel in ratio

HPQCD f_{B^+} and f_{B_s}

HPQCD uses the same MILC lattices



NRQCD used to simulate the bottom quark.

FPCP'06: Belle f_{B+}

Gray, *et al.*, Phys. Rev. Lett. **95** (2005) 2001

$$\frac{f_{B_s}}{f_{B+}} = 1.20 \pm 0.03 \pm 0.01$$

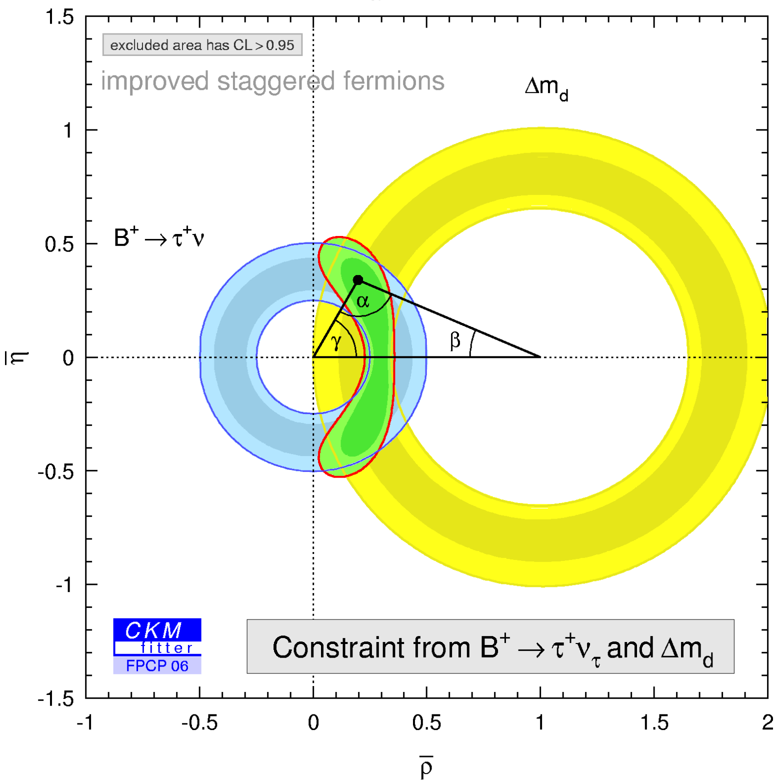
Ratio input for $\Delta M_{B_s}/\Delta M_{B_d}$ constraint

CKM constraints and f_{B^+}

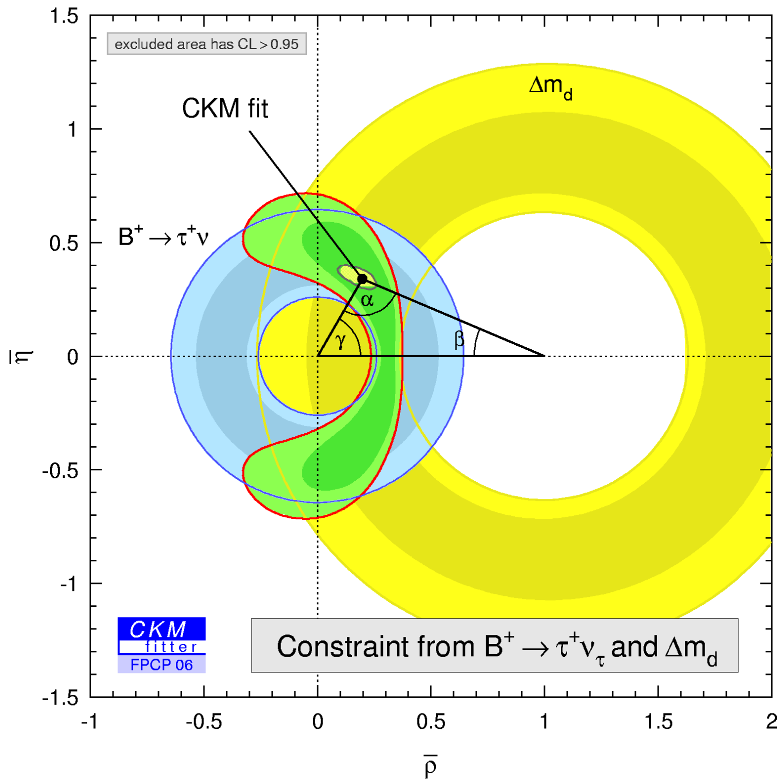
Below: constraints from ΔM_d and Belle $B \to \tau\nu$

Left: with HPQCD f_B and JLQCD $\hat{B}_{B_d}$ $(N_f = 2)$

LATTICE 05 $f^2_{B_d} B_{B_d}$



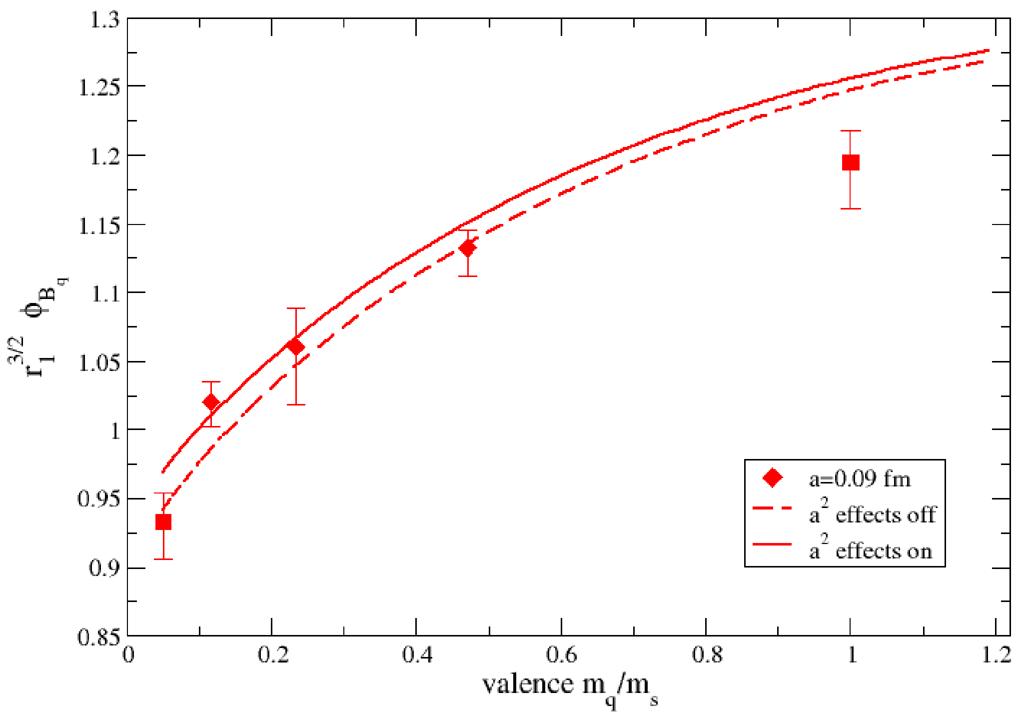
EPS05 inputs



Fermilab-MILC B meson results

Preliminary result only at lattice spacing $a = 0.09$ fm.

Calculations underway at $a = 0.12$ and 0.15 fm.



$$f_{B_s}/f_{B^+} = 1.27 \pm 0.02 \pm 0.06$$

Decay constant ratios

Preliminary ratios of decay constants at a lattice spacing $a = 0.09$ fm.

$$
\begin{aligned}
f_{D_s}/f_{D^+} &= 1.21 \pm 0.01 \pm 0.04 \\
f_{B_s}/f_{D_s} &= 0.99 \pm 0.02 \pm 0.06 \\
f_{B^+}/f_{D^+} &= 0.95 \pm 0.03 \pm 0.06 \\
R = (f_{B_s}/f_{B^+})/(f_{D_s}/f_{D^+}) &= 1.04 \pm 0.01 \pm 0.02
\end{aligned}
$$

$R - 1$ is a measure of both $SU(3)$ and HQ flavor symm. breaking. Result above indicates contributions from analytic terms are larger than just the χ-log contributions, which were estimated to be $R - 1 = -3.3\%$, [B. Grinstein, hep-ph/9308226].

More CKM physics

Lattice QCD is capable of providing form factors needed in CKM studies.

Reported at LATTICE 2006 to appear in PoS LAT06 (2006)

- $B \to D^* \ell \nu$: eliminate quenching error and reduce χ-extrap. uncertainty in $h_{A_1}(1)$
- $B \to \pi \ell \nu$: HQS and unitarity constraints applied to lattice results
- HQET matrix elements $\bar{\Lambda}$ and λ_1: appear in HQET expansion for inclusive B decay rates.
- B-$\bar{B}$ matrix elements from MILC lattices
- B_K: Mixed staggered (sea) domain wall (valence) action simplifies χ-P.Th