Recent LQCD results with improved staggered fermions from the MILC Collaboration

Ludmila Levkova
University of Utah

[DPF 2006]

Outline

- The QCD equation of state (EOS)
  - Properties of QGP from the experiment. The significance of the equation of state.
  - Nonzero temperature QCD on the lattice.
  - Integral method for the calculation of the EOS on the lattice.
  - The EOS results
  - Conclusions
- Quarkonium spectrum
  - Quarkonuim spectrum on the lattice
  - Charmonium spectrum results
  - Bottomonium spectrum results
  - Conclusions

▶ THE QCD EQUATION OF STATE (EOS)

MILC Collaboration: *C. Bernard, T. Burch, C. DeTar, Steven Gottlieb, U. Heller, L. Levkova, J. Osborn, D. Renner, R. Sugar, D. Toussaint*

QCD Matter at Extreme Conditions

- QCD – the theory of the strong interactions, as a consequence of the nonperturbative structure of the vacuum has the properties of quark confinement and dynamical chiral symmetry breaking.
- At high temperatures and/or densities the hadron constituents – quarks and gluons – are expected to be deconfined and the chiral symmetry restored. The new phase of nuclear matter is called "quark-gluon plasma" (QGP).
- Where to find QGP:
  - Early Universe
  - Early stages of supernova explosions
  - Neutron stars interior
  - Physics experiments - heavy-ion collisions (RHIC, CERN, etc.)

QGP at Experimental Conditions

- QGP's nonperturbative character at $T \approx T_c$:
  - Dimensional arguments estimate $\varepsilon_c \approx 1\,\mathrm{GeV/fm}^3$ and $T_c \approx 175\,\mathrm{MeV}$. (Density at total overlap of several light hadrons within typical hadron volume of 1-3 fm^3.)
  - $T_c/\Lambda_{QCD} \approx 0.5$, which means that at experimentally accessible temperatures $T/T_c = 1-3$ the system is still in a QCD non-perturbative regime

    $$g \equiv \sqrt{4\pi\alpha_s} = O(1).$$

    QGP $\rightarrow$ sQGP. Evidence for strong interactions.
  - The most adequate tool to study sQGP is a nonperturbative one – Lattice QCD. Perturbation theory is only a rough guide.

The significance of the EOS of QGP

- In heavy-ion collisions after thermalization the system evolves hydrodynamically and its behavior will depend on the EOS ($\varepsilon(T)$ and $p(T)$).

- The hydrodynamical models that include a QGP phase and a resonance gas for the hadronic phase connected by a first order phase transition all assume an ideal gas EOS for the QGP phase. They reproduce the low p_T proton elliptic flow.

- However still there is no consistent picture that describes the heavy-ion collisions at RHIC. A more realistic EOS from lattice calculations as an input to the hydrodynamic models is an obvious direction for comparison with data.

Nonzero Temperature Lattice QCD

The quantum statistical Gibbs ensemble partition function $Z(T)$ at temperature T and the Euclidean path integral formulation of QFT are related by

$$Z(T) = \mathrm{Tr}\, \mathrm{e}^{-\mathrm{H}/\mathrm{T}} = \int \prod_{\mathrm{x}} \mathrm{d}\phi(\mathrm{x})\, \mathrm{e}^{-\mathrm{S}_{\mathrm{E}}(\phi,\mathrm{T})},$$

where $S_E(\phi, T)$ is the classical action at imaginary time

$$t = -i/T,$$

for a field configuration $\phi(x)$ on a space-time lattice of dimensions $N^3 \times N_t$. The lattice temporal extent and temperature are related through

$$T = 1/(a_t N_t).$$

On the lattice:

$$\mathbf{S_E}(\mathbf{U}, \mathbf{\Psi}, \overline{\mathbf{\Psi}}) = \mathbf{S_G}(\mathbf{U}) + \underbrace{\mathbf{S_F}(\mathbf{U}, \mathbf{\Psi}, \overline{\mathbf{\Psi}})}_{\overline{\mathbf{\Psi}}\mathbf{M}\mathbf{\Psi}}.$$

The expectation value of an observable $\mathbf{O}(\mathbf{U}, \mathbf{\Psi}, \overline{\mathbf{\Psi}})$ is given by

$$\langle \mathbf{O} \rangle = \frac{\mathbf{1}}{\mathbf{Z}} \int [\mathbf{dU}][\mathbf{d\Psi}][\mathbf{d}\overline{\mathbf{\Psi}}]\mathbf{O}(\mathbf{U}, \mathbf{\Psi}, \overline{\mathbf{\Psi}})\mathbf{e}^{-\mathbf{S_E}(\mathbf{U}, \mathbf{\Psi}, \overline{\mathbf{\Psi}})} = \frac{\mathbf{1}}{\mathbf{Z}} \int [\mathbf{dU}]\mathbf{O}(\mathbf{U})\det(\mathbf{M})\, \mathbf{e}^{-\mathbf{S_G}(\mathbf{U})}.$$

Lattice actions

- Gauge action: 1-loop improved Symanzik action. Discretization errors – $O(\alpha_s^2 a^2, a^4)$.

$$\mathbf{S_G} = \beta \sum_{\mathbf{x},\mu<\nu} (\mathbf{1} - \mathbf{P}_{\mu\nu}) + \beta_{\rm rt} \sum_{\mathbf{x},\mu<\nu} (\mathbf{1} - \mathbf{R}_{\mu\nu}) + \beta_{\rm ch} \sum_{\mathbf{x},\mu<\nu<\sigma} (\mathbf{1} - \mathbf{C}_{\mu\nu\sigma}),$$

- Fermion action: Asqtad staggered quark action – tree level improved, taste violations suppressed. Discretization errors – $O(\alpha_s^2 a, a^4)$.

$$\begin{aligned} S_F &= \overline{\Psi} M \Psi \\ M &= 2m_f + \underbrace{\sum_i c_i (V_i - V_i^\dagger)}_{\text{fat link}} + \underbrace{w(L - L^\dagger)}_{\text{Lepage term}} + \underbrace{v(N - N^\dagger)}_{\text{Naik term}} \end{aligned}$$

- Simulation algorithm: Hybrid Molecular Dynamics R algorithm.

The EOS on the Lattice using the Integral Method

Start from the thermodynamic identities:

$$\varepsilon V = -\left.\frac{\partial \ln Z}{\partial(1/T)}\right|_V, \qquad \frac{p}{T} = \left.\frac{\partial \ln Z}{\partial V}\right|_T \approx \frac{\ln Z}{V}, \qquad I = \varepsilon - 3p = -\frac{T}{V}\frac{d \ln Z}{d \ln a},$$

where $V = N_s^3 a^3$, $T = \frac{1}{N_t a}$. The partition function is

$$Z = \int dU \exp\left\{-S_g + \sum_f (n_f/4) \mathrm{Tr} \ln[M(am_f, U, u_0)]\right\}.$$

with $M(am_f, U, u_0)$ the fermion matrix corresponding to the Asqtad quark action with 2 degenerate light quark flavors and 1 heavy quark flavor.

Thus:

$$\begin{aligned} Ia^4 &= -6\frac{d\beta_{\mathrm{pl}}}{d\ln a}\Delta\langle P\rangle - 12\frac{d\beta_{\mathrm{rt}}}{d\ln a}\Delta\langle R\rangle - 16\frac{d\beta_{\mathrm{ch}}}{d\ln a}\Delta\langle C\rangle \\ &\quad -\sum_f \frac{n_f}{4}\left[\frac{d(m_f a)}{d\ln a}\Delta\left\langle\bar\psi\psi\right\rangle_f + \frac{du_0}{d\ln a}\Delta\left\langle\bar\psi\frac{dM}{du_0}\psi\right\rangle_f\right]. \end{aligned}$$

The EOS on the Lattice using the Integral Method

$$pa^4 = \int_{\ln a_0}^{\ln a} (-Ia^4) d\ln a'$$

where $\ln a_0$ is determined by where (the zero-temperature corrected) $Ia^4 = 0$ at coarse lattice spacings.

The energy density is given by:

$$\varepsilon a^4 = (I + 3p)a^4$$

Observables to calculate: all gauge loops plus the fermion quantities in the zero- and nonzero-temperature phases

$$\begin{aligned}
\left\langle \bar{\psi}\psi \right\rangle_f &= \left\langle 2aM^{-1} \right\rangle_f \\
\left\langle \bar{\psi}\,\frac{dM}{du_0}\,\psi \right\rangle_f &= \left\langle \frac{dM}{du_0} M^{-1} \right\rangle_f .
\end{aligned}$$

Choosing the Action Parameters

- Action parameters to choose: β, m_s, m_{ud} and u_0. Changing the parameters changes the lattice scale a and the physics on the lattice.
- Simulations at different parameters and scales represent the same physics if:
  - $m_{\eta_{ss}}/m_\phi = \mathrm{const}$ - fixes the heavy quark mass
  - $m_\pi/m_\rho = \mathrm{const}$ - fixes the light quark mass
- We want a quark-gluon system for which we change the temperature ($T = 1/(aN_t)$) without changing the physics. We have to choose the parameters of the action in a way that lets us stay on a chosen constant physics trajectory at zero temperature. We approximate two such trajectories:
  - $m_{ud} \approx 0.2 m_s$, ($m_\pi/m_\rho \approx 0.4$)
  - $m_{ud} \approx 0.1 m_s$, ($m_\pi/m_\rho \approx 0.3$)

  Both trajectories have m_s tuned to the physical strange quark mass within 20 %.

Parameterizing the Constant Physics Trajectories

- Construction of a constant physics trajectory:
  - At anchor points in β, tune m_π/m_ρ and m_η/m_ϕ.
  - Between anchor points the trajectory is interpolated, using a one-loop RG inspired formula.
- The $m_{ud} = 0.2 m_s$ trajectory – 3 anchor points $\beta = 6.467$, 6.76, and 7.092:

$$am_s = \begin{cases} 0.082 \;\exp\left((\beta - 6.4674)\frac{\ln(0.050/0.0820)}{(6.76-6.4674)}\right), & \beta \in [6.467, 6.76] \\ 0.05 \;\exp\left((\beta - 6.76)\frac{\ln(0.031/0.05)}{(7.092-6.76)}\right), & \beta \in [6.76, 7.092] \end{cases}$$

$$am_{ud} = \begin{cases} 0.01675 \;\exp\left((\beta - 6.4674)\frac{\ln(0.010/0.01675)}{(6.76-6.4674)}\right), & \beta \in [6.467, 6.76] \\ 0.01 \;\exp\left((\beta - 6.76)\frac{\ln(0.00673/0.01)}{(7.092-6.76)}\right), & \beta \in [6.76, 7.092]. \end{cases}$$

Parameterizing the Constant Physics Trajectories

- The $m_{ud} = 0.1 m_s$ trajectory – 2 anchor points $\beta \in [6.458, 6.76]$:

$$am_s = 0.05 \exp\left((\beta - 6.76) \frac{\ln(0.082/0.05)}{(6.458 - 6.76)} \right)$$

$$am_{ud} = 0.005 \exp\left((\beta - 6.76) \frac{\ln(0.0082/0.005)}{(6.458 - 6.76)} \right).$$

- For both trajectories, for values of β out of the above intervals, the formulas are used as extrapolations appropriately.

Determination of the Lattice Spacing

- The lattice spacing a can be calculated from $1S - 2S$ Υ splittings
$$a = (a\Delta E)_{\rm lat}/\Delta E_{\rm exp}$$

- Measurements from about 30 zero temperature ensembles are fitted to
$$\frac{a}{r_1} = c_0 f(g^2) + c_2 g^2 f^3(g^2) + c_4 g^4 f^3(g^2),$$
where $r_1 = 0.317(7)(3)$ fm. The definition of
$$f(g^2) = (b_0 g^2)^{-b_1/(2b_0^2)} e^{-1/(2b_0 g^2)}$$
involves the universal beta-function coefficients for massless three-flavor QCD, b_0 and b_1. The coefficients c_0, c_2 and c_4 are
$$\begin{aligned} c0 &= c_{00} + c_{01}(2m_{ud} + m_s)/f(g^2) + c_{02}(2m_{ud} + m_s)^2/f^2(g^2) \\ c2 &= c_{20} + c_{21}(2m_{ud} + m_s)/f(g^2) \\ c4 &= c_{40}, \end{aligned}$$
where $c_{00} = 46.1(4)$, $c_{01} = 0.24(6)$, $c_{02} = -0.003(2)$, $c_{20} = -3.5(2) \times 10^5$, $c_{21} = 2.5(4) \times 10^3$ and $c_{40} = 2.7(1) \times 10^5$. The fit has $\chi^2/DOF \approx 1.3$ and a CL 0.14.

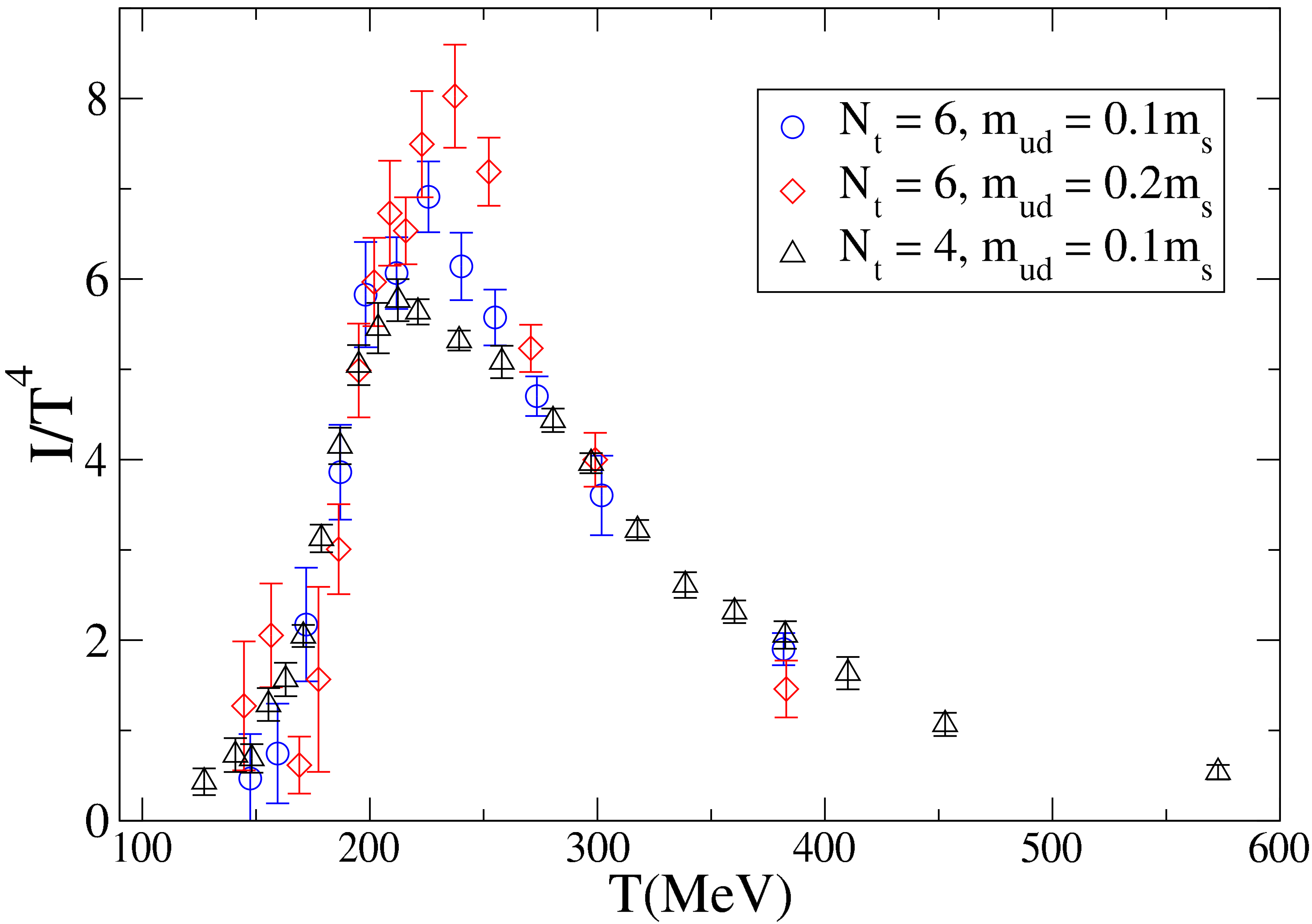
Simulations Overview

- We simulate 2+1 flavor QCD with $m_{ud} = 0.1 m_s$ and $0.2 m_s$ along trajectories of constant physics using improved gauge and quark actions. Our system is at thermal equilibrium and zero chemical potential.

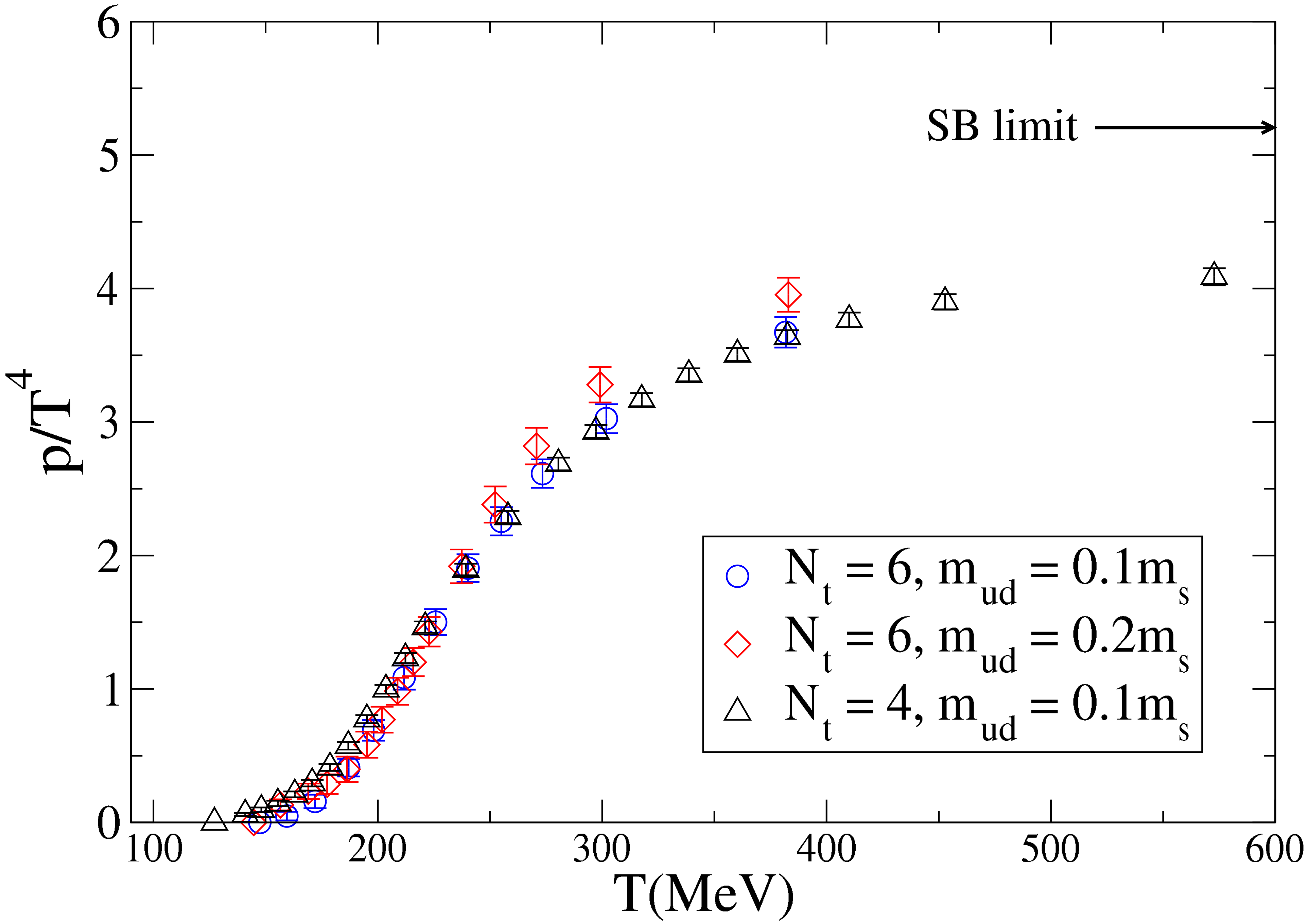
- Simulation algorithm – the inexact dynamical R-algorithm. Step-size of the equations of motion integration is the min of $2/(3m_{ud})$ and 0.02, in some cases even smaller. Estimated step-size errors are up to the size of the statistical errors.

- Temperature $1/(aN_t)$ is changed by varying a (0.09 – 0.39 fm) along the trajectory and keeping $N_t = \mathrm{const}$. We work with $N_t = 4$ and 6. These cases are interesting to compare since at smaller N_t the taste splitting in the improved staggered action is worse - we want to know how this affects the EOS.

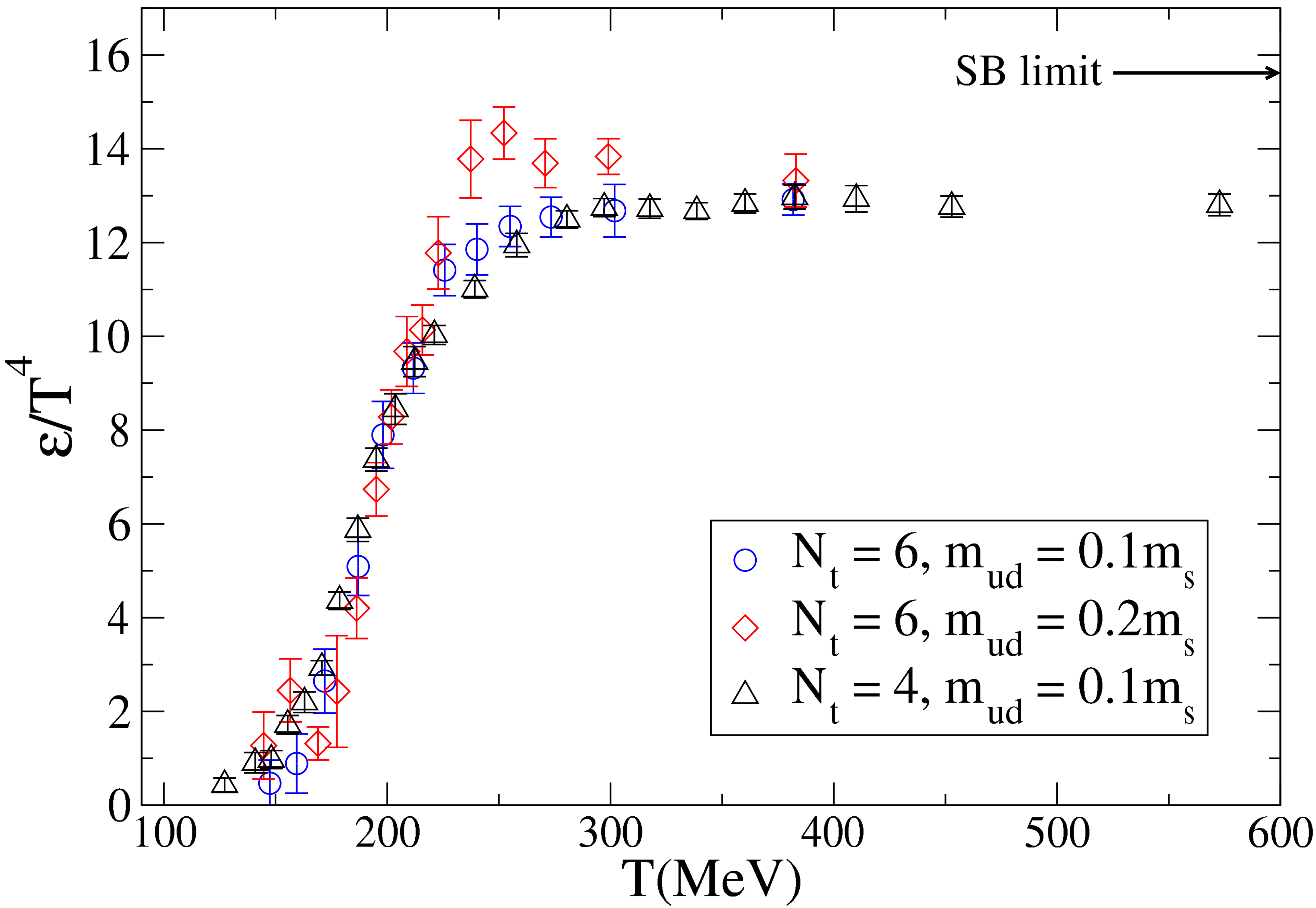
EOS results – Interaction measure



EOS results – Pressure



EOS results – Energy density



The EOS with 2+1 flavors at non-zero chemical potential

- We use the Bielefeld-Swansea Taylor expansion method (C.R. Allton *et. al*, Phys.Rev. D66(2002) 074507).
- **Pressure**:

$$\frac{p}{T^4} = \frac{\ln Z}{VT^3} = \sum_{n,m=0}^{\infty} c_{nm}(T) \left(\frac{\bar{\mu}_l}{T}\right)^n \left(\frac{\bar{\mu}_h}{T}\right)^m .$$

Due to the CP symmetry the series nonzero terms are even in $n+m$. The nonzero coefficients are

$$c_{nm}(T) = \frac{1}{n!}\frac{1}{m!}\frac{N_\tau^3}{N_\sigma^3}\frac{\partial^{n+m} \ln \mathcal{Z}}{\partial(\mu_l N_\tau)^n \partial(\mu_h N_\tau)^m}\Big|_{\mu_{l,h}=0} \quad ,$$

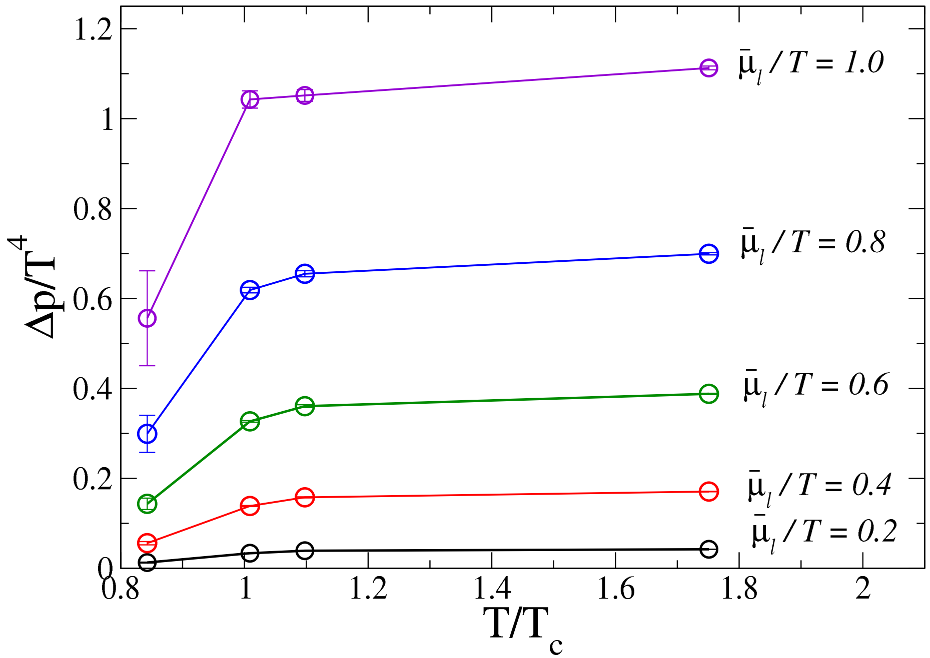
- **Interaction measure**:

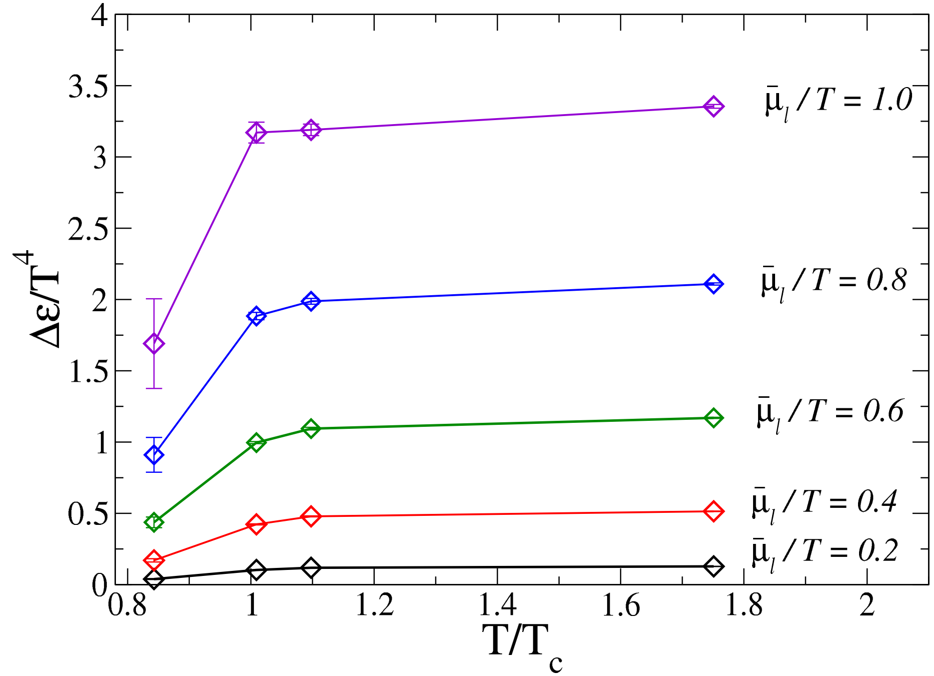
$$\frac{I}{T^4} = -\frac{N_t^3}{N_s^3}\frac{d\ln Z}{d\ln a} = \sum_{n,m}^{\infty} b_{nm}(T) \left(\frac{\bar{\mu}_l}{T}\right)^n \left(\frac{\bar{\mu}_h}{T}\right)^m ,$$

where again only even terms are nonzero and

$$b_{nm}(T) = -\frac{1}{n!m!}\frac{N_t^3}{N_s^3}\frac{\partial^{n+m}}{\partial(\mu_l N_t)^n \partial(\mu_h N_t)^m}\Bigg|_{\mu_{l,h}=0} \left(\frac{d\ln Z}{d\ln a}\right) .$$

Preliminary results for p and ε at $\mu_h = 0$ to $O(\mu^4)$





- $\Delta p = p(\mu) - p(\mu = 0)$, $\Delta\varepsilon = \varepsilon(\mu) - \varepsilon(\mu = 0)$.
- Currently the 6th order terms are too noisy to add (more statistics?).
- We intend to calculate quantities (quark number susceptibilities, quark number densities, etc) at both $\mu_{l,h} \neq 0$.

Conclusions

- We have calculated the EOS for 2+1 dynamical flavors of improved staggered quarks ($m_{ud}/m_s = 0.1$ and 0.2) along trajectories of constant physics, at $N_t = 4$ and 6.
- Our results show that the $N_t = 4$ and $N_t = 6$ results are quite similar except in the crossover region where the interaction measure is a bit higher on the finer $N_t = 6$ lattice.
- We also do not see significant differences between the EOS results from the two physics trajectories.
- We find deviations from the 3 flavor Stefan–Boltzmann limit in the temperature region that we have studied.
- Non-zero chemical potential EOS study is in progress.

▶ QUARKONIUM SPECTRUM

MILC and Fermilab Lattice Collaborations: M. Di Pierro, Steven Gottlieb,
A. El-Khadra, A.S. Kronfeld, L. Levkova, P.B Mackenzie, J. Simone

Motivations

- There are a number of stable stable to strong decay quarkonium states with narrow widths, far from decay thresholds whose masses can be determined from first principles in LQCD.

- Spectrum splittings are of special interest to LQCD since they are used to set the lattice scale due to the fact that they can be calculated very accurately.

- Calculating the quarkonium spectrum is a test for the LQCD actions used (... and good lattice actions which get the heavy quark physics right are important for B and D physics, CP violations study in the Standard Model, CKM matrix determination, etc.).

Heavy quarks on the lattice

- The fermion Clover (Sheikholeslami–Wohlert) action:

$$
\begin{aligned}
S^{\mathrm{sw}} &= \overline{\Psi} M_{\mathrm{sw}} \Psi \\
M_{\mathrm{sw}} &= M_{\mathrm{wilson}} + \underbrace{C_{\mathrm{sw}} \sigma_{\mu\nu} F_{\mu\nu}}_{\text{clover term}}
\end{aligned}
$$

  Clover term removes lattice artifacts of $O(a)$. Leading errors of $O(a^2)$.

- The Fermilab interpretation for (non-relativistic) heavy quarks: the bare quark mass in the Clover action should be tuned until the kinetic mass (M_2) of a given quarkonium state takes the physical value:
  - $M_2 \longrightarrow M_{D_s}$ for charmonium,
  - $M_2 \longrightarrow M_{B_s}$ for bottomonium.

- The Clover action is used only for the external (valence) quarks. The sea quarks are 2+1 flavors of Asqtad quarks with $m_{ud} \in [0.1 m_s, 0.6 m_s]$

- Asqtad gauge configurations: fine (0.09 fm), coarse (0.12 fm), medium coarse (0.15 fm) and extra coarse (0.18 fm).

Determination of quarkonium mass spectrum

- Meson mass determined from Bayesian multi-exponential fits to lattice correlators:

$$G_{\rm meson}(\overrightarrow{p}, t) = \sum_{k=0}^{n_{exp}} |A_k|^2 \, e^{-E_k(\overrightarrow{p})t},$$

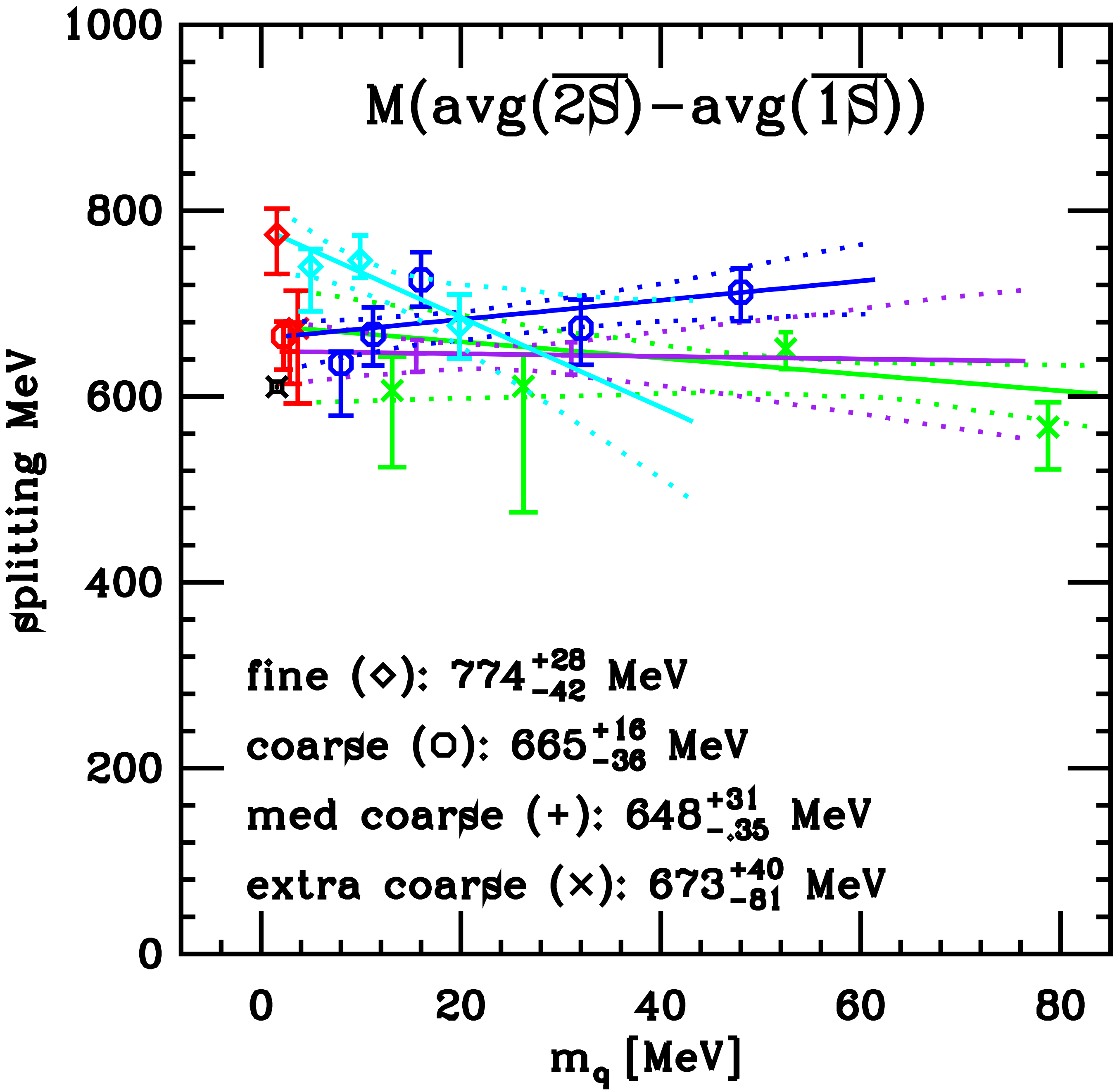
  - $E_0(0) = M$ is the rest mass.
  - The kinetic mass M_2 – extracted from fits to the dispersion relation

$$E_0(\overrightarrow{p})^2 - M^2 = c_0(\overrightarrow{p}.\overrightarrow{p}) + c_1(\overrightarrow{p}.\overrightarrow{p})^2 + c_2 \sum_j p_j^4,$$

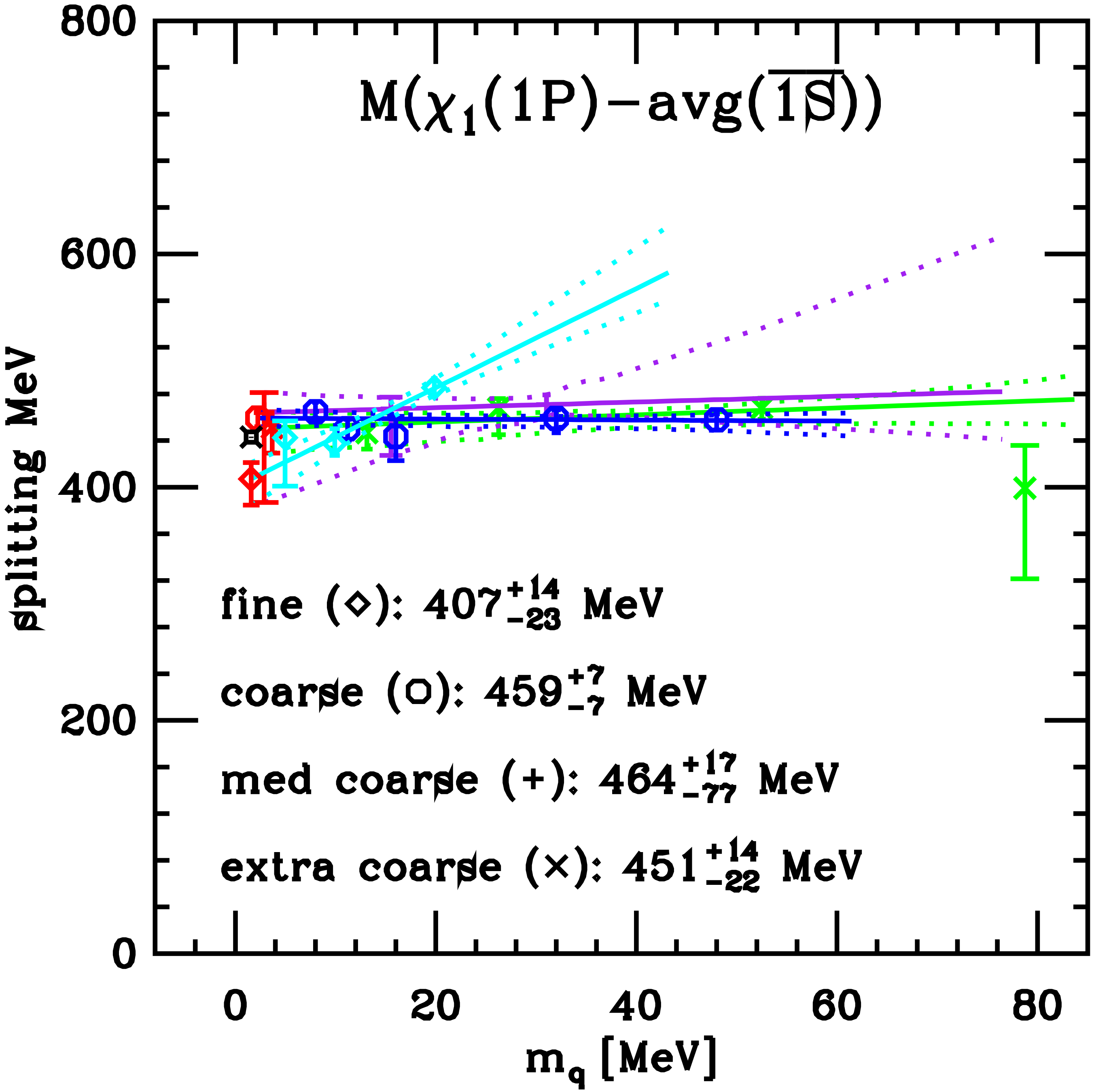
    with $M_2 = M/c_0$.

- M_2 is used to tune the bare quark mass. The rest mass M – to calculate splittings in the quarkonium spectrum.

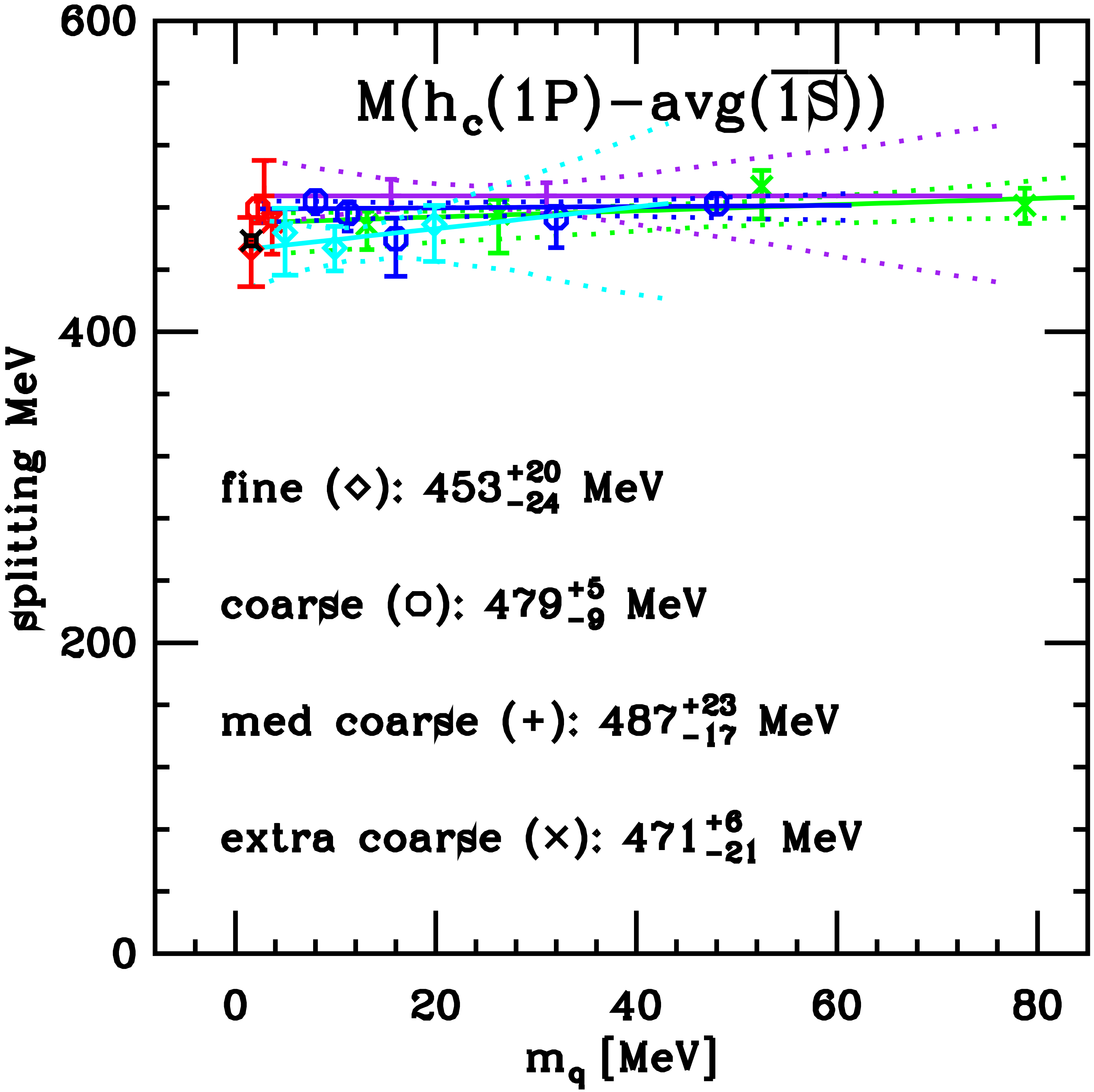
Charmonium spectrum: $\overline{2S}$ - $\overline{1S}$ splitting



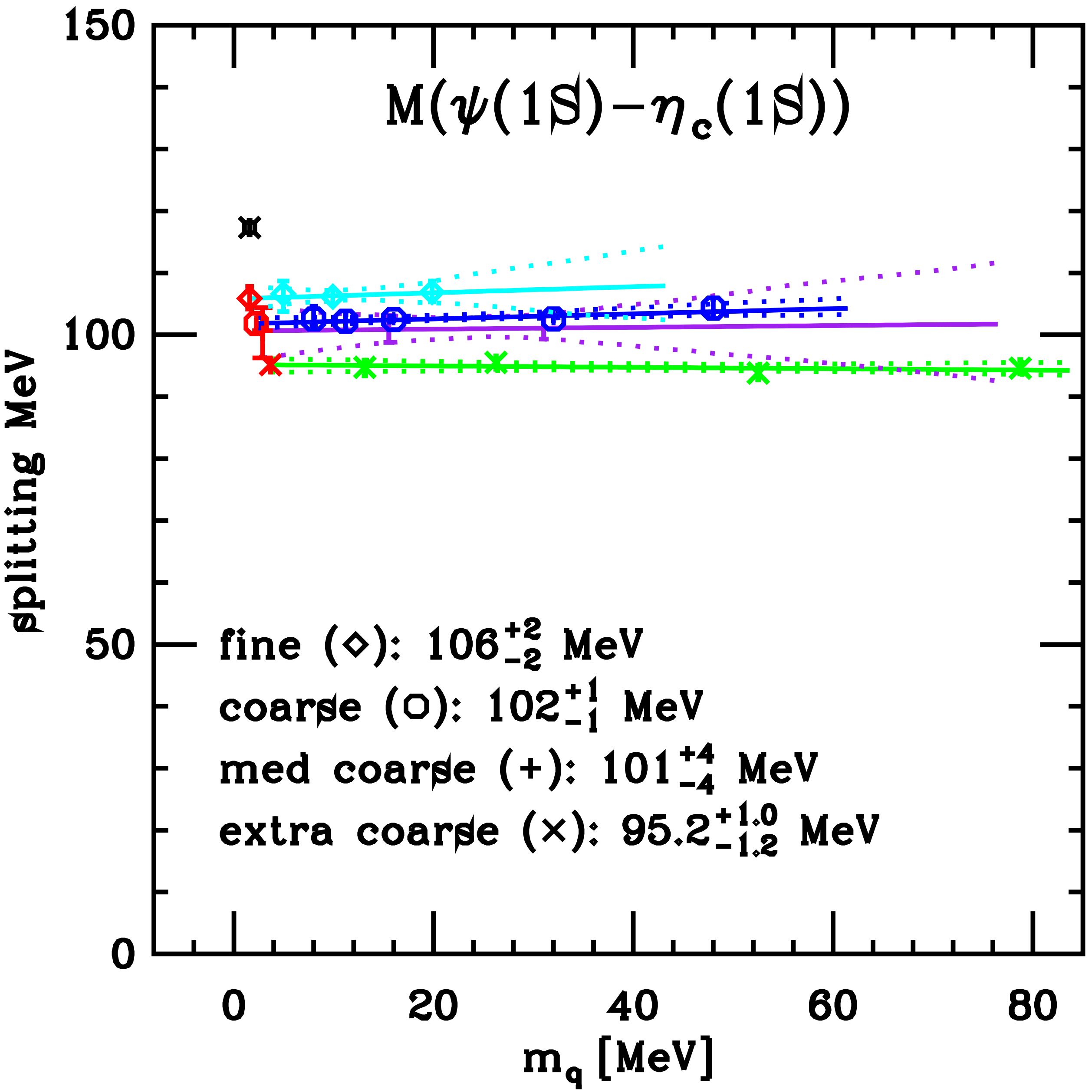
Charmonium spectrum: χ_{c_1} - $\overline{1S}$ splitting



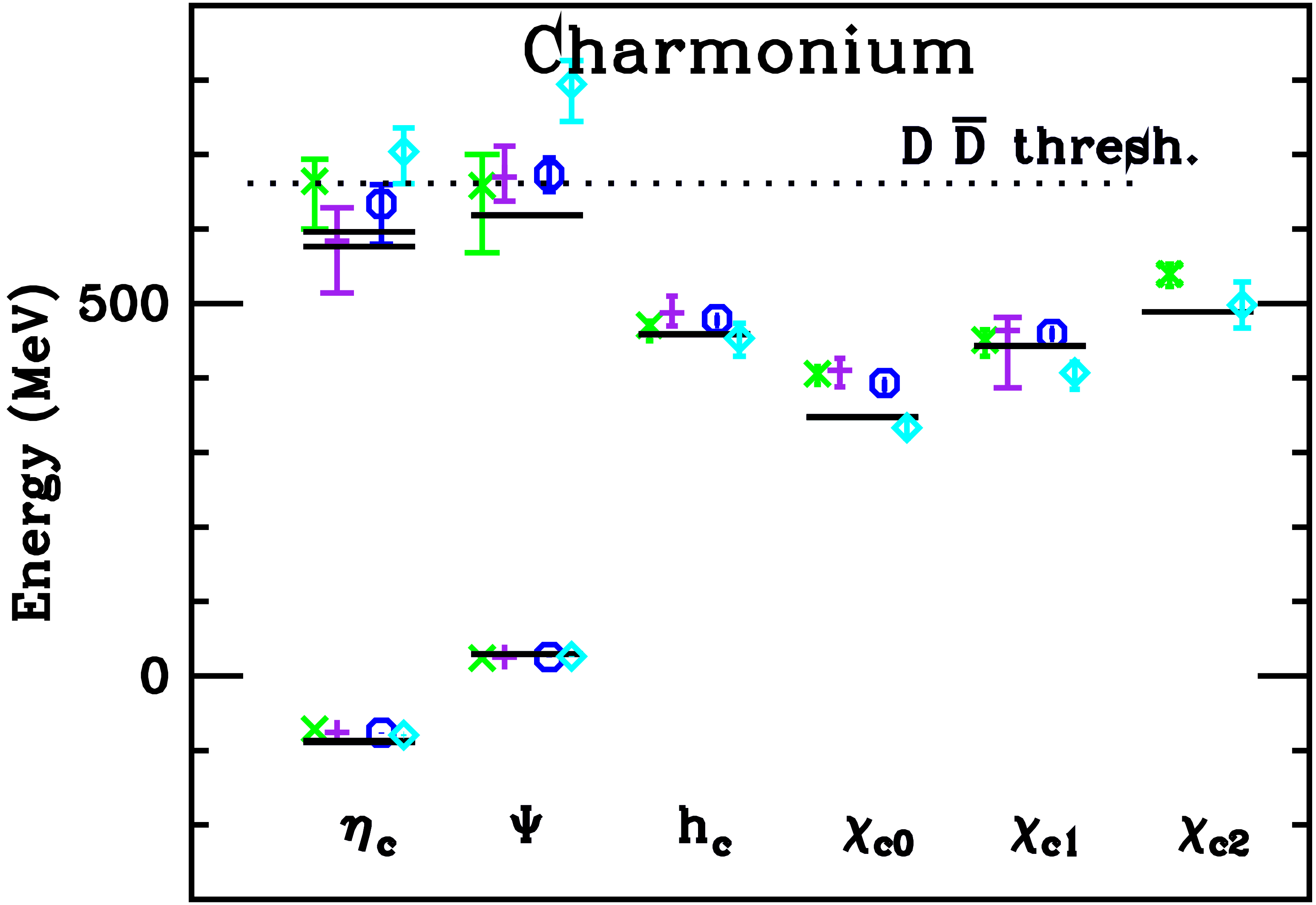
Charmonium spectrum: h_c - $\overline{1S}$ splitting



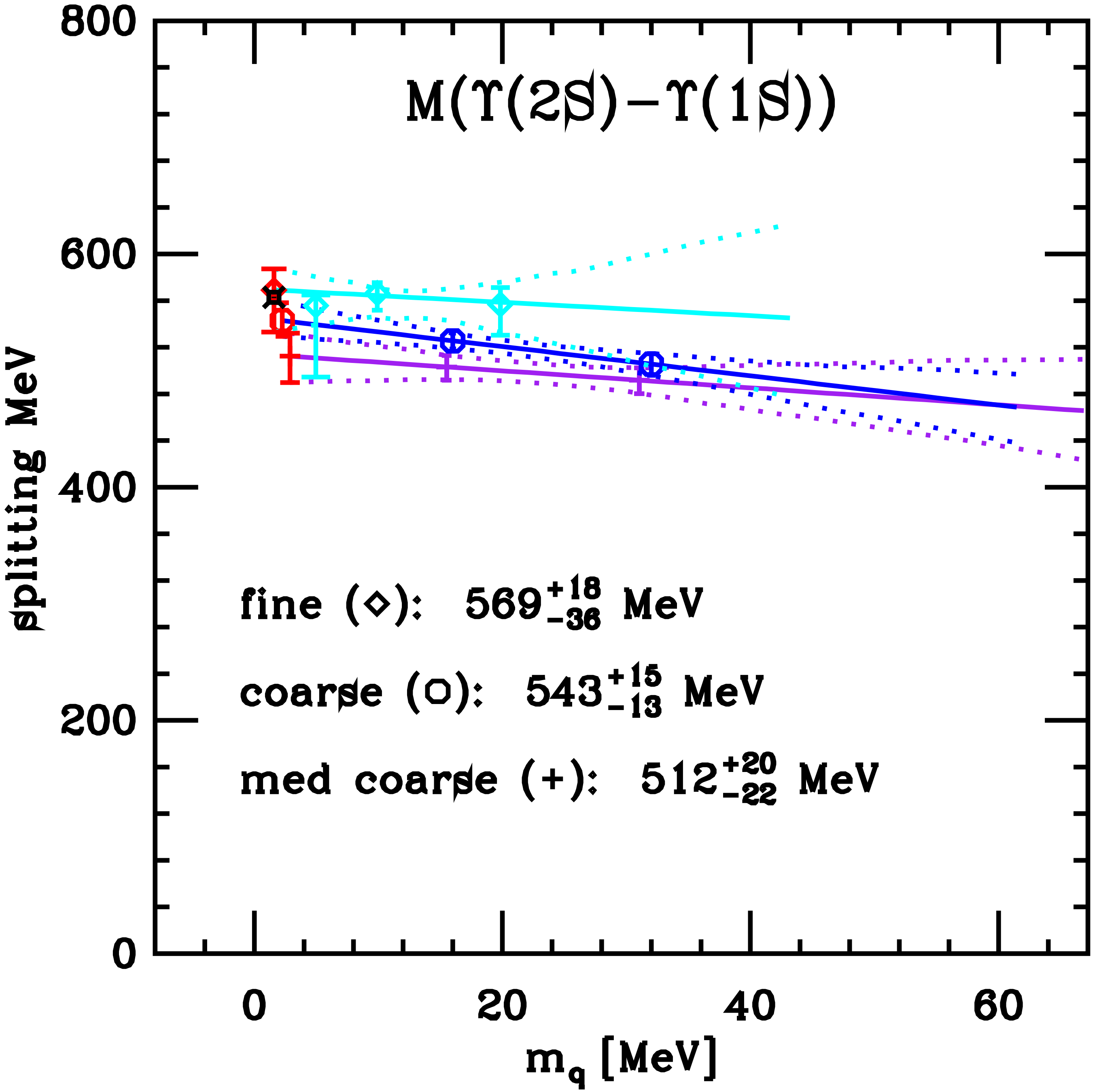
Charmonium spectrum: Hyperfine splitting $\Psi(1S)$ - $\eta_c(1S)$



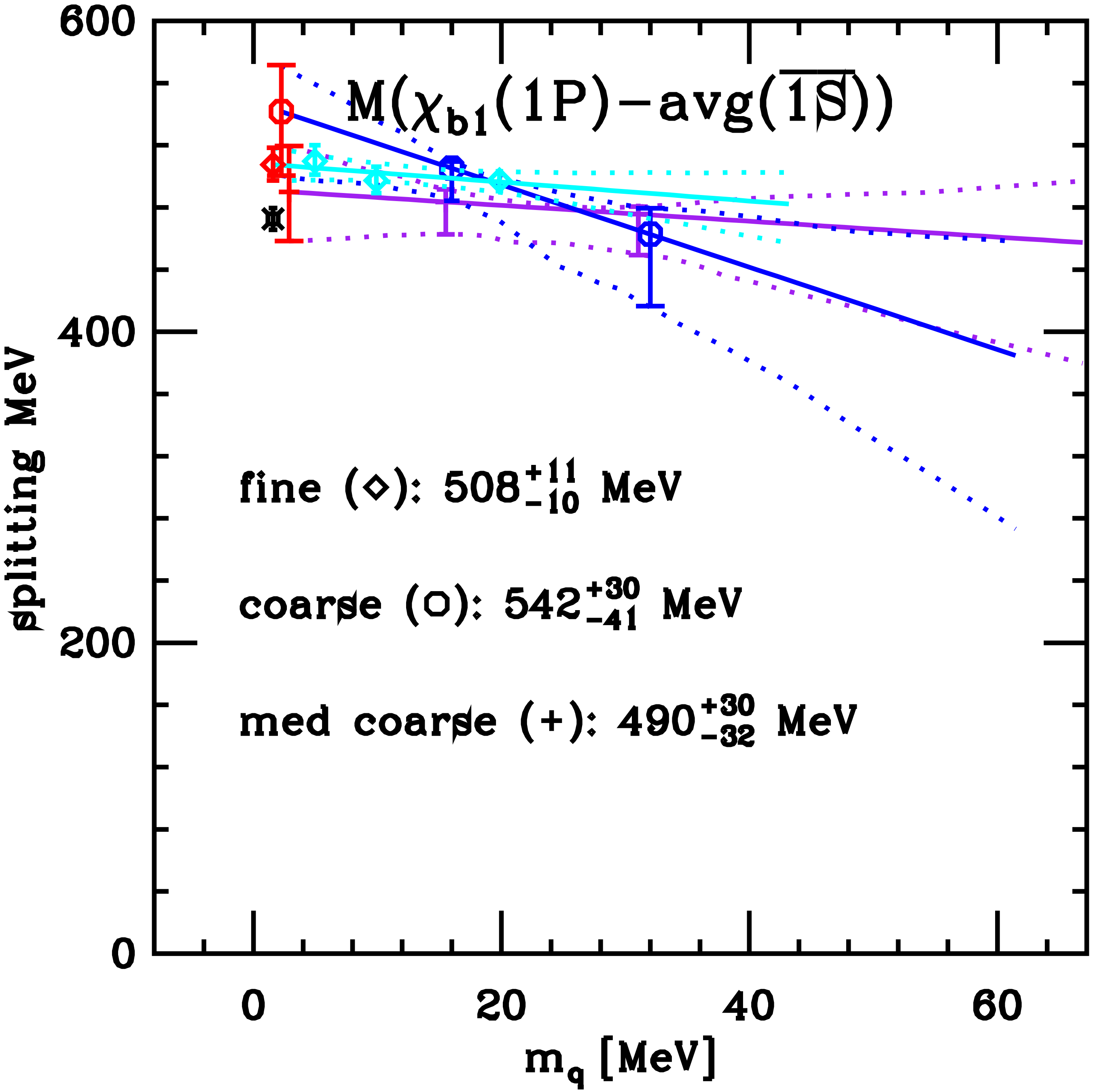
Charmonium spectrum: Summary



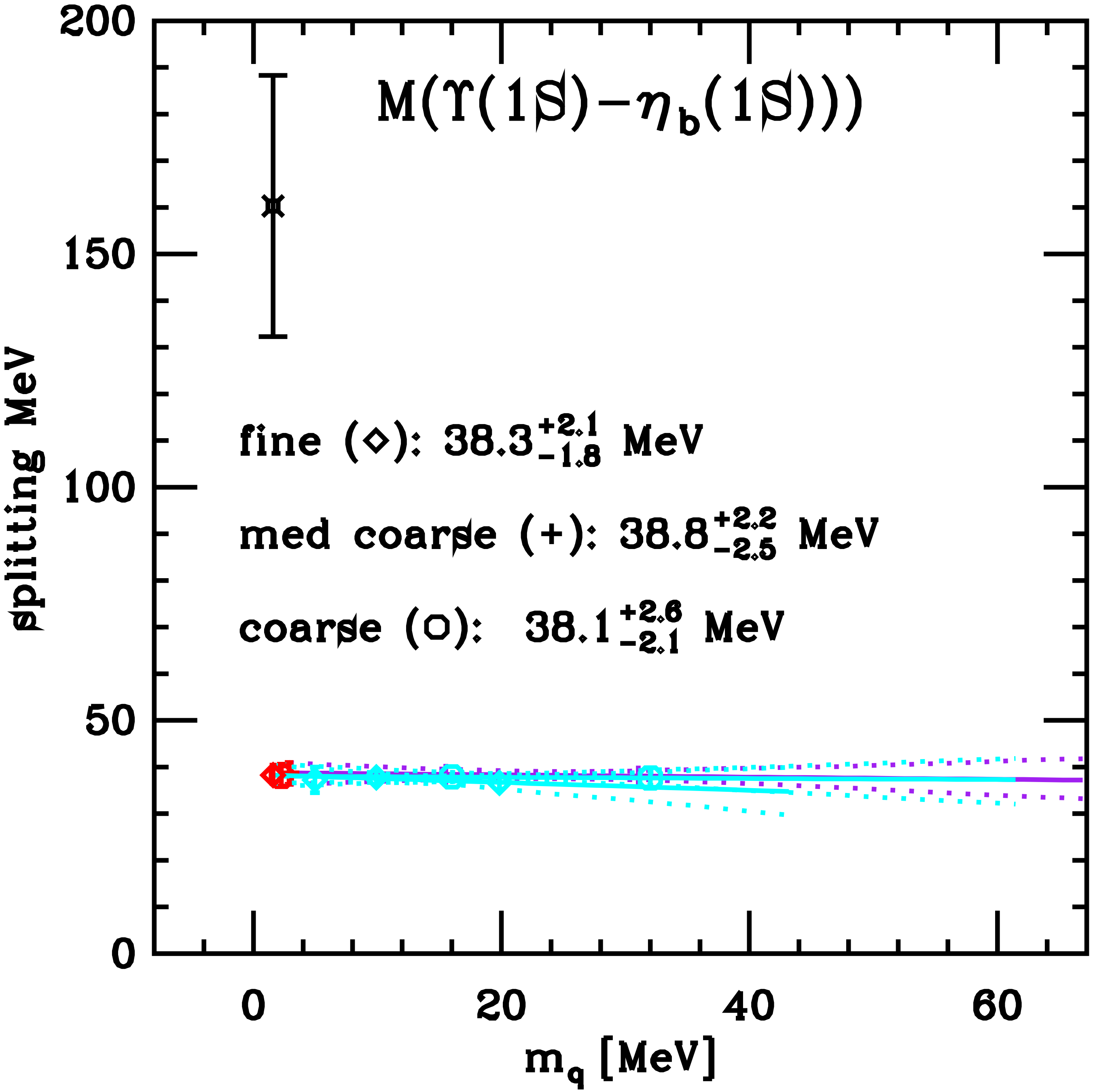
Bottomonium spectrum: $\Upsilon(2S)$ - $\Upsilon(1S)$ splitting



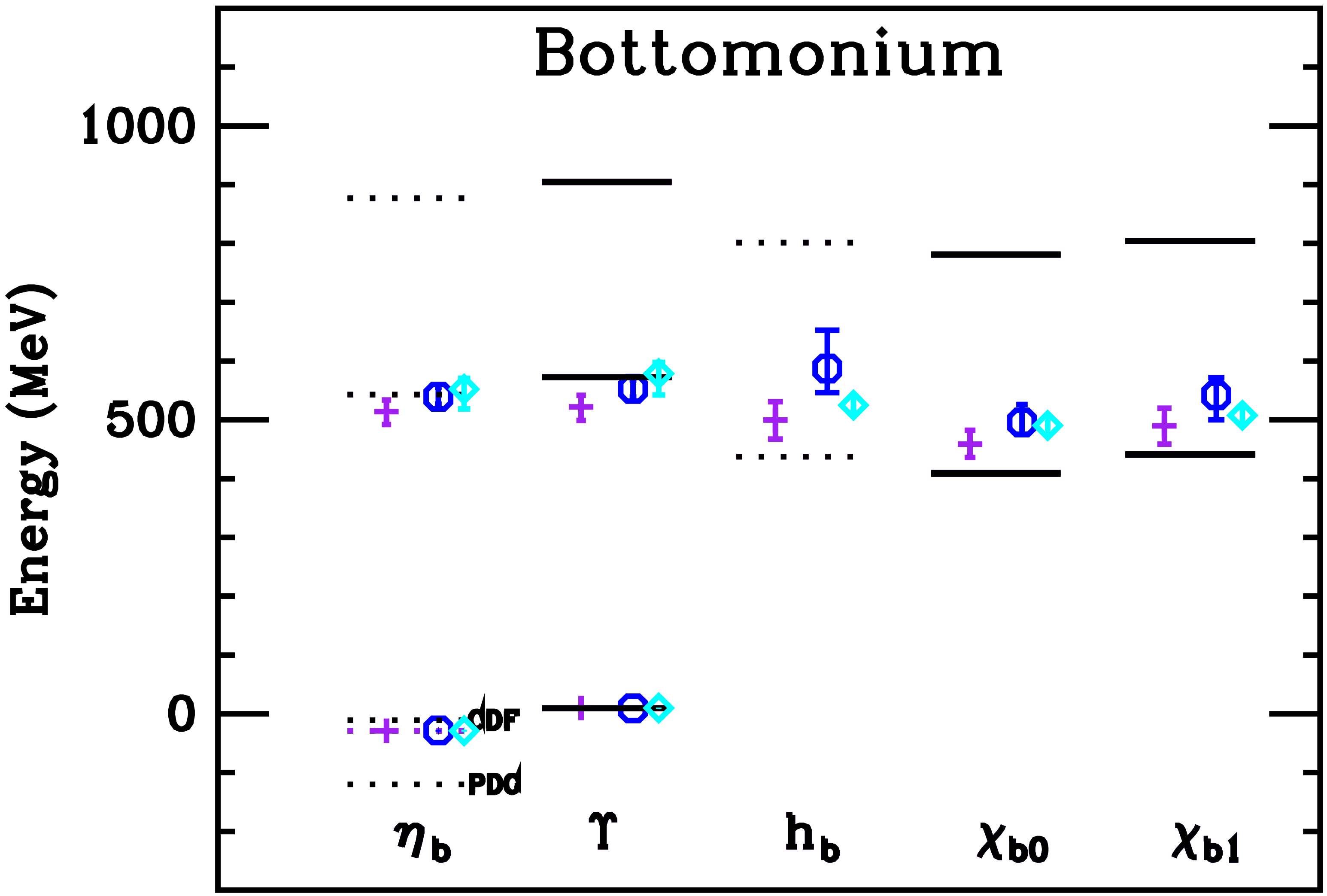
Bottomonium spectrum: $\chi_{b_1}(1P)$ - $\overline{1S}$ splitting



Bottomonium spectrum: Hyperfine splitting $\Upsilon(1S)$ - $\eta_b(1S)$



Bottomonium spectrum: Summary



Conclusions

- Charmonium spectrum:
  - h_c and χ_{c1} – well determined.
  - P-wave splitting – reasonable.
  - Hyperfine splitting – too small, but improving with $a \to 0$.
  - $2S$ states not accurately calculated (close to $D\overline{D}$ threshold).
- Bottomonium spectrum:
  - Excited states splittings look good.
  - Hyperfine splitting not yet possible to test. Compared to other LQCD results (NRQCD) it is smaller.
  - P-wave states are too heavy.
- Future:
  - Adding more statistics and new ensembles with very fine lattice spacing of 0.06 fm and possibly 0.045 fm.
  - May use a new highly improved clover quark action (by Kronfeld and Oktay) in the future.