

# THE BARYON AXIAL VECTOR CURRENT

Rubén Flores-Mendieta

*Instituto de Física, Universidad Autónoma de San Luis Potosí  
San Luis Potosí, S.L.P., Mexico*

(in collaboration with C. P. Hofmann)

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## ABSTRACT

The baryon axial vector current is computed at one-loop order in heavy baryon chiral perturbation theory in the large- $N_c$  limit, where  $N_c$  is the number of colors. Loop graphs with octet and decuplet intermediate states cancel to various orders in  $N_c$  as a consequence of the large- $N_c$  spin-flavor symmetry of QCD baryons. A comparison with conventional heavy baryon chiral perturbation theory is performed at the physical values  $N_c=3, N_f=3$ .

**Both approaches coincide**

# OUTLINE

1. Introduction
  - ▶ Heavy baryon chiral perturbation theory
  - ▶ The  $1/N_c$  expansion of QCD
2. The  $1/N_c$  chiral Lagrangian for baryons
3. One loop corrections to the baryon axial current
4. Comparison with HBCHPT in the degeneracy limit
5. Concluding remarks

## INTRODUCTION

QCD is an  $SU(3)$  gauge theory of quarks and gluons. Despite the progress achieved in the understanding of the strong interactions with QCD, analytic calculations of the spectrum and properties of hadrons are not possible because the theory is strongly coupled at low energies.

Some methods to extract the low-energy consequences of QCD are:

- ▶ Chiral perturbation theory
- ▶ The  $1/N_c$  expansion

## CHIRAL PERTURBATION THEORY

- CHPT exploits the symmetry of  $\mathcal{L}_{\text{QCD}}$  under  $SU(3)_L \times SU(3)_R \times U(1)_V$  transformations on the flavors  $u, d, s$  in the limit  $m_q \rightarrow 0$ .
- Chiral symmetry is spontaneously broken to the vector subgroup  $SU(3) \times U(1)_V$  by the QCD vacuum, resulting in an octet of pseudoscalar Goldstone bosons, the mesons.
- There is an expansion about the chiral limit in powers of  $m_q/\Lambda_\chi$ , or, equivalently, in powers of  $m_\Pi^2/\Lambda_\chi^2$ , where  $\Lambda_\chi \sim 1 \text{ GeV}$  is the scale of chiral symmetry breaking and  $m_\Pi$  is the meson mass.
- Baryons can be incorporated in a systematic way (HBCHPT).

## THE $1/N_c$ EXPANSION OF QCD

- The generalization of QCD from  $N_c = 3$  to  $N_c \gg 3$  colors, known as [the large- \$N\_c\$  limit](#), was proposed<sup>1</sup> to understand the nonperturbative dynamics of hadrons.
- Large- $N_c$  QCD is the  $SU(N_c)$  gauge theory of quarks and gluons, where  $N_c$  is a parameter of the theory.
- In the large- $N_c$  limit the meson sector consists of a spectrum of narrow resonances<sup>1</sup> and meson-meson scattering amplitudes are suppressed by powers of  $1/\sqrt{N_c}$ . The baryon sector is more subtle to analyze.<sup>2</sup>
- Physical quantities are considered in this limit, where corrections arise at relative orders  $1/N_c, 1/N_c^2, \dots$ , [the  \$1/N\_c\$  expansion](#).

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<sup>1</sup>G. 't Hooft, Nucl. Phys. B **72**, 461 (1974); B **75**, 461 (1974).

<sup>2</sup>E. Witten, Nucl. Phys. B **160**, 57 (1979).

## HEAVY BARYON CHIRAL PERTURBATION THEORY

To lowest order in the derivative expansion,  $\mathcal{L}_{\text{baryon}}$  is<sup>2</sup>

$$\begin{aligned}\mathcal{L}_{\text{baryon}} = & i \text{Tr} \bar{B}_v (v \cdot \mathcal{D}) B_v - i \bar{T}_v^\mu (v \cdot \mathcal{D}) T_{v\mu} + \Delta \bar{T}_v^\mu T_{v\mu} \\ & + 2 D \text{Tr} \bar{B}_v S_v^\mu \{ \mathcal{A}_\mu, B_v \} + 2 F \text{Tr} \bar{B}_v S_v^\mu [ \mathcal{A}_\mu, B_v ] \\ & + \mathcal{C} ( \bar{T}_v^\mu \mathcal{A}_\mu B_v + \bar{B}_v \mathcal{A}_\mu T_v^\mu ) + 2 \mathcal{H} \bar{T}_v^\mu S_v^\nu \mathcal{A}_\nu T_{v\mu} .\end{aligned}$$

$B_v$  and  $T_{abc}^\mu$  are baryon octet and decuplet fields. Octet meson fields enter into  $\mathcal{A}_\mu$  and  $\mathcal{V}_\mu$  via

$$\xi = e^{i\Pi/f}, \quad \Sigma = \xi^2 = e^{2i\Pi/f}$$

where  $f \approx 93$  MeV is the pion decay constant.

$D$ ,  $F$ ,  $\mathcal{C}$ , and  $\mathcal{H}$  are coupling constants and  $\Delta = M_\Delta - M_B$ .

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<sup>2</sup>E. Jenkins and A.V. Manohar, Phys. Lett. B **225**, 558 (1991); **259**, 353 (1991).

# CHIRAL CORRECTIONS TO THE BARYON AXIAL VECTOR CURRENT

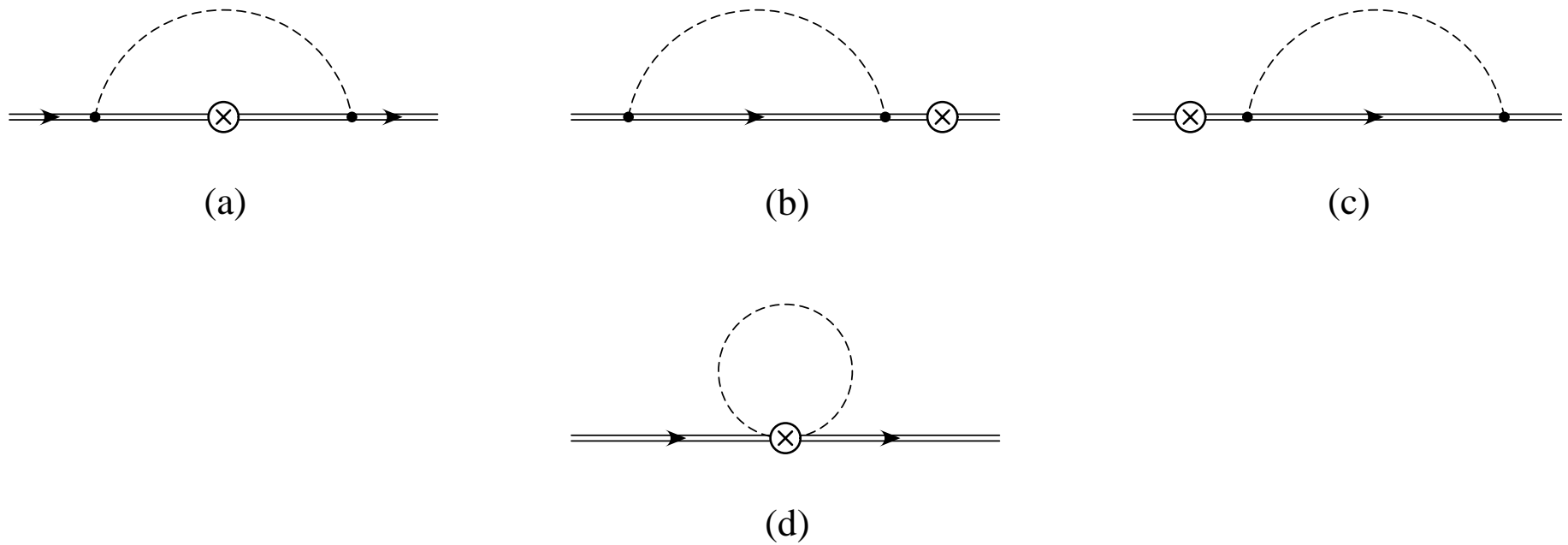


Figure 1: One-loop corrections to the baryon axial vector current.



The renormalized baryon axial vector current is<sup>3</sup>

$$\langle B_i | J_\mu^A | B_j \rangle = \left[ \alpha_{ij} - \sum_{\Pi} \left( \bar{\beta}_{ij}^{\Pi} - \bar{\lambda}_{ij}^{\Pi} \alpha_{ij} \right) F(m_{\Pi}, 0, \mu) + \sum_{\Pi} \gamma_{ij}^{\Pi} I(m_{\Pi}, \mu) \right] \\ \times \bar{u}_{B_i} \gamma_\mu \gamma_5 u_{B_j}.$$

▷  $\alpha_{ij}$ : lowest order result

▷  $\bar{\beta}_{ij}^{\Pi} = \beta_{ij}^{\Pi} + \beta'_{ij}{}^{\Pi}$ : from Fig. 1(a)

▷  $\bar{\lambda}_{ij}^{\Pi} = \lambda_{ij}^{\Pi} + \lambda'_{ij}{}^{\Pi}$ : from wave function renormalization, Figs. 1(b,c)

▷  $\gamma_{ij}^{\Pi}$ : from Fig. 1(d).

$F(m_{\Pi}, \Delta, \mu)$  and  $I(m_{\Pi}, \mu)$  are the integrals over the loops.

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<sup>3</sup>E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); **259**, 353 (1991).

## IMPORTANT RESULTS

- ▶ The  $F/D$  ratios were found to be close to their SU(6) values, with  $F/D \approx 2/3$ , value predicted by the nonrelativistic quark model.
- ▶ There were large cancellations in the corrections to the baryon axial vector current between one-loop graphs with intermediate spin-1/2 octet and spin-3/2 decuplet baryon states:
  - Corrections  $\sim 100\%$  when only octet baryon states are included
  - Corrections  $\sim 40\%$  when *both* octet and decuplet baryon states are included

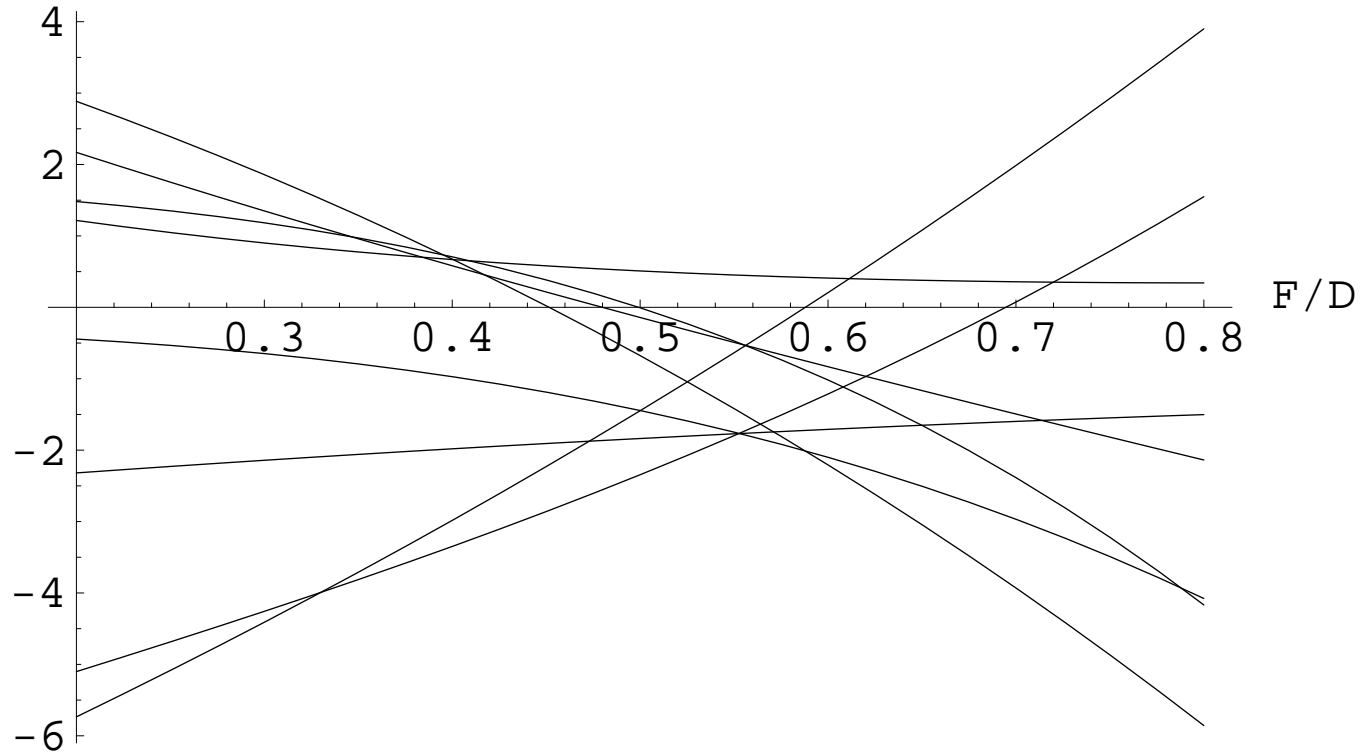
Using the  $1/N_c$  expansion it can be proved that, for pions,<sup>4</sup>

$$\frac{F}{D} = \frac{2}{3} + \mathcal{O}\left(\frac{1}{N_c^2}\right).$$

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<sup>4</sup>R. Dashen, A.V. Manohar, Phys. Lett. B **315**, 425 (1993), **315**, 438 (1993)

## NUMERICAL CANCELLATIONS IN THE ONE-LOOP CORRECTION



One-loop pion correction to the current  $\langle p | \bar{u} \gamma^\mu \gamma_5 d | n \rangle$ . The curves are  $\bar{\beta} - \bar{\lambda} \alpha$  for  $N \rightarrow N\pi$ ,  $\Xi \rightarrow \Lambda \bar{K}$ ,  $\Sigma \rightarrow \Sigma\pi$ ,  $\Xi \rightarrow \Xi\pi$ ,  $\Xi \rightarrow \Sigma \bar{K}$ ,  $\Sigma \rightarrow \Lambda\pi$ ,  $\Sigma \rightarrow N \bar{K}$ ,  $\Lambda \rightarrow N \bar{K}$ .<sup>5</sup>

<sup>5</sup>R. Flores-Mendieta, E. Jenkins, and A.V. Manohar, Phys. Rev. D **62**, 034001 (1998).

## THE $1/N_C$ EXPANSION OF QCD

In the large- $N_C$  limit the baryon sector has a contracted spin-flavor symmetry  $SU(2N_f)$ , with  $N_f$  the number of light-quark flavors.

$SU(2N_f)$  decomposes under  $SU(2) \times SU(N_f)$  into a tower of baryon states with spins  $J = \frac{1}{2}, \dots, \frac{N_C}{2}$  in the flavor representation.<sup>6</sup>

Any physical operator  $\mathcal{O}^{(m)}$  that scales as  $N_C^m$  may be written as

$$\mathcal{O}^{(m)} = N_C^m \sum_{n,p,q} c_n \left( \frac{J^i}{N_C} \right)^p \left( \frac{T^a}{N_C} \right)^q \left( \frac{G^{jb}}{N_C} \right)^{n-p-q}$$

The  $c_n$  have power series expansions in  $1/N_C$  beginning at order unity.

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<sup>6</sup>R. Dashen and A.V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993).

J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phys. Rev. D **30**, 1795 (1989).

Any QCD operator transforming according to a given  $SU(2) \times SU(N_f)$  representation can be expanded as<sup>7</sup>

$$\mathcal{O}_{\text{QCD}} = \sum_n c_n \frac{1}{N_c^{n-1}} \mathcal{O}^{(n)}$$

The spin-flavor generators  $J^i$ ,  $T^a$ , and  $G^{ia}$  of  $SU(2N_f)$  are

$$J^i = q^\dagger \left( \frac{\sigma^i}{2} \otimes I \right) q, \quad (1, 1)$$

$$T^a = q^\dagger \left( I \otimes \frac{\lambda^a}{2} \right) q, \quad (0, \text{adj})$$

$$G^{ia} = q^\dagger \left( \frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q. \quad (1, \text{adj})$$

They satisfy a Lie algebra.

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<sup>7</sup>R. F. Dashen, E. Jenkins, A.V. Manohar, Phys. Rev. D **49**, 4713 (1994).

## SOME EXAMPLES

- Baryon mass operator

$$\mathcal{M} = \sum_{n=0}^{(N_c-1)/2} m_{2n} \frac{1}{N_c^{2n-1}} (J^2)^n$$

- Baryon axial vector current

$$A^{ia} = a_1 G^{ia} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{ia} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{ia}$$

where  $m_k$ ,  $a_i$ ,  $b_j$ ,  $c_m$  are unknown coefficients and

$$\mathcal{D}_2^{ia} = J^i T^a, \quad \mathcal{O}_2^{ia} = \epsilon^{ijk} \{J^j, G^{ka}\}, \quad \mathcal{D}_3^{ia} = \{J^i, \{J^r, G^{ra}\}\}$$

$$\mathcal{O}_3^{ia} = \{J^2, G^{ia}\} - \frac{1}{2} \{J^i, \{J^r, G^{ra}\}\},$$

$$\mathcal{D}_n^{ia} = \{J^2, \mathcal{D}_{n-2}^{ia}\}, \quad \mathcal{O}_n^{ia} = \{J^2, \mathcal{O}_{n-2}^{ia}\} \quad (n \leq 4)$$

## COMBINED EXPANSION IN $m_q$ AND $1/N_c$

In the chiral limit  $m_q \rightarrow 0$

- Mesons become massless Goldstone boson states
- There is an expansion about the chiral limit in powers of  $m_q/\Lambda_\chi$

In the large- $N_c$  limit

- The nucleon and  $\Delta$  become degenerate,  $M_\Delta - M_N \propto 1/N_c \rightarrow 0$  and form a single irreducible representation of the contracted spin-flavor symmetry of baryons
- There is an expansion in powers of  $1/N_c$  about this limit

**Goal:** Consider a combined expansion in  $m_q/\Lambda_\chi$  and  $1/N_c$  about the double limit  $m_q \rightarrow 0$  and  $N_c \rightarrow \infty$ .

In the chiral limit  $m_q \rightarrow 0$  with  $\Delta$  held fixed,

$$F(m_\Pi, \Delta, \mu) = F_0 + \left(\frac{m_\Pi}{\Delta}\right) F_1 + \left(\frac{m_\Pi}{\Delta}\right)^2 F_2 + \dots$$

In the  $1/N_c \rightarrow 0$  limit with  $m_\Pi$  held fixed

$$F(m_\Pi, \Delta, \mu) = \bar{F}_0 + \left(\frac{\Delta}{m_\Pi}\right) \bar{F}_1 + \left(\frac{\Delta}{m_\Pi}\right)^2 \bar{F}_2 + \dots$$

The difference between the two expansions is referred to as the non-commutativity of the chiral and large- $N_c$  limits.<sup>8</sup>

Conditions for HBCHPT to be valid:

$$m_\Pi \ll \Lambda_\chi \quad \text{and} \quad \Delta \ll \Lambda_\chi$$

$m_\Pi/\Delta$  is not constrained ( $m_\Pi/\Delta \sim 0.5$ ).

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<sup>8</sup>T. D. Cohen, Phys. Lett. **B359**, 23 (1995).



## CHIRAL LAGRANGIAN FOR BARYONS IN THE $1/N_c$ EXPANSION

$$\mathcal{L}_{\text{baryon}} = i\mathcal{D}^0 - \mathcal{M}_{\text{hyperfine}} + \text{Tr} \left( \mathcal{A}^k \lambda^c \right) A^{kc} + \frac{1}{N_c} \text{Tr} \left( \mathcal{A}^k \frac{2I}{\sqrt{6}} \right) A^k + \dots,$$

with<sup>9</sup>

$$\xi(x) = e^{i\Pi(x)/f}, \quad \Pi(x) = \frac{\pi^a(x)\lambda^a}{2} + \frac{\eta'(x)I}{\sqrt{6}}, \quad (a = 1, \dots, 8)$$

For  $N_c = 3$

$$\mathcal{M}_{\text{hyperfine}} = m_2 \frac{1}{N_c} J^2,$$

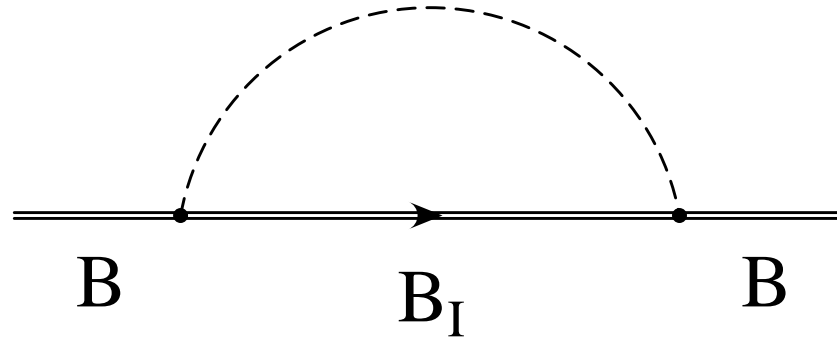
$$A^{kc} = a_1 G^{kc} + b_2 \frac{1}{N_c} J^k T^c + b_3 \frac{1}{N_c^2} \mathcal{D}_3^{kc} + c_3 \frac{1}{N_c^2} \mathcal{O}_3^{kc}$$

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<sup>9</sup>E. Jenkins, Phys. Rev. D **53**, 2625 (1996)

## RENORMALIZATION OF THE BARYON AXIAL VECTOR CURRENT

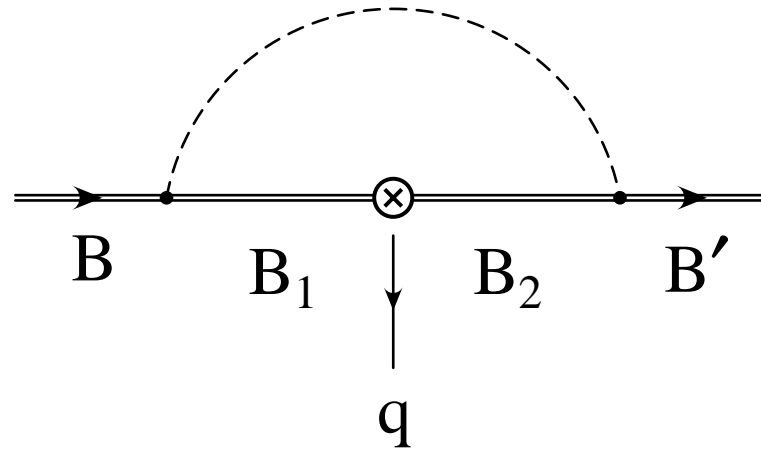
One-loop wave function renormalization graph:



$$iG_B = \sum_{j,k,b,B_I} \frac{i^2}{f^2} [A^{kb}]_{BB_I} [A^{jb}]_{B_I B} \times \int \frac{d^4 k}{(2\pi)^4} \frac{(\mathbf{k}^k)(-\mathbf{k}^j)}{(k^2 - m_b^2) [(k+p) \cdot v - (M_I - M) + i\epsilon]},$$

where  $b = 1, \dots, 9$  or  $\pi, K, \eta, \eta'$  labels the intermediate meson.

## VERTEX CORRECTION



$$\begin{aligned}
 [\delta A^{ia}]_{B'B}^{\text{vertex}} &= \sum_{j,k,b,B_1,B_2} -\frac{i}{f^2} [A^{kb}]_{B'B_2} [A^{ia}]_{B_2B_1} [A^{jb}]_{B_1B} \int \frac{d^4k}{(2\pi)^4} \\
 &\quad \frac{(\mathbf{k}^k)(-\mathbf{k}^j)}{(k^2 - m_b^2) (k \cdot v - \Delta_{M_1 M} + i\epsilon) ((k - q) \cdot v - \Delta_{M_2 M} + i\epsilon)},
 \end{aligned}$$

$q \cdot v = 0$  and  $q \cdot v = M - M'$  for octet-octet and decuplet-decuplet matrix elements, respectively.

## TOTAL CORRECTION FROM FIGS. 1(A,B,C): $\Delta/m_\Pi \rightarrow 0$ LIMIT

$$\delta A^{kc} = \frac{1}{2}[A^{ia}, [A^{ib}, A^{kc}]]\Pi^{ab}$$

where

$$\begin{aligned}\Pi^{ab} = & \frac{1}{8} [3F(\pi) + 4F(K) + F(\eta)] \delta^{ab} \\ & + \frac{2\sqrt{3}}{5} \left[ \frac{3}{2}F(\pi) - F(K) - \frac{1}{2}F(\eta) \right] d^{ab8} \\ & + \left[ \frac{1}{3}F(\pi) - \frac{4}{3}F(K) + F(\eta) \right] \left( \delta^{a8}\delta^{b8} - \frac{1}{8}\delta^{ab} - \frac{3}{5}d^{ab8}d^{888} \right)\end{aligned}$$

and

$$F(m_b, 0, \mu) = -\frac{1}{16\pi^2 f^2} m_b^2 \left( \frac{11}{3} + \ln \frac{m_b^2}{\mu^2} \right)$$

## LARGE- $N_c$ CANCELLATIONS

For baryons with spins of order unity

$$T^a \sim N_c, \quad G^{ia} \sim N_c, \quad J^i \sim 1$$

Naively

$$\left[ A^{ja}, \left[ A^{jb}, A^{kc} \right] \right] \sim \mathcal{O}(N_c^3)$$

But from large- $N_c$  consistency conditions and analytic calculations

$$\left[ A^{ja}, \left[ A^{jb}, A^{kc} \right] \right] \sim \mathcal{O}(N_c)$$

There are large- $N_c$  cancellations provided one sums over all baryon states in a complete multiplet of the large- $N_c$  SU(6) spin-flavor symmetry, *i.e.*, over both the octet and the decuplet, and uses axial coupling ratios given by the large- $N_c$  symmetry.

## SOME EXPLICIT CALCULATIONS

- Singlet contribution

$$\begin{aligned}
 & [G^{ia}, [G^{ia}, J^k T^c]] + [G^{ia}, [J^i T^a, G^{kc}]] + [J^i T^a, [G^{ia}, G^{kc}]] = \\
 & -\frac{2}{N_f}(N_c + N_f)G^{kc} + \left(\frac{9}{4}N_f - \frac{1}{N_f} + 2\right) J^k T^c \\
 & \sim \mathcal{O}(N_c)
 \end{aligned}$$

- Octet contribution

$$\begin{aligned}
 & d^{ab8}[G^{ia}, [G^{ib}, G^{kc}]] = \left(\frac{3}{8}N_f - \frac{2}{N_f}\right) d^{c8e}G^{ke} + \left(\frac{1}{2} - \frac{2}{N_f^2}\right) \delta^{c8} J^k \\
 & \sim \mathcal{O}(N_c)
 \end{aligned}$$

- **27** contribution

$$\begin{aligned}
[G^{i8}, [G^{i8}, G^{kc}]] &= \frac{1}{4} \left( -f^{c8d} f^{d8e} + 2d^{c8d} d^{d8e} \right) G^{ke} + \frac{1}{N_f} \delta^{c8} G^{k8} \\
&+ \frac{1}{2N_f} d^{c88} J^k \\
&\sim \mathcal{O}(N_c)
\end{aligned}$$

Thus

$$\delta A^{kc} = \frac{1}{2} [A^{ja}, [A^{jb}, A^{kc}]] \Pi^{ab} \sim \mathcal{O}(N_c^0)$$

$$\delta A^{kc} \sim \frac{1}{N_c} \times g_A$$

with  $g_A \propto N_c$  and  $f \propto \sqrt{N_c}$ .

## CORRECTION FROM FIG. 1(D)

$$\delta A^{kc} = -\frac{1}{2} [T^a [T^b, A^{kc}]] \Pi^{ab},$$

where  $\Pi^{ab}$  is now a function of  $I(m_\Pi)$ .

- Singlet piece  $[T^a, [T^a, A^{kc}]] = N_f A^{kc} \sim \mathcal{O}(N_c)$
- Octet piece  $d^{ab8} [T^a, [T^b, A^{kc}]] = \frac{N_f}{2} d^{c8e} A^{ke} \sim \mathcal{O}(N_c)$
- **27** piece  $[T^8, [T^8, A^{kc}]] = f^{c8d} f^{8de} A^{ke} \sim \mathcal{O}(N_c)$

Thus

$$\delta A^{kc} \sim \frac{1}{N_c} \times g_A$$



## COMPARISON BETWEEN THE TWO APPROACHES

In the limit  $\Delta/m_\Pi = 0$

$$\delta A_{\text{deg}}^{kc} = \frac{1}{2} [A^{ja}, [A^{jb}, A^{kc}]] \Pi_{(F)}^{ab} - \frac{1}{2} [T^a [T^b, A^{kc}]] \Pi_{(I)}^{ab}$$

so the renormalized current is

$$A_{\text{ren}}^{kc} = A^{kc} + \delta A_{\text{deg}}^{kc}$$

The matrix elements of the space components of the axial vector current between baryon states are

$$\langle B_j | \bar{\psi} \gamma^k \gamma_5 T^c \psi | B_i \rangle = [A_{\text{ren}}^{kc}]_{B_j B_i}$$

$B_i$  and  $B_j$  are baryons in the lowest-lying irreducible representation of contracted SU(6) spin-flavor symmetry.

The correction within HBCHPT to the axial current can be decomposed into flavor singlet, octet and **27** contributions in terms of flavor singlet, octet, and **27** linear combinations of  $F(m_\Pi)$  and  $I(m_\Pi)$ :

$$\langle B_j | J_\mu^A | B_i \rangle = \left[ \alpha_{B_j B_i} + b_{\mathbf{1}}^{B_j B_i} F_{\mathbf{1}} + b_{\mathbf{8}}^{B_j B_i} F_{\mathbf{8}} + b_{\mathbf{27}}^{B_j B_i} F_{\mathbf{27}} \right. \\ \left. + c_{\mathbf{1}}^{B_j B_i} I_{\mathbf{1}} + c_{\mathbf{8}}^{B_j B_i} I_{\mathbf{8}} + c_{\mathbf{27}}^{B_j B_i} I_{\mathbf{27}} \right] \bar{u}_{B_j} \gamma_\mu \gamma_5 u_{B_i},$$

with

$$b_{\mathbf{1}}^{B_j B_i} = -(a_{B_j B_i}^\pi + a_{B_j B_i}^K + a_{B_j B_i}^\eta), \\ a_{B_j B_i}^\Pi = \bar{\beta}_{B_j B_i}^\Pi - \bar{\lambda}_{B_j B_i}^\Pi \alpha_{B_j B_i}$$

and

$$F_{\mathbf{1}} = \frac{1}{8} [3F(\pi) + 4F(K) + F(\eta)],$$

with similar expressions for the remaining coefficients.<sup>10</sup>

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<sup>10</sup>R.F.M. and C.P. Hofmann, hep-ph/0609120

For  $N_c = 3$ , there is a one-to-one correspondence between the contributions of  $[A_{\text{ren}}^{kc}]_{B_j B_i}$  and  $\langle B_j | J_\mu^A | B_i \rangle$ .

For Figs. 1(a,b,c) one has

$$\left[ \frac{1}{2} [A^{ia}, [A^{ia}, A^{kc}]] \right]_{B_j B_i} = b_{\mathbf{1}}^{B_j B_i},$$

$$\left[ \frac{1}{2} d^{ab8} [A^{ia}, [A^{ib}, A^{kc}]] \right]_{B_j B_i} = b_{\mathbf{8}}^{B_j B_i},$$

$$\left[ \frac{1}{2} [A^{i8}, [A^{i8}, A^{kc}]] \right]_{B_j B_i} = b_{\mathbf{27}}^{B_j B_i},$$

and similar expressions occur for Fig. 1(d)

For example, for the process  $n \rightarrow pe\bar{\nu}_e$  the singlet piece is

$$\begin{aligned} \left[ \frac{1}{2} [A^{ia}, [A^{ia}, A^{kc}]] \right]_{pn} &= \frac{115}{144} a_1^3 + \frac{7}{48} a_1^2 b_2 + \frac{19}{48} a_1 b_2^2 - \frac{31}{432} a_1^2 b_3 \\ &\quad - \frac{11}{12} a_1^2 c_3 + \frac{7}{144} b_2^2 + \frac{169}{216} a_1 b_2 b_3 - \frac{37}{36} a_1 b_2 c_3 + \dots, \end{aligned} \quad (1)$$

whereas from HBCHPT

$$b_1^{pn} = -2(F + D)^3 - \frac{2}{9}(F + D)^2 \mathcal{C}^2 - \frac{50}{81} \mathcal{H} \mathcal{C}^2 \quad (2)$$

Equations (1) and (2) are found to be the same under

$$\begin{aligned} D &= \frac{1}{2} a_1 + \frac{1}{6} b_3, & \mathcal{C} &= -a_1 - \frac{1}{2} c_3, \\ F &= \frac{1}{3} a_1 + \frac{1}{6} b_2 + \frac{1}{9} b_3, & \mathcal{H} &= -\frac{3}{2} a_1 - \frac{3}{2} b_2 - \frac{5}{2} b_3. \end{aligned}$$

**Both approaches yield the same results order by order**

## SOME NUMERICAL VALUES

Including corrections to order  $\mathcal{O}(1/N_c^2)$  to  $g_A$  one has<sup>11</sup>

Table 1: Values of  $g_A$  for various semileptonic processes.

Process	Total value	Tree level	Singlet piece	Octet piece	<b>27</b> piece
$n \rightarrow pe^- \bar{\nu}_e$	1.272	1.031	0.279	-0.040	0.002
$\Sigma^+ \rightarrow \Lambda e^+ \nu_e$	0.653	0.542	0.168	-0.057	0.000
$\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$	0.624	0.542	0.113	-0.031	-0.000
$\Lambda \rightarrow pe^- \bar{\nu}_e$	-0.904	-0.720	-0.134	-0.055	0.005
$\Sigma^- \rightarrow ne^- \bar{\nu}_e$	0.375	0.298	0.080	-0.002	-0.001
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$	0.139	0.178	-0.034	-0.004	-0.001
$\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$	0.869	0.729	0.128	0.014	-0.002
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$	1.312	1.031	0.246	0.041	-0.006

Calculation of higher-order corrections is rather involved.

<sup>11</sup>R.F.M., C.P. Hofmann, hep-ph/0609120

## CONCLUSIONS

- An alternative approach to write one-loop corrections in heavy baryon chiral perturbation theory has been proposed, including the functional dependence in  $\Delta \equiv M_{\Delta} - M_N$ .
- There are large cancellations in loops containing intermediate octet and decuplet baryon states.
- These cancellations arise naturally in this approach and not as a numerical cancellation at the end of the calculation.
- The one-loop correction is very sensitive to the deviations of the axial coupling ratios from their SU(6) values.