Nucleon matrix elements from lattice QCD

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http://talks.drubryantrenner.org/dpf06.pdf

Nucleon Matrix Elements

- Moments of Parton Distributions - unpolarized, helicity and transversity distributions

$$\langle x^n \rangle_q \qquad \langle x^n \rangle_{\Delta q} \qquad \langle x^n \rangle_{\delta q}$$

- Form Factors - vector and axial vector

$$F_1 \qquad F_2 \qquad G_A \qquad G_P$$

- Generalized Form Factors / Generalized Parton Distributions -

Transverse Structure - 3D distribution of quarks in a mixed representation: 2 transverse coordinates $\vec{b}_\perp$ and 1 longitudinal momentum x

$$q(x, \vec{b}_\perp) \qquad \Delta q(x, \vec{b}_\perp)$$

Spin Decomposition - decomposition of nucleon spin into quark helicity, quark orbital and gluon contributions (ask me during the questions)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma_{u+d} + L_{u+d} + J_g$$

Comparing with Phenomenology - full determination of GPDs requires combination of experiment, lattice and phenomenology (ask me during the questions)

Mixed Action Lattice Calculation

- asqtad staggered sea quarks (MILC) with $a = 0.124$ fm

$am_{u/d}^{\text{asqtad}}$	L/a	L	m_π^{DWF}	#
		fm	MeV	
0.05	20	2.52	761	425
0.04	"	"	693	350
0.03	"	"	594	564
0.02	"	"	498	486
0.01	"	"	354	656
0.01	28	3.53	353	270

- domain wall valence quarks with HYP smearing and $L_5 = 16$
- one-loop perturbative renormalization at $\mu = 2$ GeV
- chiral perturbation theory following hep-lat/0610007
- please see hep-lat/0409130 for more details

Moments of Parton Distributions

- for example, unpolarized parton distributions

$$q(x) \;=\; \langle P,S|\int\frac{dy^-}{4\pi}\; e^{ixP^+y^-}\overline{q}(-y^-/2)\gamma^+q(y^-/2)|P,S\rangle$$

- light-cone expansion generates unpolarized twist-two operators

$$O_q^{\mu_1\cdots\mu_n} \;=\; \overline{q}iD^{(\mu_1}\ldots iD^{\mu_{n-1}}\gamma^{\mu_n)}q$$

- moments of parton distributions from forward matrix elements

$$\langle P,S|O_q^{\mu_1\ldots\mu_n}|P,S\rangle = 2\,\langle x^{n-1}\rangle_q\,P_{\mu_1}\cdots P_{\mu_n}$$

- unpolarized, helicity and transversity moments

$$\langle x^n\rangle_q \;=\; \int_{-1}^{1}dx\,x^nq(x)$$

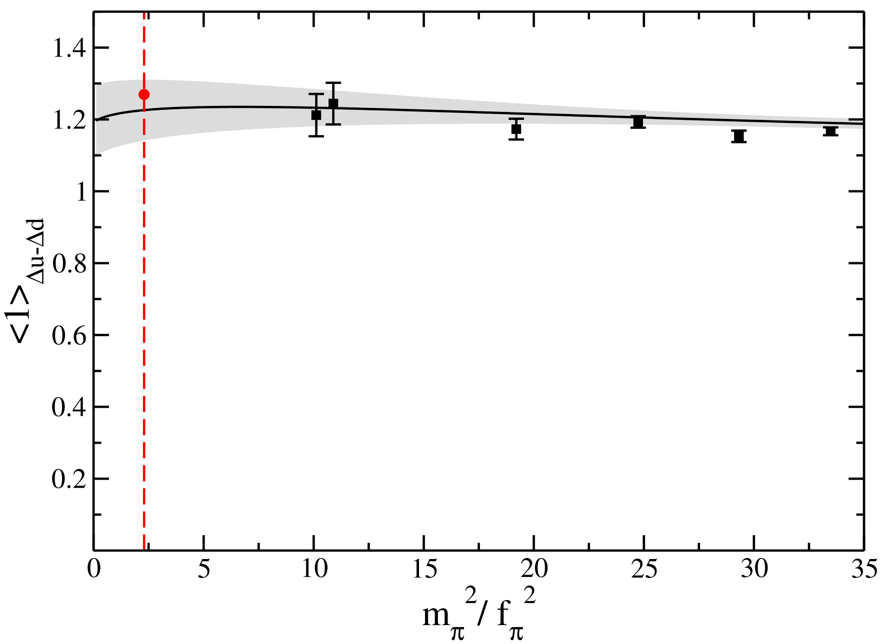
$$\langle x^n\rangle_{\Delta q} \;=\; \int_{-1}^{1}dx\,x^n\Delta q(x)$$

$$\langle x^n\rangle_{\delta q} \;=\; \int_{-1}^{1}dx\,x^n\delta q(x)$$

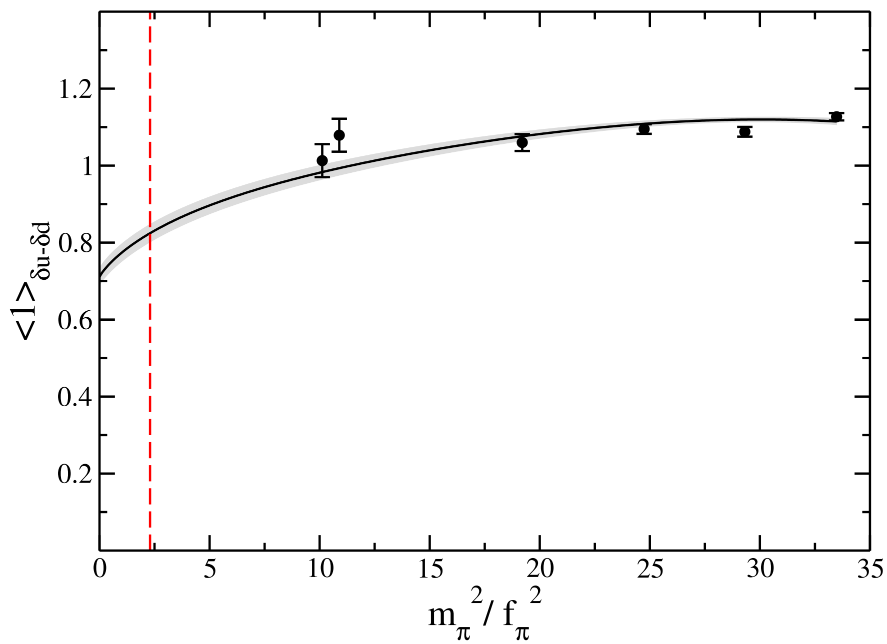
- please see, for example, hep-lat/0201021 for more details

Lowest Moments: Axial and Tensor Charges

$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$

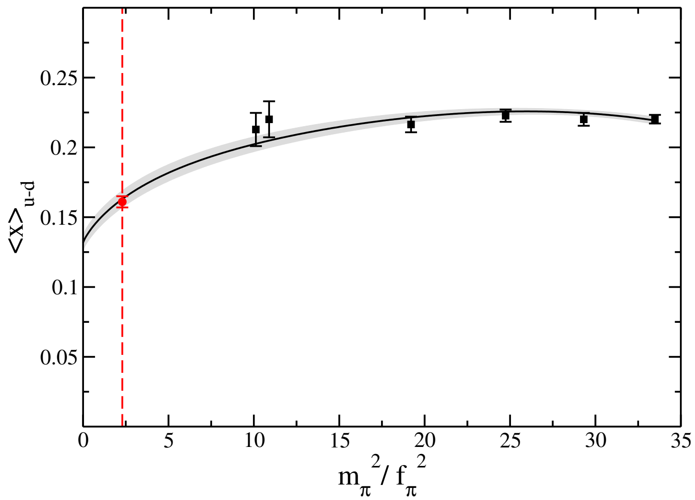


$g_T = \langle 1 \rangle_{\delta u - \delta d}$

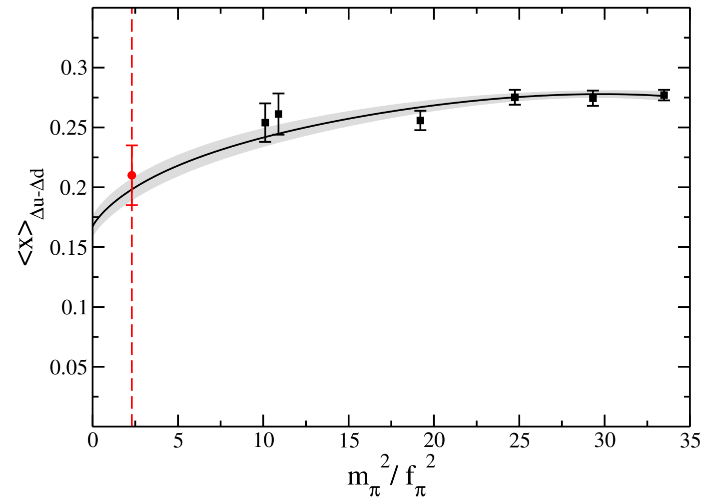


Momentum Fractions and Higher Moments

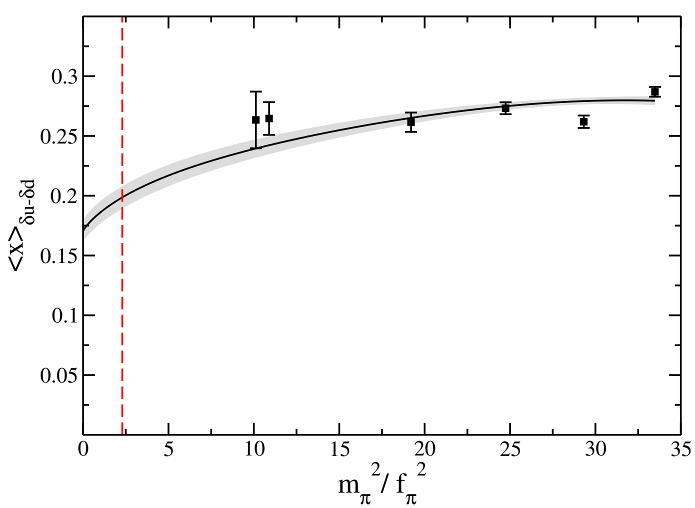
$\langle x \rangle_{u-d}$



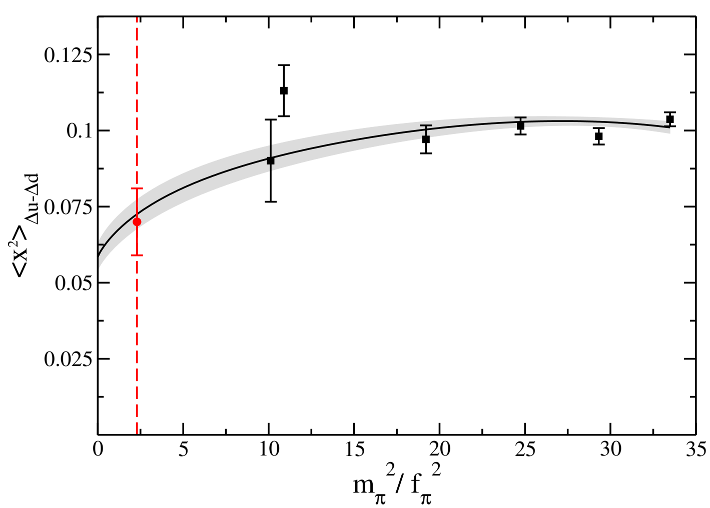
$\langle x \rangle_{\Delta u - \Delta d}$



$\langle x \rangle_{\delta u - \delta d}$

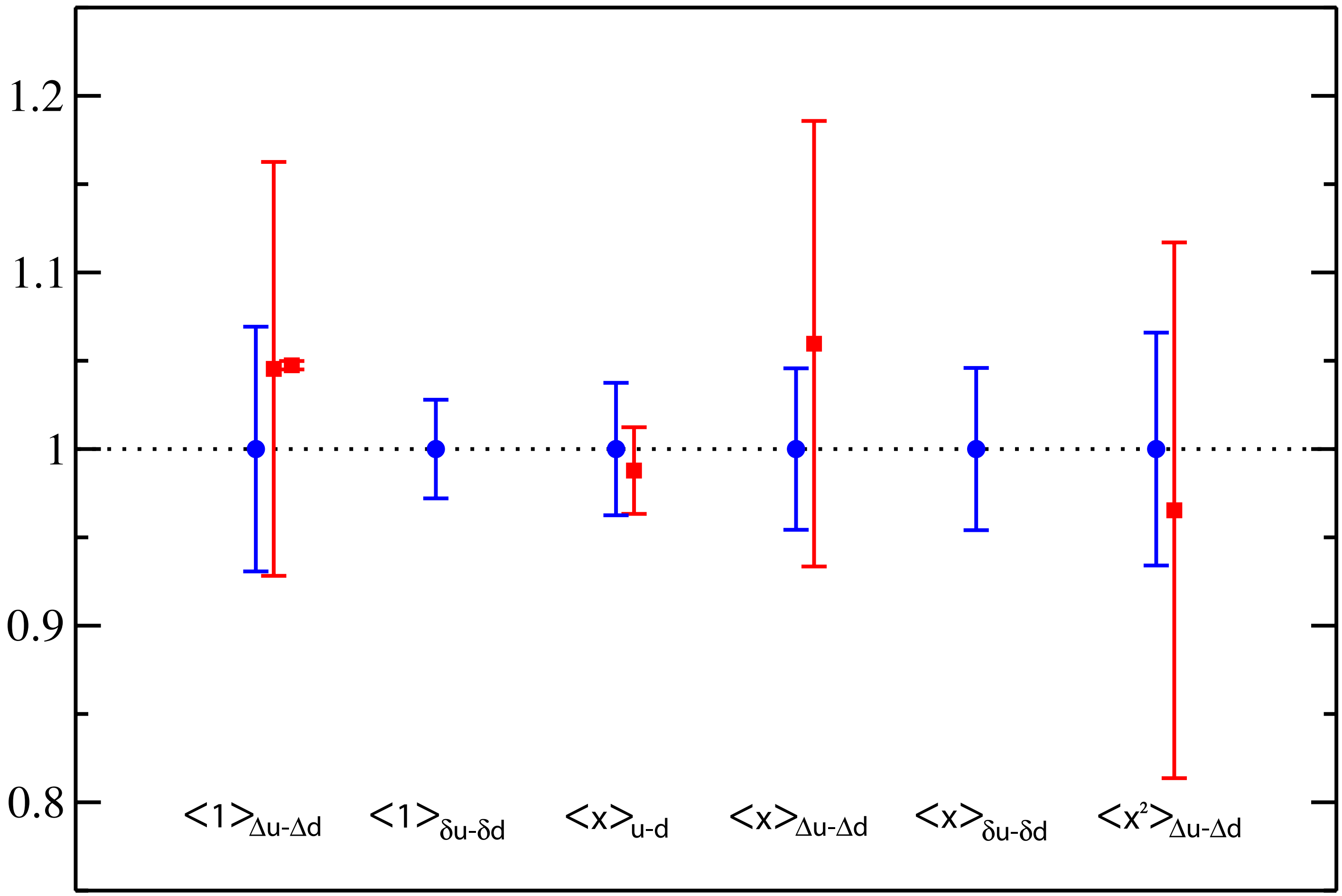


$\langle x^2 \rangle_{\Delta u - \Delta d}$



Comparing with Experiment

blue is lattice, red is experiment, both normalized to lattice



Form Factors

- vector form factors: F_1 and F_2, from off-forward matrix elements of the vector current, $\bar{q}\gamma_\mu q$, a twist-two operator

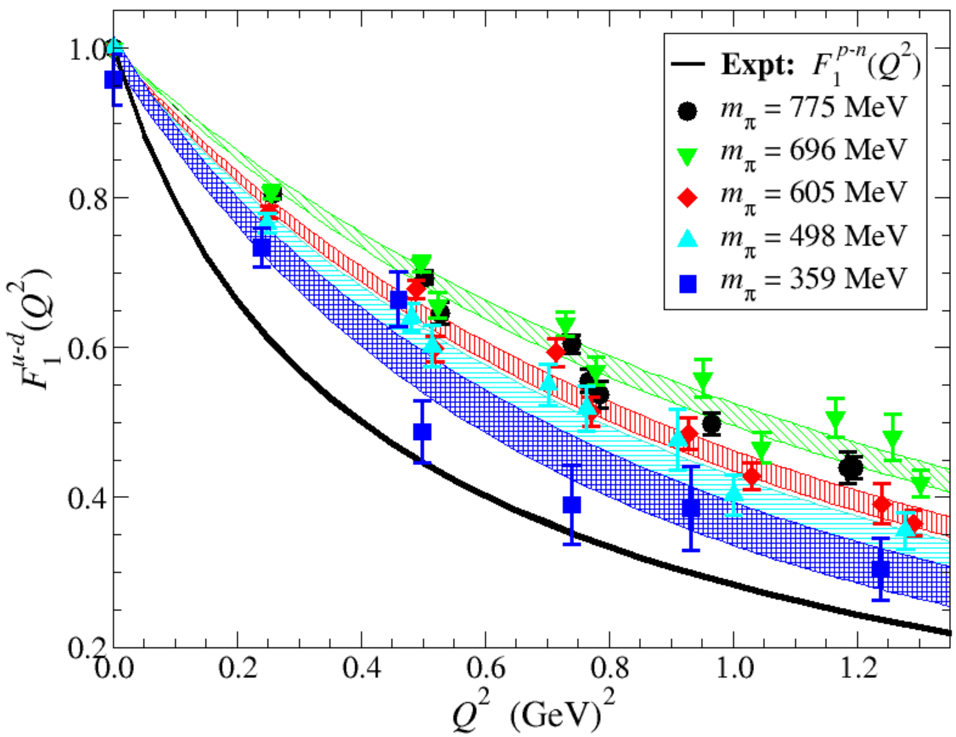
$$< P', S'|J^\mu|P, S >= \overline{U}(P', S')\left(\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}F_2(t)\right)U(P, S)$$

- axial-vector form factors: G_A and G_P, from off-forward matrix elements of the axial-vector current, $\bar{q}\gamma_5\gamma_\mu q$, again a twist-two operator

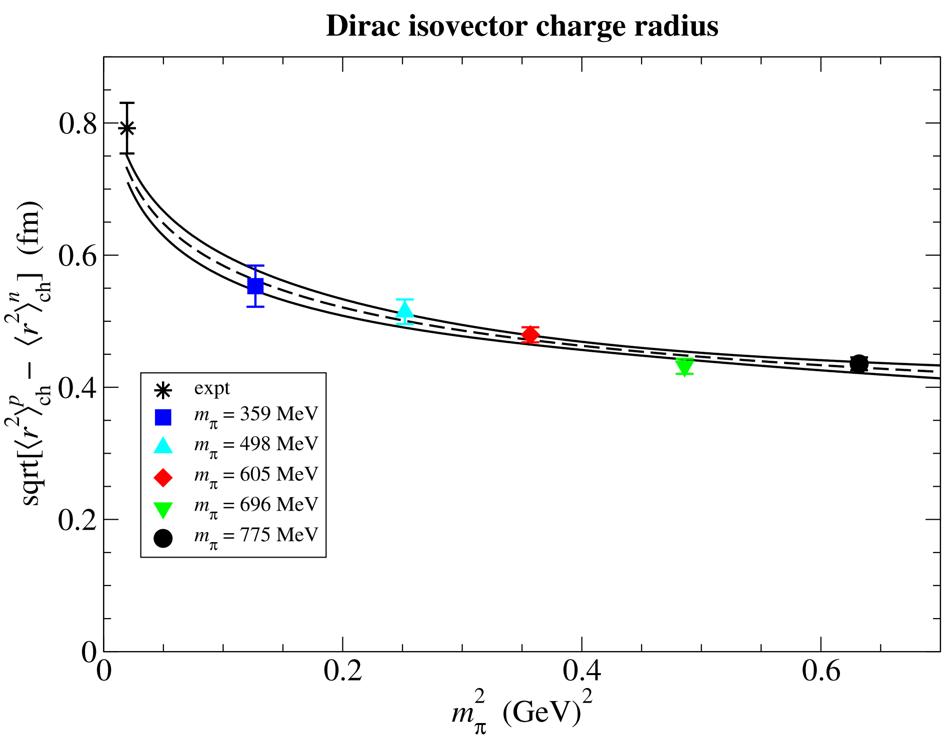
$$< P', S'|J^\mu|P, S >= \overline{U}(P', S')\left(\gamma^\mu\gamma_5 G_A(t) + \frac{\Delta^\mu\gamma_5}{2m}G_P(t)\right)U(P, S)$$

Vector Form Factors: F_1 and F_2

F_1^{u-d}

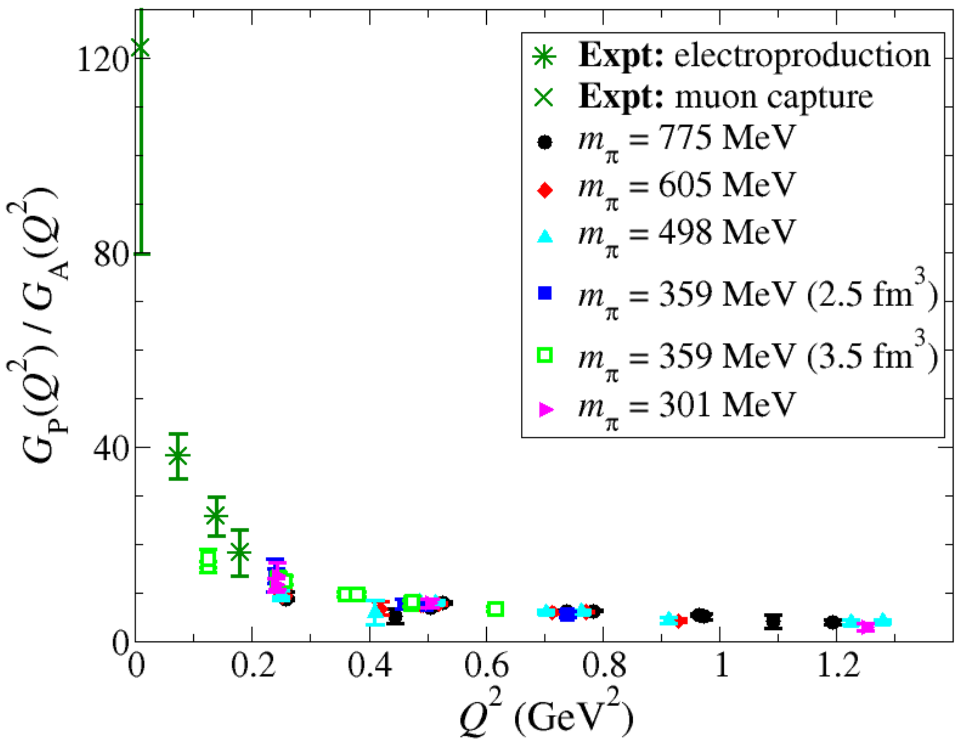


$\sqrt{\langle r^2 \rangle_\perp^{u-d}}$

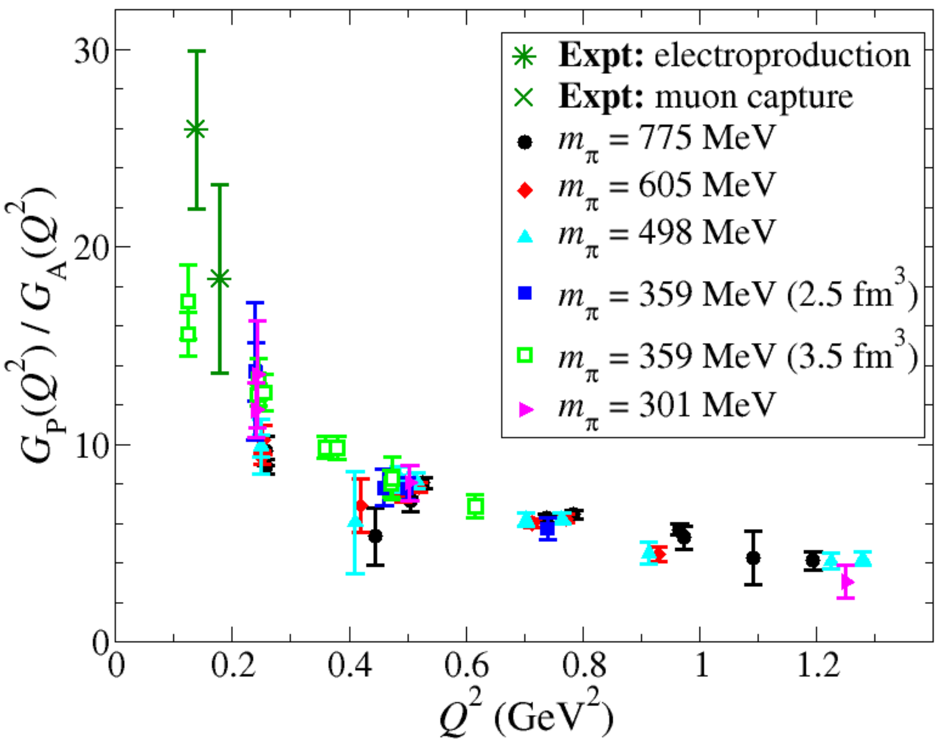


Axial Vector Form Factors: G_A and G_P

G_P^{u-d}/G_A^{u-d}



zoom of G_P^{u-d}/G_A^{u-d}



Generalized Form Factors

- for example, unpolarized twist-two operators

$$O_q^{\mu_1 \cdots \mu_n} = \bar{q} i D^{(\mu_1} \ldots i D^{\mu_{n-1}} \gamma^{\mu_n)} q$$

- off-forward matrix elements of the twist-two operators [1]

$$\langle P', S' | O_q^{\mu_1 \ldots \mu_n} | P, S \rangle = \overline{U}(P', S') [\sum_{\substack{i=0 \\ \text{even}}}^{n-1} A_{ni}^q(t) K_{ni}^A(P', P)$$
$$+ \sum_{\substack{i=0 \\ \text{even}}}^{n-1} B_{ni}^q(t) K_{ni}^B(P', P) + \delta_{\text{even}}^n C_n^q(t) K_n^C(P', P)] U(P, S)$$

- similar expression for the helicity form factors: $\tilde{A}_{ni}^q(t)$ and $\tilde{B}_{ni}^q(t)$

- examples, $A_{10}(t) = F_1(t)$, $A_{20}(0) = \langle x \rangle$, and higher moments A_{30} ...

[1] X. D. Ji hep-ph/9807358

Transverse Distributions: Q^2 Dependence

- generalized form factors, A_{n0}, are Fourier transforms of quark distributions [1]

$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx \; x^{n-1} \, q(x, \vec{b}_\perp)$$

- the slopes are related to the transverse rms radius

$$\langle b^2_\perp \rangle^q_n = -4 \, \frac{A^{q\,\prime}_{n0}(0)}{A^q_{n0}(0)}$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x\to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- higher moments A^q_{n0} weight $x \sim 1$ more heavily

$$\lim_{n\to\infty} A^q_{n0}(t) \propto \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \delta^2(\vec{b}_\perp) = \text{constant}$$

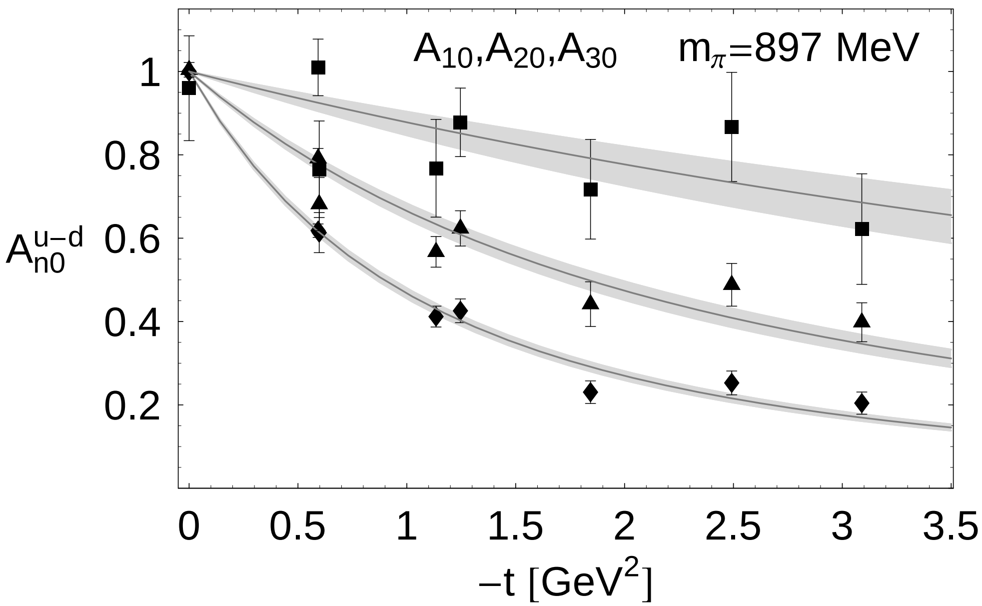
- slopes of A^q_{n0} should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q - \overline{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q + \overline{q}$

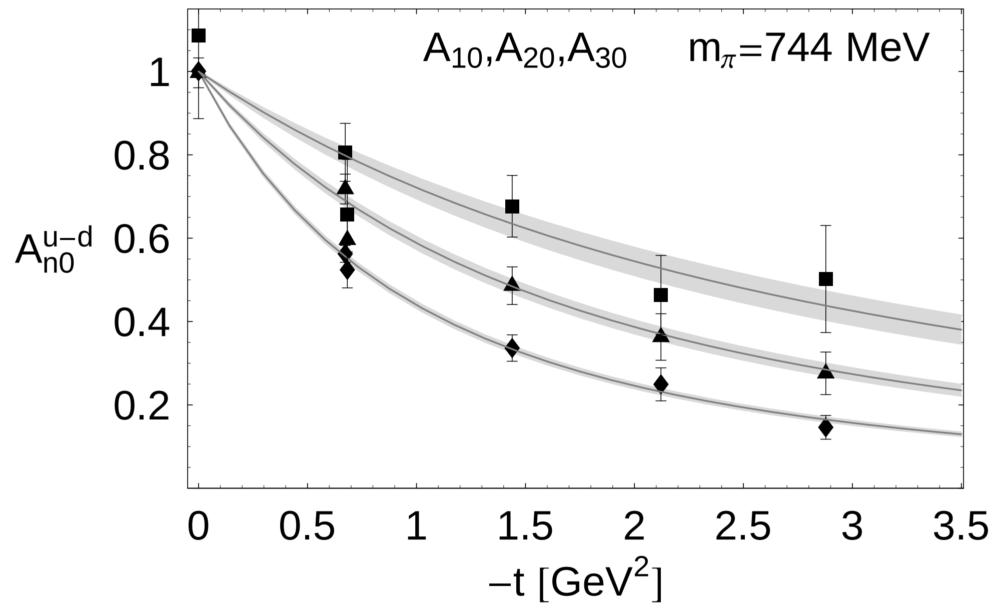
[1] M. Burkardt hep-ph/0005108

Transverse Distributions: Q^2 Dependence

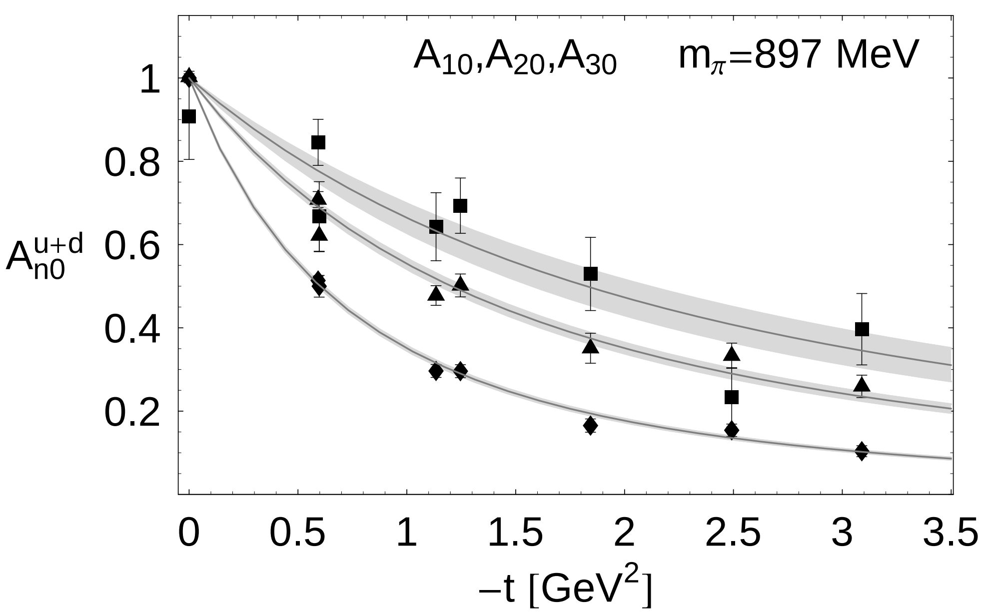
$m_\pi = 900$ MeV



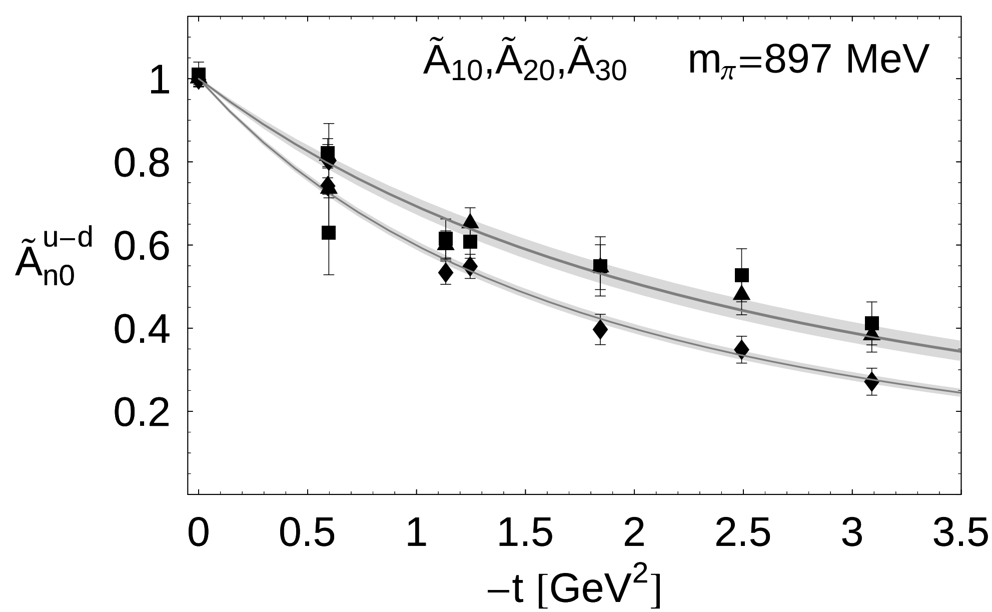
m_π dependence



flavor dependence

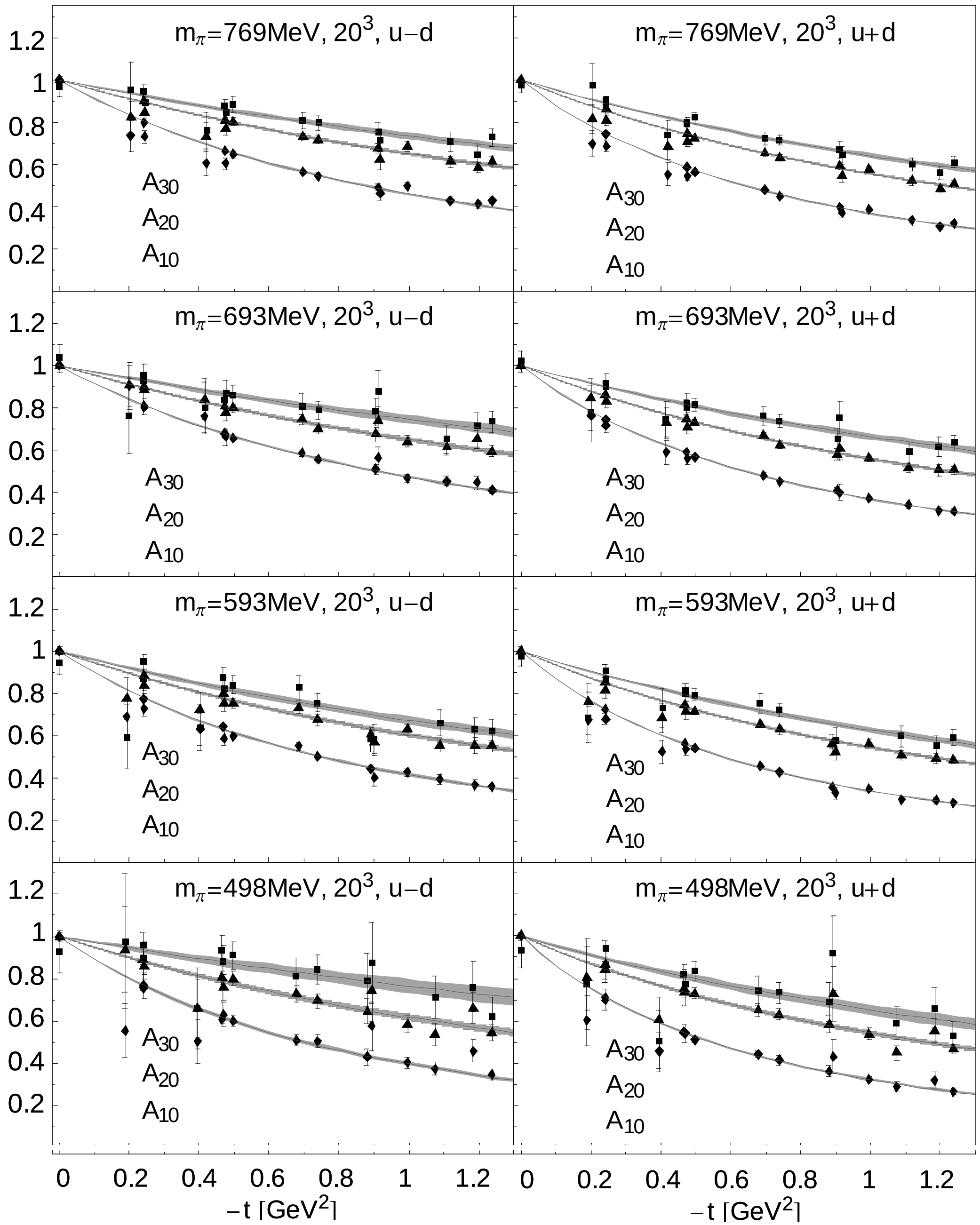


spin dependence



Transverse Distributions: Q^2 Dependence

more m_π dependence



Transverse Distributions: x and $\vec{b}_\perp$ Dependence

- transverse rms radius (momentum space)

$$\left\langle b_\perp^2 \right\rangle_x = \frac{\int d^2b_\perp \, b_\perp^2 \, q(x,\vec{b}_\perp)}{\int d^2b_\perp \, q(x,\vec{b}_\perp)}$$

- transverse rms *moment* radius (Mellin space)

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4}\left\langle b_\perp^2 \right\rangle_n \qquad \left\langle b_\perp^2 \right\rangle_n = \frac{\int d^2b_\perp \, b_\perp^2 \; \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)}{\int d^2b_\perp \; \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)}$$

- transverse distribution of quarks

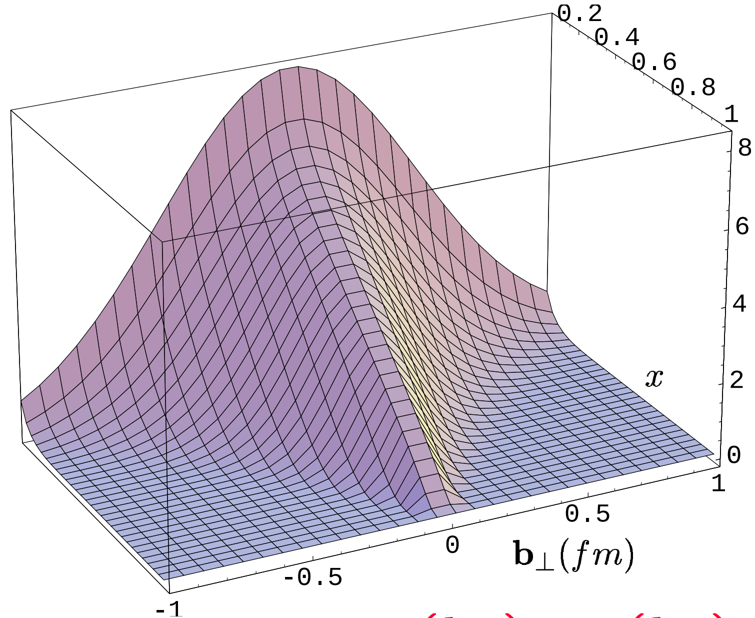
$$q_1(\vec{b}_\perp) = \int_{-1}^{1} dx \; q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{10}^q(-\vec{\Delta}_\perp^2)$$

- transverse distribution of momentum

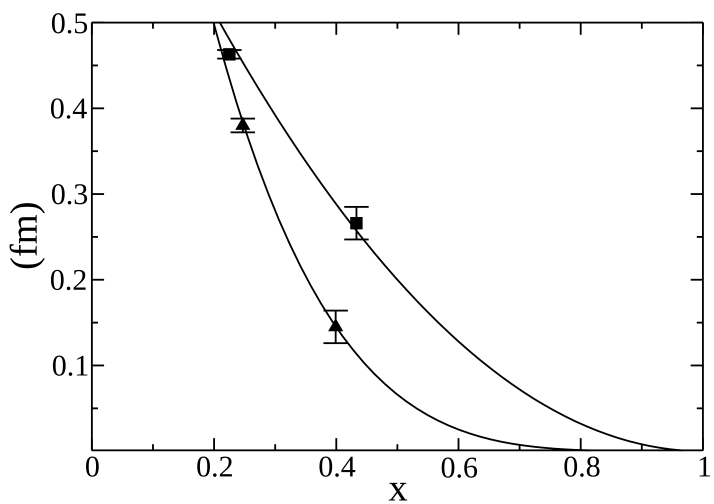
$$q_2(\vec{b}_\perp) = \int_{-1}^{1} dx \; x \, q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{20}^q(-\vec{\Delta}_\perp^2)$$

Transverse Distributions: x and $\vec{b}_\perp$ Dependence

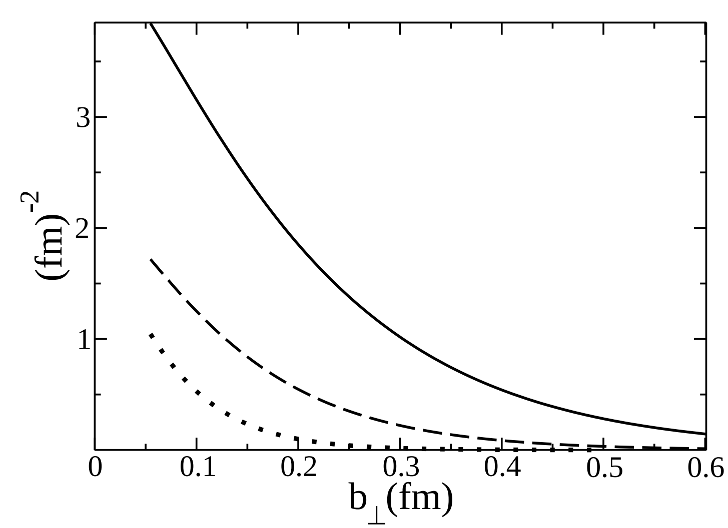
$q(x, \vec{b}_\perp)$ from model



$\left\langle b_\perp^2 \right\rangle_n$



$q_1(b_\perp)$, $q_2(b_\perp)$, $q_3(b_\perp)$



graph from M. Burkardt hep-ph/0207047

Conclusions

- challenges include: lighter pion masses, finer lattice spacings, higher q^2, disconnected diagrams, higher moments, gluon observables, and on

- chiral extrapolations give rise to results in the physical limit that agree to within statistical errors with experimental measurements of $\langle 1 \rangle_{\Delta u - \Delta d}$, $\langle x \rangle_{u-d}$, $\langle x \rangle_{\Delta u - \Delta d}$ and $\langle x^2 \rangle_{\Delta u - \Delta d}$

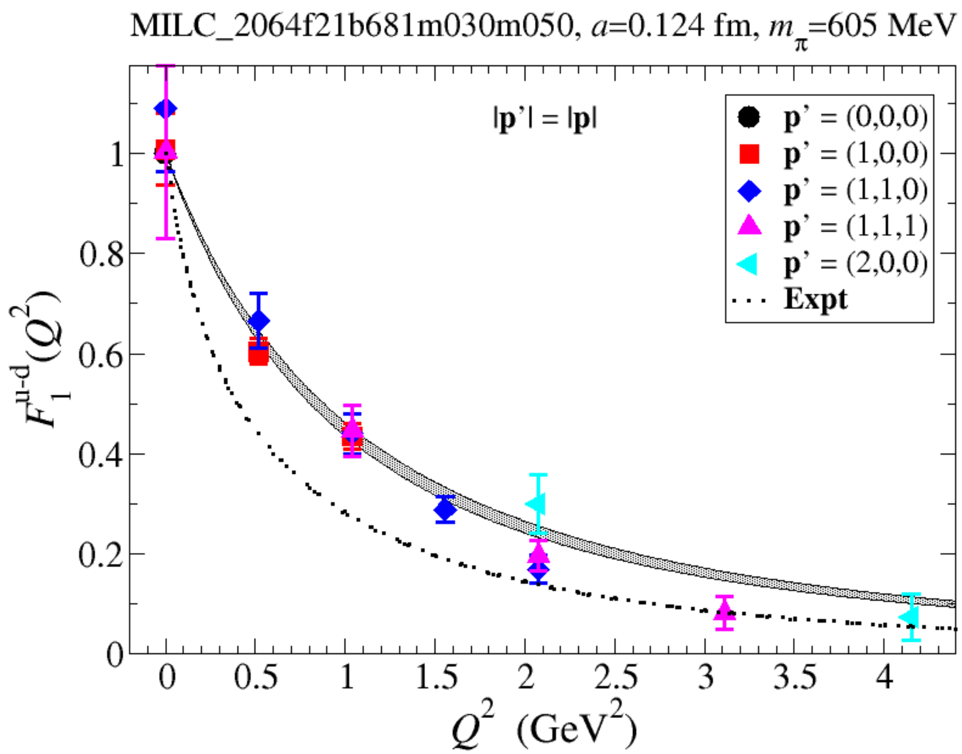
- careful study of systematic errors should lead to genuine predictions for $\langle 1 \rangle_{\delta u - \delta d}$ and $\langle x \rangle_{\delta u - \delta d}$

- accurate calculations and even predictions for moments of parton distributions will bolster the strength behind calculations of the nucleon's form factors and generalized form factors and other hadronic observables that might not be experimentally accessible

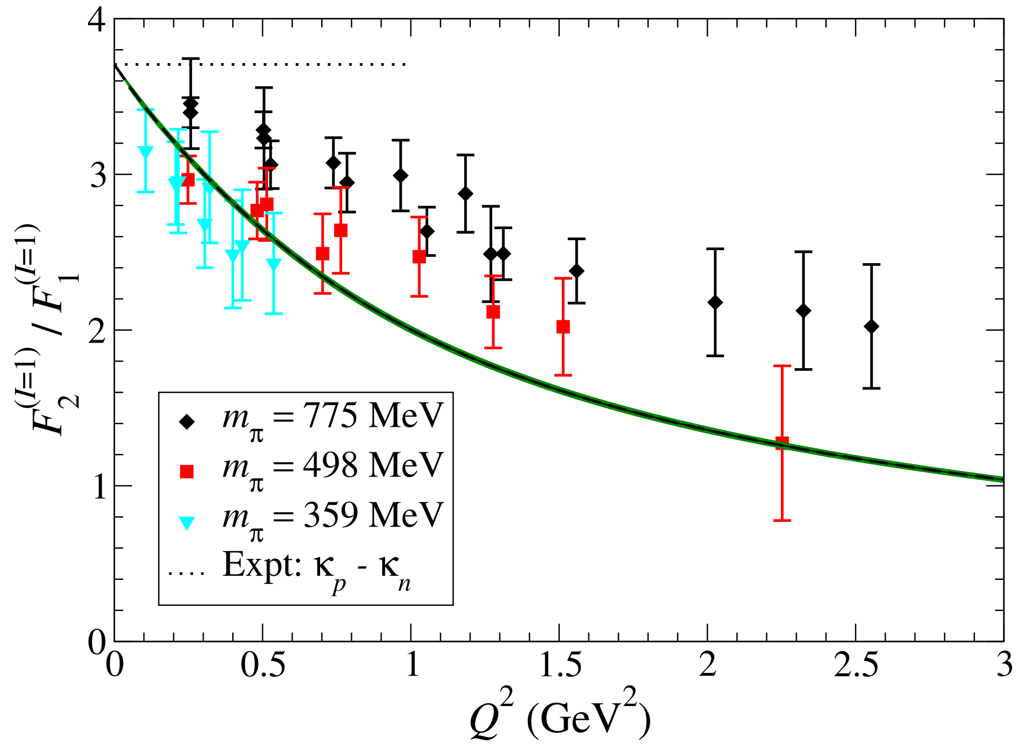
Extra Slides

Vector Form Factors: F_1 and F_2

F_1^{u-d} at higher Q^2

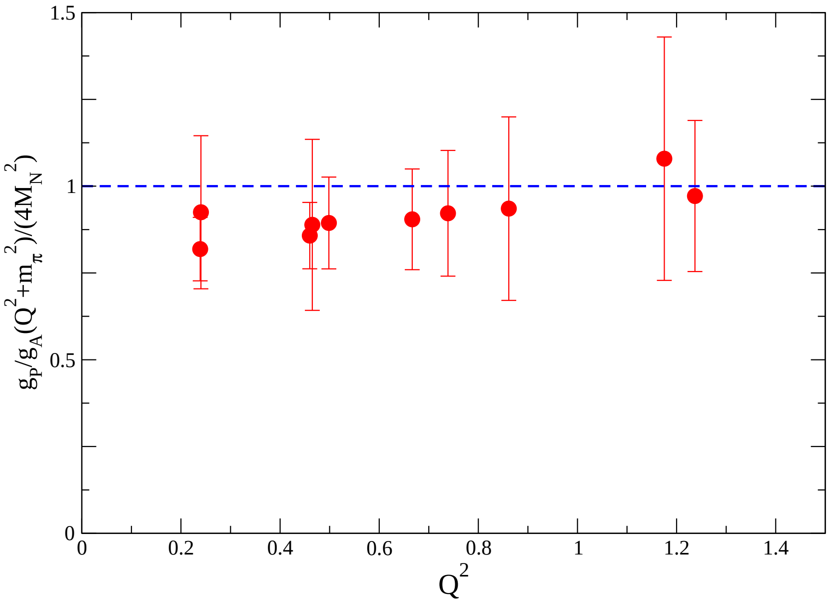


F_2^{u-d}/F_1^{u-d}

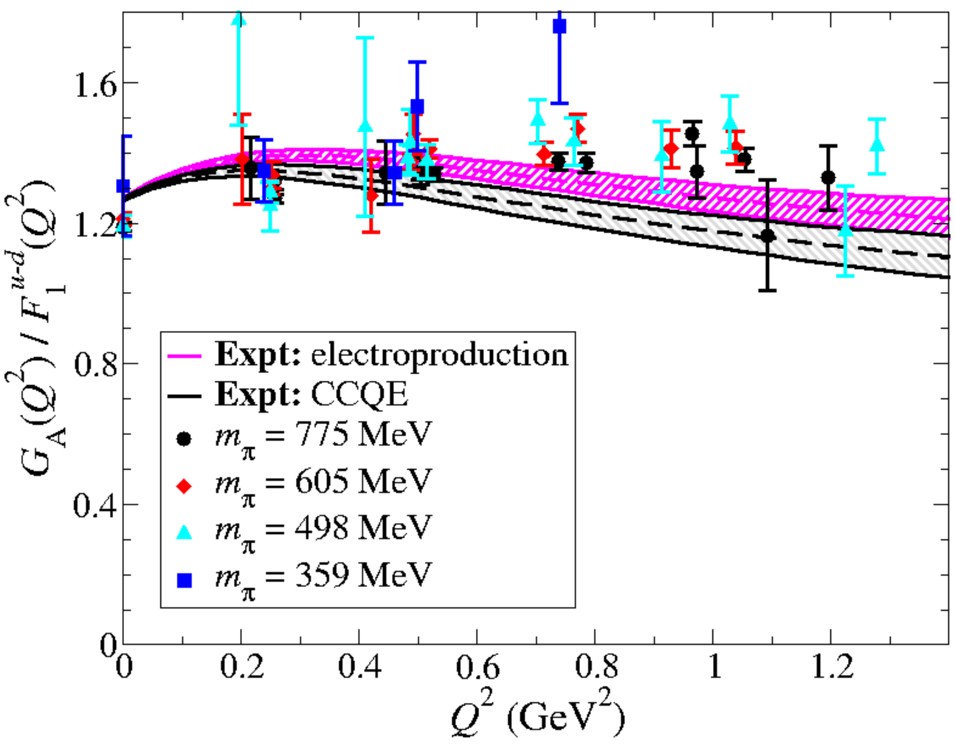


Axial Vector Form Factors: G_A and G_P

pion pole



G_P^{u-d}/F_1^{u-d}



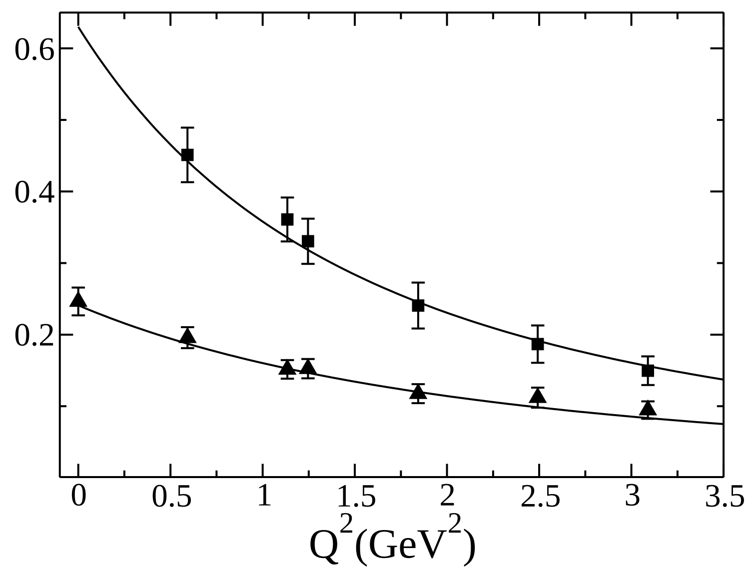
Spin Decomposition

- quark helicity, quark orbital, and gluon contributions [1]

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

$$\Delta\Sigma_q = \frac{1}{2}\tilde{A}^q_{10}(0) \qquad J_q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$$

$$L_q = J_q - \frac{1}{2}\Delta\Sigma_q \qquad J_g = \frac{1}{2} - J_{u+d}$$

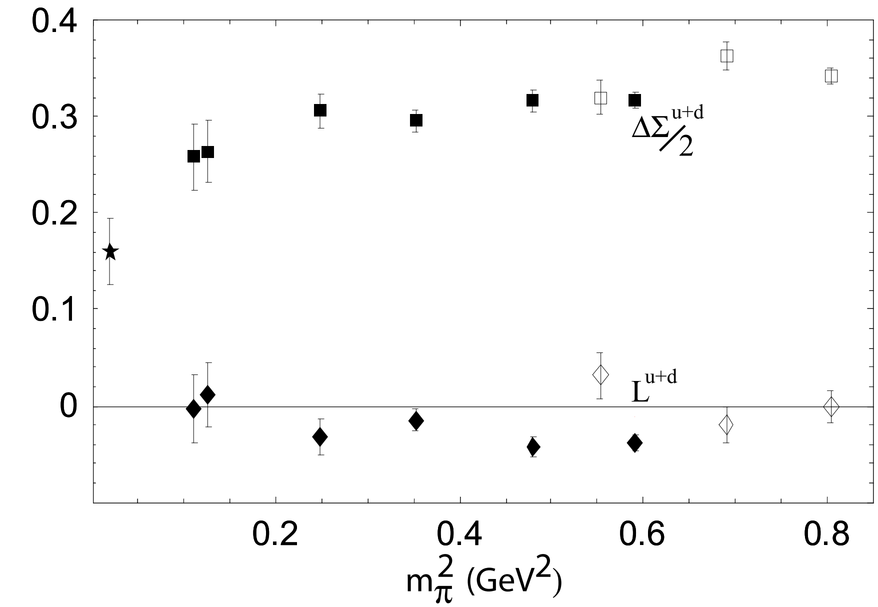


- large N_C: $B^{u-d}_{ni}/A^{u-d}_{ni} \sim N_C$

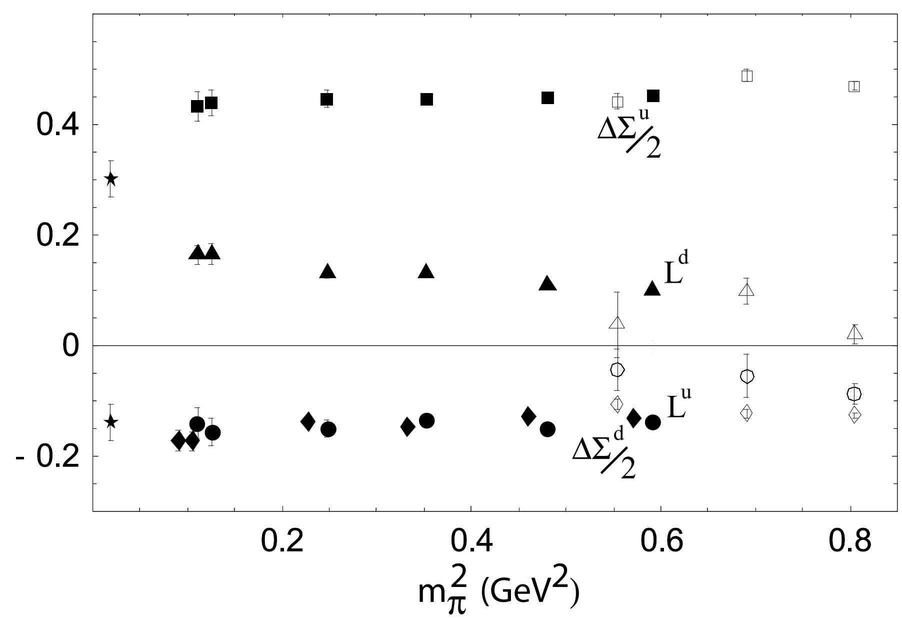
[1] X. D. Ji hep-ph/9603249

Spin Decomposition

total spin decomposition

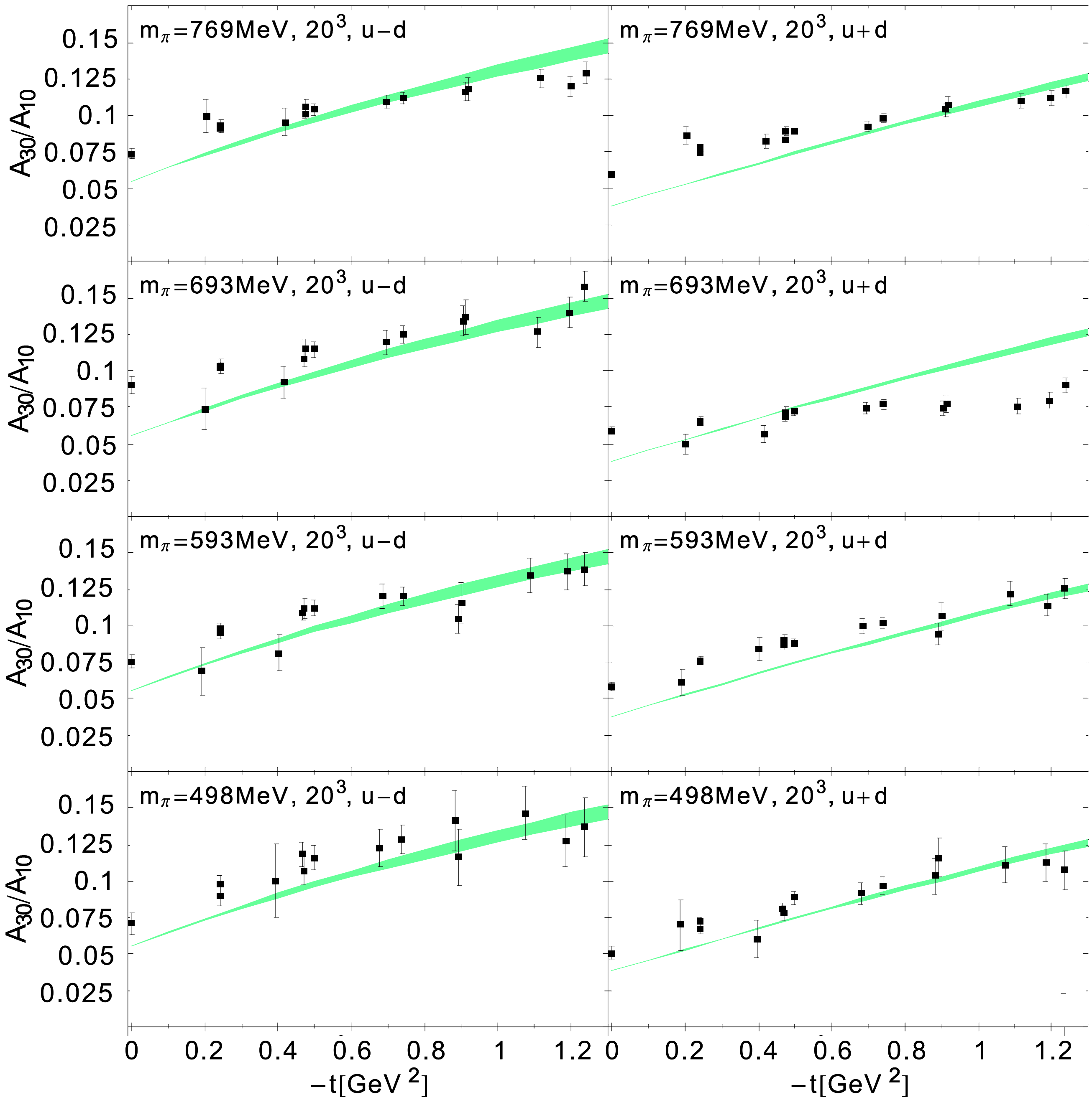


flavor contributions



Phenomenology

phenomenological curves from M. Diehl, et. al. hep-ph/0408173



Moments of Generalized Parton Distributions

- moments of generalized parton distributions

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,\xi,t) = \sum_{\substack{i=0 \\ \text{even}}}^{n-1} A^q_{ni}(t)(-2\xi)^i + \delta^n_{\text{even}} C^q_n(t)(-2\xi)^n$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,\xi,t) = \sum_{\substack{i=0 \\ \text{even}}}^{n-1} B^q_{ni}(t)(-2\xi)^i - \delta^n_{\text{even}} C^q_n(t)(-2\xi)^n$$

- transverse momentum transfer, $\xi \to 0$

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,0,t) = A^q_{n0}(t)$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,0,t) = B^q_{n0}(t)$$

- similar results relating the polarized GPDs, $\tilde{H}_q(x,0,t)$ and $\tilde{E}_q(x,0,t)$, to the polarized GFFs, $\tilde{A}^q_{n0}(t)$ and $\tilde{B}^q_{n0}(t)$

Quark Angular Momentum

- angular momentum operator

$$J_q^i = \frac{1}{2}\epsilon^{ijk} \int d^3x \left(T_q^{0k} x^j - T_q^{0j} x^k\right)$$

- energy momentum tensor

$$T_q^{\mu\nu} = \overline{q}\gamma^{\{\mu} iD^{\nu\}} q = O_q^{\mu\nu}$$

- angular momentum

$$J_q = \langle P, 1/2 | J_q^z | P, 1/2\rangle \quad \text{involves} \quad \langle P, 1/2 | O_q^{\mu\nu} | P, 1/2\rangle$$

- matrix elements of $n = 2$ twist 2 operator $O_q^{\mu\nu}$: $A_{20}(t)$, $B_{20}(t)$, $C_2(t)$

$$J_q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$$

Transverse Quark Distribution

- wave packet in transverse coordinates

$$|\psi\rangle = \int \frac{d^2 p_\perp}{(2\pi)} \frac{\psi(\vec{p}_\perp)}{\sqrt{2E_{\vec{p}}}} |\vec{p}_\perp, P_z\rangle \qquad \vec{p} = (\vec{p}_\perp, P_z)$$

- transverse quark distribution

$$q_\psi(x, \vec{b}_\perp) = \langle\psi|O_q(x, \vec{b}_\perp)|\psi\rangle \quad \text{involves} \quad \langle\vec{k}_\perp, P_z|O_q(x, \vec{b}_\perp)|\vec{p}_\perp, P_z\rangle$$

- infinite momentum limit & transverse localization

$$M \ll P_z \qquad \psi(\vec{k}_\perp) \approx \psi(\vec{p}_\perp)$$

- matrix elements of $O_q(x, \vec{b}_\perp)$: $H_q(x, \xi, t)$, $E_q(x\ , \xi, t)$

$$q(x, \vec{b}_\perp) = \int d^2\Delta_\perp \, e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H_q(x, 0, -\vec{\Delta}_\perp^2)$$

- relativistic corrections controlled by

$$\frac{1}{\sqrt{M^2 + P_z^2}} \quad \text{not} \quad \frac{1}{M}$$