Wilsonian RG analysis of Nuclear EFT

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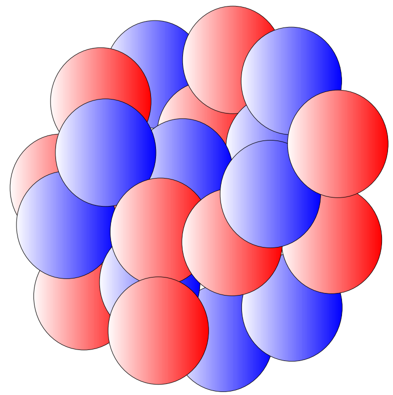
IV. Summary

This talk is based on works in collaboration with Koji Harada (Kyushu University)



Characteristic Features of Nuclear Interaction

- Very strong
- Depends on the states of nucleons (spin, isospin)
- Binding energy/nucleon ~ 8MeV (stable nuclei)

Two Nucleon System

- Very shallow ($E_b \sim 2.2$MeV) bound state; deuteron in ${}^3S_1$ channel
- Very large ($1/a^{({}^1S_0)} \sim -8.3$MeV) scattering length in ${}^1S_0$ channel

Fine-Tuning

Range of NN potential, $R \sim \frac{1}{m_\pi}$, uncertainty principle $p \gtrsim \frac{1}{R} \sim m_\pi$

$$H = T + V, \quad T \sim \frac{m_\pi^2}{M} \sim \frac{(140)^2}{1000} \sim 20\text{MeV}, \quad M : \text{nucleon mass}$$

$H \sim T$ (assuming that there are no delicate cancellations)

$2.2\text{MeV} \ll 20\text{MeV}$(energy scale)

$8\text{MeV} \ll 140\text{MeV}$(momentum scale) Finely-Tuned System !

Potential Model Approach

Few nucleon systems — **“Realistic ” NN Potential,** V_{NN}
CD Bonne, AV_{18}, Paris, Nijimegen I,II, $\cdots$ excellent agreement with experiments

Problems in Potential Model Approach

- Systematic improvement(estimation of errors) seems difficult
- Why are 3-body, 4-body,.. forces weaker than 2-body force ?
- Relation to QCD e.g. chiral symmetry, quark mass dependence of nuclear force
- Off-shell ambiguity (could be a problem in multi-nucleon systems)

We need a model-independent description !

Effective Field Theory (EFT)

Weinberg's "folk" theorem

The most general Lagrangian consistent with assumed symmetry

$\Downarrow$

The most general S-matrix consistent with general principles of QFT (analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles)

Nontrivial Assumptions

- Identification of relevant low energy degrees of freedom
- **Power Counting**: Rule to organize the contributions

Usually,

Naive Dimensional Analysis (NDA) $\sim$ Naturalness

$\longrightarrow$ **Classical** Dimensional Analysis

Contribution from an operator of mass dimension d_i: $a_i \left(\frac{Q}{\Lambda_0}\right)^{d_i}$, $\quad a_i \sim O(1)$

$\Rightarrow d_i \rightarrow$ larger, more suppressed

In the case in which quantum corrections are small, NDA should work

However,

Nuclear system is

Nonperturbative $\&$ **Finely-tuned**

$\Downarrow$

NDA would **not** work

How do we find the correct Power Counting?

What is Power Counting ? $\rightarrow$ **Dimensional Analysis (Quantum theoretical)**

Wilsonian RG allows us to determine the dimension of an operator

Legendre Flow Equation:

$$\frac{d\Gamma_\Lambda}{dt} = \frac{i}{2}\mathrm{Tr}\ \tilde{\partial}_t \left[\ln\left(\Gamma_{(2)} + R_\Lambda\right)\right], \quad \left(\Gamma_{(2)}\right)_{nm} \equiv \frac{\delta^2\Gamma_\Lambda}{\delta\Phi_n\delta\Phi_m}, \quad \Lambda = \Lambda_0 e^{-t}$$

(Effective average action, Γ_Λ: Usual effective action+IR cutoff.)

RG transformation generates all operators allowed by the symmetry

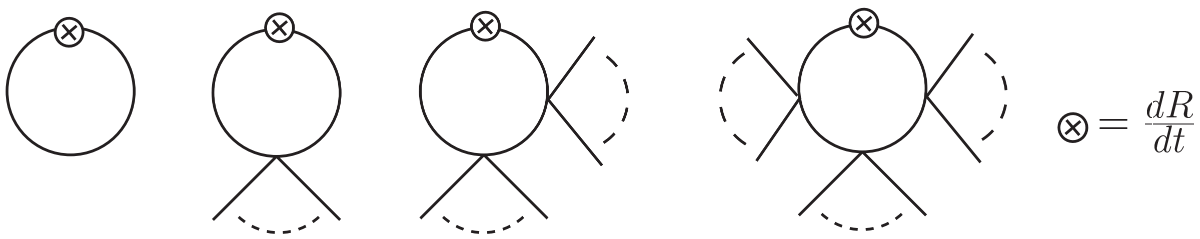
$$\Gamma_\Lambda = \int d^4x \sum_{i=all} G_i(\Lambda)\mathcal{O}_i, \quad G_i(\Lambda) \equiv \frac{g_i(\Lambda)}{\Lambda^{d_i-4}}, \quad d_i : G_i \text{ has dimension } (4-d_i).$$

One-Loop Structure

$\mathcal{P}^{-1}$: **full** propagator (with IR cutoff)

$$\frac{d\Gamma_\Lambda}{dt} = \frac{i}{2}\mathrm{Tr}\;\tilde{\partial}_t \left[\ln \mathcal{P} + \left(\mathcal{P}^{-1}\mathcal{F}\right) - \frac{1}{2}\left(\mathcal{P}^{-1}\mathcal{F}\right)^2 + \frac{1}{3}\left(\mathcal{P}^{-1}\mathcal{F}\right)^3 + \cdots \right]$$

Graphically



$\Rightarrow \beta_i(g) = -\Lambda \frac{dg_i}{d\Lambda}$, (An infinite # of coupled first-order differential equations)

Fixed Point and Scaling Dimension

- Fixed point: $\beta_i(g^\star) = 0$

Linearized RG equation: $g_i = g_i^\star + \delta g_i, \quad -\Lambda \frac{d}{d\Lambda}\delta g_i = \underbrace{\left.\frac{d\beta_i(g)}{dg_j}\right|_{g=g^\star}}_{A_{ij}(g^\star)} \delta g_j$

Diagonalize $A_{ij}(g^\star)$

Solution near the fixed point

$$\frac{d\mathbf{u_i}}{dt} = \nu_i \mathbf{u_i}, \quad \mathbf{u_i}(\Lambda) = \mathbf{u_i}(\Lambda_0)\left(\frac{\Lambda_0}{\Lambda}\right)^{\nu_i} \quad \Lambda_0 : \text{physical cutoff of EFT}$$

ν_i: Scaling Dimension (of the coupling)

$\nu > \mathbf{0} \Rightarrow$ Relevant, $\quad \nu = \mathbf{0} \Rightarrow$ Marginal, $\quad \nu < \mathbf{0} \Rightarrow$ Irrelevant

Dimensional Analysis (Λ_0-dependence)

Dimensionful coupling:

$$G_i(\Lambda) = \frac{g_i(\Lambda)}{\Lambda^{d_i - D}} \sim \underbrace{\frac{g_i^\star}{\Lambda^{d_i - D}}}_{G_i^\star} + \sum_k c_{ik} \frac{\Lambda_0^{\nu_k}}{\Lambda^{d_i - D + \nu_k}} \quad \text{small constant}$$

• Right on a fixed point ($c_{ik} = 0$): $\Rightarrow$ Scale Invariant theory (fixed point value, $G_i^\star$ itself does not contribute to Dimensional Analysis)

• Vicinity of a fixed point ($c_{ik} \neq 0$): $\Rightarrow$ k-th term in the sum acts as a coupling constant with dimension ν_k

Scaling dimensions are not prescribed, but determined by the theory itself.

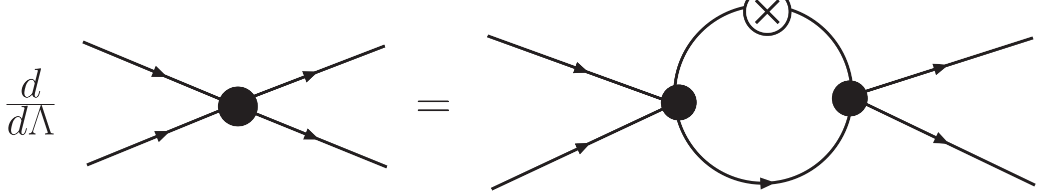
Two-Nucleon Sector

Due to the NR feature : No anti-nucleons (as explicit degrees of freedom)

$\longrightarrow$ Nucleon number is conserved.

$\longrightarrow$ $\left(N^\dagger N\right)^3, \left(N^\dagger N\right)^4, \cdots$ do not contribute to two-nucleon sector.

$$\left.\frac{d\Gamma_\Lambda}{dt}\right|_{4\text{N}} = -\frac{i}{4}\text{Tr}\ \tilde{\partial}_t \left[(\mathcal{P}^{-1}\mathcal{F})^2\right]\Big|_{4\text{N}}$$ **Exact equation**



● = four-nucleon vertices $\quad \otimes = \frac{dR}{d\Lambda}$

Truncation

Some kind of approximation is necessary for practical calculations.

$\Rightarrow$ Truncate the operator space

- Expectation: Importance of operators may be different from the one given by NDA, their orderings would not change.
  The lower an operator's dimension, the less important it is.

- Reliability of the truncation may be checked by the stability against the enlargement of the operator space.

Pionless NEFT

Truncation: by number of derivatives, $O(\nabla^2, \partial_t)$ (regarding $\nabla^2 \sim \partial_t$)

$$\Gamma_\Lambda = \int d^4x N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M}\right) N - C_0^{(S)}\mathcal{O}_0^{(S)} + C_2^{(S)}\mathcal{O}_2^{(S)} + 2B^{(S)}\mathcal{O}_2^{(SB)}$$

$$\mathcal{O}_0^{(S)} = \left(N^T P_a^{(^1S_0)} N\right)^\dagger \left(N^T P_a^{(^1S_0)} N\right), \quad \mathcal{O}_2^{(S)} = \left[\left(N^T P_a^{(^1S_0)} N\right)^\dagger \left(N^T P_a^{(^1S_0)} \overleftrightarrow{\nabla}^2 N\right) + h.c.\right],$$

$$\mathcal{O}_2^{(SB)} = \left[\underbrace{\left\{N^T P_a^{(^1S_0)} \left(i\partial_t + \frac{\nabla^2}{2M}\right) N\right\}^\dagger \left(N^T P_a^{(^1S_0)} N\right) + h.c.}_{\text{Redundant operator}}\right]$$

"Redundant Operators": operators which can be eliminated by field redefinition, or use of EOM

These must be included in Wilsonian RG analysis

RG Flow(in $^1S_0$-channel)

Dimensionless couplings: $x \equiv \frac{M\Lambda}{2\pi^2} C_0^{(S)}, \quad y \equiv \frac{M\Lambda^3}{2\pi^2} 4C_2^{(S)}, \quad z \equiv \frac{\Lambda^3}{2\pi^2} B^{(S)}$

$$\frac{dx}{dt} = -x - \left[x^2 + 2xy + y^2 + 2xz + 2yz + z^2 \right]$$

$$\frac{dy}{dt} = -3y - \left[\frac{1}{2}x^2 + 2xy + \frac{3}{2}y^2 + yz - \frac{1}{2}z^2 \right]$$

$$\frac{dz}{dt} = -3z + \left[\frac{1}{2}x^2 + xy + \frac{1}{2}y^2 - xz - yz - \frac{3}{2}z^2 \right]$$

Fixed Points

$^1S_0$ channel:

$$(x^\star, y^\star, z^\star) = (0,0,0)\,, \quad \left(-1, -\tfrac{1}{2}, \tfrac{1}{2}\right), \quad \left(-9, \tfrac{15}{2}, -\tfrac{3}{2}\right)$$

$^3S_1$–$^3D_1$ channel:

$$\left(x'^\star, y'^\star, z'^\star, w'^\star\right) = (0,0,0,0)\,, \quad \left(-1, -\tfrac{1}{2}, \tfrac{1}{2}, 0\right), \quad \left(-9, \tfrac{15}{2}, -\tfrac{3}{2}, 0\right)$$

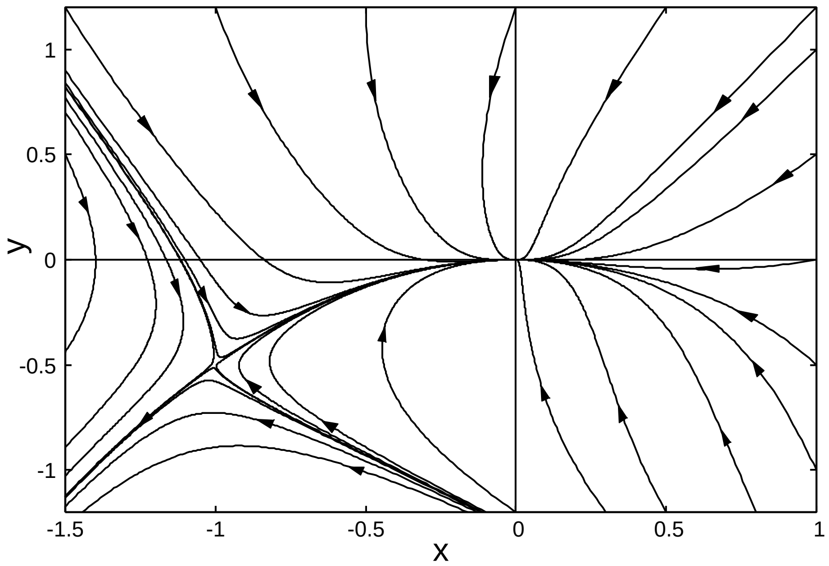


Figure 1: The RG flow for the $^1S_0$ channel projected on to the $C_0^{(S)}$–$C_2^{(S)}$ plane.

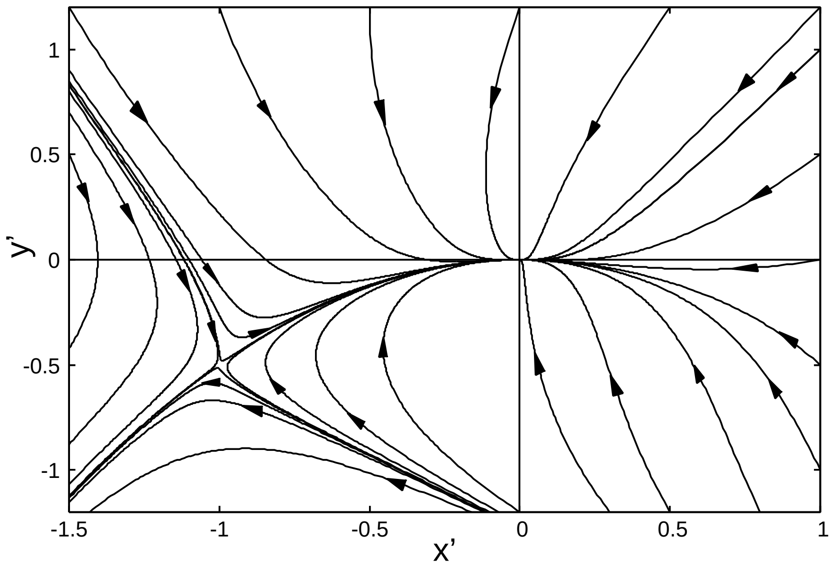


Figure 2: The RG flow for the $^3S_1$–$^3D_1$ channel projected on to the $C_0^{(T)}$–$C_2^{(T)}$ plane.

Scaling Dimension

Trivial fixed point ($^1S_0$ channel)

$$\frac{d}{dt}\begin{pmatrix}\delta x\\ \delta y\\ \delta z\end{pmatrix} = \begin{pmatrix}-1 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & -3\end{pmatrix}\begin{pmatrix}\delta x\\ \delta y\\ \delta z\end{pmatrix}$$

$$\nu_1 = \mathbf{-1} : u_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \nu_2 = \mathbf{-3} : u_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \nu_3 = \mathbf{-3} : u_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

Agree with NDA

All **Irrelevant** $\Rightarrow$ suppression $\Rightarrow$ should be treated **Perturbatively.**

Nontrivial fixed point ($^1S_0$ channel)

$$\frac{d}{dt}\begin{pmatrix}\delta x\\ \delta y\\ \delta z\end{pmatrix}=\begin{pmatrix}1 & 2 & 2\\ 2 & 0 & 1\\ -2 & -2 & -3\end{pmatrix}\begin{pmatrix}\delta x\\ \delta y\\ \delta z\end{pmatrix}$$

$$\nu_1=+\mathbf{1}:u_1=\begin{pmatrix}1\\ 1\\ -1\end{pmatrix},\quad \nu_2=-\mathbf{1}:u_2=\begin{pmatrix}0\\ -1\\ 1\end{pmatrix},\quad \nu_3=-\mathbf{2}:u_3=\begin{pmatrix}2\\ -1\\ -2\end{pmatrix}$$

- u_1 is **Relevant**$\Rightarrow$ no suppression $\Rightarrow$ should be treated **Nonperturbatively.**
- Others are **Irrelevant**$\Rightarrow$ suppression $\Rightarrow$ should be treated **Perturbatively.**

Wilsonian RG and Pionless NEFT

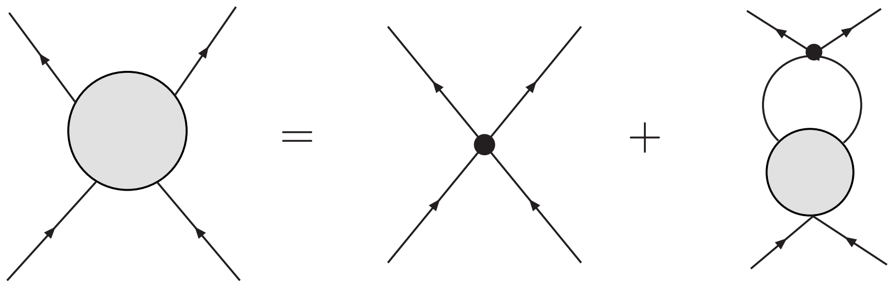
RG flows near the fixed point are very similar.

$\Rightarrow$ **Same Power Counting** in $^1S_0$ and $^3S_1 -^3 D_1$ channels.

Where do the physical differences of $^1S_0$ and $^3S_1 -^3 D_1$ channels come from?
($^3S_1 -^3 D_1$: Deuteron, $^1S_0$: No bound state)

RG Phase and Bound States

Off-shell NN scattering amplitude(four-nucleon Green's function)



$$
\begin{aligned}
-i\mathcal{A}\left(p_0, \mathbf{p_2^2}, \mathbf{p_1^2}\right) &= -iV\left(p_0, \mathbf{p_2^2}, \mathbf{p_1^2}\right) \\
&+ \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \left(-iV\left(p_0, \mathbf{p_2^2}, \mathbf{k^2}\right)\right) \frac{i}{p^0 - \frac{\mathbf{k^2}}{2M} + i\epsilon} \left(-i\mathcal{A}\left(p_0, \mathbf{k^2}, \mathbf{p_1^2}\right)\right)
\end{aligned}
$$

RG Equation (from the amplitude)

$\frac{d}{d\Lambda}\mathcal{A} = 0$ (with expansion in powers of momenta)

Same fixed points as the ones obtained from Legendre flow equation (RG equations are different)

$$(x^\star, y^\star, z^\star) = (0,0,0), \left(-1, -\tfrac{1}{2}, \tfrac{1}{2}\right)$$

Solution near the Nontrivial fixed point,

$$\begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix} = a \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \left(\frac{\Lambda}{\Lambda_0}\right)^2 + b \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \left(\frac{\Lambda}{\Lambda_0}\right) + c \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \underbrace{\left(\frac{\Lambda_0}{\Lambda}\right)}_{\textbf{Relevant}}$$

Renormalized Amplitude

Substitute the solution into $\mathcal{A}$ (theory near the nontrivial fixed point)

$$\left(\mathcal{A}(p_0, \mathbf{p'^2}, \mathbf{p^2})\right)^{-1} = -\frac{M}{4\pi}\left[\frac{2}{\pi}c\Lambda_0 - \frac{4}{\pi}b\frac{Mp_0}{\Lambda_0} - i\sqrt{Mp_0}\right] + \cdots$$

$\cdots$ denotes contributions higher order in $1/\Lambda_0$.

Analytic Structure and Bound State

$c < 0$ ("Strong coupling phase")	A physical pole do exist

$$p_0 \simeq -\frac{1}{M}\left(\frac{2}{\pi}c\Lambda_0\right)^2\left[1+\frac{16}{\pi^2}bc\right]$$

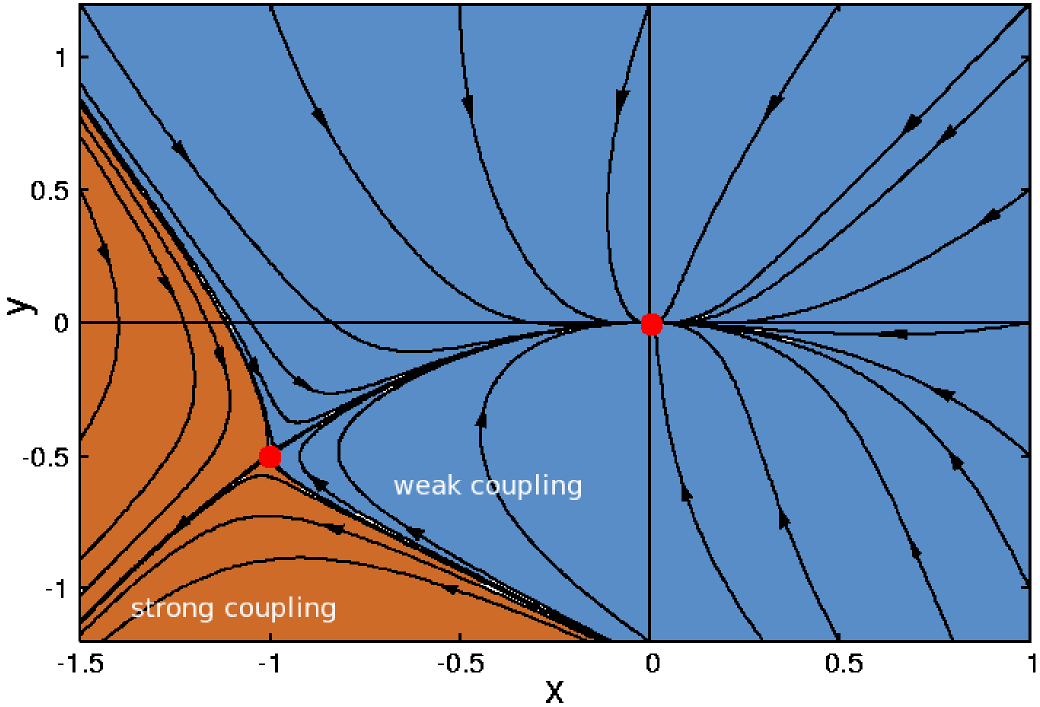
$c > 0$ ("Weak coupling phase")	A physical pole does not exist

- ${}^1S_0$ channel is in "**Weak coupling phase**"
- ${}^3S_1 - {}^3D_1$ channel is in "**Strong coupling phase**"

Summary

- We have proposed the way to determine a **Power Counting** for EFT by quantum theoretical Dimensional Analysis using Wilsonian RG. Pionless NEFT as a simplest example.

- We have found two RG phases in Pionless NEFT, and given them physical interpretations.



Future Problems

Pionfull NEFT

Our Expectations: Due to the difference of pion interactions in each channel, the RG flow is driven to

- Weak coupling phase in ${}^1S_0$ channel
- Strong coupling phase in ${}^3S_1 -{}^3 D_1$ channel

Three-Body

- Nonperturbative renormalization: each perturbative diagram has only weak cutoff dependence. However, the infinite sum of diagrams has strong cutoff dependence.

- Limit cycle behavior of D_0, $\mathcal{L}_3 = \frac{D_0}{M\Lambda_0^4}\left(N^\dagger N\right)^3$

References

Thank you for listening

- K.Harada, K. Inoue, H.K,Phys.Lett.B636(2006)305
- K.Harada, H.K, nucl-th/0605004, to be published in Nucl.Phys B