Grand Unified Theories and B_s-$\bar{B}_s$ Mixing

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Contents

1. New Results
2. Grand unification Models and lepton, quark flavor violations
3. $\tau \rightarrow \mu \gamma$ and B_s-$\bar{B}_s$ mixing
4. sin2β and V_{ub} Measurements
5. Conclusion

Introduction

Recent result of **B_s- $\bar{B}_s$ mass difference**

$\Delta M_s = 17.77 \pm 0.10 \pm 0.07 \ \text{ps}^{-1}$ **hep-ex/0609040**

Theoretical calculation: 15.5-21.8 ps^{-1} (SM)

Large uncertainties in f_{Bs} and B_{Bs}

New Physics contribution is possible.

SUSY and B_s-$\bar{B}_s$ mixing data

- The SUSY contribution is constrained
- This talk: Grand unifying SUSY models: SU(5), SO(10)
- Origin of flavor violation in these models
- $\tau \rightarrow \mu\gamma$ and B_s-$\bar{B}_s$ mixing
- $\mu \rightarrow e\,\gamma$, $\tau \rightarrow e\,\gamma$, ε_k etc.

SUSY contributions in B_s-$\bar{B}_s$ mixing

The gluino Box diagram dominates.

In the mass insertion approximation :

$$\frac{M_{12}^{\tilde{g}}}{M_{12}^{SM}} \approx a[(\delta_{\mathrm{LL}}^{d})_{32}^{2} + (\delta_{\mathrm{RR}}^{d})_{32}^{2}] - b(\delta_{\mathrm{LL}}^{d})_{32}(\delta_{\mathrm{RR}}^{d})_{32} + \ldots$$

($\Delta M = 2|M_{12}|$ and **32: Mixing among second-third generation squarks)**

Where a,b depend on squark, gluino masses.

b~O(100), a~O(1) for mSUSY~ 1 TeV

$$\delta_{\mathrm{LL,RR}}^{\mathrm{d}} = (M_{\tilde{d}}^{2})_{LL,RR} / \tilde{m}^{2}$$

$\tilde{m}$ is the average squark mass

δ^{d}_{32} can affect B_s-$\bar{B}_s$ mixing amplitude.

Universal mass and flavor violation

Suppose we start with universal scalar mass.

The flavor mixings in the squarks, slepton get induced via RGEs.

- Use MSSM RGEs: flavor changing neutral current (FCNC) is small at the weak scale (small CKM elements).
- Introduce the Neutrino couplings:
  Sizable lepton flavor violation is expected.
  (Due to large neutrino mixings.) **Borzumati'86**

Yukawa couplings and flavor violations

Sources: neutrino Yukawa and quark Yukawa coupling

✓ Neutrino Yukawa: Majorana (f) and Dirac (λ_ν), explain the two mass differences and the MNSP : Type I, Type II

$M_\nu = -\lambda_\nu f \lambda_\nu^T V^2_w/M_{pl}$ (I); $f v_L - \lambda_\nu f \lambda_\nu^T V^2_w/M_{pl}$ (II)

✓ Quark Yukawas: explain quark masses and the CKM

v_L: vev of a triplet Higgs

Grand unified models and flavor violation

In grand unified theories, the flavor violations in the quark sector and the lepton sectors can be generated via neutrino Yukawa couplings.

✓ **δ_{23} in quark and leptonic sector get correlated.**

In the lepton sector, δ_{23} is responsible for $\tau \to \mu\gamma$; [BR < 4.5×10^{-8} **[Barlow, ICHEP 06]**]

➢ What is the implication of δ_{23} (same origin) in B_s mixing? Do we have any prediction?

e.g., Phase of B_s mixing ?

Grand unified models and ...

It is possible to generate large 23 mixing terms in squarks and slepton masses in SU(5) and SO(10) models (due to large neutrino mixing) via Dirac and Majorana couplings.

We get B_s-$\overline{B}_s$ mixing and $\tau \rightarrow \mu\gamma$. **Goto etal '95**

12, 13 mixings are also generated: $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$, ε_k get contributions.

(Depends on the neutrino mass (Dirac and Majorana) hirearchies, U_{e3} etc.)

Grand unified models and ...

In SU(5): W: $\lambda_u 10\,10\,5_H + \lambda_d 10\,\overline{5}\,\overline{5}_H + \lambda_\nu \overline{5} N^c 5_H$

Down quarks (D^c) and Leptons (L): $\overline{5}$

Q, U^c and E^c : 10, Right-Neutrino: N^c

λ_ν explains neutrino mixing,

assume M_N diagonal. (Majorana mass of N^c)

D^c and L are coupled to N^c:

flavor violation in the quark sector

✓ We get large δ^d_{RR} (squarks).

Grand unified models and ...

Soft masses at the GUT scale:

$$m_{\bar{5}}^2 = m_{D^c}^2 = m_L^2 = m_0^2 (I - k\lambda_\nu \lambda_\nu^+)$$

$$k \approx 1/8\pi^2 (3 + A_0^2/\mathrm{m}_0^2 \ \ln(\mathrm{M}^*/\mathrm{M}_\mathrm{G}))$$

M* : string scale/Planck scale,

M_G: GUT scale

Even if we start with diagonal squark slepton masses, we get large flavor mixings in both squark, slepton masses.

SU(5) Model

The soft SUSY breaking masses at M_G:

$$m_{\bar{5}}^2 = m_{d^c}^2 = m_l^2 = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta \\ 0 & \delta^* & 1 \end{pmatrix}; \quad m_Q^2 = m_{u^c}^2 = m_{e^c}^2 = m_0^2 I$$

δ arises due to λ_ν

This boundary condition is in the basis:

$$\lambda_u = V_{qeL} V_{CKM}^T \lambda_u^{diag} P_u V_{CKM}, \quad \lambda_e = \lambda_e^{diag} P_e, \quad \lambda_d = V_{qeL} \lambda_d^{diag} P_d V_{qeR}^T$$

At the weak scale we use the basis

where $\lambda_{d,e}$ are real, diagonal; **We assume minimal SU(5)**

$V_{qeR} = V_{qeL} = 1$

SO(10) Model

Same effects exist in SO(10)

In SO(10): The quarks and leptons are unified in 16

(flavor violating effects are larger since both left and right quarks, leptons fee the neutrino Yukawa coupling)

The Higgs fields are in 10, $\overline{126}$

Babu, Mohapatra'92

We also have Majorana couplings:

$f\ 16\,16\ \overline{126}$

SO(10) Model ...

✓ Majorana neutrino coupling (f) introduces flavor violation in both left- and right squark mass matrices

✓ f explains the neutrino masses and mixings in the case when **$m_\nu = f\, v_L$**.

✓The scalar masses at M_G with **f**:

$$m^2_{U^c} = m^2_Q = m^2_{D^c} = m^2_{E^c} = m^2_L = m^2_0 (I - k\, f\, f^+)$$

SO(10) Model…

$$m_{16}^{2} = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta \\ 0 & \delta^* & 1 \end{pmatrix}$$

This boundary condition is in the basis:

$$\lambda_u = V_{qeL} V_{CKM}^{T} \lambda_u^{diag} P_u V_{CKM} V_{qeL}^{T}, \quad \lambda_d = V_{qeL} \lambda_d^{diag} P_d V_{qeL}^{T}$$

$$\lambda_e = \lambda_e^{diag} P_e, \qquad P_d = (e^{i\phi d}, e^{i\phi s}, e^{i\phi b}),$$

We assume $V_{qeL}=1$; (No large fine tuning in the mass fit)

In the case of 10, 126 Higgs, the Yukawa couplings are symmetric

✓ Both δ^d_{RR} and δ^d_{LL} are large.

A realistic SO(10) scenario:

✓ Recent studies show that minimal Renormalizable SO(10) models need **120** Higgs field to satisfy proton decay constraint

The Yukawa couplings are not Symmetric anymore,
we can choose them to be hermitian:

Solves SUSY CP problem

Dutta, Mimura, Mohapatra'04

The soft scalar masses at the GUT scale:

$$m_{U^c}^{2*} = m_Q^2 = m_{D^c}^{2*} = m_{E^c}^{2*} = m_L^2$$

Summary:

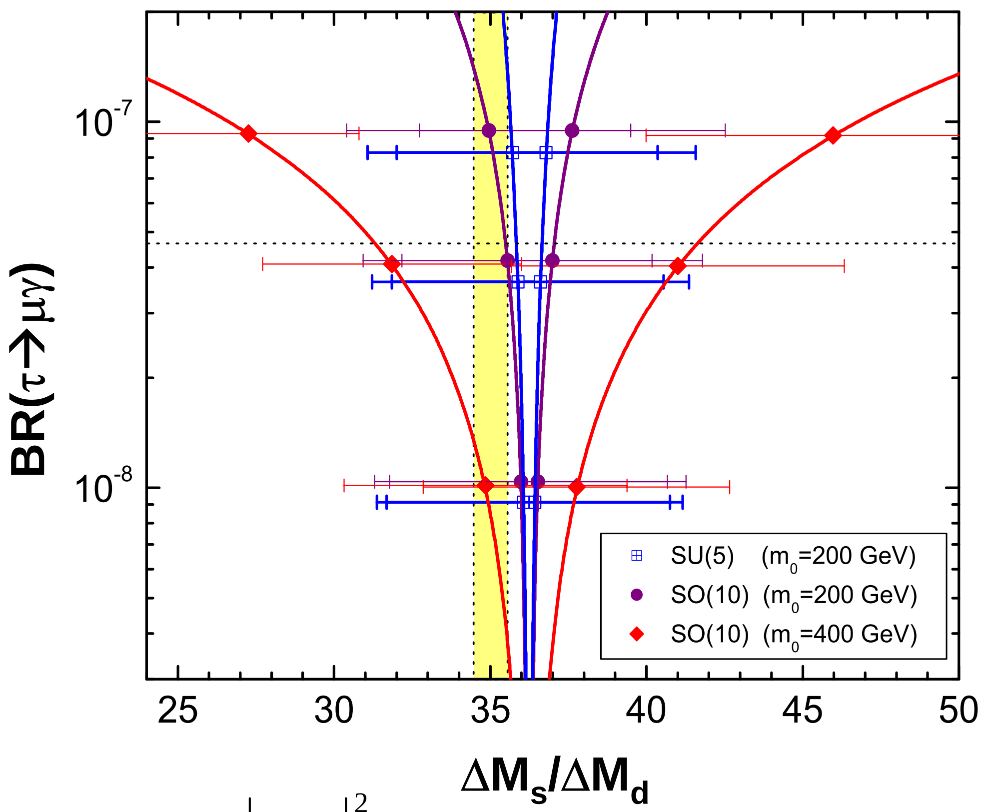
In both SU(5) and SO(10):

- 2-3 mixing in the slepton masses will introduce $\tau \rightarrow \mu\, \gamma$

- 2-3 mixing in the squark masses will introduce new contribution to B_s mixing

The phases, ϕ_s, ϕ_b enters into B_s mixing and can generate large ϕ_{Bs}

B_s-$\bar{B}_s$ and $\tau \rightarrow \mu\gamma$ in grand unified models



$M_{1/2}$=300GeV

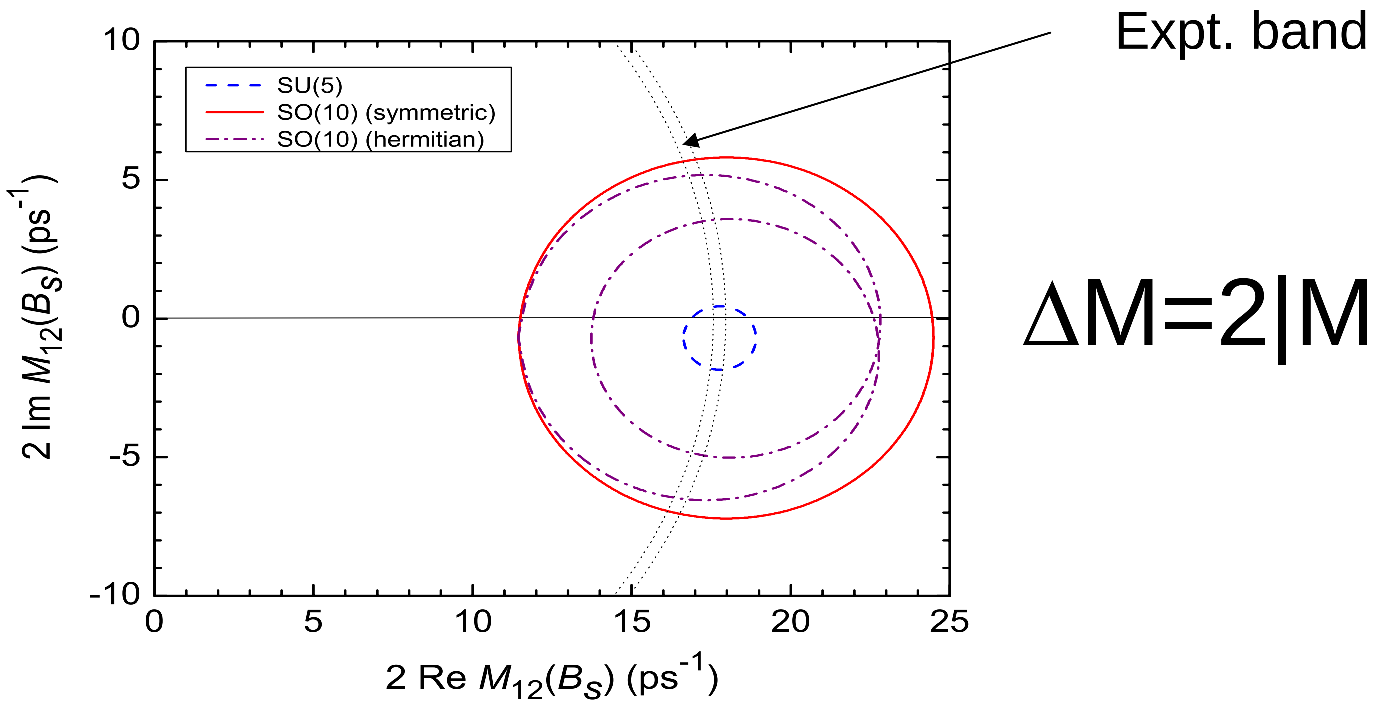
$\tan\beta$=10

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2$$

$$\xi = \sqrt{B_{B_s}} f_{B_s} / \sqrt{B_{B_d}} f_{B_d} \quad = 1.23 \pm 0.06$$

We include ξ, $|V_{ts}/V_{td}|$ errors

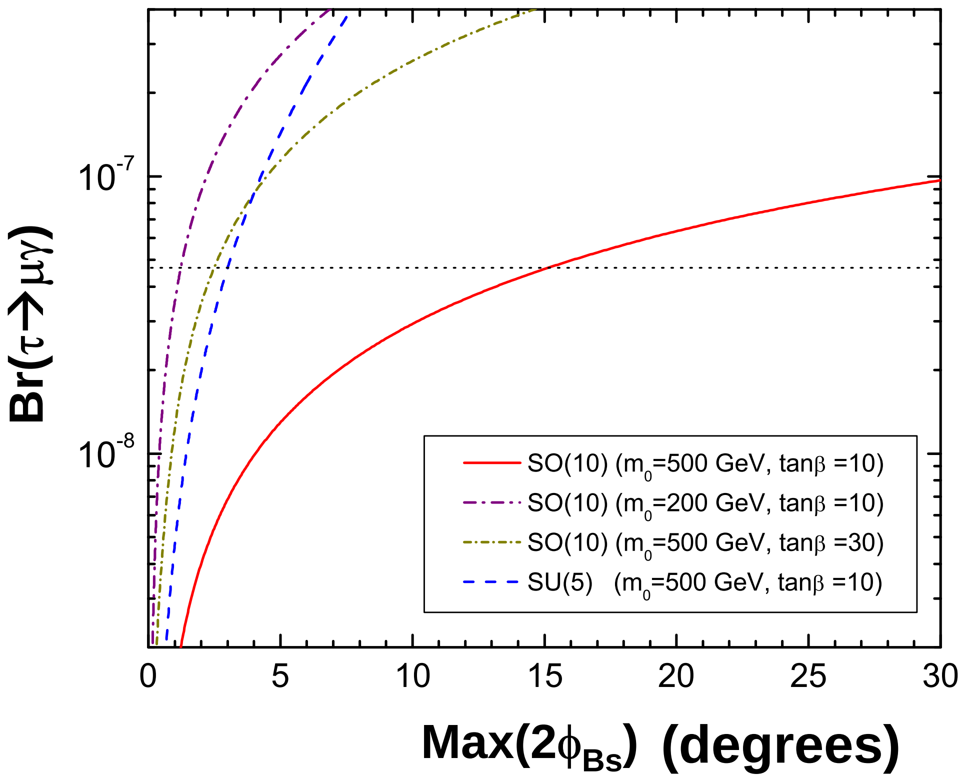
B_s-$\overline{B}_s$ mixing and $\tau \rightarrow \mu\gamma$ in grand unified…



$\Delta M = 2|M_{12}|$

Re-Im plot for $2M_{12}(B_s)$ when $Br(\tau \rightarrow \mu\gamma)$ saturates the experimental bound.

B_s-$\bar{B}_s$ mixing phase and future



$$C_{Bs} e^{2i\phi_{Bs}} = \frac{M_{12}(B_s)^{full}}{M_{12}(B_s)^{SM}}$$

The phase ϕ_{Bs} is large for SO(10)

This phase can be measured in the CP asymmetry of $B_s \rightarrow \psi\ \phi$, CP asymmtry of semileptonic B_s decays

$\mathrm{Sin}2\beta$ and V_{ub} measurement

Recent measurements:

$\mathrm{Sin}2\beta = 0.674 \pm 0.026$ (WA) **[Hazumi, ICHEP 06]**

$V_{ub} = (4.49 \pm 0.19 \pm 0.27) \times 10^{-3}$ (Inclusive)

[Kowelewski, ICHEP 06]

Exclusive determination has large theoretical error (**WA):** $3.84^{+0.67}_{-0.49}$ $\mathbf{x10^{-3}}$ **[PDG]**

- $V_{ub} = (3.49 \pm 0.17) \times 10^{-3}$ using $\sin 2\beta$ (WA)

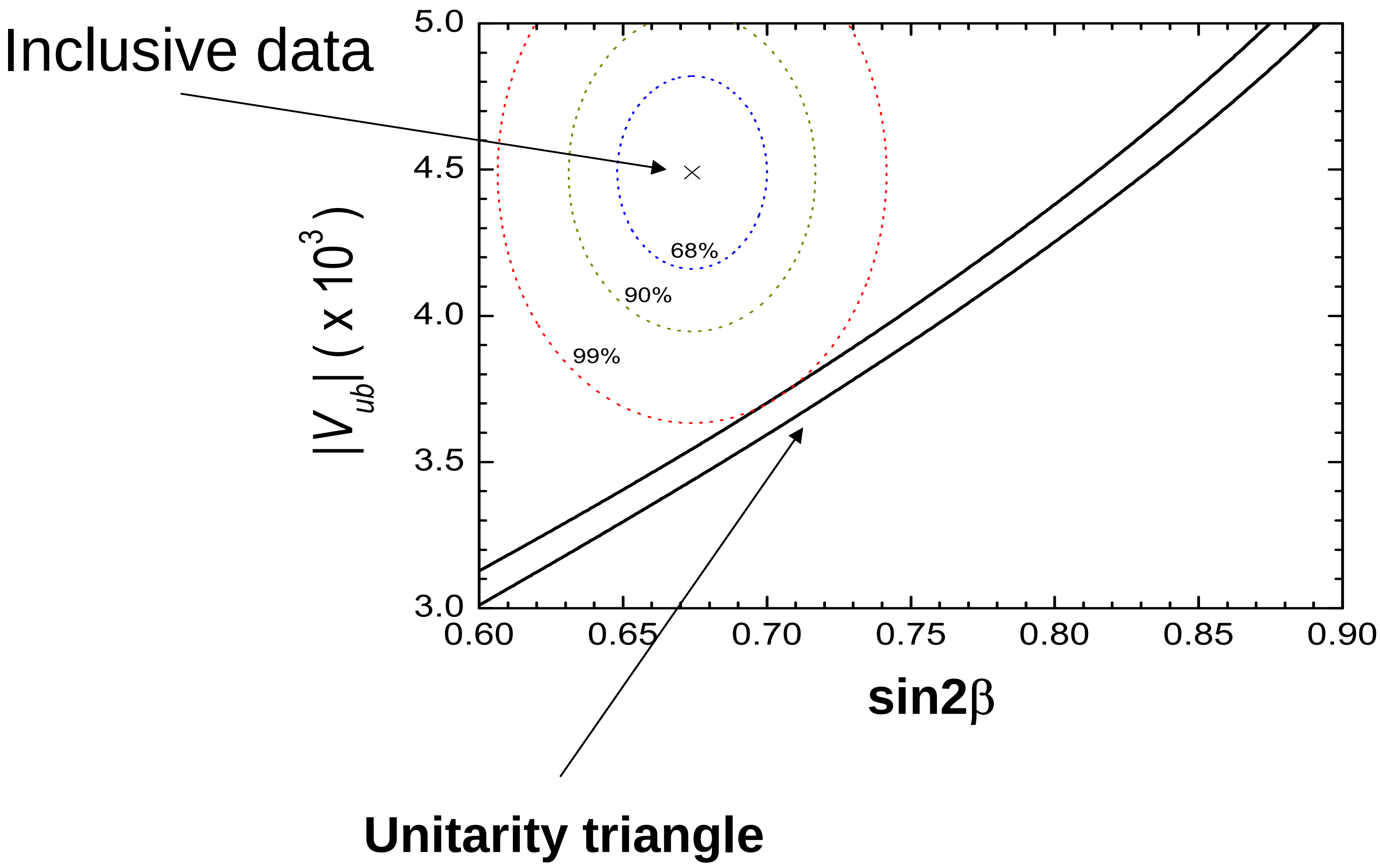
With $|V_{td}/V_{ts}| = 0.206^{+0.008}_{-0.006}$

Sin2β and V_{ub} measurement

There exists tension in the fit.

- Can SUSY grand unified models explain this discrepancy?
- What are the predictions?
- We need δ_{13} terms - mixing bewteen first-third generations in the scalar mass matrices to modify Sin2β.

V_{ub} vs sin2β in SM



The δ_{13} term (mixing between the first and the third generation) gets induced from the neutrino mixing: SU(5) case:

$$m^2_{D^c} = m^2_L = m^2_0 (I - k\lambda_\nu \lambda_\nu^{\ +})$$ (**κ involves ln M*/M_G**)

$$\lambda_\nu \lambda_\nu^+ = y_3^2 P_l U^*_{MNSP} \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^T_{MNSP} P_l^*$$

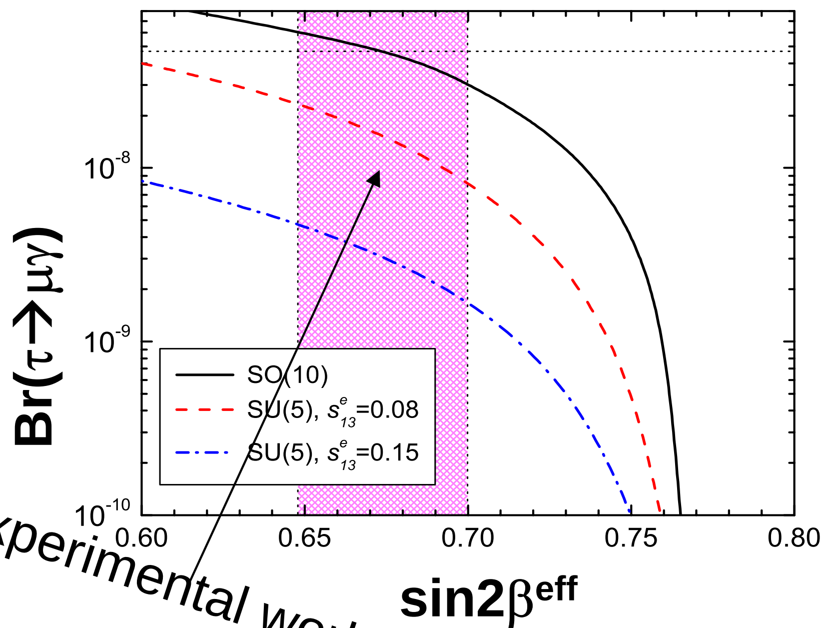
where $k_2 \approx \sqrt{\Delta m^2_{sol} / \Delta m^2_{atm}} M_2 / M_3$ **M_i:Majorana mass**

Small k_2, U_{e3} ➔ small δ_{13} and δ_{12}

✓ δ_{13} term affects the B_d-$\overline{B}_d$ mixing → sin2β.

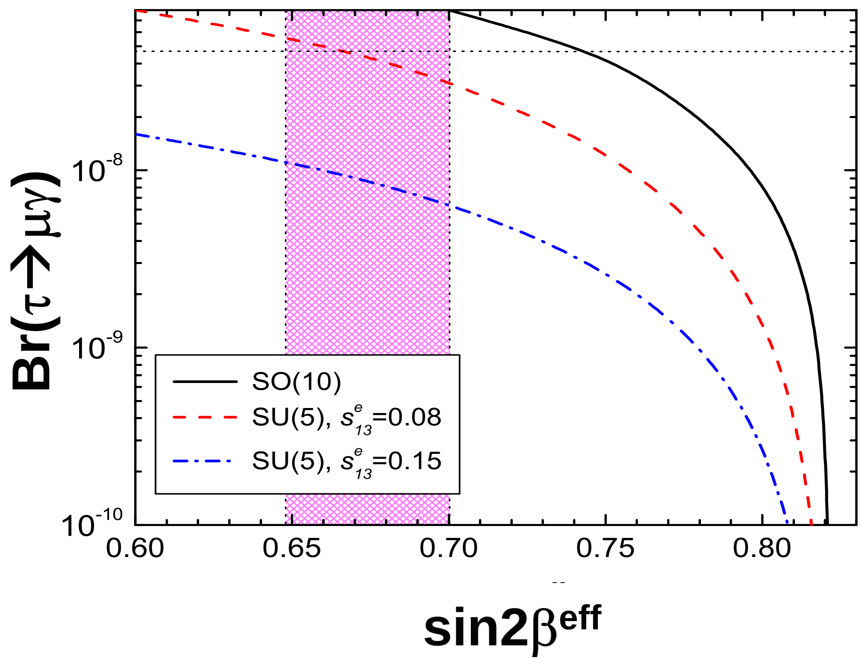
Br(τ→μγ) vs sin2β in SO(10) and SU(5)

V_{ub}=0.0041



Experimental world average

V_{ub}=0.00449



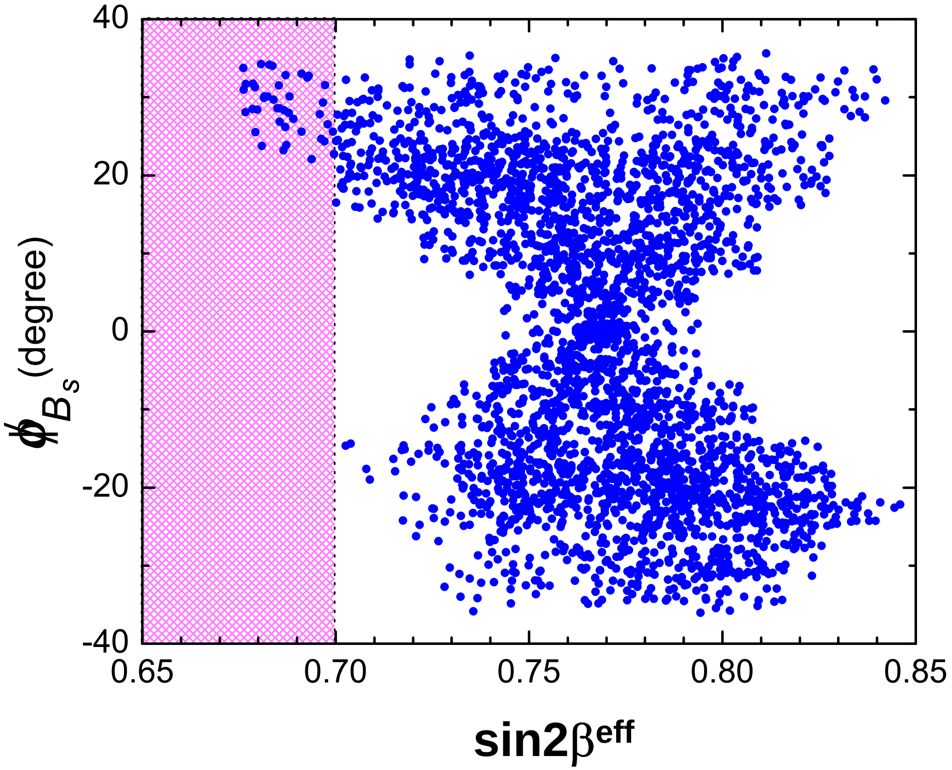
Br(μ→eγ) is around 10^{-12} by our choice of U_{e3} and k_2

k_2 and U_{e3} are free for SU(5), U_{e3}~0.02 for SO(10)

We choose m_0=1 TeV, $m_{1/2}$=300 GeV tanβ=10, A_0=0

ϕ_{Bs} vs sin2β in SO(10) and SU(5)

SO(10)



ϕ_{Bs} is very small ~ ± 3° for SU(5).

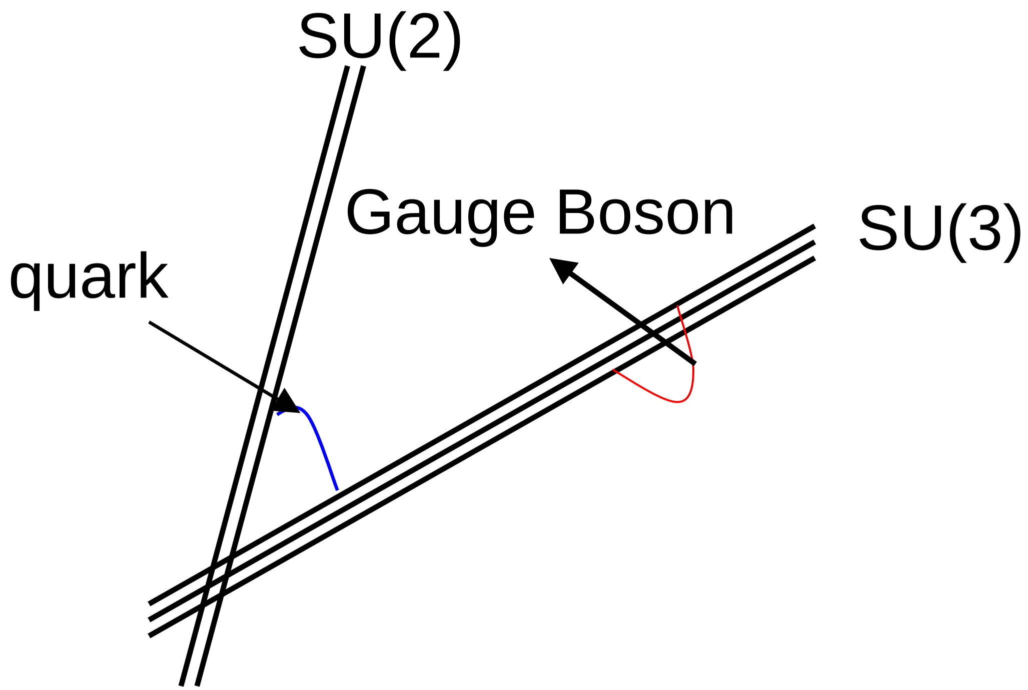
Non-proportional A terms

Non-proportional A terms can also reduce the tension of the V_{ub}, sin2β fit

$$A^{u,d,e}_{i,j} = A_0(\lambda^{u,d,e} + \Delta^{u,d,e})_{i,j}$$

- These terms can arise naturally in Intersecting D-brane models

Non-proportional A terms in intersecting D-brane Models



Bi-fundamental Quark & lepton fields are zero modes of open strings stretched at the intersection.

Non-proportional A terms…

Flavor universality of SUSY breaking scalar mass is maintained

U moduli derivative generates non-proportional term in scalar trilinear coupling

$$A_{ij} = A_0(Y_{ij} + c\,\partial_U Y_{ij})$$

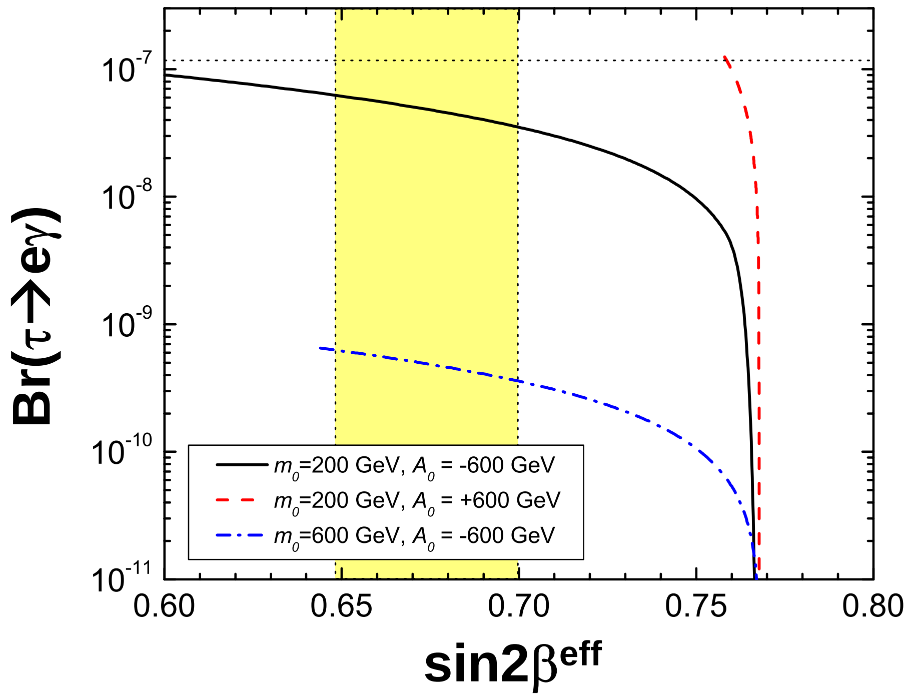
Lust, reffert, Steibergerl;
Camara,Ibanez,Urangal;
Cvetic, papadimitru;
Nath, Kors l; Abel,Owen

Yukawa couplings depend on U moduli only

We need A_{13} to resolve the tension

We get large Br($\tau \rightarrow e\gamma$)

Br(τ→eγ) vs sin2β



Assume:

Δ_{13} and Δ_{31} are non-zero ($A_{i,j}^{u,d,e} = A_0(\lambda^{u,d,e} + \Delta^{u,d,e})_{i,j}$), all other are 0 in the non-proportional part

Conclusion

- Recent results on B_s mixing allows SUSY contribution
- Large neutrino mixing gives rise to large $Br(\tau \rightarrow \mu\gamma)$ and B_s-$\bar{B}_s$ mixing in these models

- In grand unifying models, e.g., SU(5), SO(10), the flavor violations in quark and lepton sectors are related.

- Experimental upper limit on $Br(\tau \rightarrow \mu\gamma)$ constraints the phase of the B_s mixing.

Conclusion…

- The phase of B_s mixing can be large, for SO(10) model and small for SU(5)
- The tension between V_{ub} and $\sin 2\beta$ can be reduced in these models
- The non-proportional A terms can also reduce this tension with a prediction of large $Br(\tau \rightarrow e\gamma)$