$B \to \pi^+\pi^-\pi^0$ time-dependent Dalitz analysis from Belle

DPF/JPS 2006

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  - $\pi\pi$ Lineshape determination.
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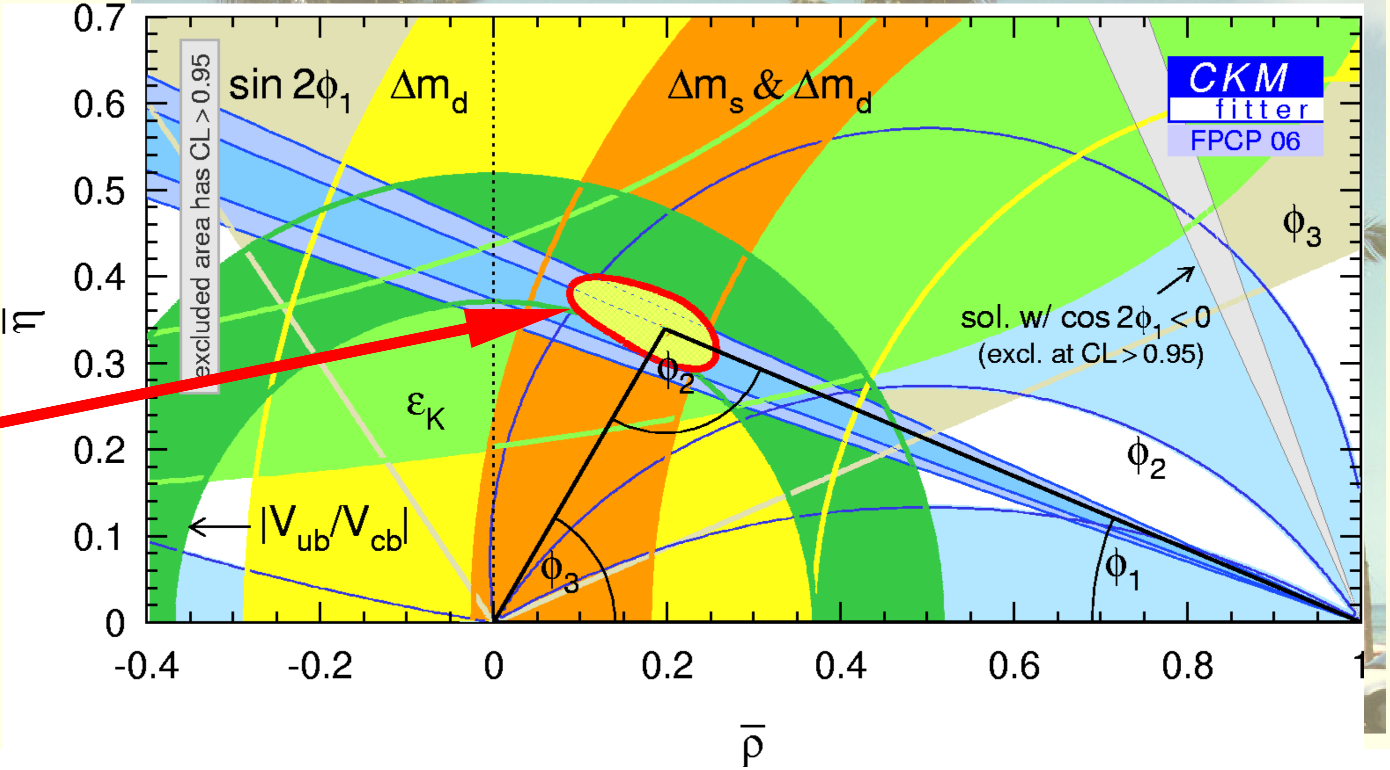
Introduction

CKM Matrix:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Unitarity Triangle

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0$$



- The angle ϕ_2 can be constrained by b→u transitions.
- By B→$\rho\pi$, ϕ_2 can be constrained without ambiquity.

Formulism for time-dependent Dalitz analysis

B→($\rho\pi$)^0→$\pi^+\pi^-\pi^0$ time-dependent Dalitz plot decay width:

$$|A(\Delta t; S_+, S_-)|^2$$

$$= e^{-\Gamma|\Delta t|}\{(|A_{3\pi}|^2 + |\overline{A}_{3\pi}|^2) - q_{tag}[(|A_{3\pi}|^2 - |\overline{A}_{3\pi}|^2)\cos(\Delta M \Delta t) - 2 Im[\frac{q}{p} A^*_{3\pi}\overline{A}_{3\pi}]\sin(\Delta M \Delta t)]\}$$

$$A_{3\pi} = A_{3\pi}(s_+, s_-) = f_+(s_+, s_-)A^+ + f_-(s_+, s_-)A^- + f_0(s_+, s_-)A^0$$

$$\frac{q}{p}\overline{A}_{3\pi} = A_{3\pi}(s_+, s_-) = \overline{f}_+(s_+, s_-)\overline{A}^+ + \overline{f}_-(s_+, s_-)\overline{A}^- + \overline{f}_0(s_+, s_-)\overline{A}^0$$

For better fitting performance, we redifined the PDF by

$$|A_{3\pi}|^2 \pm |\overline{A}_{3\pi}|^2 = \sum_{\kappa\in\{+,-,0\}} |f_\kappa|^2 U^\pm_\kappa + 2\sum_{\kappa<\sigma\in\{+,-,0\}} (Re[f_\kappa f^*_\sigma] U^{\pm,Re}_{\kappa\sigma} - Im[f_\kappa f^*_\sigma] U^{\pm,Im}_{\kappa\sigma})$$

$$Im(\frac{q}{p} A^*_{3\pi}\overline{A}_{3\pi}) = \sum_{\kappa<\sigma\in\{+,-,0\}} (Re[f_\kappa f^*_\sigma] I^{Im}_{\kappa\sigma} + Im[f_\kappa f^*_\sigma] I^{Re}_{\kappa\sigma})$$

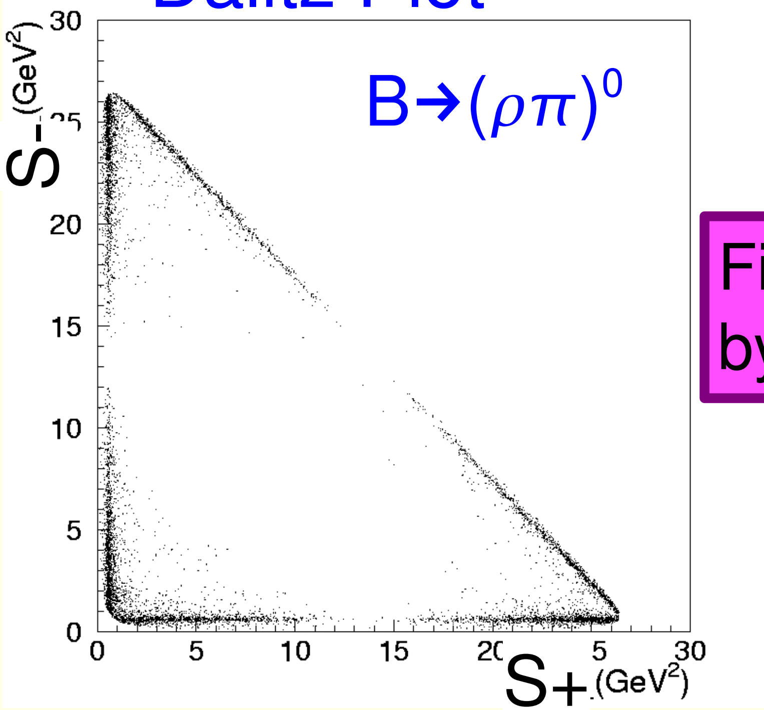
27 parameters

U and I to be obtained.

Square Dalitz Plot (SDP)

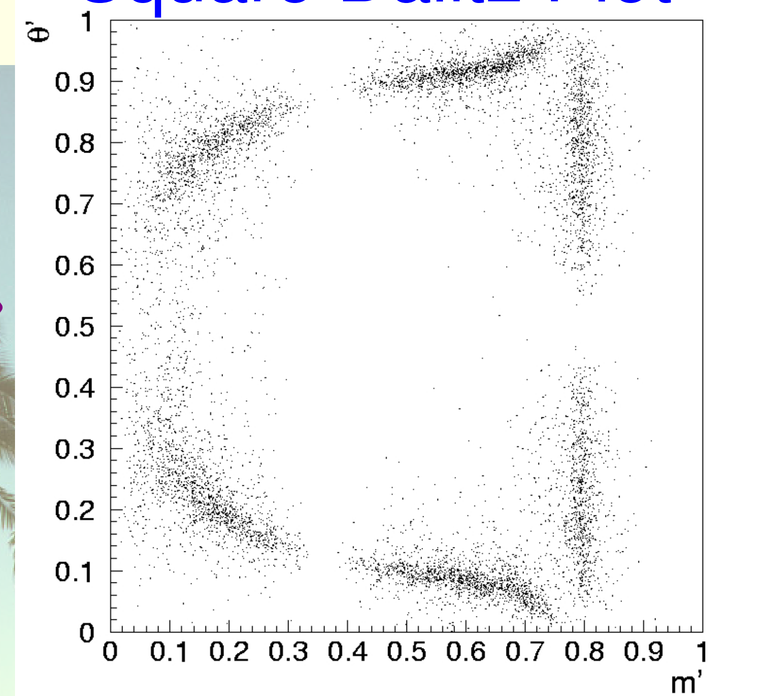
Jacobian: $|\det J| = 4|\vec{p}_+||\vec{p}_0|\, m_0 \frac{m_0^{max} - m_0^{min}}{2} \pi \sin(\pi m') \pi \sin(\pi \theta')$

Dalitz Plot



First propoesed by Babar

Square Dalitz Plot

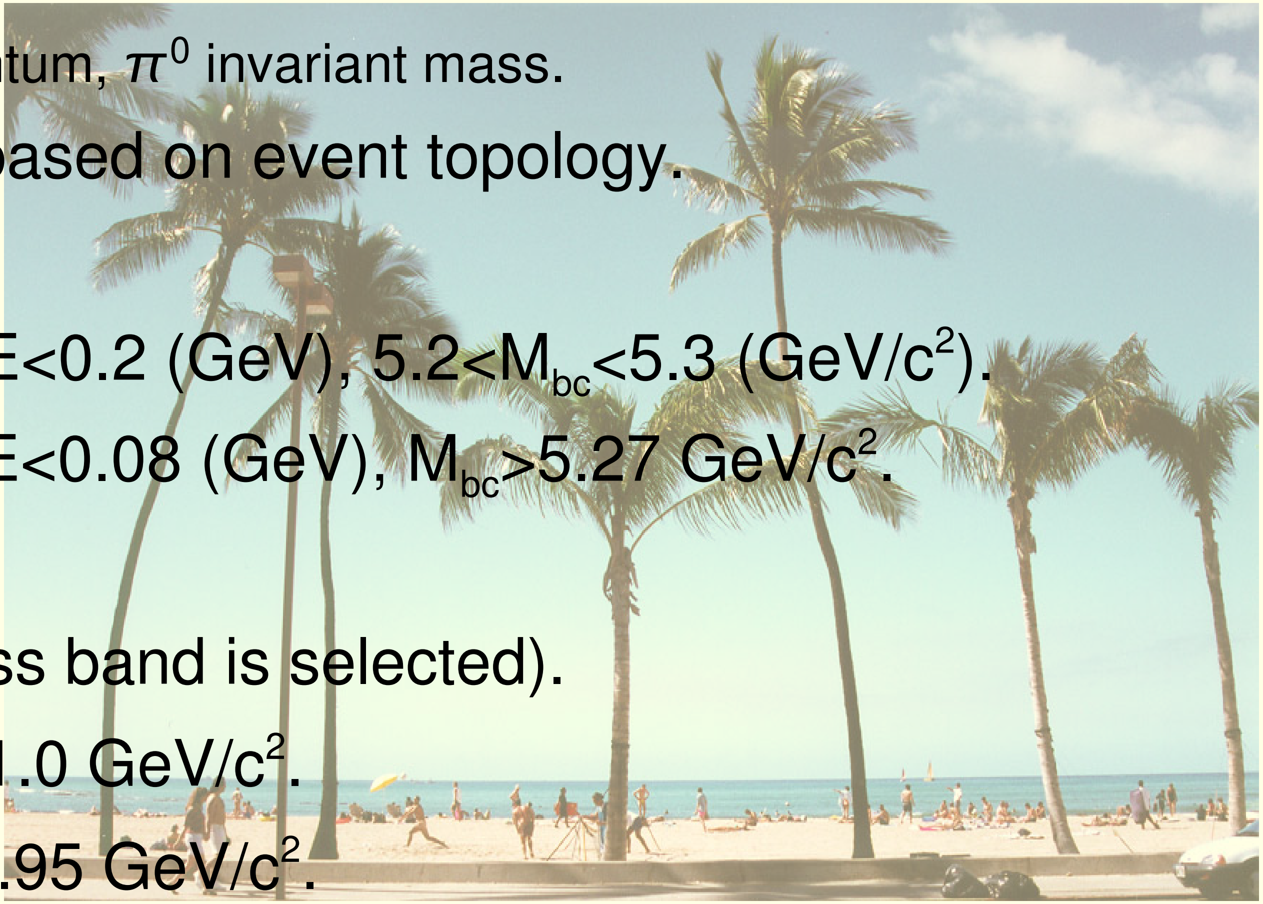


$$m' \equiv \frac{1}{\pi} \arccos\left(2 \frac{m_0 - m_0^{min}}{m_0^{max} - m_0^{min}} - 1\right)$$

$$\theta' \equiv \frac{1}{\pi} \theta_0$$

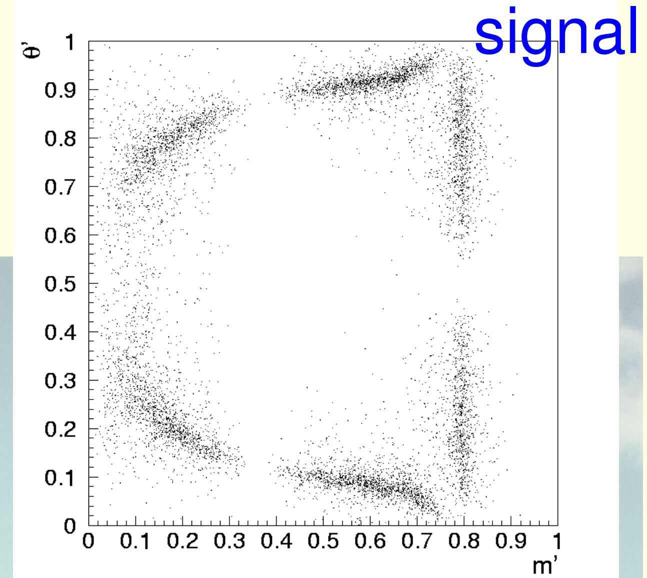
Event Selection

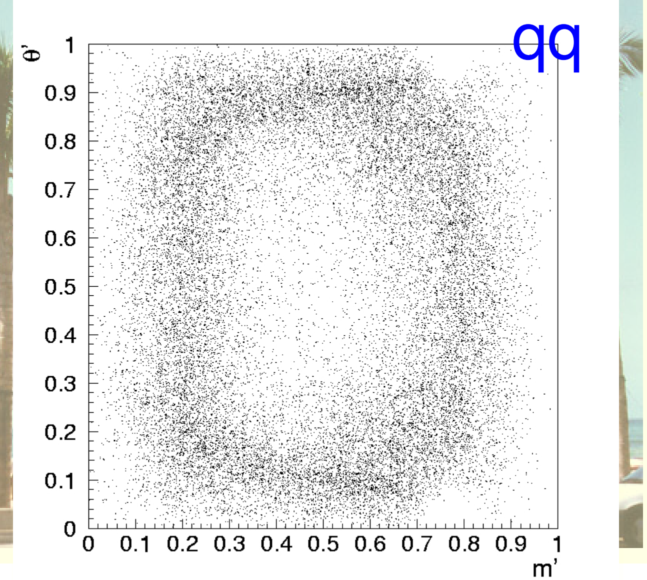
- Charged track ($\pi^{\pm}$) selection
  - Interaction point, transverse momentum, particle identification.
- Neutral track (π^0) selection
  - Photon energy, π^0 momentum, π^0 invariant mass.
- Continuum suppression based on event topology.
- Kinematic selection:
  - Grand region: $-0.2<\Delta E<0.2$ (GeV), $5.2<M_{bc}<5.3$ (GeV/c^2).
  - Signal region: $-0.1<\Delta E<0.08$ (GeV), $M_{bc}>5.27$ GeV/c^2.
- Best candidate selection.
- Veto on Dalitz plot (ρ mass band is selected).
  - 0.55 GeV/$c^2$$<M_{\pi^{\pm}\pi^0}<$$1.0$ GeV/c^2.
  - 0.55 GeV/$c^2$$<M_{\pi^+\pi^-}<$$0.95$ GeV/c^2.



Event Category

- Signal:
  - Correctly reconstructed.
  - Self-cross-feed (SCF):
    - ~20% to correctly reconstructed events.
    - $\pi^{\pm}$ replaced (~6%).
    - π^{0} replaced (~14%).
- Background:
  - Continuum $q\bar{q}$ (q=u,d,s)
    - Dominant source (~60% in signal region).
  - $B\bar{B}$
    - ~8% in signal region.





$\pi\pi$ lineshape determination

$\pi\pi$ lineshape:

$$f(s) = BW_{\rho}(s) + \beta\, BW_{\rho'}(s) + \gamma\, BW_{\rho''}(s)$$

- $\pi\pi$ lineshape could be different for 6 decay amplitudes.
- Neither good theoretical estimation nor other experimental result to constrain β and γ.

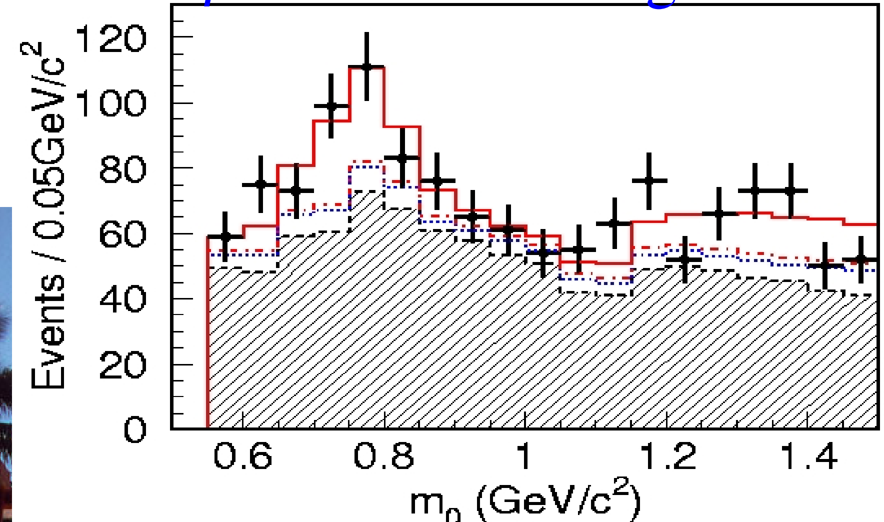
⇩

- We determined the ρ lineshape from data.
  - The lineshape is fitted with looser Dalitz veto.
  - Averaged β and γ are used in the nominal fit.
  - The systematic error is reduced to an acceptable level.

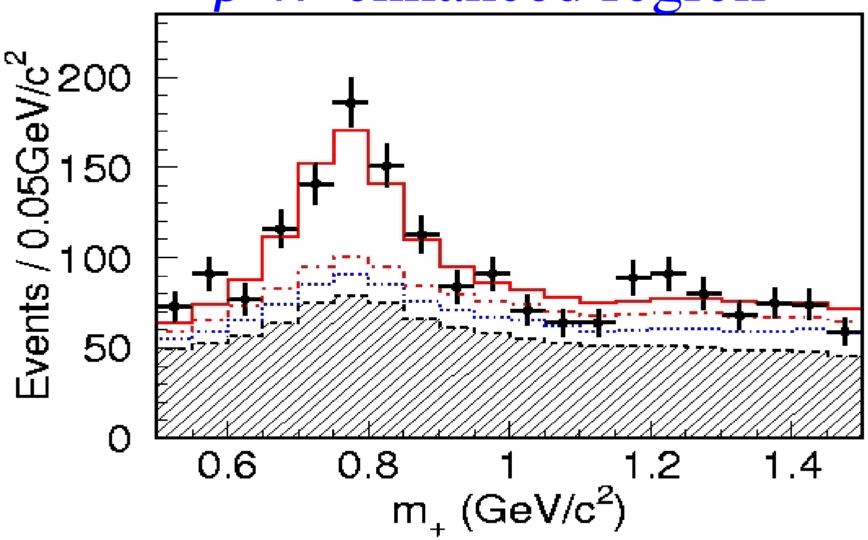
$\pi\pi$ lineshape determination

red solid: correctly reconstructed signal
red dashed: SCF
hatched:qq
blue dashed: BB

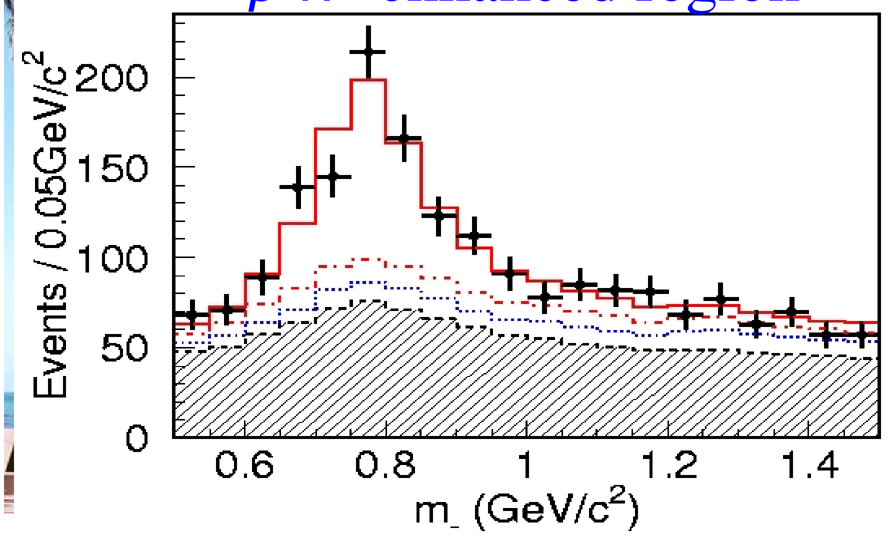
$\rho^0\pi^0$ enhanced region



$\rho^+\pi^-$ enhanced region



$\rho^-\pi^+$ enhanced region



$\pi\pi$ lineshape determination

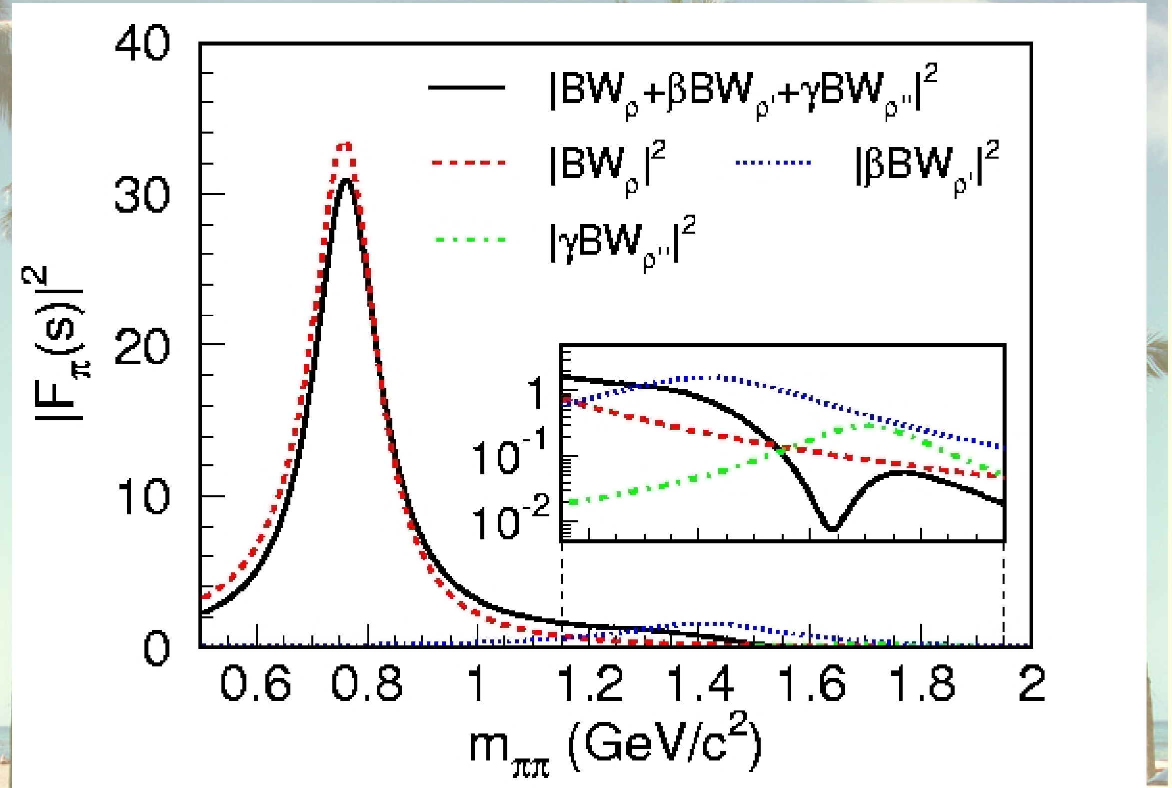
- ρ' and ρ'' mass and width are based on PDG2005 values.
- We obtained:

$$|\beta| = 0.30^{+0.06}_{-0.05}$$

$$arg\beta = (213^{+15}_{-19})^o$$

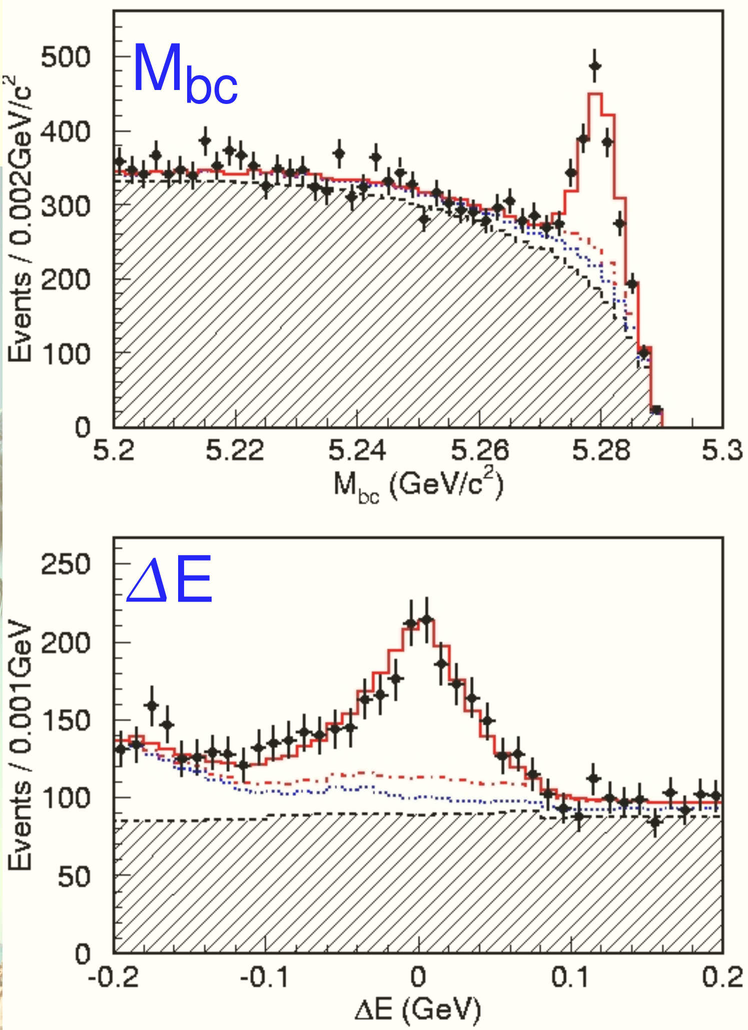
$$|\gamma| = 0.07 \pm 0.03$$

$$arg\gamma = (91^{+27}_{-32})^o$$



Signal fraction determination

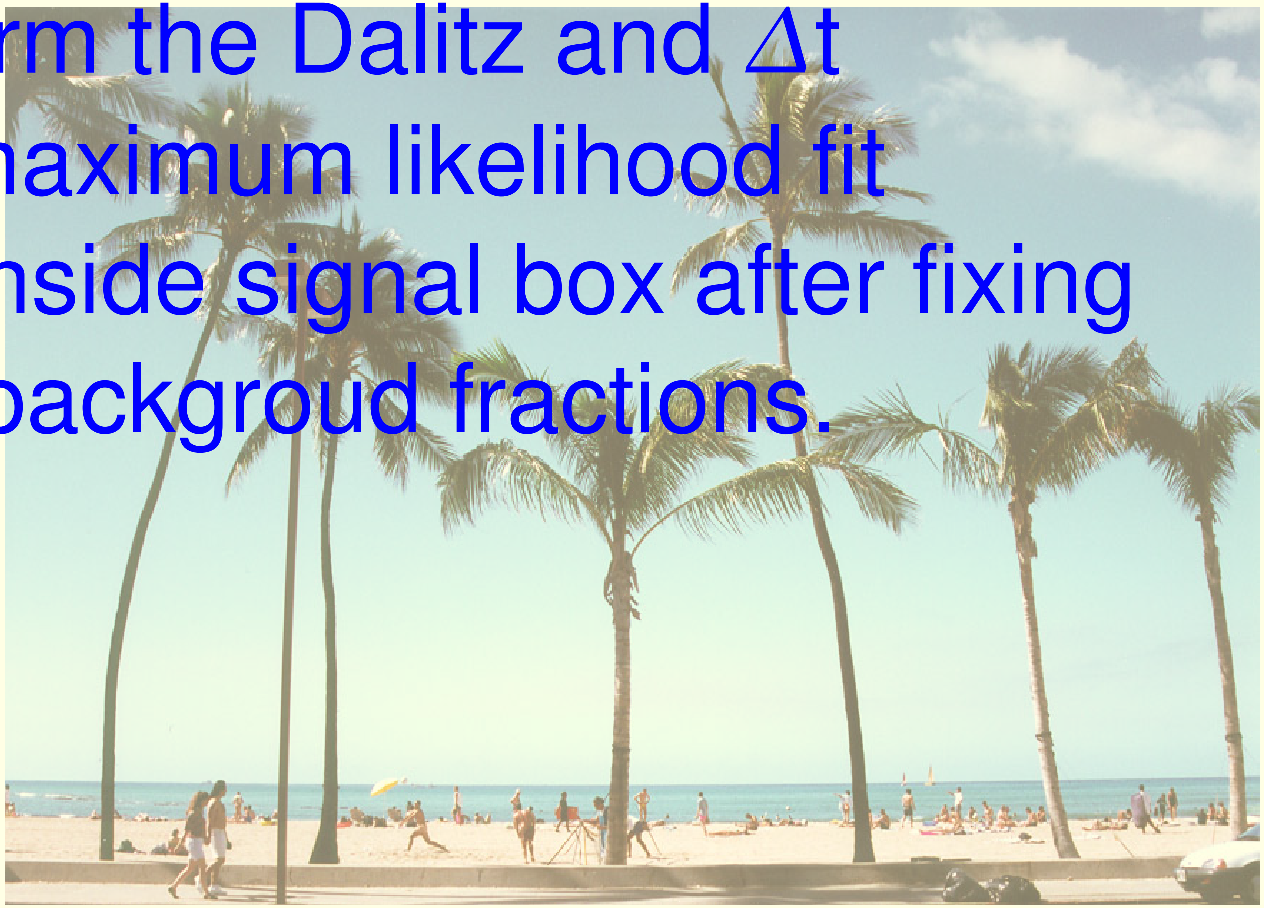
- ΔE-M_{bc} and Dalitz simultaneous fit.
- From 414fb^{-1}:
  - 987±42 signal yield inside the signal box.



red solid: correctly reconstructed signal
red dashed: SCF
hatched:qq
blue dashed: BB

Dalitz and Δt simultaneous fit

Then, we perform the Dalitz and Δt simultaneous maximum likelihood fit for the events inside signal box after fixing the signal and backgroud fractions.



Dalitz and Δt simultaneous fit result

hep-ex/0609003

U_{+}^{+}	$+1(\text{fixed})$
U_{-}^{+}	$+1.28 \pm 0.13(\text{stat.}) \pm 0.08(\text{syst.})$
U_{0}^{+}	$+0.30 \pm 0.06(\text{stat.}) \pm 0.05(\text{syst.})$
$U_{+-}^{+,\text{Re}}$	$+0.62 \pm 0.80(\text{stat.}) \pm 0.57(\text{syst.})$
$U_{+0}^{+,\text{Re}}$	$+0.41 \pm 0.52(\text{stat.}) \pm 0.46(\text{syst.})$
$U_{-0}^{+,\text{Re}}$	$+0.49 \pm 0.65(\text{stat.}) \pm 0.44(\text{syst.})$
$U_{+-}^{+,\text{Im}}$	$+0.86 \pm 0.83(\text{stat.}) \pm 0.49(\text{syst.})$
$U_{+0}^{+,\text{Im}}$	$-0.53 \pm 0.39(\text{stat.}) \pm 0.47(\text{syst.})$
$U_{-0}^{+,\text{Im}}$	$-1.72 \pm 0.69(\text{stat.}) \pm 0.53(\text{syst.})$

Dalitz and Δt simultaneous fit result (2)

hep-ex/0609003

U_{+}^{-}	$+0.22 \pm 0.15(\mathrm{stat.}) \pm 0.10(\mathrm{syst.})$
U_{-}^{-}	$-0.62 \pm 0.17(\mathrm{stat.}) \pm 0.09(\mathrm{syst.})$
U_{0}^{-}	$+0.14 \pm 0.11(\mathrm{stat.}) \pm 0.09(\mathrm{syst.})$
$U_{+-}^{-,\mathrm{Re}}$	$-1.70 \pm 1.59(\mathrm{stat.}) \pm 0.77(\mathrm{syst.})$
$U_{+0}^{-,\mathrm{Re}}$	$-2.46 \pm 1.39(\mathrm{stat.}) \pm 0.86(\mathrm{syst.})$
$U_{-0}^{-,\mathrm{Re}}$	$-0.70 \pm 1.59(\mathrm{stat.}) \pm 0.86(\mathrm{syst.})$
$U_{+-}^{-,\mathrm{Im}}$	$-2.21 \pm 1.71(\mathrm{stat.}) \pm 1.03(\mathrm{syst.})$
$U_{+0}^{-,\mathrm{Im}}$	$-0.83 \pm 0.98(\mathrm{stat.}) \pm 0.65(\mathrm{syst.})$
$U_{-0}^{-,\mathrm{Im}}$	$-0.79 \pm 1.59(\mathrm{stat.}) \pm 1.05(\mathrm{syst.})$

Dalitz and Δt simultaneous fit result (3)

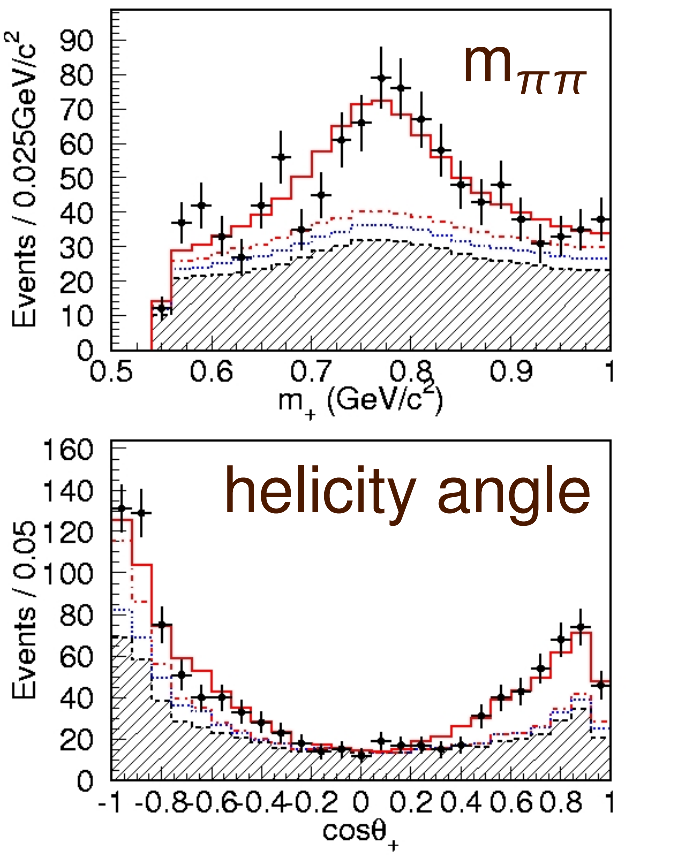
hep-ex/0609003

I_+	$-0.03 \pm 0.11(\text{stat.}) \pm 0.06(\text{syst.})$
I_-	$+0.11 \pm 0.11(\text{stat.}) \pm 0.05(\text{syst.})$
I_0	$+0.02 \pm 0.09(\text{stat.}) \pm 0.06(\text{syst.})$
I_{+-}^{Re}	$+1.62 \pm 2.65(\text{stat.}) \pm 1.23(\text{syst.})$
I_{+0}^{Re}	$+1.45 \pm 2.41(\text{stat.}) \pm 1.12(\text{syst.})$
I_{-0}^{Re}	$-0.65 \pm 1.63(\text{stat.}) \pm 1.49(\text{syst.})$
I_{+-}^{Im}	$-1.76 \pm 2.42(\text{stat.}) \pm 1.31(\text{syst.})$
I_{+0}^{Im}	$+0.00 \pm 2.06(\text{stat.}) \pm 1.15(\text{syst.})$
I_{-0}^{Im}	$-2.58 \pm 1.72(\text{stat.}) \pm 1.33(\text{syst.})$

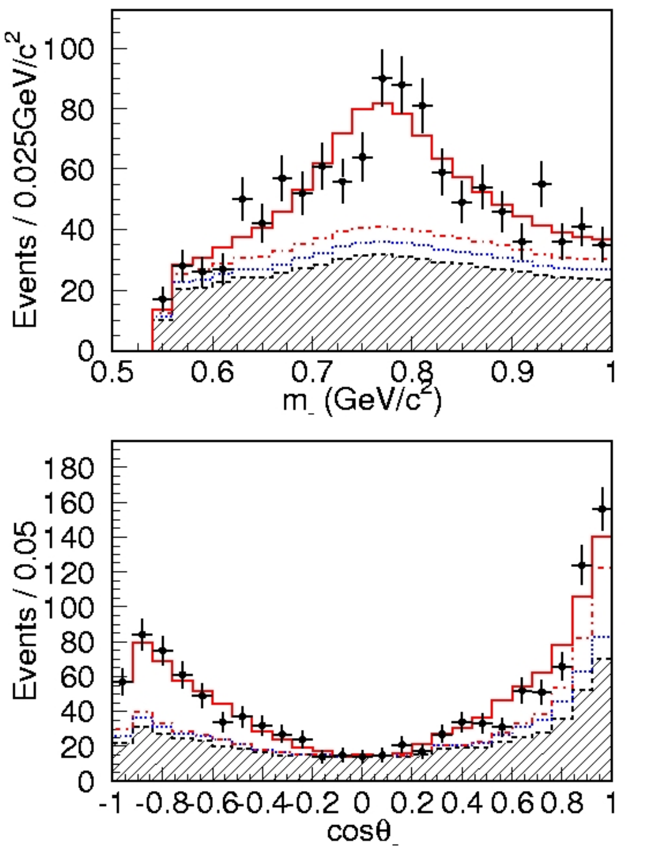
Fit result (Dalitz plot)

hep-ex/0609003

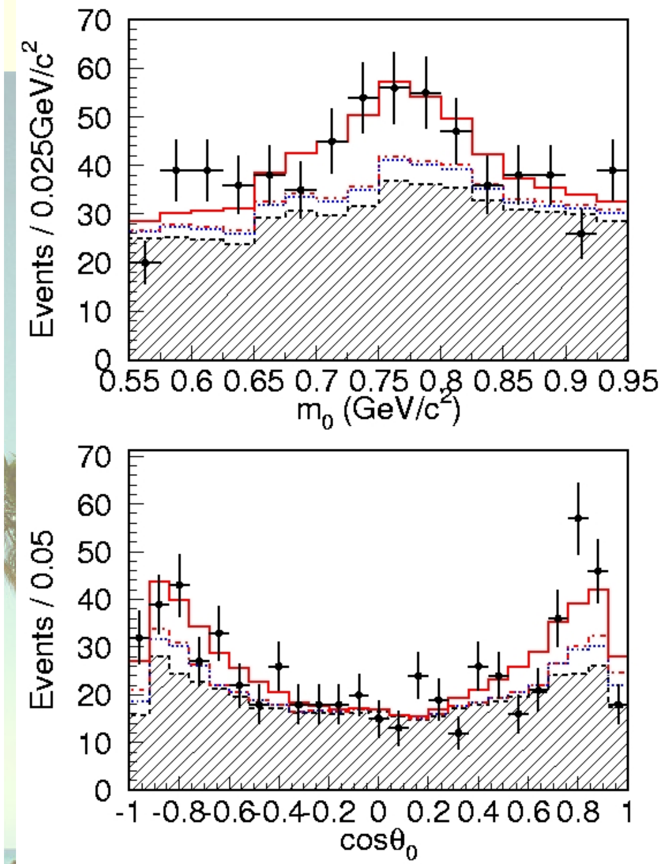
$\rho^+\pi^-$ enhanced region



$\rho^-\pi^+$ enhanced region



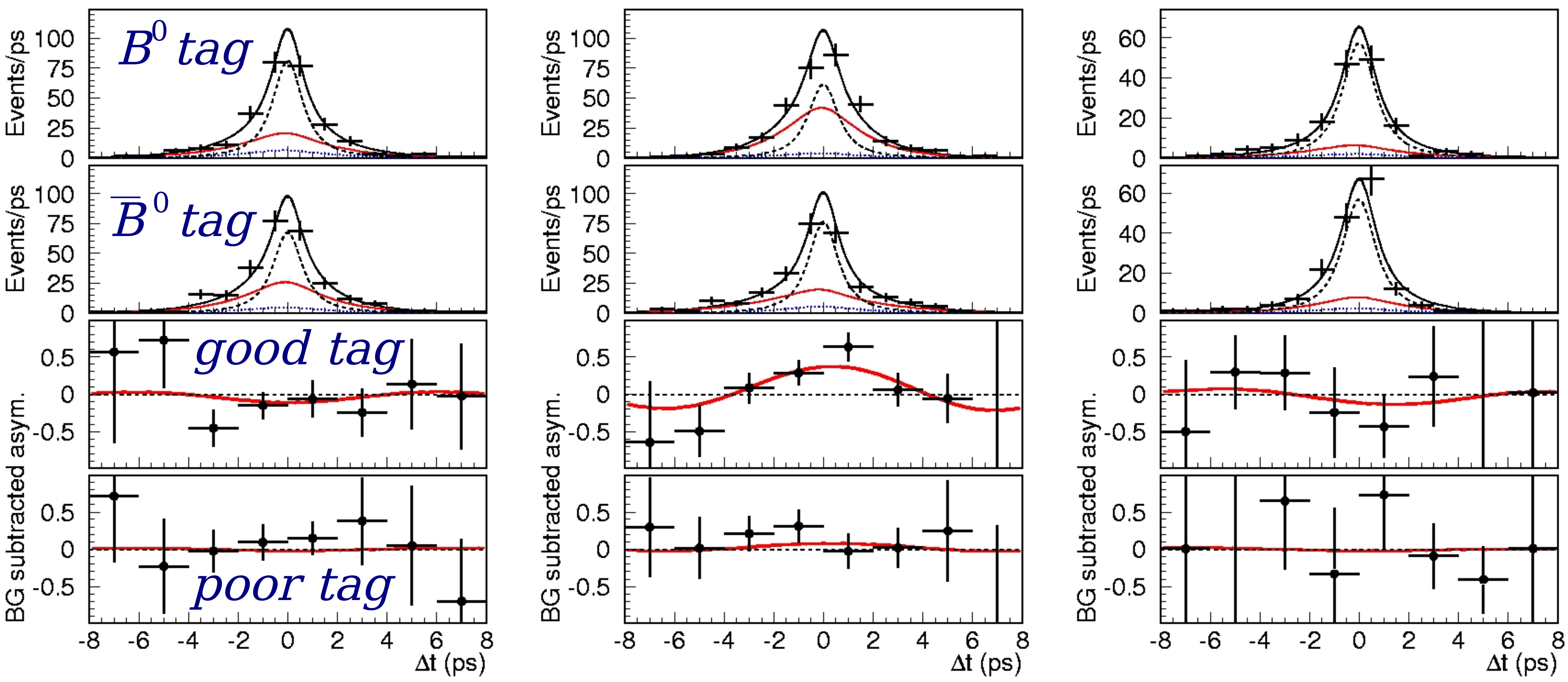
$\rho^0\pi^0$ enhanced region



Fit result (Δt distribution)

hep-ex/0609003

$\rho^+\pi^-$ enhanced region $\rho^-\pi^+$ enhanced region $\rho^0\pi^0$ enhanced region



Quasi-two-body parameters

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$$A^{CP}_{\rho\pi}=\frac{U^+_+ - U^+_-}{U^+_+ + U^+_-}$$

$$C=\left(\frac{U^-_+}{U^+_+}+\frac{U^-_-}{U^+_-}\right)/2$$

$$\Delta C=\left(\frac{U^-_+}{U^+_+}-\frac{U^-_-}{U^+_-}\right)/2$$

$$S=\left(\frac{I_+}{U^+_+}+\frac{I_+}{U^-_-}\right)$$

$$\Delta S=\left(\frac{I_+}{U^+_+}-\frac{I_+}{U^-_-}\right)$$

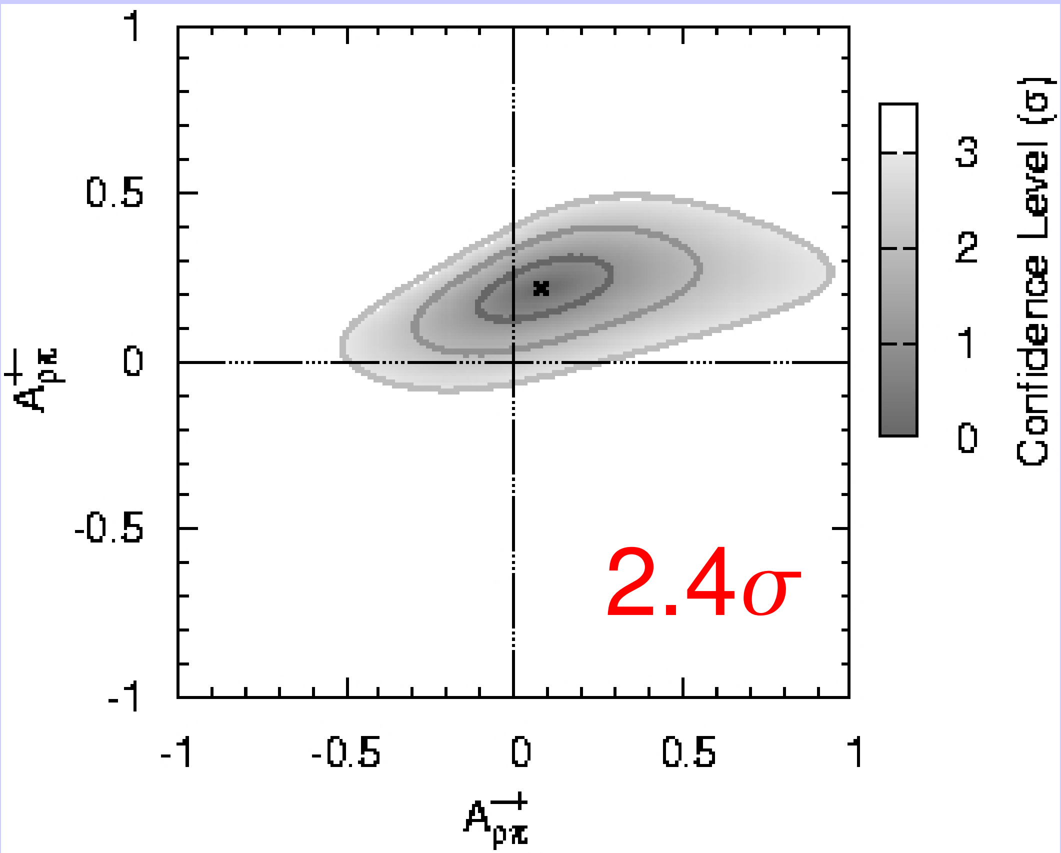
$$A_{\rho^0\pi^0}=-\frac{U^-_0}{U^+_0}$$

$$S_{\rho^0\pi^0}=\frac{2I_0}{U^+_0}$$

	Belle result	Babar ICHEP06
$A^{CP}_{\rho\pi}$	$-0.12\pm0.05\pm0.03$	$-0.14\pm0.04\pm0.02$
C	$-0.13\pm0.09\pm0.06$	$+0.15\pm0.09\pm0.04$
ΔC	$+0.35\pm0.10\pm0.06$	$+0.38\pm0.09\pm0.02$
S	$+0.06\pm0.13\pm0.07$	$+0.01\pm0.12\pm0.03$
ΔS	$-0.12\pm014\pm0.07$	$+0.06\pm013\pm0.03$
$A_{\rho^0\pi^0}$	$-0.45\pm0.35\pm0.32$	
$S_{\rho^0\pi^0}$	$+0.15\pm0.57\pm0.43$	

Direct CP violation

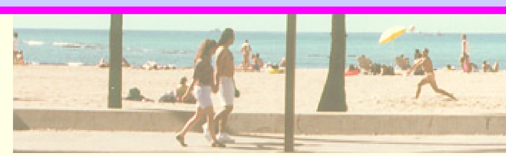
hep-ex/0609003



Belle result

$$A^{+-}_{\rho\pi} = +0.22 \pm 0.08 \pm 0.05$$

$$A^{-+}_{\rho\pi} = +0.08 \pm 0.17 \pm 0.12$$



Babar ICHEP06

$$A^{+-}_{\rho\pi} = +0.03 \pm 0.07 \pm 0.03$$

$$A^{-+}_{\rho\pi} = -0.38^{+0.15}_{-0.16} \pm 0.07$$

ϕ_2 constraint from B→$\pi^+\pi^-\pi^0$

b→u CP eigenstate

(B→$\pi\pi$, $\rho\rho$)

If no penguin:

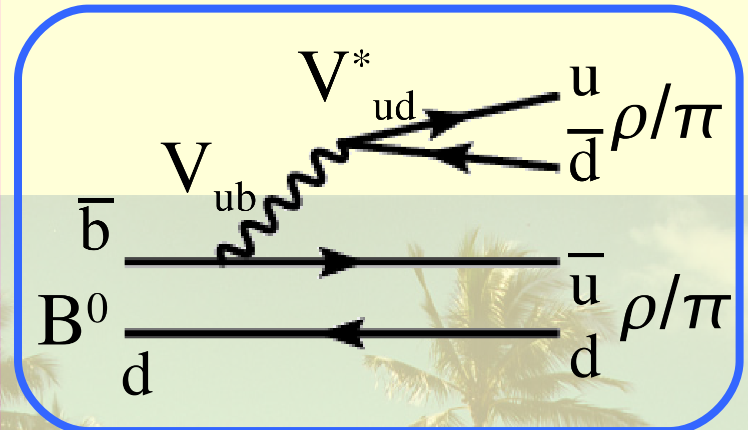
measure time-dependent decay rates

||

measure ϕ2

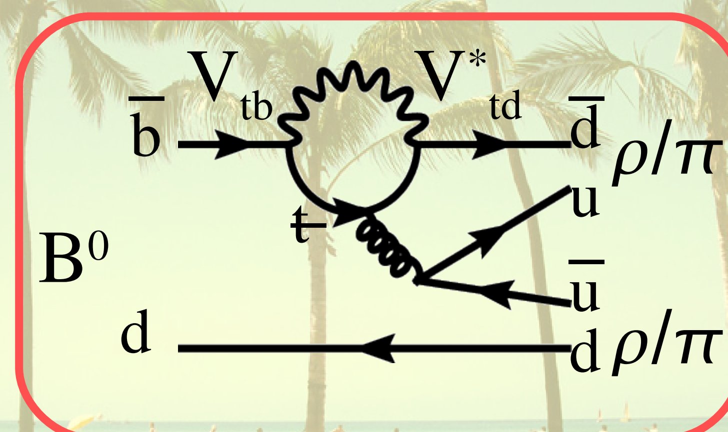
with penguin:

Isospin analysis



ϕ_2

Tree diagram



Penguin diagram

ϕ_2 constraint from B→$\pi^+\pi^-\pi^0$

Another method
non-CP-eigenstate B→$\rho\pi(\pi^+\pi^-\pi^0)$

ϕ_2 can be constrainted by time-dependent Dalitz analysis without ambiquities

First proposed by A.E.Snyder and H.R.Quinn
Phys. Rev. D48. 2139 (1993)

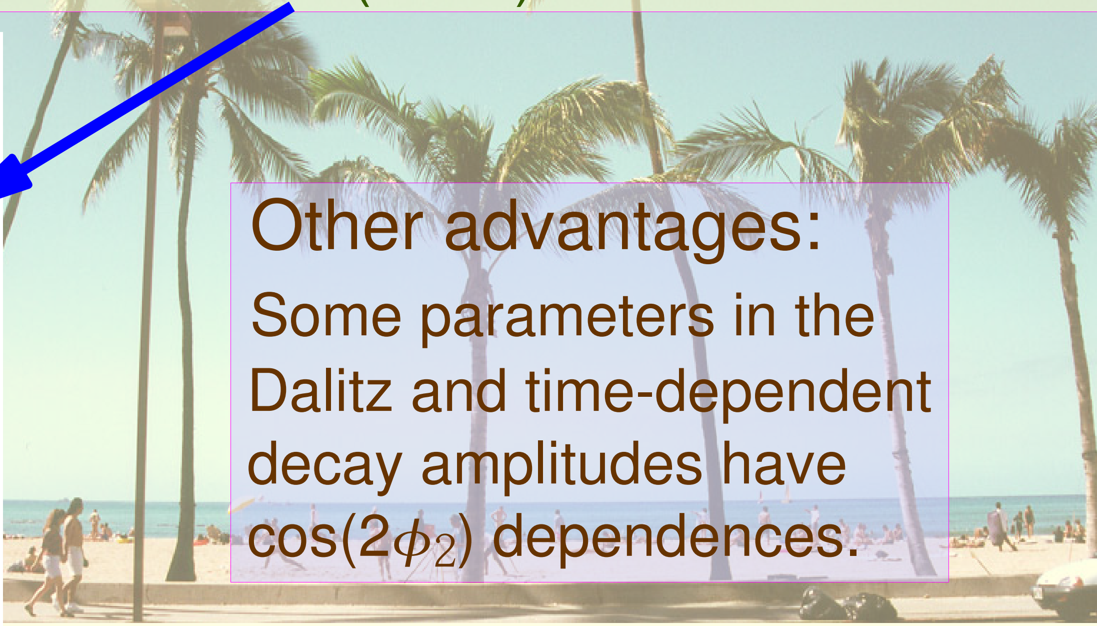


TABLE I. The time and kinematic dependence of contributions to the distribution of events.

Time dependence	Kinematic form	Amplitude measured	α dependence (all $P_i=0$)
1	f^+f^{+*}	$S_3S_3^* + \bar{S}_4\bar{S}_4^*$	1
$\cos(\Delta M t)$	f^+f^{+*}	$S_3S_3^* - \bar{S}_4\bar{S}_4^*$	1
$\sin(\Delta M t)$	f^+f^{+*}	$\mathrm{Im}(q\bar{S}_4S_3^*)$	$\sin(2\alpha)$
1	f^-f^{-*}	$S_4S_4^* + \bar{S}_3\bar{S}_3^*$	1
$\cos(\Delta M t)$	f^-f^{-*}	$S_4S_4^* - \bar{S}_3\bar{S}_3^*$	1
$\sin(\Delta M t)$	f^-f^{-*}	$\mathrm{Im}(q\bar{S}_3S_4^*)$	$\sin(2\alpha)$
1	f^0f^{0*}	$(S_5S_5^* + \bar{S}_5\bar{S}_5^*)/4$	1
$\cos(\Delta M t)$	f^0f^{0*}	$(S_5S_5^* - \bar{S}_5\bar{S}_5^*)/4$	1
$\sin(\Delta M t)$	f^0f^{0*}	$\mathrm{Im}(q\bar{S}_5S_5^*)/4$	$\sin(2\alpha)$
1	$\mathrm{Re}(f^+f^{-*})$	$\mathrm{Re}(S_3S_4^* + \bar{S}_4\bar{S}_3^*)$	1
$\cos(\Delta M t)$	$\mathrm{Re}(f^+f^{-*})$	$\mathrm{Re}(S_3S_4^* - \bar{S}_4\bar{S}_3^*)$	1
$\sin(\Delta M t)$	$\mathrm{Re}(f^+f^{-*})$	$\mathrm{Im}(q\bar{S}_4S_4^* - q^*S_3\bar{S}_3^*)$	$\sin(2\alpha)$
1	$\mathrm{Im}(f^+f^{-*})$	$\mathrm{Im}(S_3S_4^* + \bar{S}_4\bar{S}_3^*)$	1
$\cos(\Delta M t)$	$\mathrm{Im}(f^+f^{-*})$	$\mathrm{Im}(S_3S_4^* - \bar{S}_4\bar{S}_3^*)$	1
$\sin(\Delta M t)$	$\mathrm{Im}(f^+f^{-*})$	$\mathrm{Re}(q\bar{S}_4S_4^* - q^*S_3\bar{S}_3^*)$	$\cos(2\alpha)$
1	$\mathrm{Re}(f^+f^{0*})$	$\mathrm{Re}(S_3S_5^* + \bar{S}_4\bar{S}_5^*)/2$	1
$\cos(\Delta M t)$	$\mathrm{Re}(f^+f^{0*})$	$\mathrm{Re}(S_3S_5^* - \bar{S}_4\bar{S}_5^*)/2$	1
$\sin(\Delta M t)$	$\mathrm{Re}(f^+f^{0*})$	$\mathrm{Im}(q\bar{S}_4S_5^* + q^*S_3\bar{S}_5^*)/2$	$\sin(2\alpha)$
1	$\mathrm{Im}(f^+f^{0*})$	$\mathrm{Im}(S_3S_5^* + \bar{S}_4\bar{S}_5^*)/2$	1
$\cos(\Delta M t)$	$\mathrm{Im}(f^+f^{0*})$	$\mathrm{Im}(S_3S_5^* - \bar{S}_4\bar{S}_5^*)/2$	1
$\sin(\Delta M t)$	$\mathrm{Im}(f^+f^{0*})$	$\mathrm{Re}(q\bar{S}_4S_5^* - q^*S_3\bar{S}_5^*)/2$	$\cos(2\alpha)$
1	$\mathrm{Re}(f^-f^{0*})$	$\mathrm{Re}(S_4S_5^* + \bar{S}_3\bar{S}_5^*)/2$	1
$\cos(\Delta M t)$	$\mathrm{Re}(f^-f^{0*})$	$\mathrm{Re}(S_4S_5^* - \bar{S}_3\bar{S}_5^*)/2$	1
$\sin(\Delta M t)$	$\mathrm{Re}(f^-f^{0*})$	$\mathrm{Im}(q\bar{S}_3S_5^* - q^*S_4\bar{S}_5^*)$	$\sin(2\alpha)$
1	$\mathrm{Im}(f^-f^{0*})$	$\mathrm{Im}(S_4S_5^* + \bar{S}_3\bar{S}_5^*)/2$	1
$\cos(\Delta M t)$	$\mathrm{Im}(f^-f^{0*})$	$\mathrm{Im}(S_4S_5^* - \bar{S}_3\bar{S}_5^*)/2$	1
$\sin(\Delta M t)$	$\mathrm{Im}(f^-f^{0*})$	$\mathrm{Re}(q\bar{S}_3S_5^* - q^*S_4\bar{S}_5^*)/2$	$\cos(2\alpha)$

Other advantages:

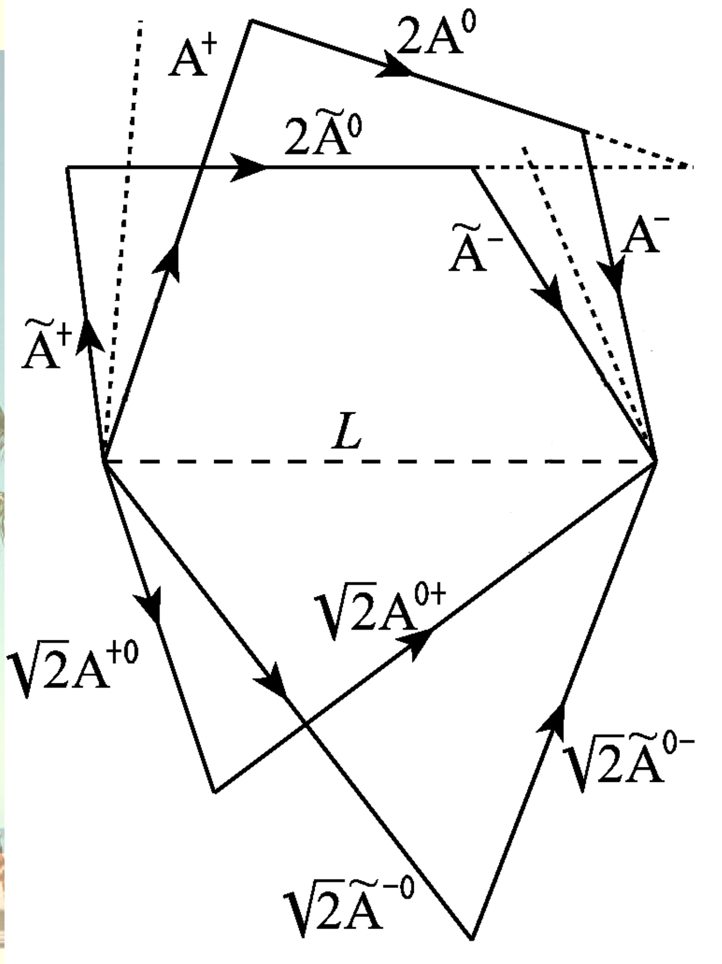
Some parameters in the Dalitz and time-dependent decay amplitudes have $\cos(2\phi_2)$ dependences.

Isospin and Time-depedent Dalitz analysis

$$A^{+} \equiv A(B^0 \to \rho^{+}\pi^{-})$$
$$A^{-} \equiv A(B^0 \to \rho^{-}\pi^{+})$$
$$A^{0} \equiv A(B^0 \to \rho^{0}\pi^{0})$$
$$A^{+0} \equiv A(B^{+} \to \rho^{+}\pi^{0})$$
$$A^{0+} \equiv A(B^{+} \to \rho^{0}\pi^{+})$$

ϕ_2 extraction can be improved by isospin and time-dependent Dalitz combined analysis

$$e^{2i\phi_2}\tilde{A}^{+} \equiv \overline{A}^{+} \equiv (p/q)A(\overline{B}^0 \to \rho^{+}\pi^{-})$$
$$e^{2i\phi_2}\tilde{A}^{-} \equiv \overline{A}^{-} \equiv (p/q)A(\overline{B}^0 \to \rho^{-}\pi^{+})$$
$$e^{2i\phi_2}\tilde{A}^{0} \equiv \overline{A}^{0} \equiv (p/q)A(\overline{B}^0 \to \rho^{0}\pi^{0})$$
$$e^{2i\phi_2}\tilde{A}^{-0} \equiv \overline{A}^{-0} \equiv (p/q)A(B^{-} \to \rho^{-}\pi^{0})$$
$$e^{2i\phi_2}\tilde{A}^{0-} \equiv \overline{A}^{0-} \equiv (p/q)A(B^{-} \to \rho^{0}\pi^{-})$$



$$Br(B \to \rho^{\pm}\pi^{\mp}) = (24.0 \pm 2.5) \times 10^{-6}$$
$$Br(B \to \rho^{+}\pi^{0}) = (10.8^{+1.4}_{-1.5}) \times 10^{-6}$$
$$Br(B \to \rho^{0}\pi^{+}) = (8.7^{+1.0}_{-1.1}) \times 10^{-6}$$
$$A_{\rho^{+}\pi^{0}} = 0.01 \pm 0.11$$
$$A_{\rho^{0}\pi^{+}} = -0.07^{+0.12}_{-0.13}$$

HFAG2006
hep-ex/0603003

Fit result

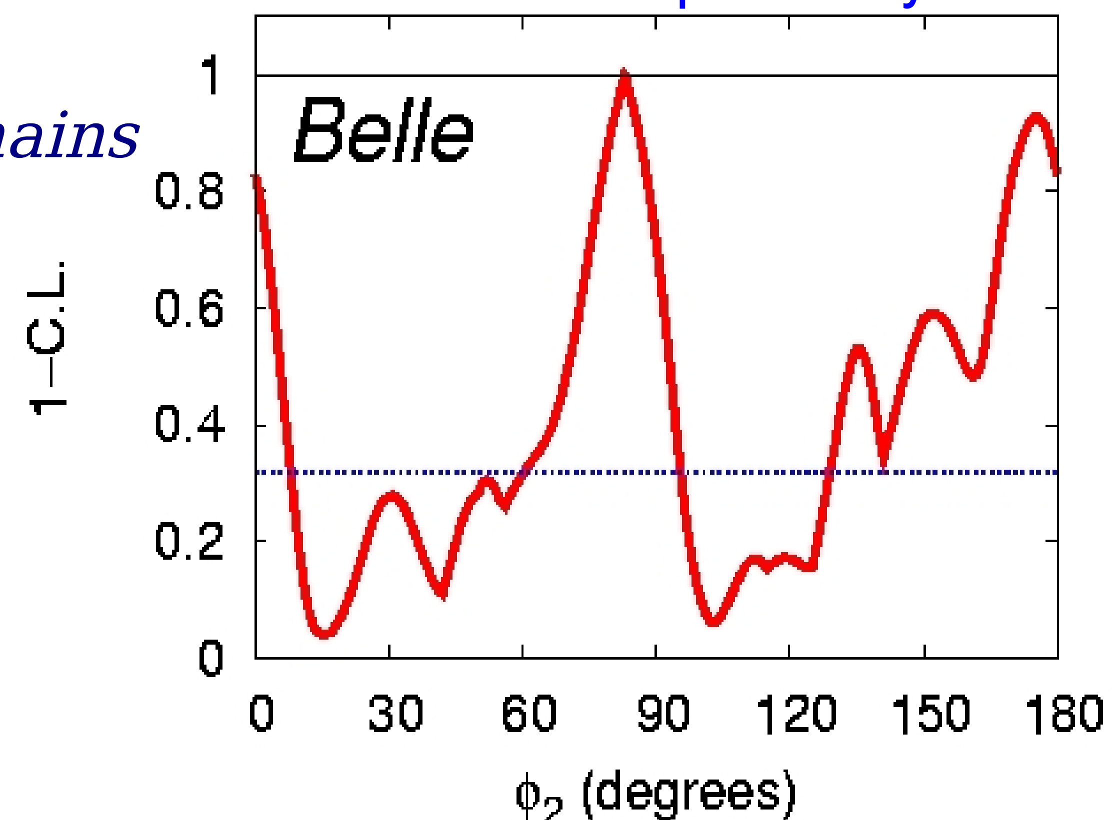
$\phi_2 = (83^{+12}_{-23})^o$

CKM disfavored region

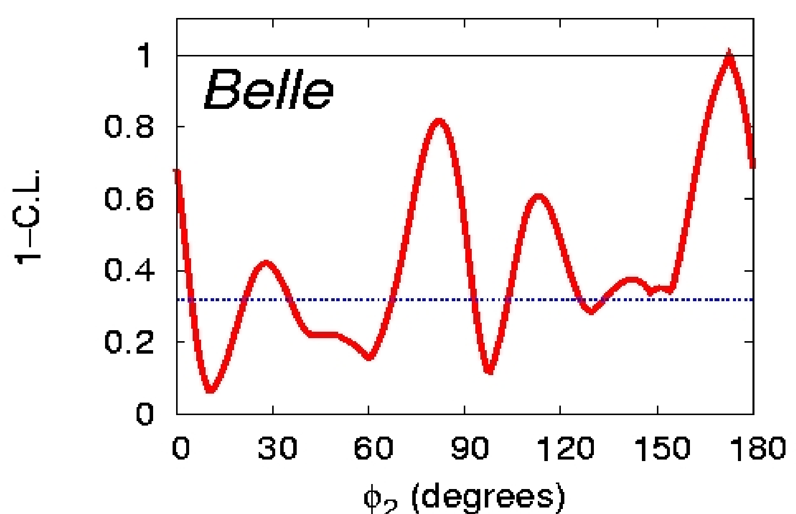
$(\phi_2 < 8^o$ *and* $\phi_2 > 129^o)$ *remains*

hep-ex/0609003

Dalitz and isospin analysis



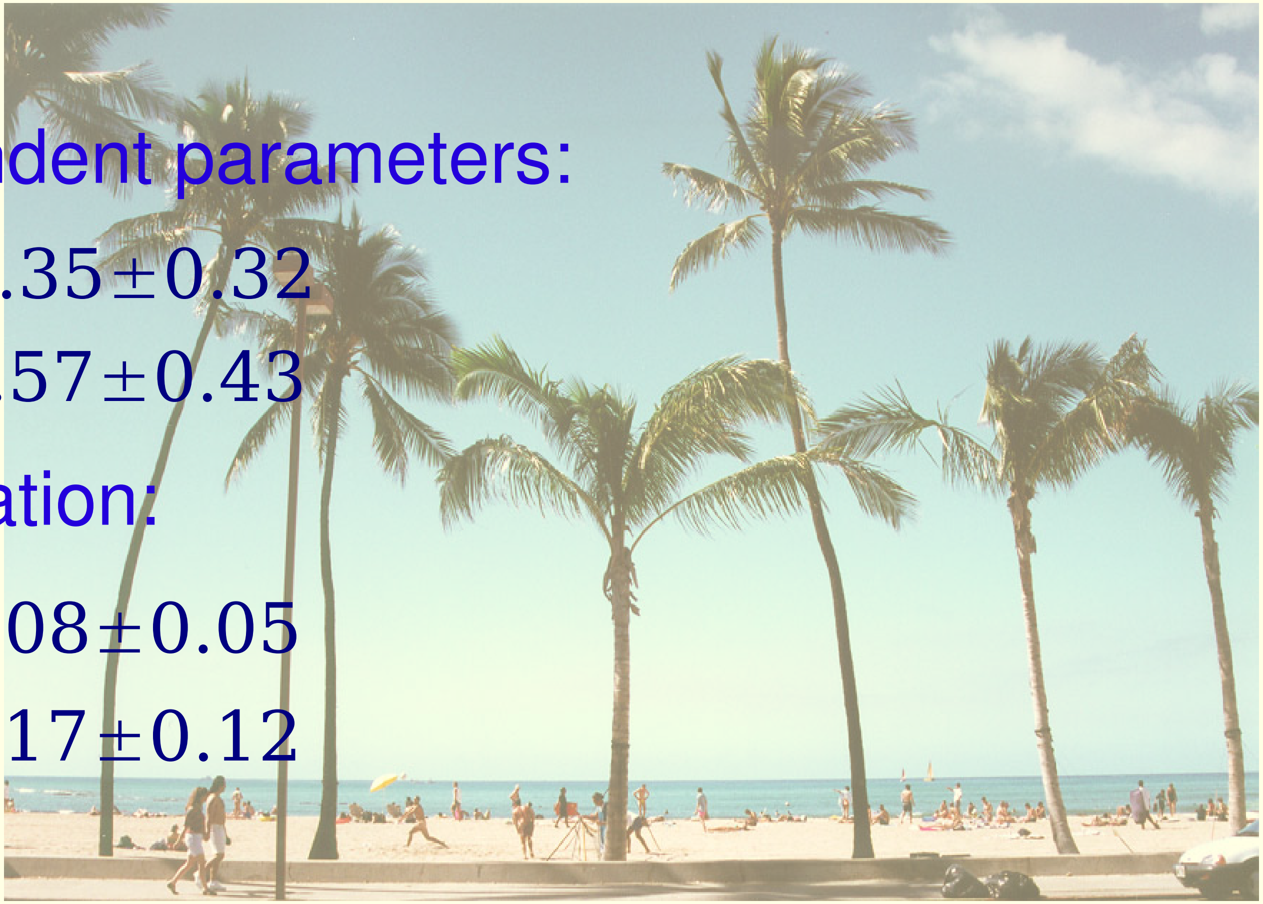
Dalitz analysis only



First result from isospin and time-dependent combined fit.

Summary

- First time-dependent-Dalitz and isospin analysis for ϕ_2 extraction.
- $\phi_2 = (83^{+12}_{-23})^o$
- $B \to \rho^0\pi^0$ time-dependent parameters:
  - $A_{\rho^0\pi^0} = -0.45 \pm 0.35 \pm 0.32$
  - $S_{\rho^0\pi^0} = +0.15 \pm 0.57 \pm 0.43$
- 2.4σ direct CP violation:
  - $A_{\rho\pi}^{+-} = +0.22 \pm 0.08 \pm 0.05$
  - $A_{\rho\pi}^{-+} = +0.08 \pm 0.17 \pm 0.12$



Backup slides

Formalism

$$|A_{3\pi}|^2 \pm |\overline{A}_{3\pi}|^2 = \sum_{\kappa\in\{+,-,0\}} |f_\kappa|^2 U_\kappa^\pm + 2\sum_{\kappa<\sigma\in\{+,-,0\}} \left(Re[f_\kappa f_\sigma^*]\, U_{\kappa\sigma}^{\pm,Re} - Im[f_\kappa f_\sigma^*]\, U_{\kappa\sigma}^{\pm,Im}\right)$$

$$Im\left(\frac{q}{p} A_{3\pi}^* \overline{A}_{3\pi}\right) = \sum_{\kappa<\sigma\in\{+,-,0\}} \left(Re[f_\kappa f_\sigma^*]\, I_{\kappa\sigma}^{Im} + Im[f_\kappa f_\sigma^*]\, I_{\kappa\sigma}^{Re}\right)$$

$$U_\kappa^\pm = \left(|A^\kappa|^2 \pm |\overline{A}^\kappa|^2\right)/N$$

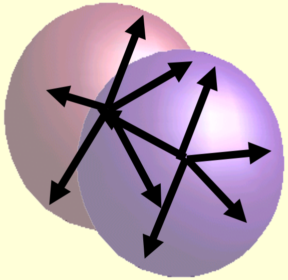
$$U_{\kappa\sigma}^{\pm,Re(Im)} = Re(Im)\left[A^\kappa A^{\sigma *} \pm \overline{A}^\kappa \overline{A}^{\sigma *}\right]/N$$

$$I_\kappa = Im\left[\overline{A}^\kappa A^{\kappa *}\right]/N$$

$$I_{\kappa\sigma}^{Re} = Re\left[\overline{A}^\kappa A^{\sigma *} - A^\sigma A^{\kappa *}\right]/N$$

$$I_{\kappa\sigma}^{Im} = Im\left[\overline{A}^\kappa A^{\sigma *} + A^\sigma A^{\kappa *}\right]/N$$

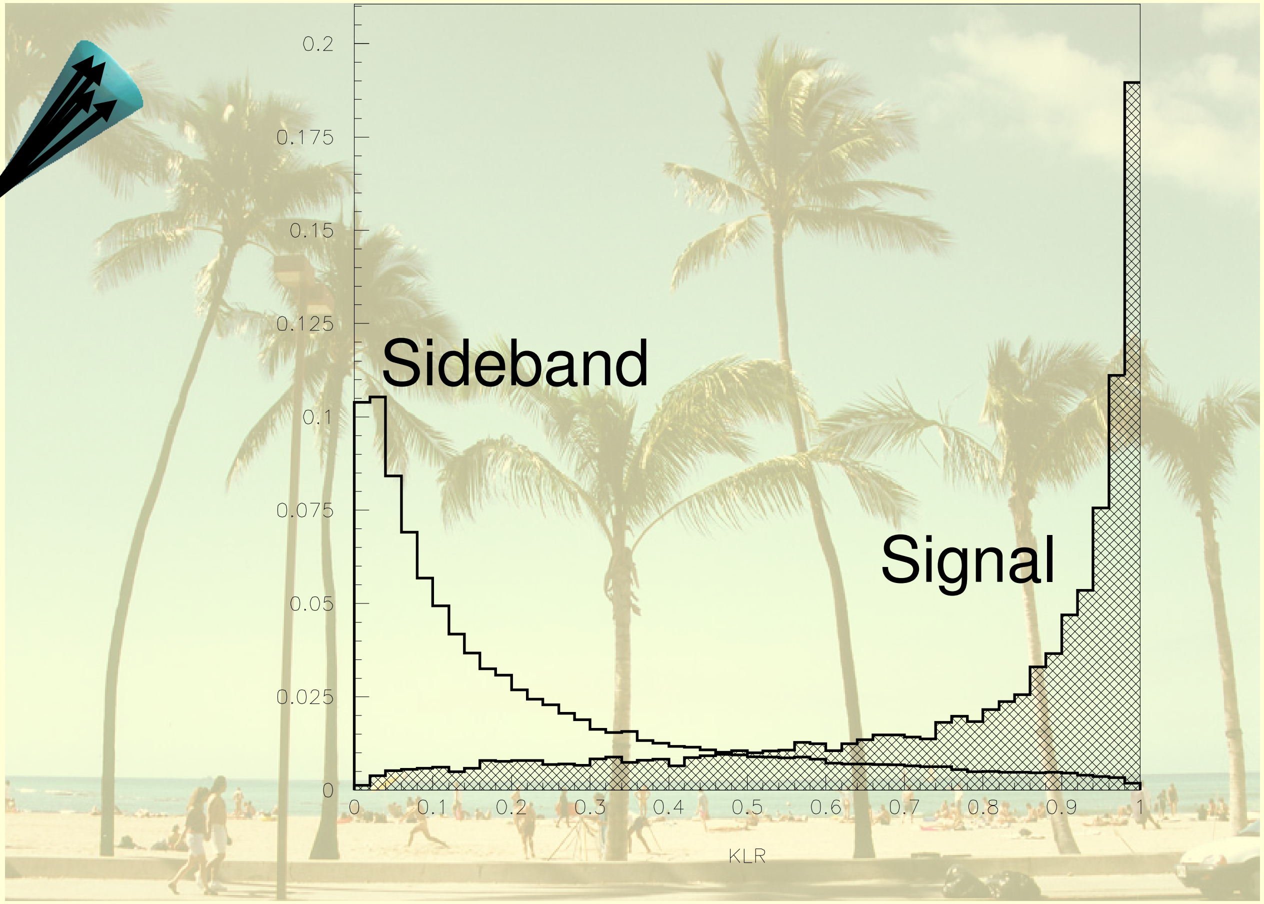
Continuum Suppression



$B\bar{B}$ events

Continuum events

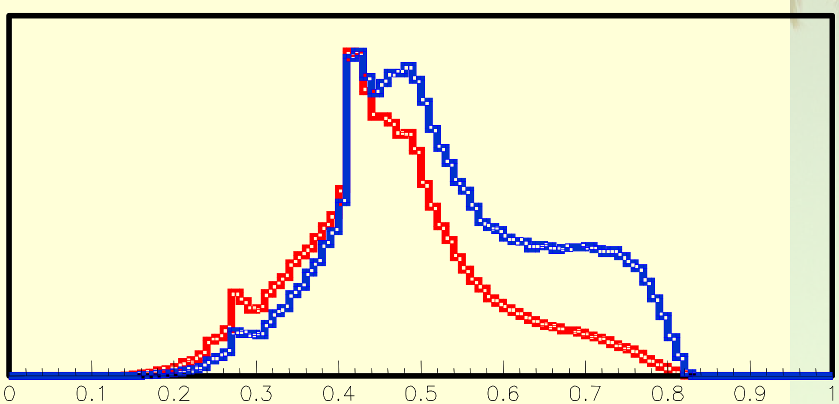
Likelihood ratio



Best candidate selection

Using $m_{\pi 0}$ and likelihood ratio

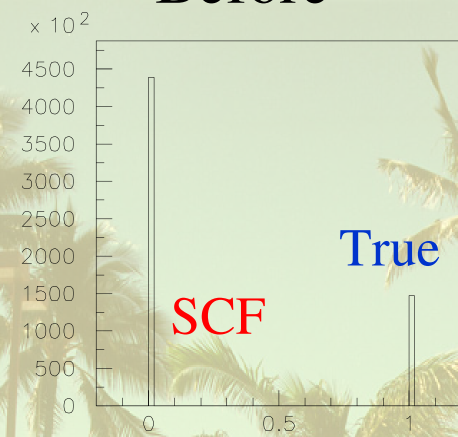
Likelihood



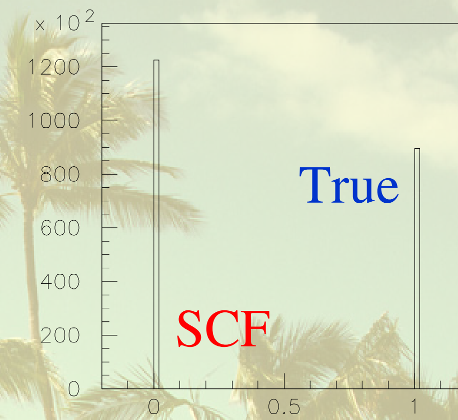
Truly reconstructed

Self Cross Feed

Before



After



~70% reduction of SCF

~30% loss of True recon

for multi-candidate events

ΔE and M_{bc}

$$M_{bc} \equiv \sqrt{E_{beam}^2 - P_B^2}$$

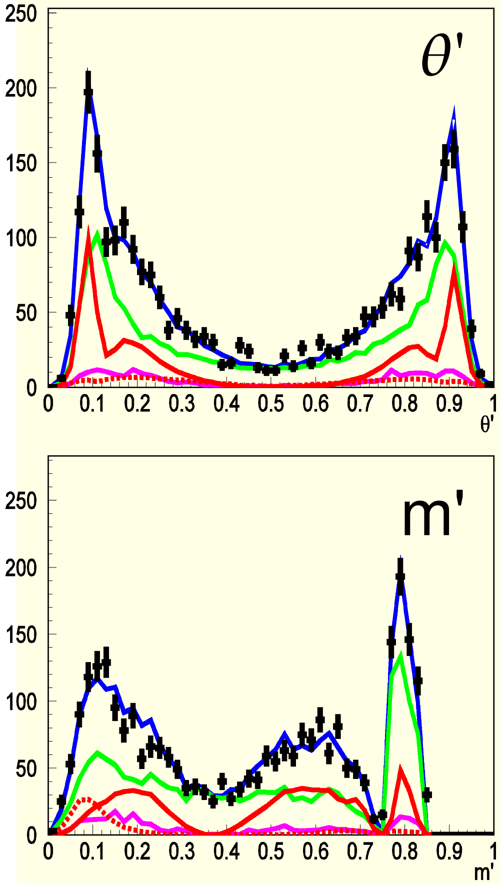
$$\Delta E \equiv E_B - E_{beam}$$

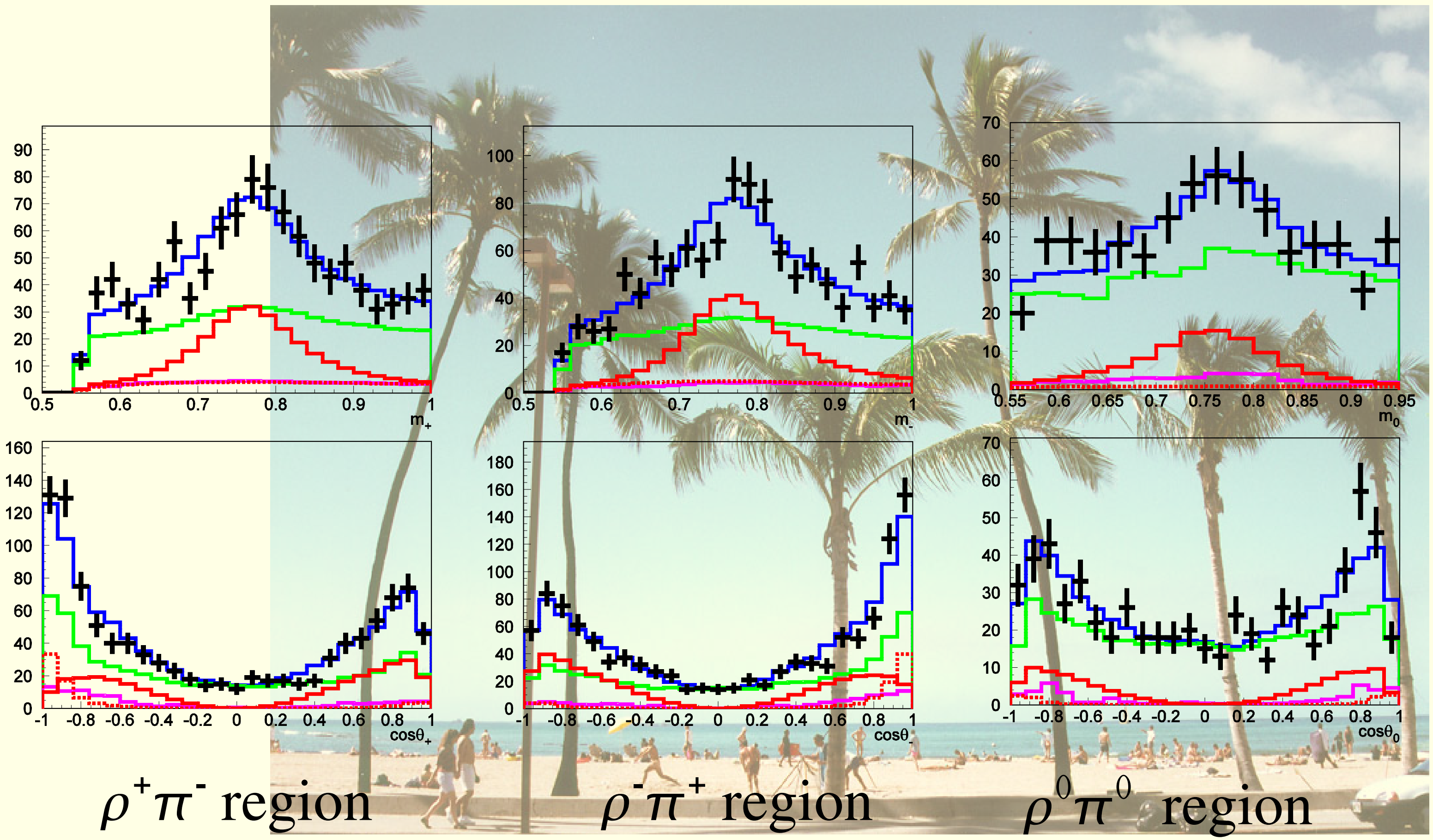
E_{beam}: *beam energy*

P_B: *B meson momentum*

E_B: *B meson energy*

ΔE-M_{bc} and Dalitz simultaneous fit





Systematic Uncertainties

	U_-^+	U_0^+	$U_{+-}^{+,\mathrm{Re}}$	$U_{+0}^{+,\mathrm{Re}}$	$U_{-0}^{+,\mathrm{Re}}$	$U_{+-}^{+,\mathrm{Im}}$	$U_{+0}^{+,\mathrm{Im}}$	$U_{-0}^{+,\mathrm{Im}}$
ρ' and ρ''	0.01	0.01	0.29	0.19	0.26	0.32	0.37	0.29
SCF	0.01	0.02	0.31	0.14	0.17	0.03	0.03	0.10
Signal Dalitz	0.02	0.01	0.24	0.15	0.19	0.13	0.06	0.13
BG Dalitz	0.02	0.01	0.16	0.12	0.14	0.14	0.12	0.22
Other $\pi\pi\pi$	0.06	0.03	0.10	0.08	0.10	0.15	0.10	0.08
BG fraction	0.03	0.02	0.14	0.19	0.13	0.23	0.07	0.22
Physics	0.02	< 0.01	0.01	0.02	0.02	0.01	0.01	0.02
BG Δt	< 0.01	< 0.01	0.03	0.01	0.02	0.02	0.01	0.02
Vertexing	0.02	0.02	0.02	0.16	0.11	0.08	0.08	0.09
Resolution	< 0.01	< 0.01	0.04	0.07	0.03	0.04	0.03	0.02
Flavor tagging	< 0.01	< 0.01	< 0.01	< 0.01	0.01	< 0.01	< 0.01	0.01
Fit bias	0.01	0.01	0.16	0.22	0.07	0.09	0.22	0.24
TSI	< 0.01	< 0.01	0.01	0.01	0.01	0.03	0.01	0.01
Total	0.08	0.05	0.57	0.46	0.44	0.49	0.47	0.53

Systematic Uncertainties

	U_+^-	U_-^-	U_0^-	$U_{+-}^{-,\mathrm{Re}}$	$U_{+0}^{-,\mathrm{Re}}$	$U_{-0}^{-,\mathrm{Re}}$	$U_{+-}^{-,\mathrm{Im}}$	$U_{+0}^{-,\mathrm{Im}}$	$U_{-0}^{-,\mathrm{Im}}$
ρ' and ρ''	0.02	0.02	0.05	0.42	0.31	0.41	0.77	0.45	0.36
SCF	0.02	0.03	0.03	0.29	0.27	0.32	0.09	0.25	0.17
Signal Dalitz	0.01	0.02	0.02	0.28	0.32	0.32	0.38	0.15	0.53
BG Dalitz	0.04	0.03	0.02	0.29	0.36	0.30	0.31	0.22	0.41
Other $\pi\pi\pi$	0.05	0.05	0.03	0.12	0.11	0.14	0.15	0.11	0.13
BG fraction	0.03	0.04	0.02	0.31	0.30	0.32	0.38	0.22	0.49
Physics	0.01	0.01	< 0.01	0.04	0.03	0.04	0.04	0.02	0.06
BG Δt	< 0.01	< 0.01	< 0.01	0.03	0.04	0.02	0.04	0.02	0.05
Vertexing	0.04	0.02	0.05	0.17	0.45	0.16	0.08	0.10	0.27
Resolution	0.01	0.01	0.01	0.16	0.17	0.32	0.11	0.10	0.29
Flavor tagging	0.01	0.01	< 0.01	0.03	0.04	0.04	0.05	0.03	0.03
Fit bias	0.01	0.03	< 0.01	0.05	0.07	0.12	0.18	0.02	0.23
TSI	0.04	0.04	0.01	0.05	0.07	0.03	0.02	0.06	0.01
Total	0.10	0.09	0.09	0.77	0.86	0.86	1.03	0.65	1.05

Systematic Uncertainties

	I_+	I_-	I_0	$I^{\rm Re}_{+-}$	$I^{\rm Re}_{+0}$	$I^{\rm Re}_{-0}$	$I^{\rm Im}_{+-}$	$I^{\rm Im}_{+0}$	$I^{\rm Im}_{-0}$
ρ' and ρ''	0.03	0.02	0.04	0.95	0.59	1.32	0.89	0.84	0.89
SCF	0.01	0.01	0.01	0.09	0.64	0.07	0.50	0.08	0.65
Signal Dalitz	0.01	0.01	0.01	0.33	0.29	0.30	0.31	0.35	0.31
BG Dalitz	0.01	0.01	0.01	0.34	0.38	0.30	0.32	0.34	0.33
Other $\pi\pi\pi$	0.03	0.03	0.02	0.17	0.15	0.18	0.22	0.15	0.20
BG frac.	0.02	0.01	0.01	0.44	0.34	0.33	0.32	0.37	0.29
Physics	0.01	0.01	< 0.01	0.05	0.06	0.03	0.05	0.05	0.05
BG Δt	< 0.01	< 0.01	< 0.01	0.05	0.04	0.04	0.05	0.04	0.11
Vertexing	0.02	0.02	0.04	0.16	0.28	0.14	0.42	0.37	0.28
Resolution	0.01	0.01	0.01	0.30	0.21	0.18	0.35	0.25	0.28
Flavor tagging	< 0.01	< 0.01	< 0.01	0.04	0.07	0.04	0.03	0.07	0.03
Fit bias	< 0.01	0.01	< 0.01	0.12	0.01	0.27	0.09	0.09	0.22
TSI	0.01	< 0.01	0.01	0.09	0.07	0.04	0.04	0.03	0.07
Total	0.06	0.05	0.06	1.23	1.12	1.49	1.31	1.15	1.33

Isospin relation

$$A^{+} \equiv A(B^0 \rightarrow \rho^{+}\pi^{-}) = e^{-i\phi_2} T^{+} + P^{+}$$

$$A^{-} \equiv A(B^0 \rightarrow \rho^{-}\pi^{+}) = e^{-i\phi_2} T^{-} + P^{-}$$

$$A^{0} \equiv A(B^0 \rightarrow \rho^{0}\pi^{0}) = e^{-i\phi_2} T^{0} - \frac{1}{2}(P^{+} + P^{-})$$

$$A^{+0} \equiv A(B^{+} \rightarrow \rho^{+}\pi^{0}) = [e^{-i\phi_2} T^{+0} + P^{+} - P^{-}]/\sqrt{2}$$

$$A^{0+} \equiv A(B^{+} \rightarrow \rho^{0}\pi^{+}) = [e^{-i\phi_2}(T^{+} + T^{-} + 2\mathrm{T}^{0} - T^{+0}) - P^{+} + P^{-}]/\sqrt{2}$$

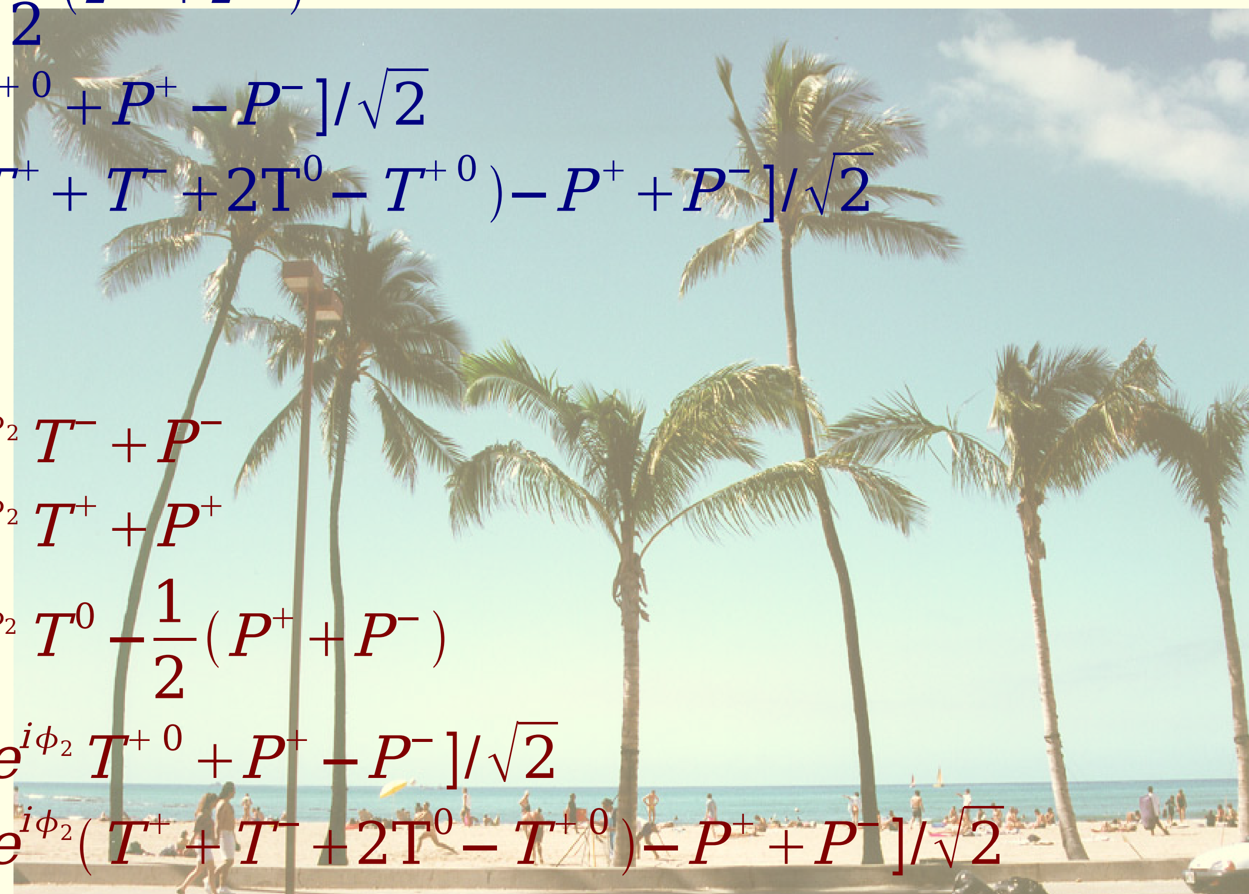
$$\overline{A}^{+} \equiv (p/q) A(\overline{B}^0 \rightarrow \rho^{+}\pi^{-}) = e^{i\phi_2} T^{-} + P^{-}$$

$$\overline{A}^{-} \equiv (p/q) A(\overline{B}^0 \rightarrow \rho^{-}\pi^{+}) = e^{i\phi_2} T^{+} + P^{+}$$

$$\overline{A}^{0} \equiv (p/q) A(\overline{B}^0 \rightarrow \rho^{0}\pi^{0}) = e^{i\phi_2} T^{0} - \frac{1}{2}(P^{+} + P^{-})$$

$$\overline{A}^{-0} \equiv (p/q) A(B^{-} \rightarrow \rho^{-}\pi^{0}) = [e^{i\phi_2} T^{+0} + P^{+} - P^{-}]/\sqrt{2}$$

$$\overline{A}^{0-} \equiv (p/q) A(B^{-} \rightarrow \rho^{0}\pi^{-}) = [e^{i\phi_2}(T^{+} + T^{-} + 2\mathrm{T}^{0} - T^{+0}) - P^{+} + P^{-}]/\sqrt{2}$$



World average from CKM fitter

Summer 2006
without Belle
B→$\rho\pi$ result