Chiral Phase Transition from String Theory

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Introduction

Over past few years there has been considerable progress in studying QCD like theories using holography.

In this talk I will discuss a particular model for χSB

$$U(N_f) \times U(N_f) \rightarrow U(N_f)$$

Outline

1. The model: The weak coupling

2. Gravity description (at $T=0$ and $T\neq 0$)

3 Thermodynamics and chiral symmetry restoration

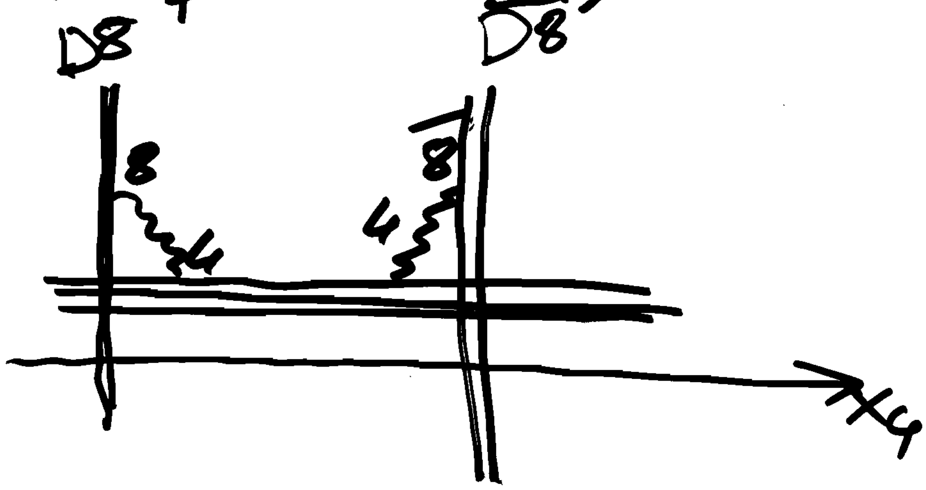
4 Finite chemial potential

5 Photon Emission

The system:

N_f - D8 $\quad N_f$ - $\overline{D8}$ $\quad N_c$ D4

($N_f \ll N_c$)



Particle content:

4 - 8 and 4 - $\bar{8}$ strings

Massless excitations are fermionic: q_L (4-8) & q_R (4 - $\bar{8}$)

The interaction:
Mediated by five dim gauge field A_μ

$$V_{eff} \sim \frac{g_5^2}{4\pi} \int d^4x\, d^4y \;\; G(x-y, L) \cdot$$

$$[\, q_L^+(x)\, q_R(y) \,] \; [\, q_R^+(y) \cdot q_L(x) \,]$$

$$g_5^2 = (2\pi)^3 g_s l_s^2 \qquad \lambda = \frac{g_5^2}{4\pi} N_c$$

The symmetry: $U(N_f) \times U(N_f)$

Gravity description
$(T=0)$ $\quad N_c \gg N_f$

$$ds_4^2 = \left(\frac{u}{R}\right)^{3/2}\left[\eta_{\mu\nu}\,dx^{\mu}dx^{\nu} + (dx^4)^2\right]$$

$$+\left(\frac{R}{u}\right)^{3/2}\left[du^2 + u^2\, d\Omega_4^2\right]$$

$$e^{\phi} = g_s\left(\frac{u}{R}\right)^{3/4} \; ; \quad R^3 = \pi g_s N_c\, l_s^3$$

D8 & $\overline{\text{D8}}$ are probes in this geometry the DBI action for

$D8 - \overline{D8}$; $u = u(x^4)$

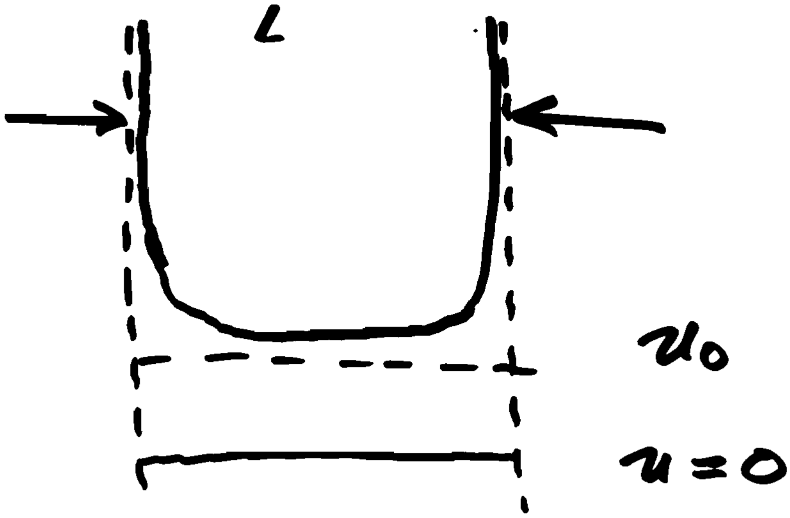
$$S_{D8} \sim \int dx^4\, u^4 \sqrt{1 + \left(\frac{R}{u}\right)^3 u'^2}$$

$(\tau \equiv x^4)$

The E.O.M.

$$\frac{u^4}{\sqrt{1+\left(\frac{R}{u}\right)^3}} = u_0^4$$

2 types of solutions



Gravity description:

The near horizon geom:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(f(u)\, dt^2 + dx_i dx^i + (dx^4)^2\right) +$$

$$\left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2\, d\Omega_4^2\right)$$

$$e^{\phi} = g_s \left(\frac{u}{R}\right)^{3/4} \quad ; \quad f(u) = 1 - \frac{u_T^3}{u^3}$$

Here

$$R^3 = \pi g_s N_c = \pi \lambda \quad ; \quad u_T = \left(\frac{4\pi T}{3}\right)^2 R^3$$

$$t \approx t + \frac{1}{T} .$$

The DBI action for D8 $-\overline{D8}$

$\tau(u) \equiv x^4(u)$

$$S = \int d^3x \int du\, u^{5/2} \sqrt{1 + \left(\frac{u}{R}\right)^3 f(u) (\partial_u \tau)^2}$$

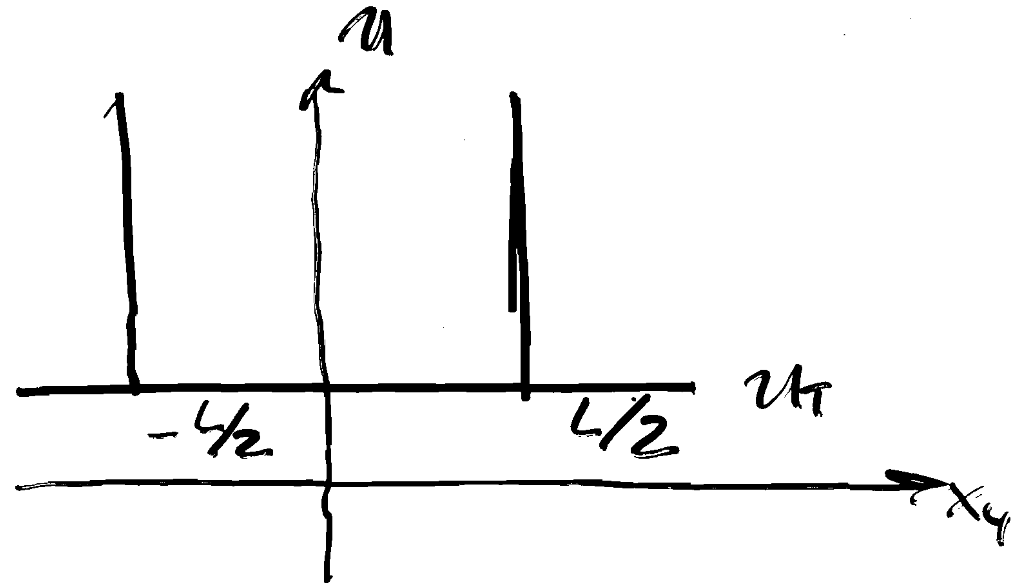
E. O. M.

$$\frac{d}{du}\left[\frac{\delta S}{\delta \tau_u}\right] = 0$$

$$\frac{d}{du}\left[\frac{\left(\frac{u}{R}\right)^{\frac{11}{2}} f(u)\, \partial_u \tau}{\sqrt{1 + \left(\frac{u}{R}\right)^3 f(u) (\partial_u \tau)^2}}\right] = 0$$

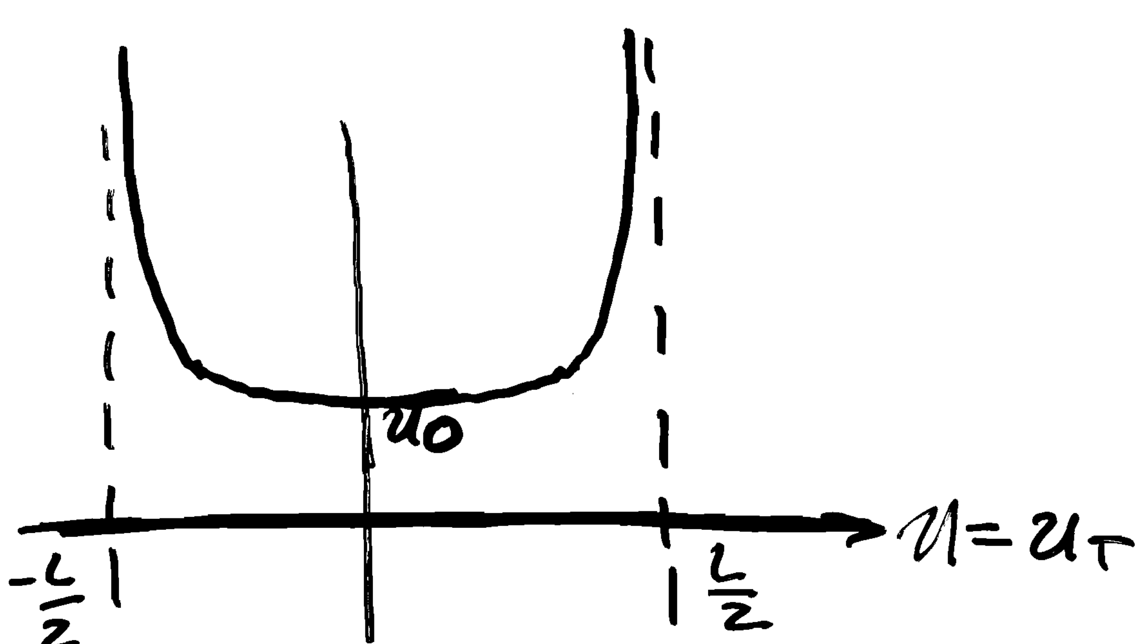
Two types of solutions:

(1) $\tau = \text{const}$; unbroken $U(N_f)\times U(N_f)$



(2)

$$\tau(u) = \pm\sqrt{f(u_0)}\, u_0^4 \int_{u_0}^{u} \frac{du}{\sqrt{u^3 f(u)\left(f(u)u^8 - f(u_0)u_0^8\right)}}$$



$U(N_f)\times U(N_f) \longrightarrow U(N_f)$

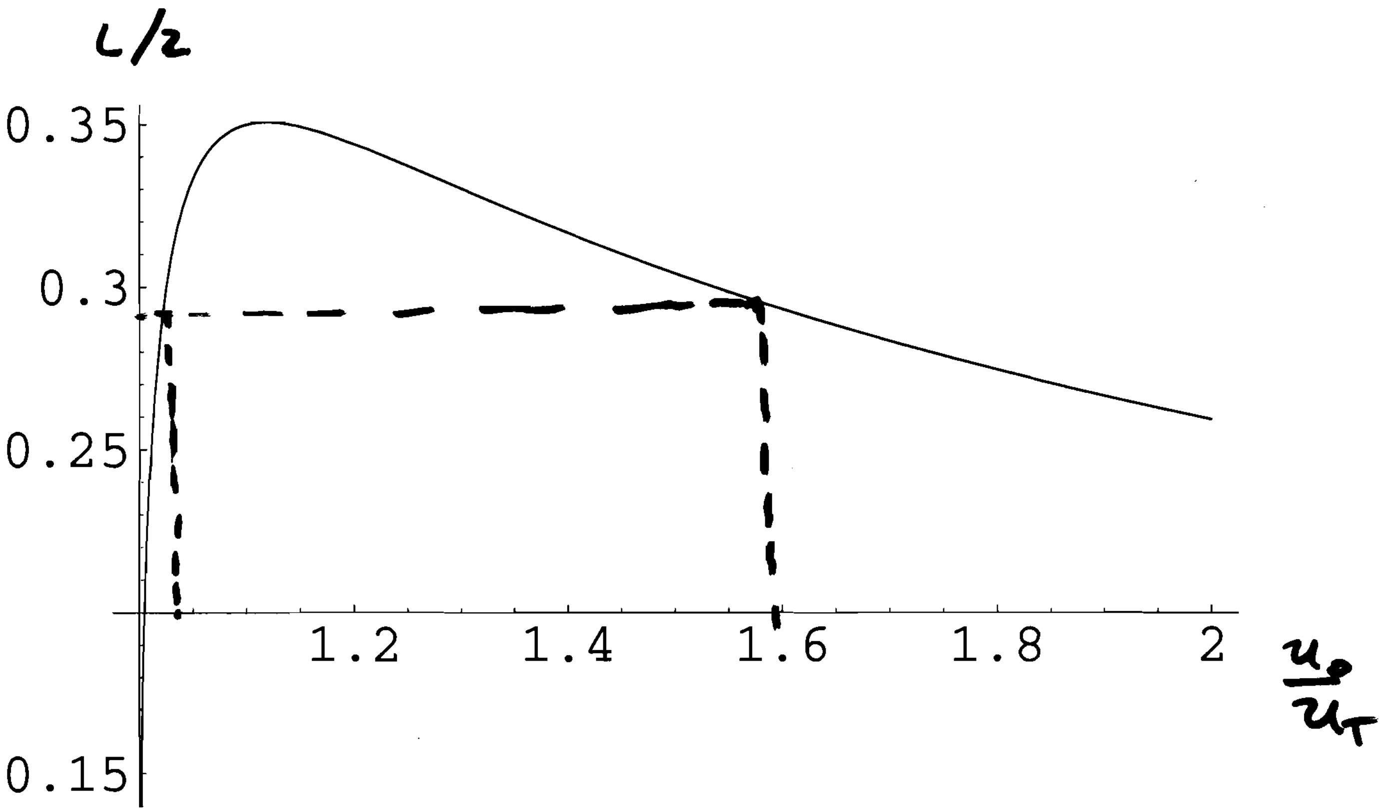
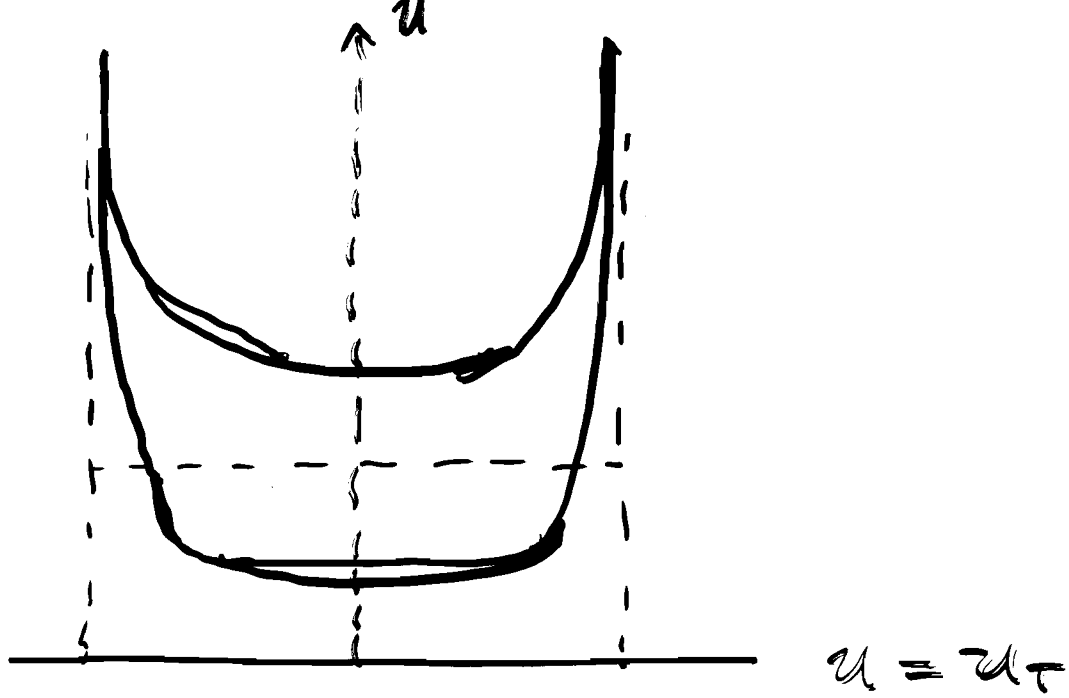


Figure 1: $L/2$ in the units of $3/4\pi T$ as a function of U/U_T.

Two solutions of second type (fixed L and T)



For $T > T_* = .17\, L^{-1}$ there is no curved solution

Thermodynamics of (χSB)

Solution with lowest free energy is preffered

$$\frac{1}{T}(F_{st} - F_{curv}) = S_{st} - S_{curv} \sim$$

$$\int_{u_T}^{u_0} du\, u^4 \left(\frac{R}{u}\right)^{3/2} +$$

$$+ \int_{u_0}^{u_T} du\, u^4 \left(\frac{R}{u}\right)^{3/2} \left[1 - \sqrt{1 + \frac{f(u_0)u_0^8}{f(u)u^8 - f(u_0)u_0^8}}\right.$$

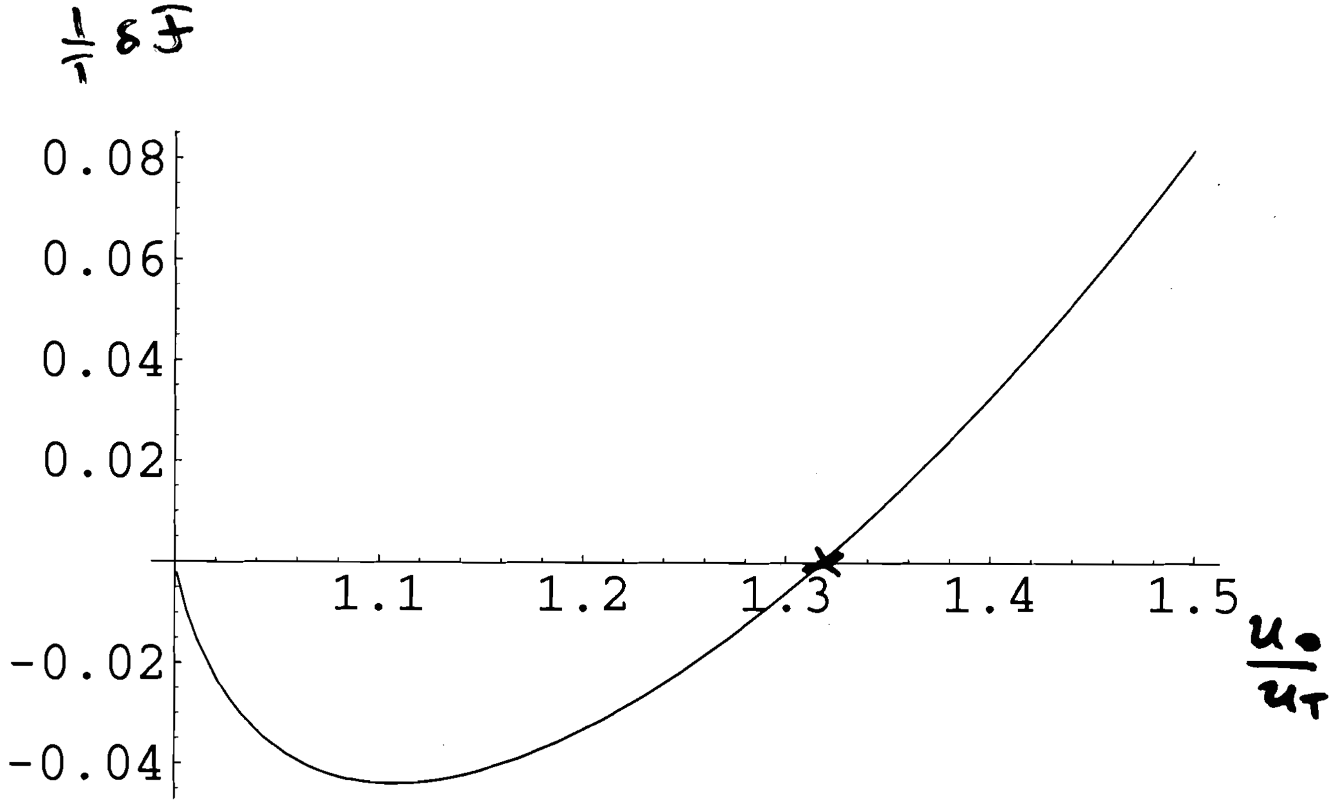


Figure 2: $(1/T)(\mathcal{F}_{cur} - \mathcal{F}_{st})$ as function of U_0/U_T.

The phase transition is of 1^{st} order and happens at

$T_c = .15\, L^{-1}$;

$C_L = \frac{T_c}{V_3}(S_{st} - S_{curv}) = .03\pi^5 N_c \lambda^3 T_c^7$

The finite chemical potential.

Chemical potential $\mu \simeq$ turning on imaginary A_0

The action:

$$S \approx \int du\, u^{5/2} \sqrt{1 + 4\pi F_{0u}^2 + \left(\frac{u}{R}\right)^3 f \cdot \dot{\tau}^2}$$

The thermal exp. value for charge:

$$\rho = iT \left. \frac{\delta S}{\delta A_0^{(0)}(\infty)} \right|_{EOM} = iT \lim_{u\to\infty} \frac{\delta S}{\delta A_0'(u)}$$

E.O.M.

$$\frac{d}{du}\left[\frac{2\pi u^{5/2} A_0^{\prime(0)}}{\sqrt{1-(2\pi A_0')^2+\left(\frac{u}{R}\right)^3 f(u)\tau'^2}}\right]=0$$

$$\frac{d}{du}\left[\frac{u^{11/2} f(u)\,\tau'}{\sqrt{1-(2\pi A_0^{\prime(0)})^2+\left(\frac{u}{R}\right)^3 f(u)\tau'^2}}\right]=0$$

$A_0^{\prime(0)}$ is a monotonic function

so for the phase w/no chiral symmetry, there are no non-trivial solutions.

Unbroken phase

E.O.M.

$$\frac{2\pi\, u^{5/2} A_0'}{\sqrt{1 - 4\pi^2 A_0'^2}} = c.$$

$$\mu = A_0(\infty) - A_0(u_T) \quad ; \quad A_0(u_T) = 0$$

$$\mu = \frac{1}{2\pi}\int_{u_T}^{\infty} du \sqrt{\frac{c^2}{c^2 + u^5}} =$$

$$\frac{c^{2/5}}{2\pi}\left[\frac{\Gamma\left(\frac{3}{10}\right)\Gamma\left(\frac{6}{5}\right)}{\sqrt{\pi}} - \frac{u_T}{c^{2/5}}\, {}_2F_1\left(\frac{1}{5}, \frac{1}{2}, \frac{6}{5}, -\frac{u_T}{c^2}\right)\right]$$

Asymptotics:

$$\rho \simeq \frac{1}{3\sqrt{2\pi}}\left(\frac{\Gamma\left(\frac{3}{10}\right)\Gamma\left(\frac{6}{5}\right)}{\sqrt{\pi}}\right)^{-5/2} N_c\, \lambda^{-\frac{1}{2}}\mu^{5/2}$$

$\mu >> (\lambda T)T$

$$\rho \simeq N_c\left[\frac{\pi}{9}(\lambda T)T^2\mu + \frac{13}{9\pi^2}\frac{\mu^3}{\lambda T}\right]$$

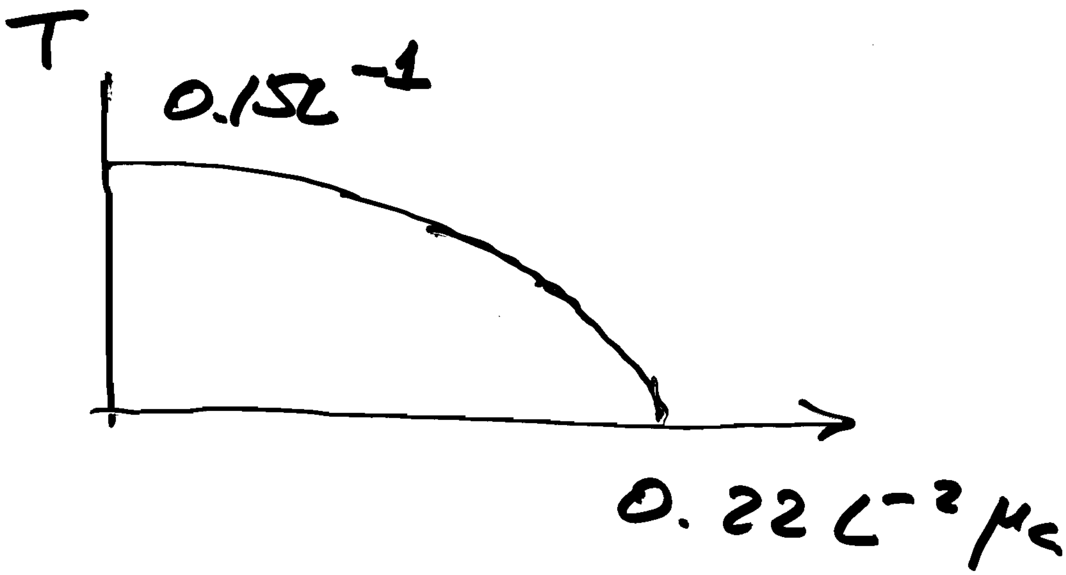
$\mu << (\lambda T)T$

Cf. Free fermion gas:

$$\rho_{free} = \frac{N_c}{6\pi^2}\left[\mu\pi^2T^2 + \mu^3\right]$$

Phase transition at $T = 0$, μ finite

$\mu_c = 0.22\, L^{-2}\, \lambda$



Conclusions

We discussed χSB phase transition

* Found $T_c = .15\,L^{-1}$
* Showed that for $T > T_{**} = .17\,L^{-1}$ the phase with broken symmet. does not exist
* Showed that the transition is of 1^{st} order and computed the latent heat
* Discussed finite μ.