Violations of the Narrow Width Approximation in BSM Physics

- Assumptions of the NWA
- Violations from Matrix Element effects
- Violations from PDF effects
- Conclusions

Daniel Berdine

Nikolas Kauer, Dave Rainwater

University of Rochester

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Narrow Width Approximation

- Production of unstable particles
- Calculate $2 \to 2$, multiply BR
- Empirical Success in SM
- Assumptions:
    - Separability of the Propagator
        - ➞ Resonant Diagrams Only
    - Massless Final State
    - $\sqrt{\hat{s}} - m \gg \Gamma$

$$\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \left| \frac{1}{q^2 - m^2 + im\Gamma} \right|^2$$

$$= \int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{dq^2}{(q^2 - m^2)^2 + (m\Gamma)^2}$$

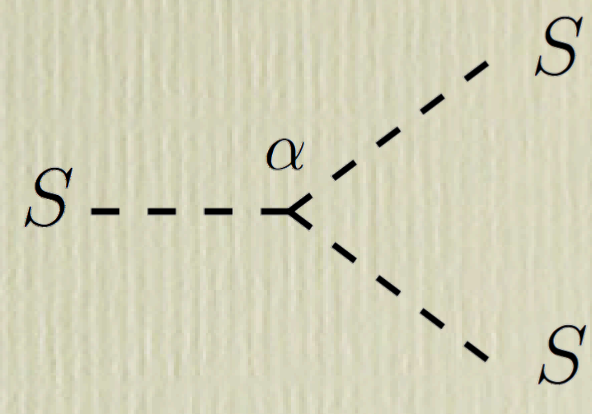
$$= \int_{q^2_{\rm min} - m^2}^{q^2_{\rm max} - m^2} \left\{ \frac{dx}{x^2 + (m\Gamma)^2} \right\}$$

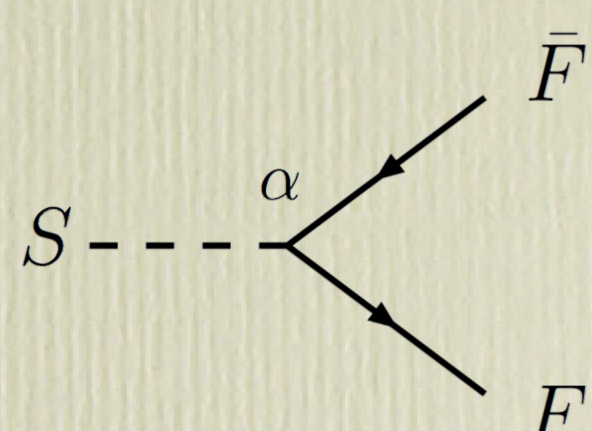
$$\cong \int_{-m^2}^{s - m^2} \left\{ \right\} \begin{array}{l} q^2_{max} \longrightarrow s \\ q^2_{min} \longrightarrow 0 \end{array}$$

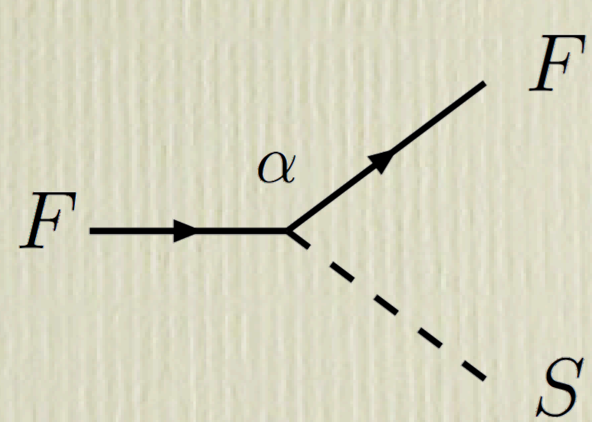
$$\approx \int_{-\infty}^{\infty} \left\{ \right\} \begin{array}{l} s \longrightarrow \infty \\ m^2 \longrightarrow \infty \end{array} = \frac{\pi}{m\Gamma}$$

Types of Matrix Element Verticies

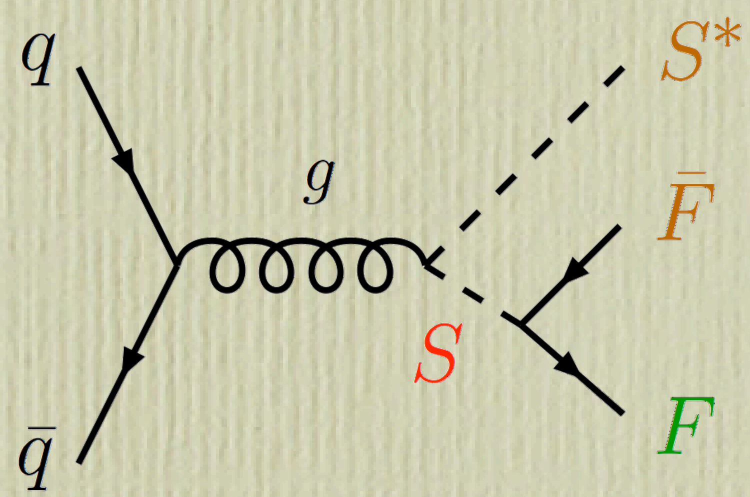
- SSS → Scalar - Scalar - Scalar
    - No Dirac/momentum structure
    - Exactly Separable ME
        - True *only* for SSS
- SFF → Scalar - Fermion - Fermion
    - S:FF → Scalar parent
    - F:SF → Fermion parent







Scalar ⟶ Fermion-Fermion



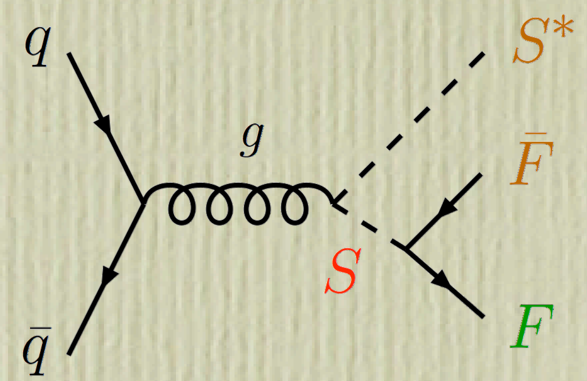
$$\frac{\sigma_{OFS}}{\sigma_{NWA}} =$$

Ratio of cross section with complete propagator to NWA value. Any difference from 1 is a correction to the NWA

Conventional wisdom: corrections will be $\mathcal{O}\left(\frac{\Gamma_S}{m_S}\right)$

- Keep only leading terms in $^1/_s$ from σ_{OFS}
- Take limit: $m_{\bar{F}} \to 0, m_{S^*} \to 0$
- Frequently use fixed quark beams

S:FF - Complete Analytic Expression



$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim \frac{m_S\Gamma_S}{2\pi\left(m_S^2-m_F^2\right)^2\left(m_S^2+\Gamma_S^2\right)} \cdot$$

$$\times\left(m_S^2\left(m_S^2+\Gamma_S^2\right)\left(-\frac{11}{3}+\log\left(\frac{s^2}{\left(m_S^2-m_F^2\right)^2+m_S^2\Gamma_S^2}\right)\right)\right.$$

$$+\frac{m_F^4}{m_S^4}\log\left(\frac{\left(m_S^2-m_F^2\right)^2+m_S^2\Gamma_S^2}{m_F^4}\right)$$

$$\left.+\frac{m_S}{\Gamma_S}\left(\left(m_S^2-m_F^2\right)^2+\left(m_S^2-2m_F^2\right)\Gamma_S^2\right)\left(\pi+2\cot^{-1}\left(\frac{m_S\Gamma_S}{m_S^2-m_F^2}\right)\right)\right)$$

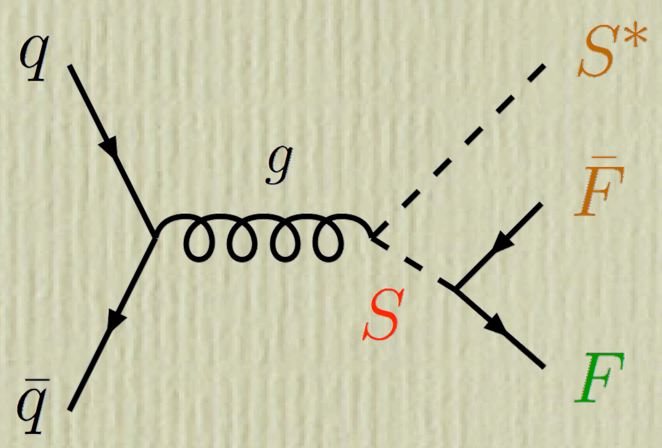
- Expected:
  - $\mathcal{O}\left(\frac{\Gamma_S}{m_S}\right)$ corrections
  - $\cot^{-1}\left(\Gamma_S\right)$ dep.
- Unexpected
  - $\log\left(s\right)$ dependance
  - $\log\left(m_F\right)$ dep.

S:FF - All Massless Final State $(m_F \to 0)$

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \xrightarrow{m_F \to 0} \frac{1}{2} + \frac{1}{\pi}\cot^{-1}\left(\frac{\Gamma_S}{m_S}\right) - \frac{11}{6\pi}\frac{\Gamma_S}{m_S} + \frac{1}{2\pi}\frac{\Gamma_S}{m_S}\log\left(\frac{s^2}{m_S^2\left(m_S^2+\Gamma_S^2\right)}\right)$$

- $\cot^{-1}(\Gamma_S)$, $\log(s)$ corrections remain
- All m_F dependence is now in Γ_S
- For small $\frac{\Gamma_S}{m_S}$, $\frac{\sigma_{OFS}}{\sigma_{NWA}} \approx 1 + \mathcal{O}\left(\frac{\Gamma_S}{m_S}\right)$

Scalar ⟶ Fermion-Fermion: Small Γ_S



- $\Gamma \ll m$ limit
- Works in Standard Model

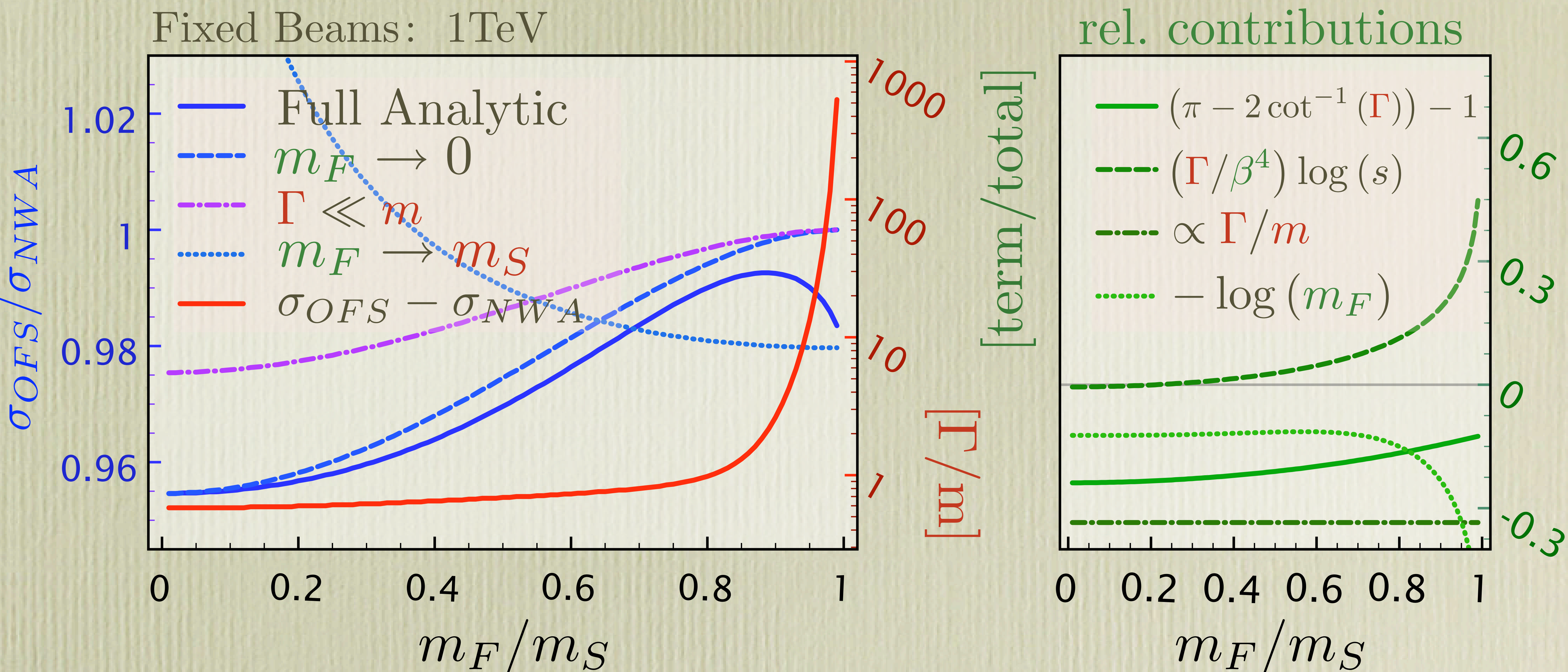
$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \xrightarrow[\Gamma_S \ll m_S]{m_F \to 0} 1 - \frac{17}{6\pi}\frac{\Gamma_S}{m_S} + \frac{1}{\pi}\frac{\Gamma_S}{m_S}\log\left(\frac{s}{m_S^2}\right)$$

- Deviation from NWA even with small Γ_S, light daughter Fermion

➡ Expected $\mathcal{O}\left(\frac{\Gamma_S}{m_S}\right)$ corrections

➡ Unexpected $\log(s)$ dependence

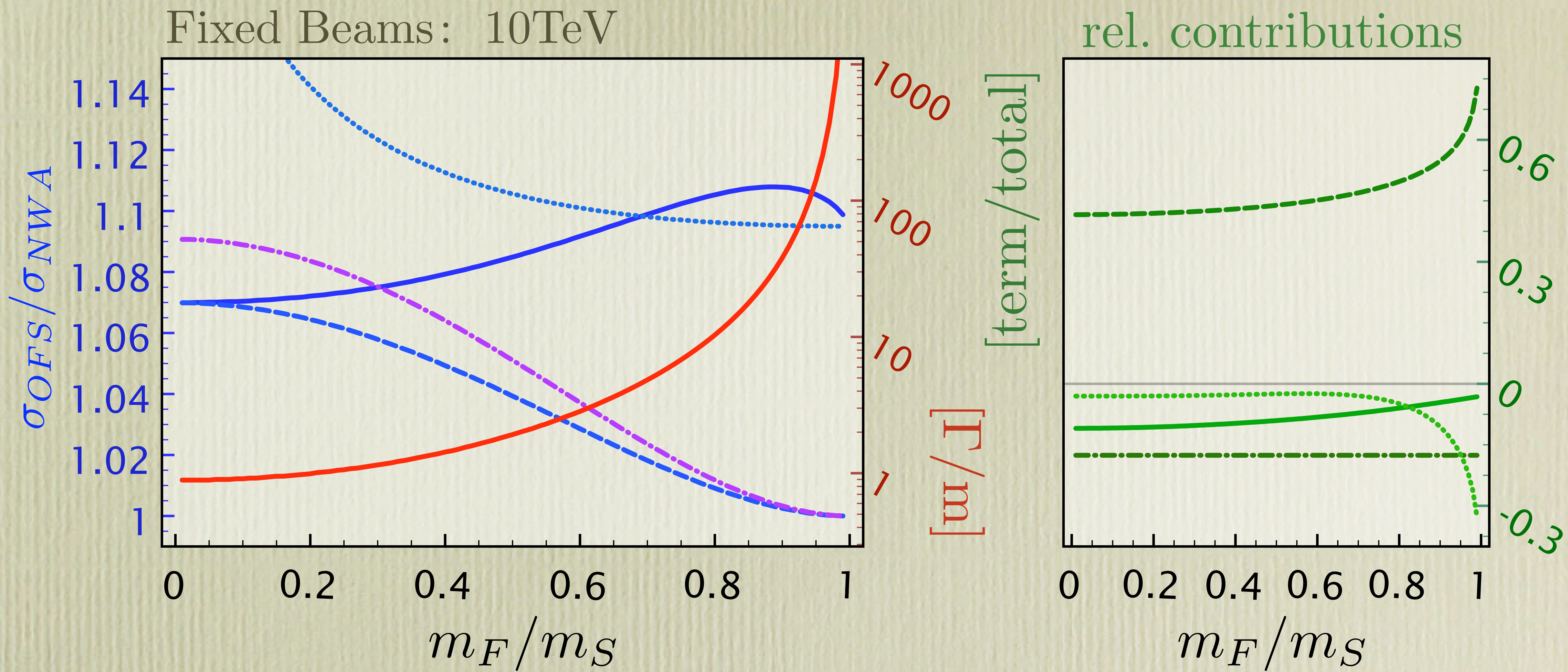
SFF: Single Decay Mode ($S \to F\bar{F}$ only)



- $m_F \to 0$ usually good
- $\Gamma \ll m$ poor
- $m_F \to m_S$ almost useless

- $\log(\cdots)$ terms - cancellations OK
- Corrections *many* $\times \Gamma_S/m_S$
- Small Γ, moderate corrections

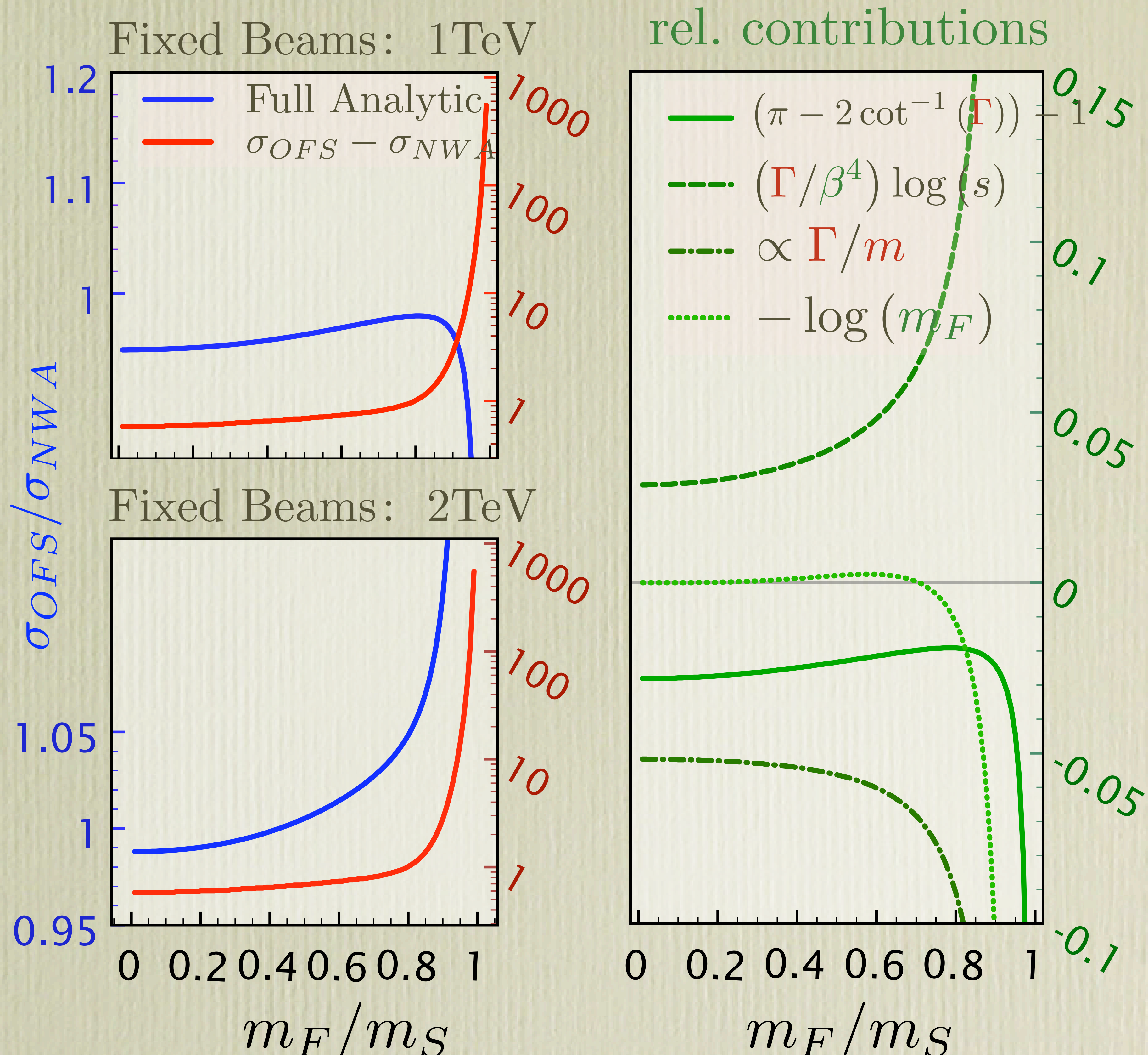
SFF: Single Decay Mode ($S \to F\bar{F}$ only)



- All approximations - useless
- Corrections still *many* $\times \Gamma_S/m_S$
- $(\Gamma/\beta^4)\log(s)$ begins to dominate
- Large corrections

S:FF Multimode Decay

- "BR" drops w/ β_F^4.
- β_F^4 do not cancel
- Absolute corrections also large here
- 1TeV beams $(11/3)$ dominates $\log(s)$
- "Rare" decays can get *huge* BR enhancement
- Enhance/Suppress by $\times\Gamma_S/m_S$, but Γ now has minimum



S:FF - β Cancelations

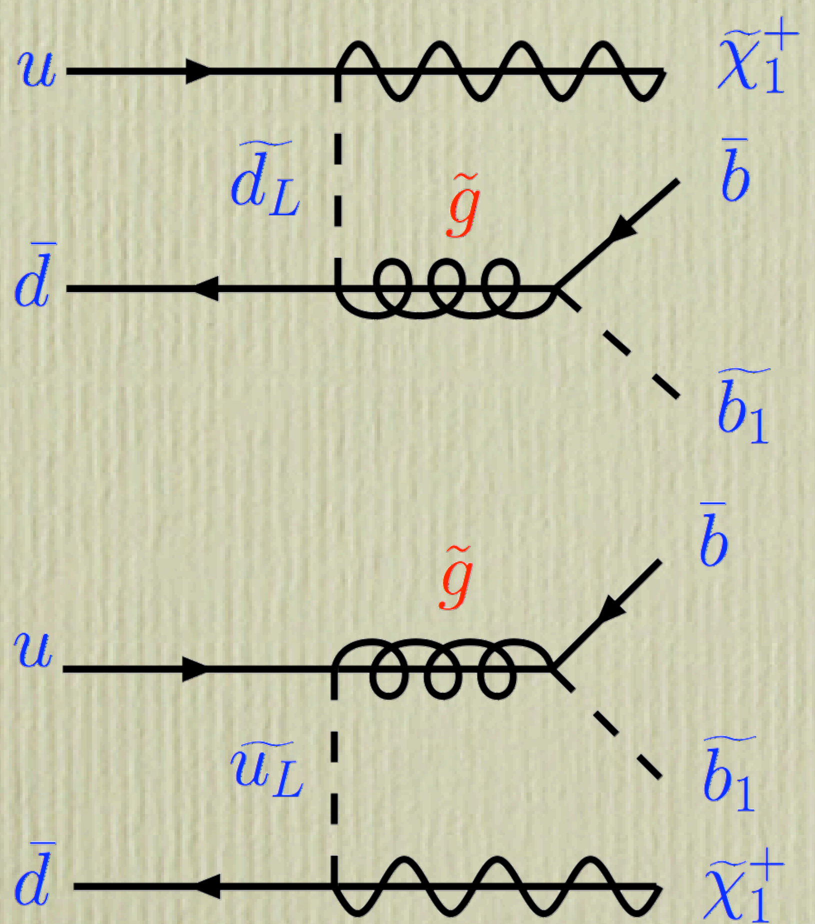
$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim \frac{1}{2\pi}\left(\beta_F^{-4}\frac{\Gamma_S}{m_S}\left(-\frac{11}{3}+\log\left(\frac{s^2}{m_S^4\left(\beta_F^{-4}+\frac{\Gamma_S^2}{m_S^2}\right)}\right)+\frac{m_F^4}{m_S^4}\log\left(\frac{m_S^4\left(\beta_F^{-4}+\frac{\Gamma_S^2}{m_S^2}\right)}{m_F^4}\right)\right)\right.$$
$$\left.\frac{\beta_F^{-4}+\left(1-2\frac{m_F^2}{m_S^2}\right)\frac{\Gamma_S^2}{m_S^2}}{\beta_F^{-4}\left(1+\frac{\Gamma_S^2}{m_S^2}\right)}\left(\pi+2\cot^{-1}\left(\beta_F^{-4}\frac{\Gamma_S}{m_S}\right)\right)\right)$$

$$\Gamma_{S:F\bar{F}} = \frac{g^2}{6\pi}m_S\left(1-\frac{m_F^2}{m_S^2}\right)$$
$$= \frac{g^2}{6\pi}m_S\beta_F^4$$

- β_F^4 cancelations
- $\Gamma_{1-\text{mode}} \sim \Gamma_{S:F\bar{F}}$
- $\Gamma_{\text{n}-\text{mode}} \sim \Gamma_\alpha + \Gamma_{S:F\bar{F}}$

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \xrightarrow{m_F\to m_S} \frac{1}{2\pi}\left(\beta_F^{-4}\frac{\Gamma_S}{m_S}\left(-\frac{11}{3}+\log\left(\frac{s^2}{(m_S^4)}\right)\right)\right.$$
$$\left.+\left(1-\left(1+\beta_F^{-4}\right)\frac{\Gamma_S^2}{m_S^2}\right)\left(\pi+2\cot^{-1}\left(\beta_F^{-4}\frac{\Gamma_S}{m_S}\right)\right)\right)$$

Fermion ⟶ Scalar - Fermion



- SUSY example, formulae still to generic F:FS
- New $m_T = m_{\widetilde{u_L}}$ dep., otherwise similar to S:FF

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim (\cdots)\left((\cdots)\left(\pi - 2\cot^{-1}\left(\frac{m_F\Gamma_F}{m_S^2 - m_F^2}\right)\right)\right.$$

$$(\cdots)\left(-6 + \log\left(\frac{s^4}{m_T^4\left((m_S^2 - m_F^2)^2 + m_F^2\Gamma_F^2\right)}\right)\right)$$

$$\left. + m_S^4\Gamma_F \log\left(\frac{(m_S^2 - m_F^2)^2 + m_F^2\Gamma_F^2}{m_S^4}\right)\right)$$

F:SF - Monte Carlo

$u\bar{d} \to \bar{q}\tilde{q}\tilde{\chi}_1^+$

$m_{\tilde{q}}$	$m_{\tilde{q}}/m_{\tilde{g}}$	$\sigma_{\rm NWA}$	$\sigma_{\rm OFS}$	$\sigma_{\rm OFS}/\sigma_{\rm NWA}$
210	0.30	14.7	17.0	1.16
350	0.50	11.5	13.1	1.14
560	0.80	8.6	10.0	1.17
630	0.90	7.87	9.20	1.17
660	0.94	7.67	8.91	1.16
680	0.97	7.72	8.55	1.11

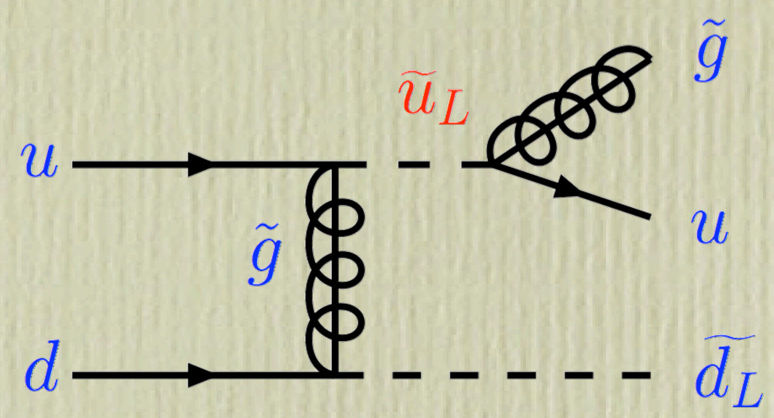
- 2TeV Fixed Beams
- 700GeV $\tilde{g}$
- Competing terms evident in full event

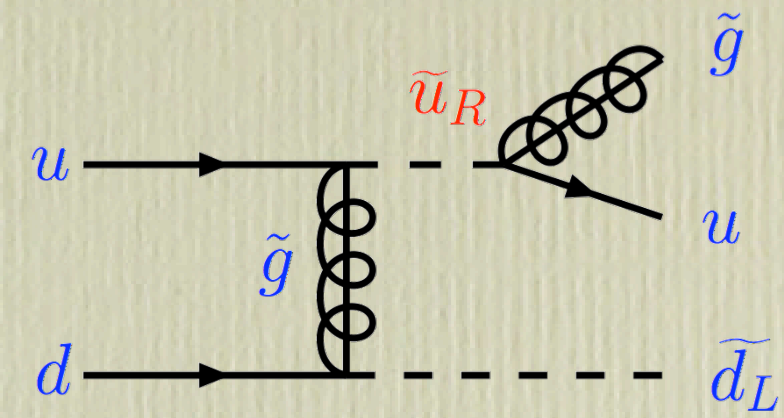
$u\bar{d} \to (\tilde{g})\,\tilde{\chi}_1^+ \to (\tilde{c}_R\bar{c})\,\tilde{\chi}_1^+$

no decay	253 fb
σ_{OFS}	12.7 fb
σ_{ONS}	11.7 fb
correction	8.3%

- LHC, SPS1a-mod2
- slightly reduced $m_{\tilde{q}}/m_{\tilde{g}}$
- Realistic, observable process
- Corrections near $\mathcal{O}$ (QCD NLO)

PDF Effects





- Steeply falling PDFs distort Breit Wigner
- Only at high Feynman x
  - TeV scale mass
- Need large width to test NWA

- Double heavy squark, small rate
  - Simplest for example
  - Effect exists for more dominant processes
- Use resonant diagrams only
  - Interferences small (~2%)

PDF Effects: Heavy Squarks

	SPS2m1		SPS2		SPS2m2		SPS2m3	
decays	$\widetilde{u}_L$ only	$\widetilde{u}_L,\widetilde{d}_L$	$\widetilde{u}_L$ only	$\widetilde{u}_L,\widetilde{d}_L$	$\widetilde{u}_L$ only	$\widetilde{u}_L,\widetilde{d}_L$	$\widetilde{u}_L$ only	$\widetilde{u}_L,\widetilde{d}_L$
ONS	3.11	1.28	4.83	1.88	5.85	1.67	2.98	0.248
OFS res	2.76	0.96	4.36	1.48	5.60	1.50	2.95	0.278
shift	-11%	-25%	-9.7%	-22%	-4.3%	-10%	-1.0%	+12%

- Custom MSSM points based on SPS2
- Tests with FB: SPS2m3 has 12% ME enhancement
  - Multiple FW effects important simultaneously
- 2-diagram case → suppression worse than multiplicative
- Large masses favor mixed $\widetilde{q} - \widetilde{g}$
  - gives effect $\mathcal{O}\,(1 - \text{diagram})$
- Effect applies to *any* decay: depends only on width

Conclusions

- In any theory with massive final states, any/all of the following may be important:
  - ME integration NWA violations
  - PDF integration NWA violations
  - Standard interference effects
- ME effect can have unexpected results:
  - Effective BR changes
  - Radically different behavior above/below threshhold
- Really can't get away with NWA in BSM with massive final states (e.g. Dark Matter) or near-degenerate spectra.