Oblique Corrections in Extended Electroweak Theories

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- Oblique Corrections
- Gauge (Non)-Invariance: The Pinch Technique
- Pinch Contributions in 3-site Model
- Conclusions

Joint Meeting of Pacific Region Particle Physics Communities
(APS-DPF2006 + JPS2006...)

RSC, S. Matsuzaki, E. Simmons, hep-ph/0608020 & M. Tanabashi

“Oblique” Electroweak Corrections

Electroweak Parameters

EW corrections $(S,\ T,\ \Delta\rho,\ \delta)$ defined from amplitudes for "on-shell" 4-fermion processes

$$-\mathcal{A}_{NC} = e^2 \frac{\mathcal{Q}\mathcal{Q}'}{Q^2} \quad + \quad \frac{(I_3 - s^2\mathcal{Q})(I_3' - s^2\mathcal{Q}')}{\left(\frac{s^2c^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2} - \alpha T\right)}$$

$$+ \quad \sqrt{2}G_F\,\frac{\alpha\delta}{s^2c^2}\,I_3 I_3' + 4\sqrt{2}G_F\,(\Delta\rho - \alpha T)\,(\mathcal{Q} - I_3)(\mathcal{Q}' - I_3')$$

$$-\mathcal{A}_{\rm CC} = \frac{(I_+ I_-' + I_- I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F\,\frac{\alpha\delta}{s^2c^2}\,\frac{(I_+ I_-' + I_- I_+')}{2}\ .$$

$\Delta\rho$ & δ small in this class of Higgsless models

S,T: Peskin & Takeuchi

Altarelli, et. al. and Hagiwara, et. al.

RSC, Kurachi, He, Simmons & Tanabashi hep-ph/0408262 & 0410154

Electroweak Parameters

$$-\mathcal{A}_{NC} = e^2 \frac{\mathcal{Q}\mathcal{Q}'}{Q^2} + \frac{(I_3 - s^2\mathcal{Q})(I_3' - s^2\mathcal{Q}')}{\left(\frac{s^2c^2}{e^2} - \frac{S}{16\pi}\right) Q^2 + \frac{1}{4\sqrt{2}G_F}(1-\alpha T)} + \ flavor\ dependent$$

Universal Corrections Depend only
on External Quantum Numbers!

S and T defined in terms of position of
Z- and W-boson poles and corresponding residues!

Gauge-Invariant, to all orders, as defined here!

Hagiwara, Matsumoto, Haidt, & Kim: hep-ph/9409380

Calculation at one-loop

Assuming relevant univesal corrections arise from self-energy corrections, consider matrix propagator:

$$\frac{1}{Q^2 + M^2 + \Pi(Q^2)} = \frac{1}{(1 - \Pi'(Q^2))Q^2 + M^2 + \Pi(0)}$$

$$\Pi'(Q^2) \equiv -\left(\frac{\Pi(Q^2) - \Pi(0)}{Q^2}\right)$$

$$(M^2 + \Pi(0))\,\vec{v}(Q^2) = -\mathsf{m}^2(Q^2)(1 - \Pi'(Q^2))\,\vec{v}(Q^2)$$

Poles: $\quad Q^2 = -\mathsf{m}^2(Q^2)$

Standard Formulae

In perturbation theory, we may extract the position of the pole and the corresponding residues, yielding:

$$\frac{\alpha S}{4s^2c^2} = \Pi'_{ZZ} - \Pi'_{AA} - \left(\frac{c^2 - s^2}{cs}\right)\Pi'_{ZA}$$

$$\alpha T = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

However, gauge-boson self-energies are not gauge-invariant!

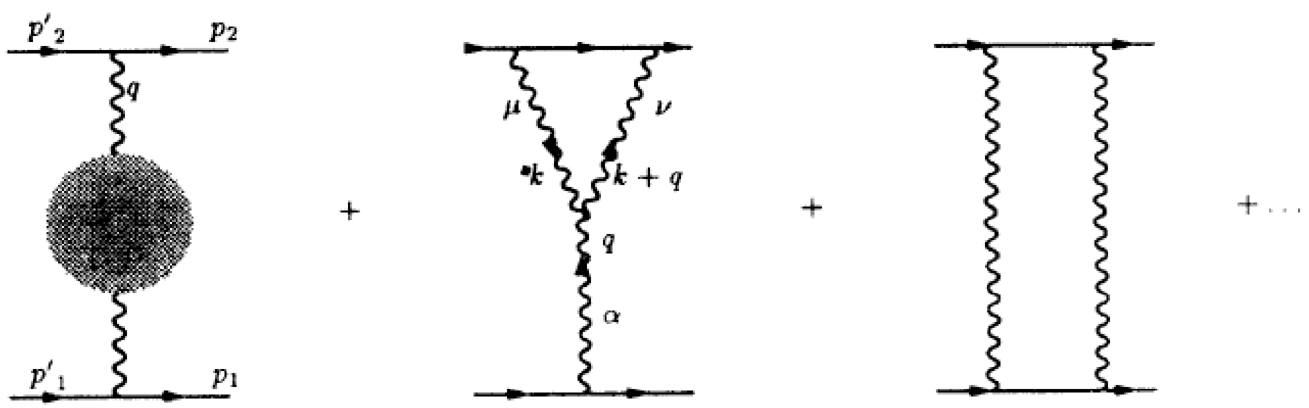
The "Pinch Technique"

Cornwall, 1982

Cornwall and Papavassiliou, 1989

Vertex and Box Corrections

- Gauge-invariance of scattering amplitudes arises by addition of vertex and box corrections



(a) (b) (c)

Gauge-Dependent Box and Vertex Contributions

Arise, for example, via longitudinal part of R_ξ-gauge propagator:

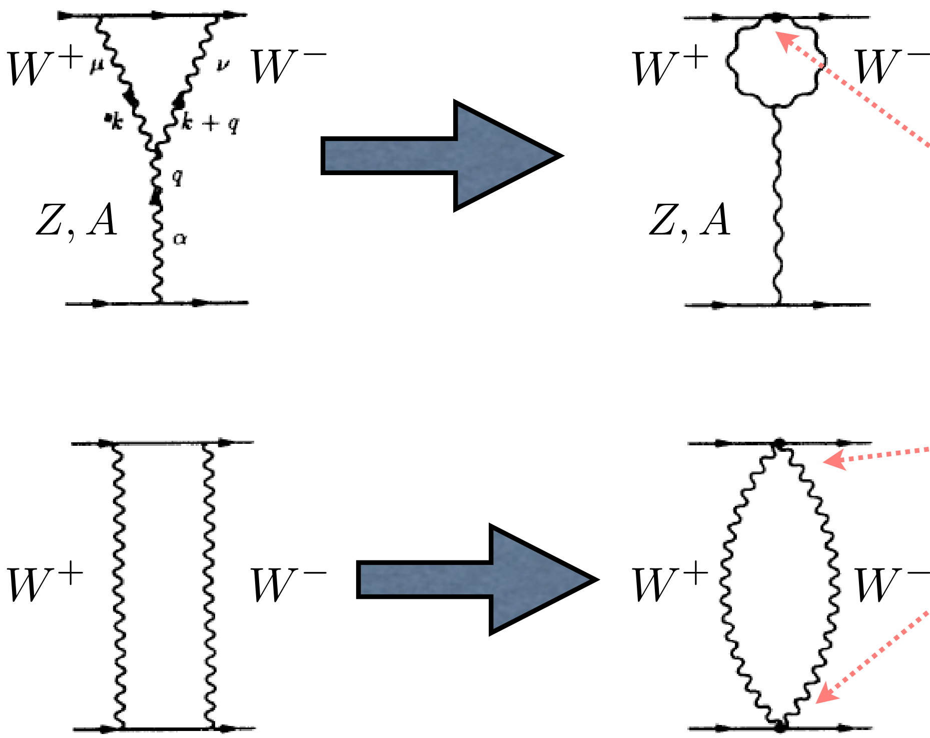
$$D_W^{\mu\nu}(k) = -i\,\frac{1}{k^2 - M_W^2}\left[g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2 - \xi M_W^2}\right]$$

$$k^\mu \gamma_\mu P_L = S^{-1}(p+k)P_L - P_R S^{-1}(p)$$

For massless fermions, this yields universal gauge dependent pinch contributions through the Ward identity.

Pinch Technique: collect all such contributions in an effective self-energy function

Pinch Box and Vertex Contributions



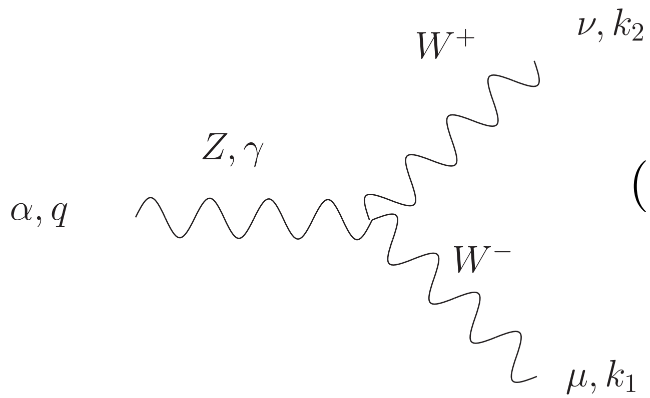
Contributions universal:

$$\propto [gT^+, gT^-] = 2g^2 T_3$$

Degrassi and Sirlin, 1992

`t Hooft-Feynman Gauge

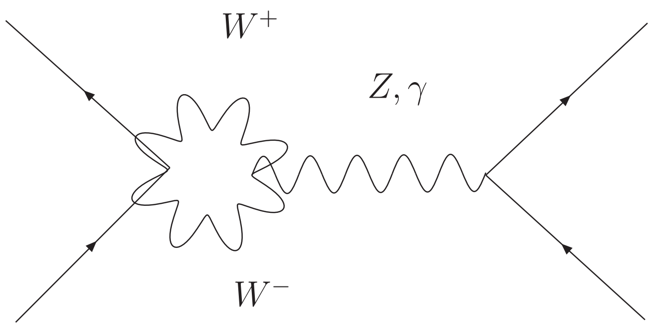
Pinch contribution arises only from three-boson vertex:



$$(q - k_1)_\nu g_{\alpha\mu} + (k_1 - k_2)_\alpha g_{\mu\nu} + (k_2 - q)_\mu g_{\alpha\nu} =$$
$$k_{2\nu} g_{\alpha\mu} - k_{1\mu} g_{\alpha\nu} + non - pinch$$

Photon Pinch in SM

$$\begin{pmatrix} J_Z^\mu \\ J_A^\mu \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} \frac{e}{s} J_3^\mu \\ \frac{e}{c} J_Y^\mu \end{pmatrix} \qquad 2g^2 J_3^\mu = 2\left(e\frac{c}{s} J_Z^\mu + e J_A^\mu\right)$$



+ other vertex

$$\mathcal{M}|^{\gamma-pinch}_{one-loop} \propto \mathcal{A}\cdot\frac{1}{p^2}\cdot e\,G(p^2, M_W^2)\cdot 2(e\frac{c}{s}\mathcal{Z}'+e\mathcal{A}')+2(e\frac{c}{s}\mathcal{Z}+e\mathcal{A})\cdot e\,G(p^2, M_W^2)\cdot\frac{1}{p^2}\cdot\mathcal{A}'$$

Compare: $\mathcal{M}|^{AA}_{one-loop} \propto \mathcal{A}\cdot\frac{1}{p^2}\cdot\Delta\Pi_{AA}(p^2)\cdot\frac{1}{p^2}\cdot\mathcal{A}'$

$$\Delta\Pi_{AA}(p^2) = 4e^2 p^2 F_2(M_W, M_W; p^2)$$

$$F_2(M_A, M_B; p^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - M_A^2]\,[(k+p)^2 - M_B^2]}$$

SM Pinch Contributions

$$\Delta\Pi_{AA} = 4e^2 p^2 F_2(M_W, M_W; p^2)$$

$$\Delta\Pi^{\gamma}_{AZ} = 2e^2 \frac{c}{s}(p^2 - M_Z^2) F_2(M_W, M_W; p^2)$$

$$\Delta\Pi^{Z}_{ZA} = 2e^2 \frac{c}{s} p^2 F_2(M_W, M_W; p^2)$$

$$\Delta\Pi_{ZZ} = 4e^2 \frac{c^2}{s^2}(p^2 - M_Z^2) F_2(M_W, M_W; p^2)$$

$$F_2(M_A, M_B; p^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - M_A^2]\,[(k+p)^2 - M_B^2]}$$

Degrassi and Sirlin, 1992

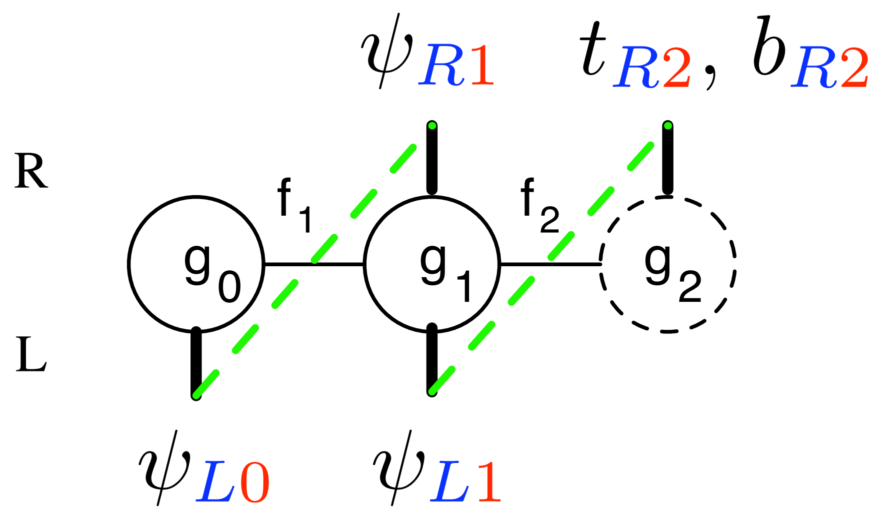
The 3-site Model:

new pinch contributions

3-Site Model: basic structure

$SU(2) \times SU(2) \times U(1)$

$g_0, \; g_2 \ll g_1$



$$x = \frac{g_0}{g_1} \ll 1$$

Gauge boson spectrum: photon, Z, Z', W, W'

Fermion spectrum: t, T, b, B (ψ is an SU(2) doublet)

and also c, C, s, S, u, U, d, D plus the leptons

3-Site Model: Gauge Currents

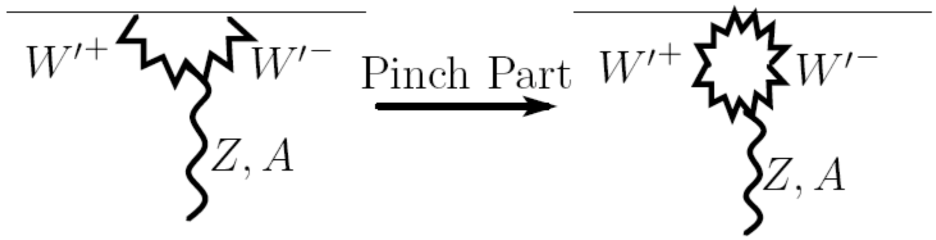
$$\begin{pmatrix} J^\mu_Z \\ J^\mu_A \\ J^\mu_{W'} \end{pmatrix} = \begin{pmatrix} c & -s & \frac{c^2-s^2}{2c}x \\ s & c & sx \\ -\frac{x}{2} & -\frac{sx}{2c} & 1 \end{pmatrix} \begin{pmatrix} (1-x_1)\frac{e}{s}J^\mu_3 \\ \frac{e}{c}J^\mu_Y \\ \frac{e}{sx}(J^{\mu\,\prime}_3 + x_1 J^\mu_3) \end{pmatrix}$$

Concentrate on light fermons: $\varepsilon_R \to 0$

"Integrate out" heavy fermons: $x_1 = \varepsilon_L^2$

$$\mathcal{L}_f = \vec{J}^\mu_L \cdot ((1-x_1)L_\mu + x_1 V_\mu) + J^\mu_Y B_\mu \qquad \mathcal{L'}_f = x_1 \cdot \bar{\psi}_L (i\not{D}\Sigma_{(1)}\Sigma^\dagger_{(1)})\psi_L$$

3-Site Model: Pinch Contributions



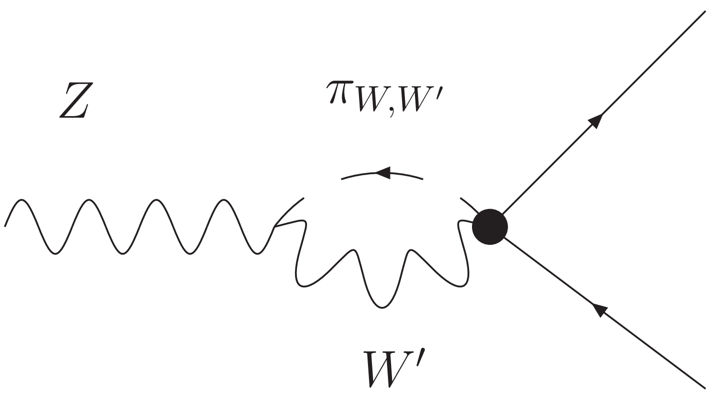
$$\propto [g_1 T^{+'}, g_1 T^{-'}] = 2 g_1^2 T_3' + \mathcal{O}(x^2)$$

$$g_1^2 J_3^{\mu\prime} = g_1 \left(1 - \frac{x_1}{2}\right) J^{\mu}_{W'} + e \left(1 - \frac{x_1}{x^2}\right) J^{\mu}_{A} + e \frac{c^2 - s^2}{2cs} \left(1 - \frac{2c^2}{c^2 - s^2} \frac{x_1}{x^2}\right) J^{\mu}_{Z}$$

Gives rise to new pinch contributions!

3-Site Model: Delocalization Contributions

$$\mathcal{L}'_f = x_1 \cdot \bar{\psi}_L (i \not{D} \Sigma_{(1)} \Sigma^\dagger_{(1)}) \psi_L$$



$$\Delta\Pi^{delocal}_{AA} = 4e^2p^2 \frac{x_1}{x^2} F_2(M_\rho, M_\rho; 0)$$

$$\Delta\Pi^{delocal}_{ZA} = \frac{2e^2c}{s} \frac{x_1}{x^2} (2p^2 - M_Z^2) F_2(M_\rho, M_\rho; 0)$$

$$\Delta\Pi^{delocal}_{ZZ} = \frac{4e^2c^2}{s^2} \frac{x_1}{x^2} (p^2 - M_Z^2) F_2(M_\rho, M_\rho; 0)$$

3-site Model Checks

$$\Delta\Pi_{AA}|_{x_1} = 0$$

$$\Delta\Pi_{ZA}|_{x_1} = \frac{e^2}{cs}\frac{x_1}{x^2}p^2 F_2(M_\rho, M_\rho; 0)$$

$$\Delta\Pi_{ZZ}|_{x_1} = \frac{2e^2}{s^2}\frac{x_1}{x^2}(p^2 - M_Z^2)F_2(M_\rho, M_\rho; 0)$$

$$\Delta\Pi_{WW}|_{x1} = \frac{2e^2}{s^2}\frac{x_1}{x^2}(p^2 - M_W^2)F_2(M_\rho, M_\rho; 0)$$

- Electromagnetic Gauge-Invariance
- Delocalization Independence of T
- Results for S ... see next talk