Phase Transitions in High Density QCD

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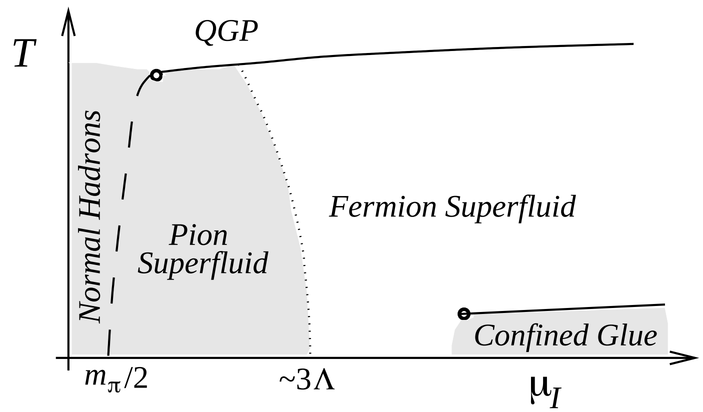
I. Introduction

1. The phase diagram of QCD at nonzero temperature and isospin chemical potential μ_I should look like....



First and second order phase transitions are depicted by solid and dashed curves, respectively. The confined phases are shaded.

Remark: lattice simulations can be performed for μ_I rather than for μ_B (there is no sign problem with μ_I) Therefore, our results, in principle, can be tested on the lattice.

• **Part I** of this talk : confinement- deconfinement phase transition at $\mu \sim \Lambda_{QCD}, T \sim 0.$

• **Part II** of this talk: transition of the normal to the superfluid (still confined) phase at $\mu \sim m_\pi,\ T \sim 0.$

2. Few Preliminary Remarks:

a)At large $\mu \gg \Lambda_{QCD}$ the system in the deconfined phase; at small $\mu \simeq 0$ the system in the hadronic (or superfluid) confined phase. Something should occur on the way from $(\mu \simeq 0) \Rightarrow (\mu \gg \Lambda_{QCD})$;

b) Indeed, the transition has been recently seen on the lattice for $SU(2)$, at $\mu \sim 600 MeV$, see Simon Hands et al, hep-lat/0604004.

3. On the phenomenological side: The development of the **instanton liquid model, ILM** (Shuryak and Co.) has encountered successes: chiral symmetry breaking, resolution of the $U(1)$ problem, spectrum etc and failures:

a) confinement can not be described by well separated lumps with integer topological charges;

b) lattice calculations suggest that T_c for confinement and chiral phase transitions are very close to each other (*which is difficult to interpret if two phenomena originated at vastly different scales*).

c) Additional criticism of ILM is based on study of topological vacuum structure using overlaped or chirally improved fermions (e.g. Horvath et al.).

4. Questions we address in this talk:

a) What is the driving force of the confinement -de confinement phase transition when μ changes?
b) Are these problems *(confinement/χSB)* originated from the same physics?

Main strategy:
We attack the problem by introducing isotopical chemical potential μ_I (to make contact with the lattice simulations) as external parameter. We keep θ parameter to analyze the changes in topological properties of the system.

Main Observations:
a) The variation of μ_I changes the theory from "Higgs-like" gauge theory ($\mu_I \gg \Lambda_{QCD}$, weak coupling regime) to "Non-Higgs-like" gauge theory ($\mu_I = 0$, strong coupling regime).
b) Drastic changes of vacuum energy (topological characteristics) with μ is interpreted as confinement -de confinement phase transition.

II. Main Goals and Results

1. We argue that the instantons is the driving force for confinement-deconfinement phase transition at nonzero μ (they are not necessary small well-localized lumps, see below).
2. We argue that the low-energy effective chiral Lagrangian corresponds to a statistical system of interacting pseudo-particles with fractional $1/N_c$ charges. (dual representation)
3. We shall identify these objects with **instanton quarks** suspected long ago (demonstration of a link between confinement and instantons in 2d): V.Fateev et al, B.Berg and M.Luscher, A. Belavin et al, (1979).
4. We make some very specific predictions which can be tested with traditional Monte Carlo techniques, by studying QCD at nonzero isospin chemical potential μ_I where there is no sign problem. In particular we predict that the confinement-deconfinement transition and the topological charge density distribution must experience sharp changes exactly at the same critical value $\mu_c(T)$. We estimate $\mu_c(T)$ for different N_c, N_f.
5. The key observation here is there existence of the free parameter θ (it plays the crucial role in mapping of one problem to another). The θ plays the role of the messenger between colorless (chiral effective Lagrangian fields) and colorful (instanton quarks) objects.

III. Main Logic of the Presentation

1. I use a TRICK which allows me to represent the low energy effective lagrangian in terms of dual variables (a statistical system of some interacting pseudo-particles).
2. I test this trick in the weak coupling regime in QCD (large chemical potential, Color Superconductor) where all calculations are under complete theoretical control.
3. I observe that the instanton-instanton(II) and instanton -anti-instanton ($\bar{I}I$) interactions at large distances are very different from the naive semiclassical calculations.
4. I apply the same TRICK to QCD at zero chemical potential and $T = 0$. I advocate the picture that in the strongly coupled theories the instantons and anti-instantons lose their individual properties (instantons will "melt") their sizes become very large and they overlap.
5. The description in terms of the instantons and anti-instantons is not appropriate any more, and alternative degrees of freedom should be used to describe the physics. The relevant description is that of instanton-quarks with fractional topological charges $1/N_c$.
6. I approach the phase transition region from the high density side where instanton calculations under complete theoretical control.
7. For $\mu_I \neq 0$ our predictions can be tested with traditional Monte Carlo techniques, by studying QCD at nonzero isospin chemical potential.

IV. Color Superconductivity for Pedestrians, $\mu \gg \Lambda_{QCD}, \theta \neq 0$.

1. If there is a channel in which the quark-quark interaction is attractive, than the true ground state of the system will be a complicated coherent state of Cooper pairs like in **BCS** theory (ordinary superconductor).

2. Diquark condensates break color symmetry (CFL phase, $N_c = N_f = 3$):

$$\langle \psi^{ia}_{L\alpha} \psi^{jb}_{L\beta} \rangle^* \sim \epsilon_{\alpha\beta\gamma} \epsilon^{ij} \epsilon^{abc} X^{\gamma}_{c},$$
$$\langle \psi^{ia}_{R\alpha} \psi^{jb}_{R\beta} \rangle^* \sim \epsilon_{\alpha\beta\gamma} \epsilon^{ij} \epsilon^{abc} Y^{\gamma}_{c}$$

3. $SU(3)_c \times U(1)_{EM} \times SU(3)_L \times SU(3)_R \times U(1)_B$

$$\Downarrow$$

$$SU(3)_{c+L+R} \times U(1)^*_{EM}$$

4. Pattern:
a) Color gauge group is completely broken;
b) $U(1)_B$ is spontaneously broken;
c) $U(1)_{EM}$ is not broken;
d) $U(1)_A$ is broken spontaneously and explicitly (by instantons)
5. Goldstone fields are the phases of the condensate

$$\Sigma_\gamma^\beta = \sum_c X_c^\beta Y_\gamma^{c*} \sim e^{i\lambda^a \pi^a} e^{i\varphi_A}. \tag{1}$$

6. $U(1)_A$ is spontaneously broken. The symmetry is broken also explicitly by the instantons. Effective lagrangian is

$$L_A \sim f_A^2[(\partial_0 \varphi_A)^2 - u^2(\partial_i \varphi_A)^2] + a\mu^2\Delta^2 \cos(\varphi_A - \theta)$$

Coefficient a can be explicitly calculated from the t'Hooft formula *(Son, Stephanov, AZ, 2001)*.

7. To compute a we start from the instanton induced effective four-fermion interaction,

$$\begin{aligned} L_{\rm inst} &= e^{-i\theta}\int d\rho\, n_0(\rho)\left(\frac{4}{3}\pi^2\rho^3\right)^2\Big\{(\bar u_R u_L)(\bar d_R d_L) + \\ &+ \frac{3}{32}\Big[(\bar u_R\lambda^a u_L)(\bar d_R\lambda^a d_L) \\ &- \frac{3}{4}(\bar u_R\sigma_{\mu\nu}\lambda^a u_L)(\bar d_R\sigma_{\mu\nu}\lambda^a d_L)\Big]\Big\} + \text{H.c.} \end{aligned} \qquad (2)$$

where $n_0(\rho)$ in the presence of $\mu \neq 0$ is given by

$$n_0(\rho) = C_N\left(\frac{8\pi^2}{g^2}\right)^{2N_c}\rho^{-5}\exp\left(-\frac{8\pi^2}{g^2(\rho)}\right)e^{-N_f\mu^2\rho^2} \qquad (3)$$

with

$$C_N = \frac{0.466e^{-1.679N_c}1.34^{N_f}}{(N_c-1)!(N_c-2)!}\,, \qquad (4)$$

8. Averaging Eq. (2) in the superconducting background, we find

$$V_{\text{inst}}(\varphi) = -\int d\rho\, n_0(\rho) \left(\frac{4}{3}\pi^2\rho^3\right)^2 12|X|^2 \cos(\varphi_A - \theta).$$

where

$$|X| = \frac{3}{2\sqrt{2}\pi} \frac{\mu^2\Delta}{g}.$$

Using formula for $n_0(\rho)$ we get the final result

$$a(\mu \gg \Lambda_{QCD}) = 5 \times 10^4 \left(\ln \frac{\mu}{\Lambda_{QCD}}\right)^7 \left(\frac{\Lambda_{QCD}}{\mu}\right)^{29/3} \ll 1$$

9. Weak coupling regime: dilute gas approximation leads exactly to the combination $\left(e^{i(\varphi_A-\theta)} + e^{-i(\varphi_A-\theta)}\right)$ which is expected from the very beginning.

V. Instanton interactions in dense QCD, $\mu \gg \Lambda_{QCD}, \theta \neq 0$

1. Partition function for φ_A is: $Z = \int \mathcal{D}\varphi_A \, e^{-f^2 u \int d^4x (\partial\varphi_A)^2} \, e^{a' \int d^4x \cos(\varphi_A(x)-\theta)}$,

2. The dual Coulomb Gas (CG) representation for the partition function Z is

$$Z = \sum_{M_\pm=0}^{\infty} \frac{(a'/2)^M}{M_+! M_-!} \int d^4x_1 \ldots \int d^4x_M \; e^{-i\theta \sum_{a=0}^{M} Q_a} \; . \qquad (5)$$

$$e^{-\frac{1}{2f^2u} \sum_{a>b=0}^{M} Q_a Q_b G(x_a - x_b)} \; , \quad G(x_a - x_b) = \frac{1}{4\pi^2 (x_a - x_b)^2} \, .$$

3. Physical Interpretation:

a) Since $Q_{\rm net} \equiv \sum_a Q_a$ is the total charge and it appears in the action multiplied be the parameter θ, one concludes that $Q_{\rm net}$ *is* the total topological charge of a given configuration.

b) Each charge Q_a in a given configuration should be identified with an integer topological charge well localized at the point x_a. This, by definition, corresponds to a small instanton positioned at x_a.

4. The following hierarchy of scales exists: The typical size of the instantons $\bar{\rho} \sim \mu^{-1}$ is much smaller than the short-distance cutoff of our effective low-energy theory, Δ^{-1},

$$\begin{array}{ccccccc} \text{(size)} & \ll & \text{(cutoff)} & \ll & (II \text{ distance}) & \ll & \text{(Debye)} \\ \mu^{-1} & \ll & \Delta^{-1} & \ll & (\sqrt{a}\mu\Delta)^{-1/2} & \ll & (\sqrt{a}\Delta)^{-1} \end{array}$$

Due to this hierarchy, ensured by large $\mu/\Lambda_{\rm QCD}$, we acquire analytical control.
5. The starting low-energy effective Lagrangian contains only a colorless field φ_A, we have ended up with a representation of the partition function in which objects carrying color (the instantons) can be studied.
6. In particular, II and $I\bar{I}$ interactions (at very large distances) are exactly the same up to a sign, order g^0, and are Coulomb-like. This is in **contrast with semiclassical expressions** when II interaction is zero and $I\bar{I}$ interaction is order $1/g^2$.
7. Very complicated picture of the **bare** II and $I\bar{I}$ interactions becomes very simple for **dressed** instantons/anti-instantons when all integrations over all possible sizes, color orientations and interactions with background fields are properly accounted for!
8. As expected, the ensemble of small $\rho \sim 1/\mu$ instantons can not produce confinement.

VI.Chiral Lagrangian ($\mu = 0 \quad \theta \neq 0$)

1. Effective lagrangian for the singlet combination is defined as $\phi = \mathrm{Tr}\ U$ is given by

$$L_\phi = f^2(\partial_\mu \phi)^2 + E\cos\left(\frac{\phi - \theta}{N_c}\right) + \sum_{a=1}^{N_f} m_a \cos\phi_a \quad (6)$$

(Conjecture. Veneziano, 1979).

2. A Sine-Gordon structure for the singlet combination corresponds to the following behavior of the $(2k)^{\mathrm{th}}$ derivative of the vacuum energy in pure gluodynamics (Veneziano, 1979)

$$\left.\frac{\partial^{2k} E_{vac}(\theta)}{\partial\, \theta^{2k}}\right|_{\theta=0} \sim \int \prod_{i=1}^{2k} dx_i \langle Q(x_1) ... Q(x_{2k})\rangle \sim (\frac{i}{N_c})^{2k},$$

where $Q = \frac{g^2}{32\pi^2} G_{\mu\nu}\widetilde{G}_{\mu\nu}$ is topological density. *(Veneziano originally interpreted this relation as an evidence that the periodicity should be $2\pi N_c$ rather than 2π)*.

VII. Dual Representation for the Chiral Lagrangian ($\mu = 0 \quad \theta \neq 0$)

1. One can represent the Sine Gordon effective field theory in terms of CG representation,

$$Z = \sum_{Q_a^{(0)} = \pm\frac{1}{N_c}} \frac{(\frac{E}{2})^{M_0}}{M_0!} \int (dx_1^{(0)} \ldots dx_{M_0}^{(0)})\; e^{-S_{CG}}$$

$$S_{CG} = i\theta Q_T^{(0)} + \frac{1}{2f^2}\left\{ \sum_{b,c=1}^{M_0} Q_b^{(0)}\; G(x_b^{(0)} - x_c^{(0)})\; Q_c^{(0)} \right\}, \quad Q_T^{(0)} = \sum_{b=1}^{M_0} Q_b^{(0)}$$

2. One can identify $Q_T^{(0)}$ as the total topological charge of the given configuration
3. The fundamental difference in comparison with the previous case: while the total charge in integer, the individual charges are fractional $\pm 1/N_c$. The fact that species $Q_i^{(0)}$ have charges $\sim 1/N_c$ is a direct consequence of the θ/N_c dependence in the underlying QCD.
4. Due to the 2π periodicity of the theory, only configurations which contain an integer topological number contribute to the partition function. Therefore, the number of particles for each given configuration $Q_i^{(0)}$ with charges $\sim 1/N_c$ must be proportional to N_c..

5. The number of integrations over $d^4x_i^{(0)}$ exactly equals $4N_ck$, where k is integer. This number, $4N_ck,$ exactly corresponds to the number of zero modes in the k-instanton background, and we *conjecture* that at low energies (large distances) the fractionally charged species-$Q_i^{(0)}$ pseudo-particles are the **instanton-quarks** suspected long ago...

6. There is an interesting connection between the CG statistical ensemble and the $2d\ CP^{N_c}$ models. An exact accounting and resummation of the n-instanton solutions maps the original problem to a $2d$-CG with fractional charges (dubbed in 1979 as the instanton-quarks). These pseudo-particles do not exist separately as individual objects; rather, they appear in the system all together as a set of $\sim N_c$ instanton-quarks so that the total topological charge of each configuration is always integer.

7. One immediate objection: it has long been known that instantons can explain most low energy QCD phenomenology (chiral symmetry breaking, resolution of the $U(1)$ problem, spectrum, etc) with the exception confinement; and we claim that confinement can arise in this picture: how can this be consistent?

8. In dilute gas approximation quark confinement can not be described. However, in strongly coupled theories the instantons and anti-instantons lose their individual properties their sizes become very large, they overlap. Confinement is the possibility.

VIII. Conjecture and Results

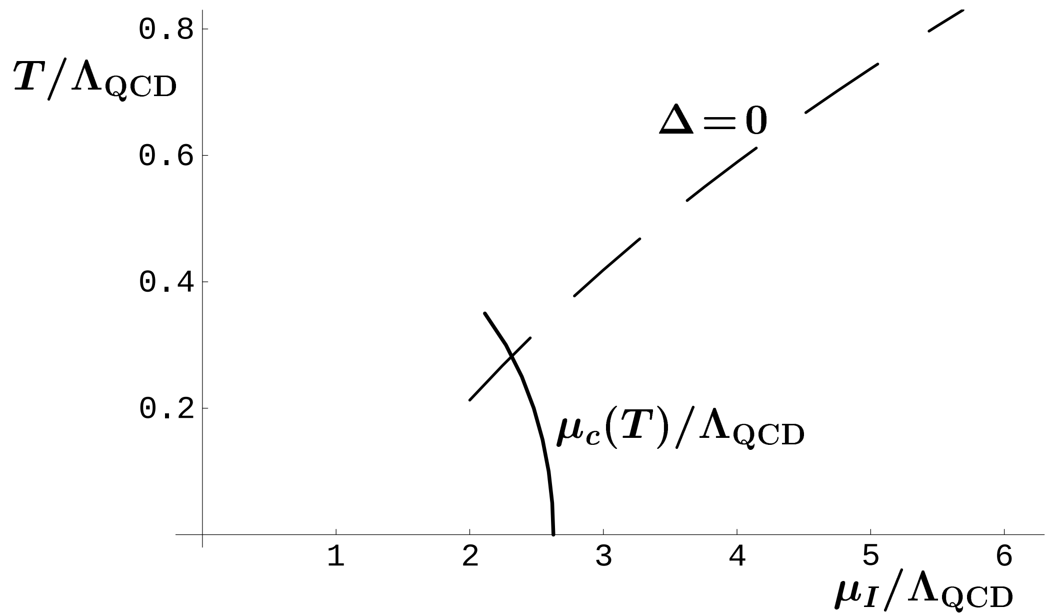
1. We thus **conjecture** that the confinement-deconfinement phase transition takes place precisely at the value where the dilute instanton calculation breaks down:
a) At low $\mu \ll \Lambda_{QCD}$ color is confined (because of the instanton-quarks), θ dependence comes through $\sim \cos\left(\frac{\phi-\theta}{N_c}\right)$
b)At large $\mu \gg \Lambda_{QCD}$ color is not confined (because of dilute instantons), θ dependence comes through $\sim \cos\left(\phi-\theta\right)$

2. We have determined the critical chemical potential where the dilute instanton calculation breaks down. We identify this point with the position of the phase transition when $\cos\left(\phi-\theta\right) \Rightarrow \cos\left(\frac{\phi-\theta}{N_c}\right)$

3. For different cases of nonzero baryon or isospin chemical potential μ_c is given by (assuming $m_s = 150\ MeV$ for $N_f = 3$ case)

	$N_c=3, N_f=2$	$N_c=N_f=3$	$N_c=N_f=2$
μ_{Bc}/Λ	2.3	1.4	3.5
μ_{Ic}/Λ	2.6	1.5	3.5

4. One can generalize the calculations for $T \neq 0$ when gap is still large and the instanton calculations are still justified.



Critical isospin chemical potential $\mu_I(T)$ for the confinement-deconfinement phase transition as a function of temperature (solid curve) at $N_c = 3, N_f = 2$ where direct lattice calculation are possible.

IX. Future Directions

1. We claim that the topological charge density distribution measured as a function of μ_I will experience sharp changes at the same critical value $\mu_I = \mu_c(T)$ where the phase transition occurs.

a). There are well- established lattice methods which allow to measure the topological density distribution.

(See e.g. E.M.Ilgenfritz et al , I.Horvath et al, 2002, 2005; Gattringer, 2003....)

b). Independently, there are well- established lattice method which allow to introduce μ_I into the system.

(See e.g. Kogut et al, Sinclair et al 2002; S. Hands et al, 2006; M. Lombardo et al, 2006)

Combine these two lattice measurements!

2. Such an analysis gives an unique opportunity

a). to study a transition from "Higgs -like" gauge theory to "Non -Higgs" like gauge theory by varying the external parameter μ_I;

b). to understand what is happening with small size instantons, $\rho \sim 1/\mu_I$ (which are under complete theoretical control at large μ_I) when transition from weak coupling regime to strong coupling regime occurs.

X. Relation to other Studies

1. Relation with 't Hooft and Mandelstam picture of the confinement (dynamical monopoles exist and Bose condense) $\Leftrightarrow$ The instanton-quarks carry the magnetic charges. In this case both pictures could be the two sides of the same coin.

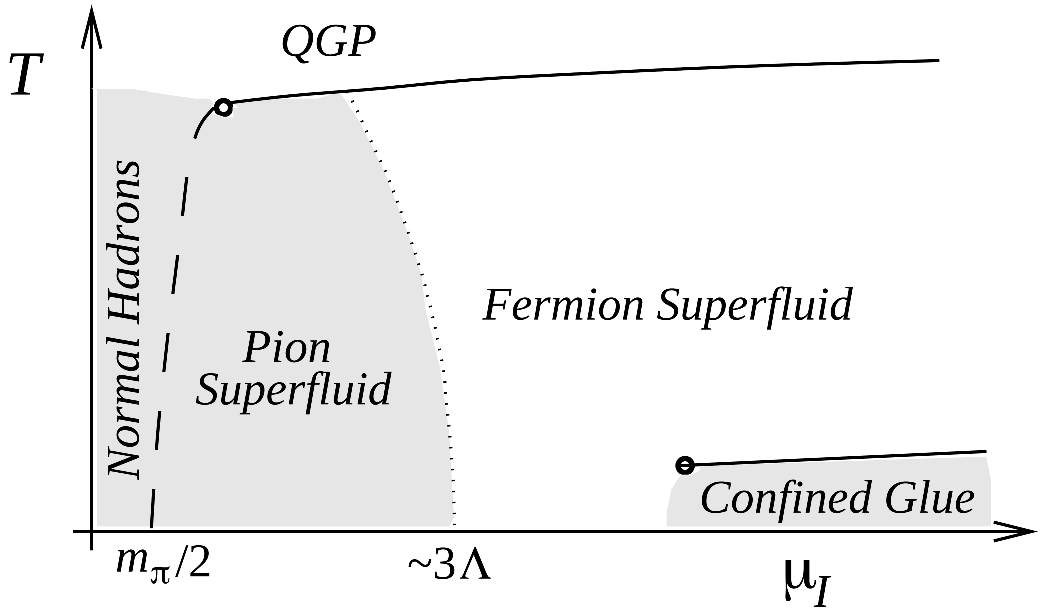
2. There is a close relation between instanton quarks and the "periodic instantons" or carolons. Carolons have the internal structure resembling the instanton-quarks. Particularly, in both cases the constituents carry the magnetic charges and appear as set of N_c constituents (see e.g.Pierre van Baal, Gattringer, Diakonov, E.M.Ilgenfritz, ...);

3. *Difference:* The instanton quarks have pure quantum origin. There is a fundamental difference between any semiclassical objects (merons, carolons, center vortices and nexuses with fractional fluxes $1/N_c$ etc) and instanton quarks. In particular, the interactions at large distances between different instanton quarks $\sim Q_i \frac{1}{\partial^2} Q_j$ are the same as between instanton quark- instantanton antiquark $\sim \bar{Q}_i \frac{1}{\partial^2} Q_j$ in contrast with semiclassical picture.

4. There is an interesting recent development (I. Horvath et al) when the topological charge fluctuations can be studied without any assumptions or guidance. One of the interesting observation in these calculations: the relevant 4D structures should shrink to mere points in the continuum limit. It is tempting to identify these points with 4D instanton quarks classified by 4 translational zero modes.

Part II

Transition of the normal to the superfluid (still confined) phase at $\mu \sim m_\pi,\ T \sim 0$.



XI Gluon condensate in dense matter.

1. The system is under complete theoretical control in the regime $\mu_I \sim m_q \ll \Lambda_{QCD}$ for $SU(N_c = 3)$ or $\mu_B \sim m_q \ll \Lambda_{QCD}$ for $SU(N_c = 2)$ (due to the extended symmetry for $N_c = 2$).

2 The system exhibits the phase transition from normal to superfluid phase at $\mu = m_\pi$;

3. The μ dependence in this regime can be calculated exactly using the effective lagrangian for the Goldstone fields Σ,

$$\mathcal{L} = \frac{F^2}{2} Tr \nabla_\nu \Sigma \nabla_\nu \Sigma^\dagger, \quad \nabla_0 \Sigma = \partial_0 \Sigma - \mu \left[B\Sigma + \Sigma B^T \right].$$

4. Knowing $\mathcal{L}(\mu)$ allows us to calculate the chiral $\langle \bar{\psi}\psi \rangle$ and diquark $\langle \psi\psi \rangle$ condensates as function of μ *(it has been done in 2000 by J. B. Kogut, M. A. Stephanov, D. Toublan, J. J. M. Verbaarschot and A. Zhitnitsky)*

5. New result: Knowing $\mathcal{L}(\mu)$ allows us to calculate the change of the gluon condensate due to a finite chemical potential $\mu_B \ll \Lambda_{QCD}$

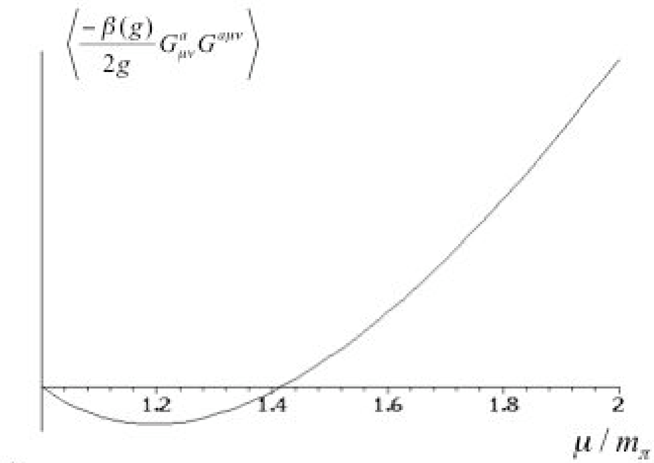
6. The energy density ϵ and pressure p are obtained from the free energy density $\epsilon = \mathcal{F} + \mu\rho_B,\ p = -\mathcal{F}$. Therefore, the conformal anomaly implies,

$$\langle \frac{bg^2}{32\pi^2} G^a_{\mu\nu} G^{\mu\nu a} \rangle_\mu - \langle \frac{bg^2}{32\pi^2} G^a_{\mu\nu} G^{\mu\nu a} \rangle_0 = 4F^2(\mu^2 - m_\pi^2)\left(1 - 2\frac{m_\pi^2}{\mu^2}\right)$$

7.

a)For $\mu \le m_\pi$, the condensate **does not change**

b)For $\mu \ge m_\pi$, the condensate **decreases** with μ for $m_\pi < \mu_B < 2^{1/4} m_\pi$ and **increases** afterwards.



8. Simple explanation: Immediately after the phase transition point $\mu = m_\pi$, the baryon density ρ_B is small and our system can be understood as a weakly interacting gas of diquarks. Picture is similar to the nuclear matter case: $\langle G^2_{\mu\nu} \rangle$ decreases when baryon density increases at small densities when the interaction does not play a role.

Conclusion

1. Analysis of the vacuum energy as function of μ and θ allows us to identify two different phase transitions when μ_I varies.
2. Confinement -de confinement phase transition occurs at $\mu_c \simeq \Lambda_{QCD}$ and it is related to sudden changes of the topological properties [$\cos(\theta/N_c)$ dependence for $\mu_I \leq \mu_c$; versus $\cos(\theta)$ dependence for $\mu_I \geq \mu_c$].
3. Transition of the normal to the superfluid (still confined) phase at $\mu_I \sim m_\pi,\ T \sim 0$ is under complete theoretical control. This phase transition is similar to transition to nuclear matter when confinement still persists.