Monte Carlo Studies of Matrix Theory at Finite Temperature

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based on collaboration with
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0. Introduction

- matrix models

(or dimensionally reduced large N gauge theories)

- non-pert. formulation of superstring/M theories

e.g.) Matrix Theory — BFSS('96)

IIB matrix model — IKKT('96)

- gauge/gravity duality

e.g.) AdS/CFT — Maldacena('97)

➢ quantum description of blackholes, big-bang singularity, ...

technical problem:
strongly coupled gauge theory

(perturbative approach is valid only in limited situations e.g., protected by SUSY)

non-perturbative approaches

1) Gaussian Expansion Method (GEM)

or (improved) mean-field approximation

S : action of the model

S_{G} : Gaussian action → incl. various param. $\{v_i\}$

expansion w.r.t. $(S - S_{\mathrm{G}})$

truncate at some order

⇒ $F(v_i)$

determine v_i by

Principle of Minimal Sensitivity

$$\frac{\partial}{\partial v_i} F(v_i) = 0$$

- Matrix Theory ⇒ blackhole entropy

Kabat-Lifschytz-Lowe ('00)

- IIB matrix model ⇒ 4d space-time

J.N.-Sugino ('01),
Kawai-Kawamoto-Kuroki-Matsuo-Shinohara ('02),
......, Aoyama-Shibusa ('06)

2) Monte Carlo simulation of matrix models

- calculations from first principles
- required CPU time grows with *N*

$$\begin{cases} \sim O(N^3) & \text{bosonic} \\ \sim O(N^6) & \text{incl. fermions (det>0)} \\ \sim O(\mathrm{e}^{cN^2}) & \text{incl. fermions (otherwise)} \end{cases}$$

Matrix Theory
IIB matrix model

various simplification is needed
to achieve large *N*

- bosonic IIB matrix model — Hotta-J.N.-Tsuchiya ('98)
- 4d version (SUSY) of the IIB matrix model
- IIB matrix model (phase of det omitted, off-diagonal elements integrated out at one-loop)

Ambjorn-Anagnostopoulos-Bietenholz-Hotta-J.N. ('00)

- bosonic Matrix Theory at finite temperature

justified at high *T*

Janik-Wosiek ('00), this work

aim of the present work :

Matrix Theory at finite temperature

Janik-Wosiek ('00)	this work
$D=3+1$ $N \le 8$ discretized time $(N_t = 4)$	$D=9+1$ (as in BFSS) large N limit $(N = 16, 32)$ continuum limit $(a = 0.02)$ $(N_t = 13 \sim 125)$

calculations from first principles
(except for omitting fermions)

- provide a firm ground for a study including fermions

- comparison with the prediction from blackhole entropy (Klebanov-Tseytlin '99)
  based on gauge/gravity duality

  (Itzhaki-Maldacena-Sonnenschein-Yankielowicz '98)

- a clear test of GEM within the bosonic model

  (Kabat-Lifschytz '99)

Plan

0. Introduction
1. Bosonic BFSS model
2. Phase diagram

 - dim. reduction in high *T* lim. (equiv. to bosonic IKKT)

 - Eguchi-Kawai reduction in the confined phase

3. Comparison with blackhole
4. Testing Gaussian Expansion Method
5. Summary and discussions

1. Bosonic BFSS model

$$S = \frac{1}{g^2} \int_0^\beta dt \ \mathrm{tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\}$$

periodic b.c.

$$DX_j(t) = \frac{d}{dt} X_j(t) - i\,[A(t), X_j(t)]$$

$$j = 1, \cdots, 9$$

- $T = \beta^{-1}$ temperature
- $\lambda \equiv \dfrac{g^2 N}{T^3}$ dim'less 't Hooft coupling const.
  - low T ➡ strong coupling
    - relevant to gauge/gravity duality
  - high T ➡ weak coupling
    - except for the zero modes ← bosonic IKKT model
- In what follows, we set $g^2 N = 1$

2. Phase diagram

order param. for the deconf. transition

1) Polyakov line

$$P = \frac{1}{N}\left\langle \mathrm{tr}\,\mathcal{P}\exp\left(i\int_0^\beta dt\,A(t)\right)\right\rangle \sim \exp\left(-\frac{F_q}{T}\right)$$

deconfined phase $(T > T_{\mathrm{c}})$

$P \neq 0$
$(F_q < \infty)$

SSB of U(1) sym.

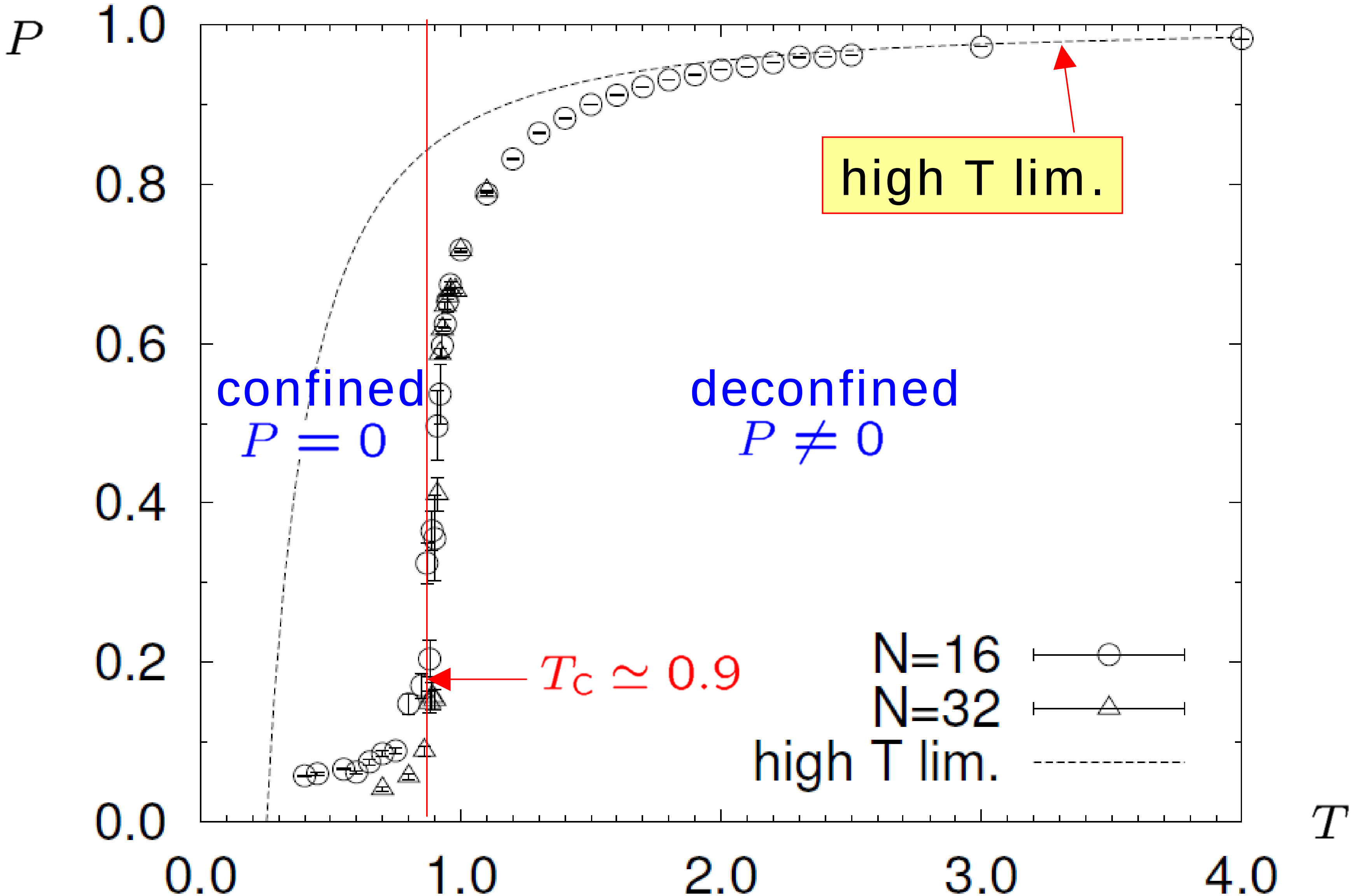
$A(t) \to A(t) + \alpha\,(\mathrm{const.})$

2) free energy

$$F = -\frac{1}{\beta}\log Z \;; \qquad Z = \int \mathcal{D}A\mathcal{D}X\,\mathrm{e}^{-S}$$

$$\sim \begin{cases} \mathcal{O}(1) & \text{confined phase} \quad (T < T_{\mathrm{c}}) \\ \mathcal{O}(N^2) & \text{deconfined phase} \quad (T > T_{\mathrm{c}}) \end{cases}$$

Polyakov line



dim. reduction in the high T lim.

$$X_i(t) = \sum_{n=-\infty}^{\infty} \tilde{X}_i^{(n)} \, \mathrm{e}^{i\omega_n t} \; ; \qquad \omega_n \equiv \frac{2\pi}{\beta} n \; ; \qquad n \in \mathbb{Z}$$

(Matsubara frequencies)

$T \to \infty$ ⇨ Only zero modes $\tilde{X}_i^{(0)}$ survive.

$$\begin{aligned} S &\simeq \beta N \, \mathrm{tr} \left(-\frac{1}{2} [A, \tilde{X}_i^{(0)}]^2 - \frac{1}{4} [\tilde{X}_i^{(0)}, \tilde{X}_j^{(0)}]^2 \right) \\ &= N \, \mathrm{tr} \left(-\frac{1}{4} [\mathcal{A}_\mu, \mathcal{A}_\nu]^2 \right) \equiv S_{\mathrm{bIKKT}} \end{aligned}$$

$$\mathcal{A}_\mu \equiv \begin{cases} \beta^{1/4} A & \text{for } \mu = 0 \\ \beta^{1/4} \tilde{X}_i^{(0)} & \text{for } \mu = i \end{cases}$$

high *T* lim. of Matrix Theory

⟺ bosonic IKKT model

MC sim., 1/D exp.

Hotta-J.N.-Tsuchiya ('98)

Some explicit results in the high T lim.

- $$\left\langle \frac{1}{\beta}\int_0^\beta dt\, \frac{1}{N}\mathrm{tr} X_i(t)^2 \right\rangle \simeq \left\langle \frac{1}{N}\mathrm{tr}\,(\mathcal{A}_\mu)^2 \right\rangle_{\mathrm{bIKKT}} \times \beta^{-1/2} \times \frac{9}{10}$$

- $$\left\langle \frac{1}{\beta}\int_0^\beta dt\, \frac{1}{N}\mathrm{tr} F_{ij}(t)^2 \right\rangle \simeq \left\langle \frac{1}{N}\mathrm{tr}\,(\mathcal{F}_{\mu\nu})^2 \right\rangle_{\mathrm{bIKKT}} \times \beta^{-1} \times \frac{{}_9C_2}{{}_{10}C_2}$$

$$F_{ij}(t) \equiv -i\,[X_i(t), X_j(t)] \qquad \mathcal{F}_{\mu\nu} \equiv -i\,[\mathcal{A}_\mu, \mathcal{A}_\nu]$$

- $$P = \left\langle \frac{1}{N}\mathrm{tr}\exp\left(i\int_0^\beta dt\, A(t)\right) \right\rangle$$

$$\simeq 1 - \frac{1}{2N}\beta^{3/2}\left\langle \frac{1}{N}\mathrm{tr}\,(\mathcal{A}_\mu)^2 \right\rangle_{\mathrm{bIKKT}} \times \frac{1}{10}$$

next leading order terms : calculable perturbatively

Eguchi-Kawai reduction in confined phase

$(T < T_c)$

U(1) sym. : $A(t) \to A(t) + \alpha\,(\text{const.})$

is NOT spontaneously broken

⇨ VEV of a single-trace operator $\left\langle \frac{1}{N} \mathrm{tr}\,\mathcal{O} \right\rangle$

has no T dependence in the $N \to \infty$ limit

c.f.) Eguchi-Kawai reduction ('82)

in D-dim. large *N* gauge theory :

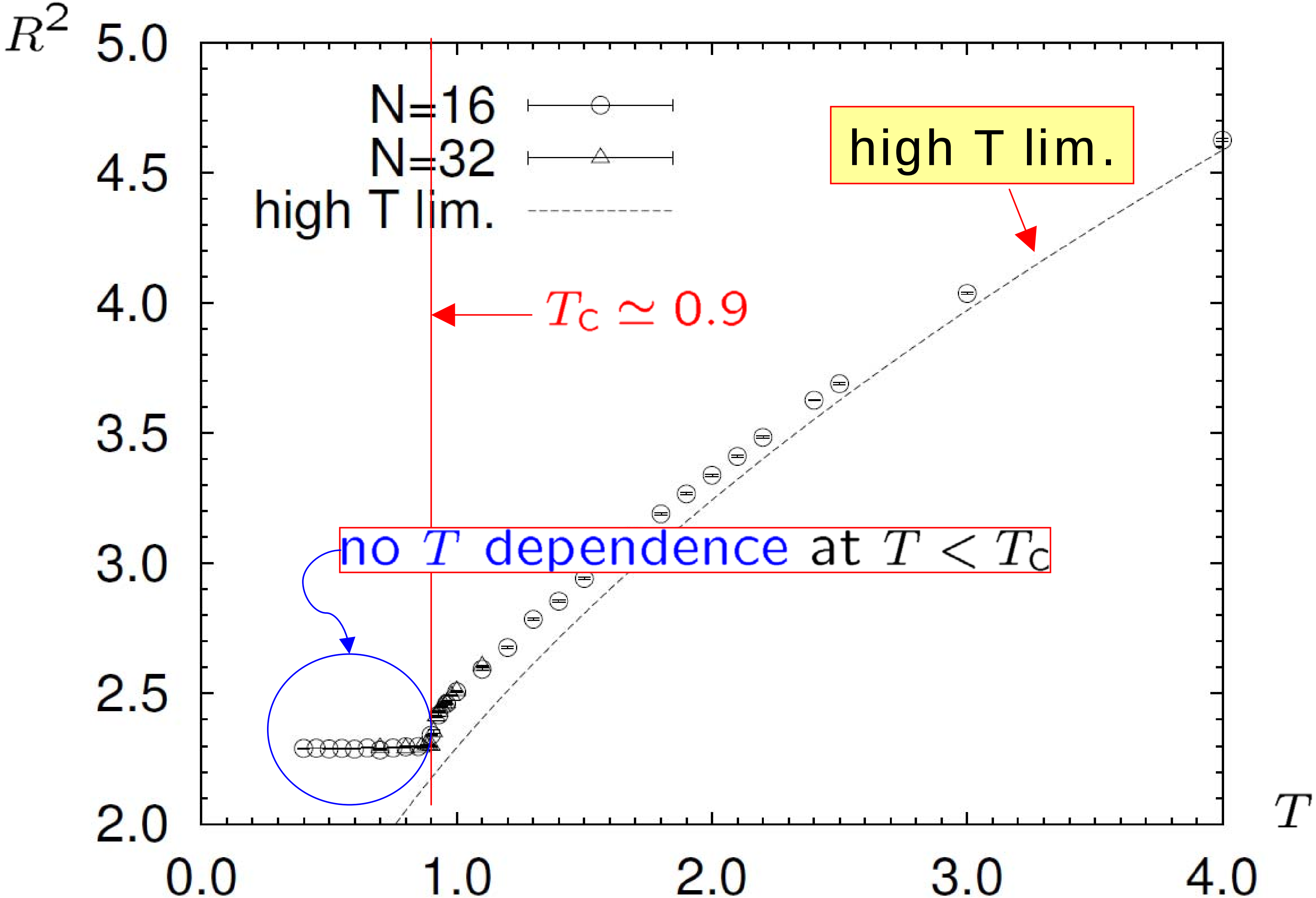
no vol. dependence

if $U(1)^D$ is unbroken

e.g.) the extent of space-time

$$R^2 = \left\langle \frac{1}{\beta} \int_0^\beta dt \, \frac{1}{N} \mathrm{tr} X_i(t)^2 \right\rangle$$

the extent of space-time



3. Comparison with blackhole

- Blackhole in the dual geometry predicts :

$$F = -C\,T^p\,N^2$$

(Klebanov-Tseytlin '99)

$$C = \left(\frac{2^{21}3^2 5^7 \pi^{14}}{7^{19}}\right)^{1/5} = 4.115\cdots$$

$$p = \frac{14}{5} = 2.8$$

- In MC sim., we calculate the energy:

$$E \equiv \frac{\partial}{\partial\beta}(\beta F) \overset{\text{bBFSS}}{=} \frac{3N^2}{4}\left\langle \frac{1}{N\beta}\int_0^\beta dt\,\mathrm{tr}\,F_{ij}(t)^2 \right\rangle$$

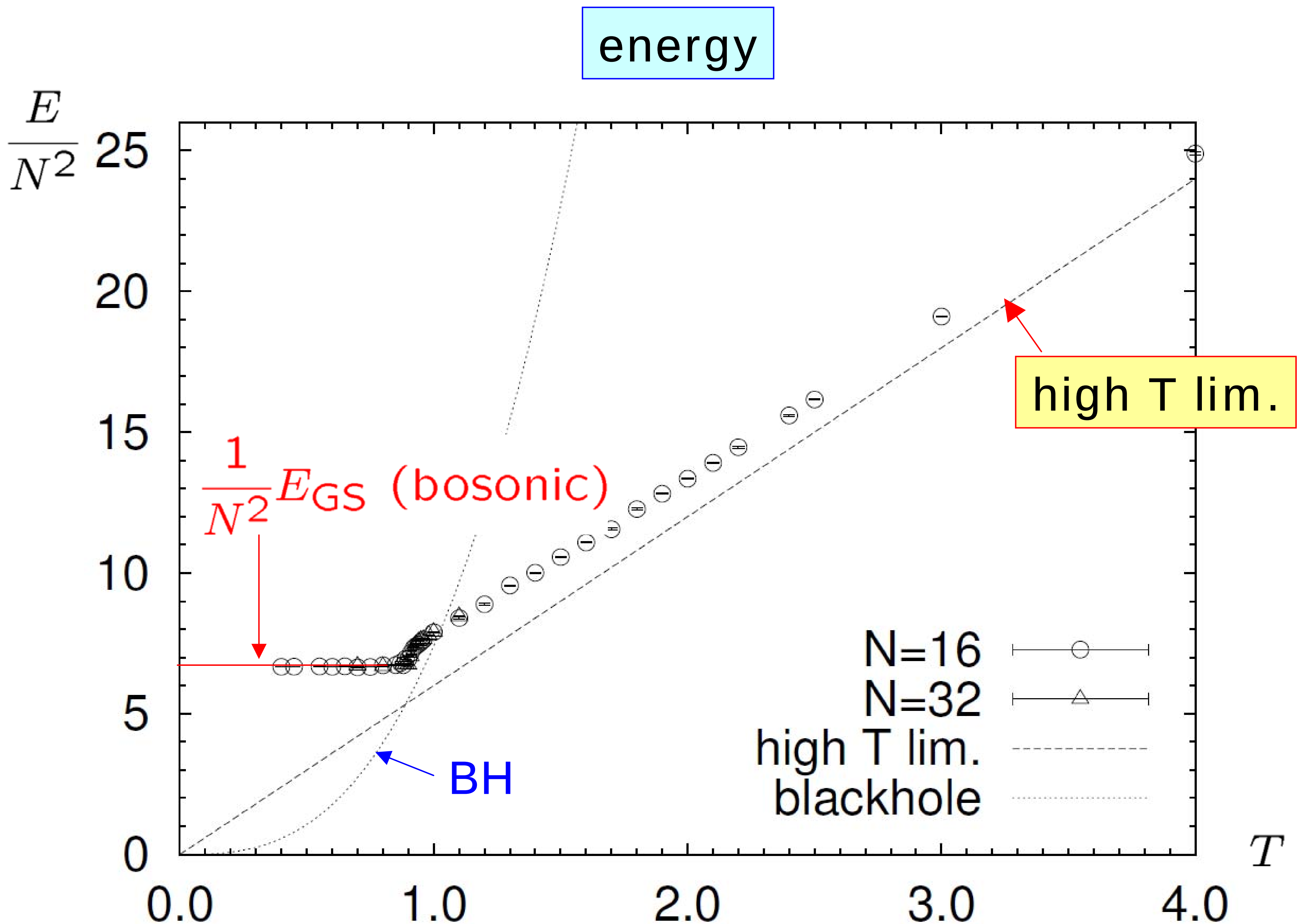
$$F_{ij}(t) \equiv -i\,[X_i(t), X_j(t)]$$

$$\begin{aligned} E &= (p-1)\,C\,T^p \times N^2 \\ &= 7.4\,T^{2.8} \times N^2 \end{aligned}$$

c.f.) in the confined phase : $(T < T_\mathrm{c})$

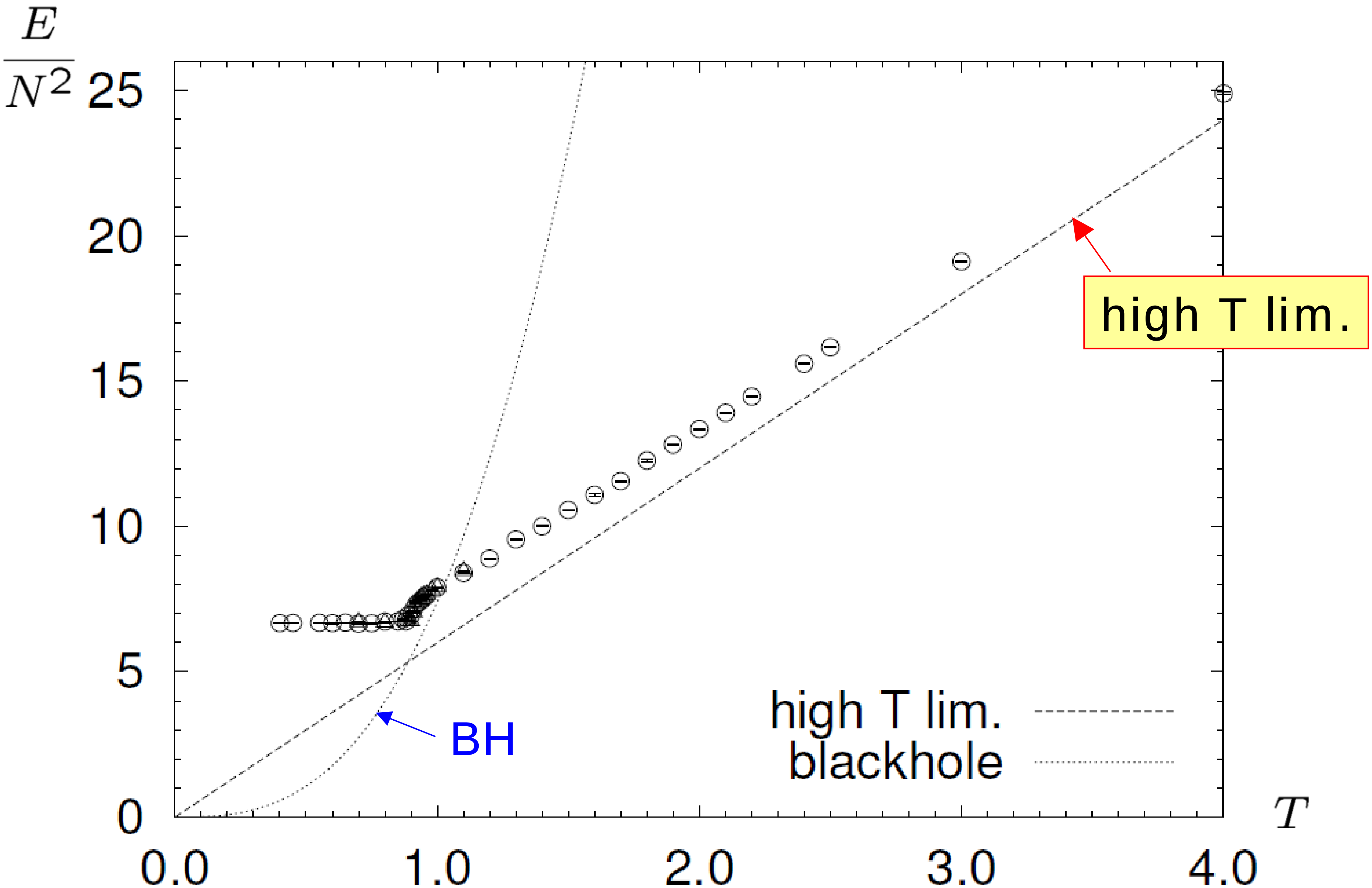
$$E = E_\mathrm{GS} \begin{cases} > 0 & \text{bBFSS} \\ = 0 & \text{BFSS (if SUSY is unbroken)} \end{cases}$$

only singlet states in the low-energy spectrum

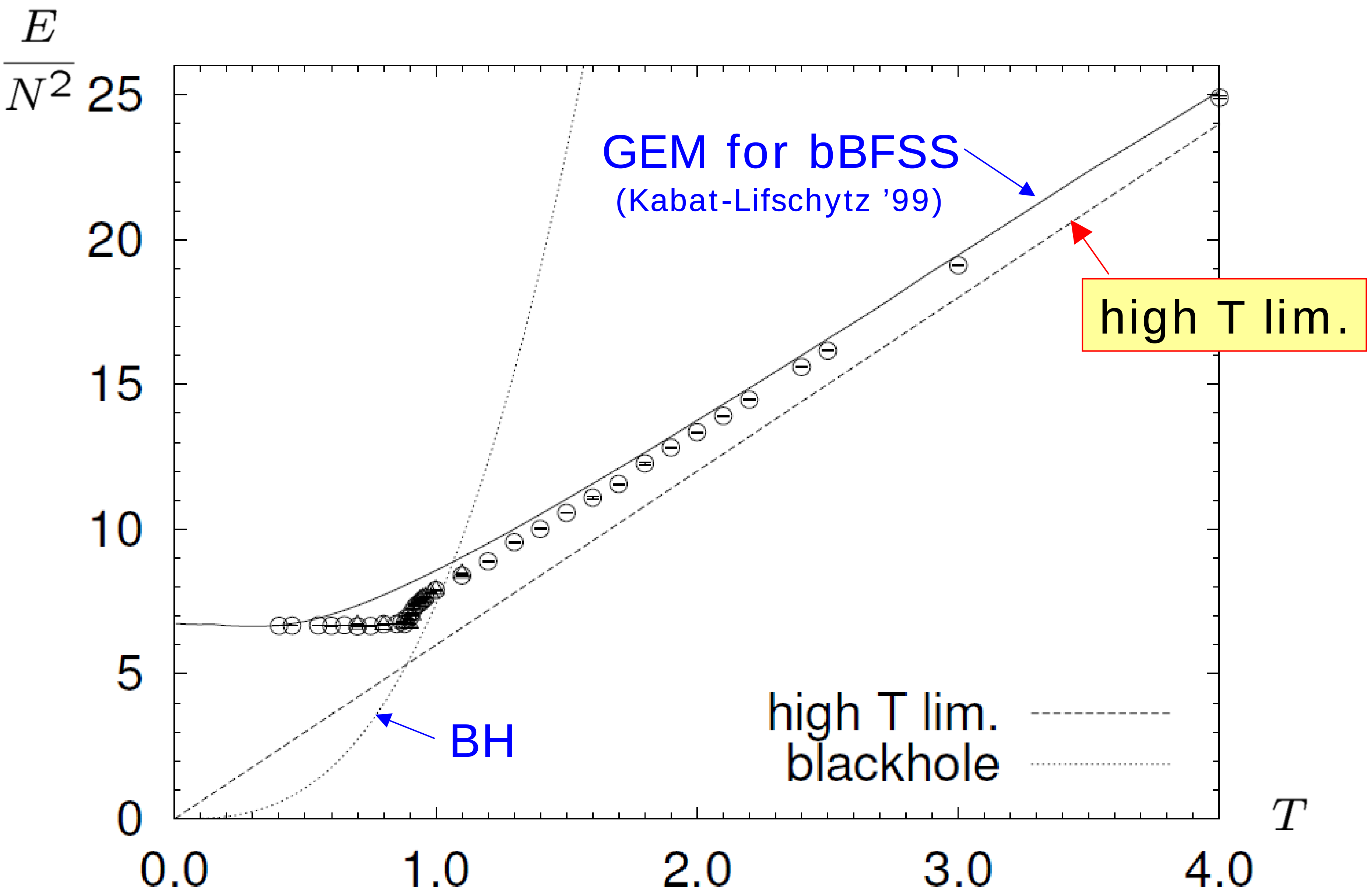


energy
$\frac{E}{N^2}$
25
20
15
10
5
0
0.0
1.0
2.0
3.0
4.0
T
$\frac{1}{N^2}E_{GS}$ (bosonic)
high T lim.
BH
N=16
N=32
high T lim.
blackhole

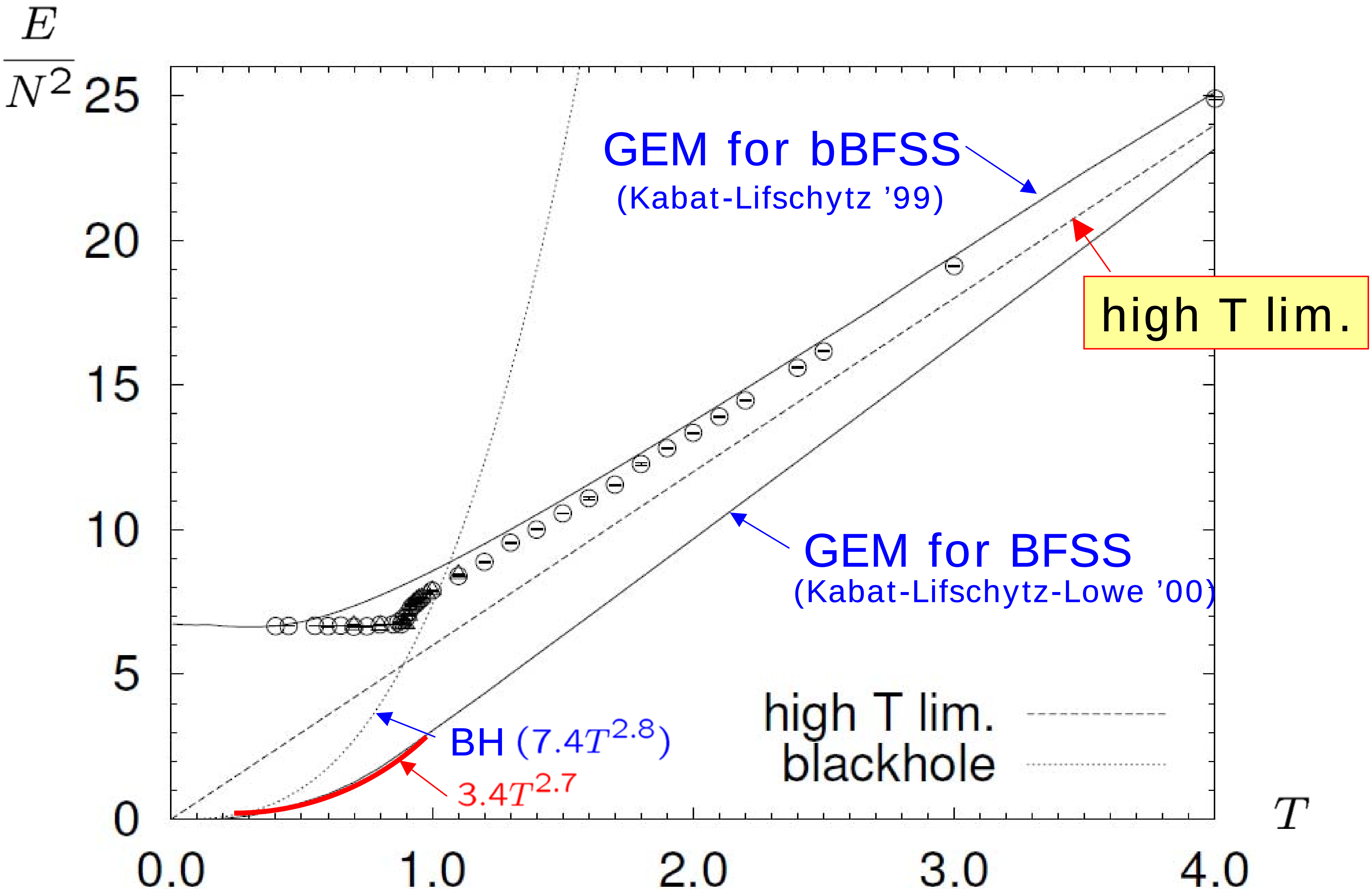
4. Testing GEM



4. Testing GEM



4. Testing GEM



5. Summary and discussions

- Monte Carlo studies of bosonic BFSS model

  (D=9+1, large N, continous time)

  c.f.) MC studies of bosonic IKKT model

  Hotta-J.N.-Tsuchiya ('98)

  $T=\infty$ limit of BFSS model

  extension to finite T including only bosons

- deconf. phase transition at $T \sim 0.9$
  - 2nd order (E, R^2 : continuous at $T = T_c$)
  - comparison with BH entropy : promising
  - GEM fails to reproduce the transition

    ⇨ SUSY case?

- effects of fermions :
  - lower T_c (attractive force between eigenval's of A)
  - order of the transition? (Hawking-Page transition)