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Origin of lower-dimensional pure-spinor superstrings

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(Talk based on:
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Pure-spinor Superstrings

Berkovits(2000)

Fields

$X^\mu(\mu = 0, \ldots, 9)$: spacetime coordinates

$\theta^\alpha, p_\alpha(\alpha = 1, \ldots, 16)$: spacetime Majorana-Weyl spinors (worldsheet scalars) and conjugates

$\lambda^\alpha, w_\alpha(\alpha = 1, \ldots, 16)$: spacetime bosonic spinor ghosts (worldsheet scalars) and conjugates

"Pure-spinor" constraints : $\lambda\gamma^\mu\lambda = 0$

γ^μ: 16x16 symmetric matrices; off-diagonal blocks of gamma matrices in chiral representation

BRST charge: $Q = \int \lambda^\alpha d_\alpha$

$$d_\alpha = p_\alpha - \frac{1}{2}(\partial x^\mu + \frac{1}{4}\theta\gamma^\mu\partial\theta)(\gamma_\mu\theta)_\alpha$$

Lower-dimensional pure-spinor superstrings

Wyllard; Grassi, Wyllard hep-th/0509140

	D=4	D=6	D=10
Pure-spinor constraints	$\lambda\Gamma^\mu\lambda = 0$	$\lambda^I\gamma^\mu\lambda_I = 0$	$\lambda\gamma^\mu\lambda = 0$
Spinor type	Weyl	symplectic-Weyl	Weyl

Independent components	D=4	D=6	D=10
X^μ	4	6	10
θ^α, p_α	2x4	2x8	2x16
λ^α, w_α	2x2	2x5	2x11

- Constructed by just mimicking the D=10 procedure
- Super-Poincare covariant (but the pure spinor constraints)
- No compact matter couplings

What are these theories?

- Are these lower-dimensional pure-spinor models equivalent to (or at least related to) the Green-Schwarz superstrings, as has been shown in D=10?
- If yes, there should be some inconsistency, but where?
- Why do its spectra coincide with those of noncritical strings? Adam, Grassi, Mazzucato, Oz, Yankielowicz hep-th/0605118

↓

Use Aisaka-Kazama's double-spinor formalism

Doulbe-spinor formalism

Aisaka,Kazama(hep-th/0502208)

1. Start from Green-Schwarz-like action with an extra set of fermionic coordinates $\tilde{\theta}^{\alpha}$, in addtion to θ^{α}. The action has a local femionic symmetry (besides κ -symmetry) which can be gauge-fixed to ordinary Green-Schwarz action.
2. Impose the semi-light cone gauge condition to $\tilde{\theta}^{\alpha}$, while keeping θ^{α} covariant. Compute the Dirac bracket.
3. Find a new set of fields so that they are canonical w.r.t. the Dirac bracket. Write the remaining 1st class constraints using these fields.

Doulbe-spinor formalism(cont'd)

3.

4. Quantization. Replace Dirac brackets with OPEs.

5. Remove double-contraction singularities by appropriately fixing normal-ordering ambiguities of the constraints.

$\Downarrow$

- Closed constraint algebra → Standard BRST charge
- Energy-momentum tensor is automatically modified to vanishing central charge
- BRST charge is automatically nilpotent and cohomologically equivalent to the pure-spinor BRST charge

Strategy

- Consider the D=4 analogue of the double-spinor action
- Take the same steps as Aisaka-Kazama and see if there are any problem:
  - Free fields really found?
  - OPEs close?
  - Central charge vanishes?
  - BRST charge equivalent to the pure-spinor one?

D=4 Aisaka-Kazama double-spinor superstring (type IIB)

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_{WZ}$$
$$\mathcal{L}_K = -\frac{1}{2}\sqrt{-g}g^{ab}\Pi_a\Pi_b$$
$$\mathcal{L}_{WZ} = \epsilon^{ab}\left(\Pi_a(W_b - \hat{W}_b) - W_a\hat{W}_b\right)$$

$$\Pi_a^\mu = \partial_a X^\mu - i\sum_{A=1,2}\left(W_a^{A\mu} + i\partial_a(\theta^A\sigma^\mu\bar{\tilde{\theta}}^A - \tilde{\theta}^A\sigma^\mu\bar{\theta}^A)\right)$$
$$W_a^{A\mu} = i(\Theta^A\sigma^\mu\partial_a\bar{\Theta}^A - \partial_a\Theta^A\sigma^\mu\bar{\Theta}^A) \qquad \Theta^A = \tilde{\theta}^A - \theta^A$$

- $a, b, ...$: worldsheet, $\mu, \nu, ... = 0, 1, 2, 3$: spacetime indices
- Spinors are all two-component, complex Weyl
- $A = 1, 2$ → (eventually) left, right
- Local fermionic symmetry : $\delta\theta = \delta\tilde{\theta} = \epsilon(t, \sigma)$
- Can be gauge-fixed to $\Theta^A = \tilde{\theta}^A, \quad \theta^A = 0$ → GS action

Constraints

$$P_\mu \equiv \frac{\partial}{\partial \dot{X}^\mu}\mathcal{L} \qquad P^A_\alpha \equiv \frac{\partial}{\partial \dot{\theta}^{A\alpha}}\mathcal{L} \qquad \bar{P}^A \equiv \frac{\partial}{\partial \dot{\bar{\theta}}^{A\bar{\alpha}}}\mathcal{L}$$

$$D^A_\alpha \equiv P^A_\alpha + i(\not{P}\bar{\tilde{\theta}}^A)_\alpha + i(P_\mu + \eta^A(\Pi_{\mu\sigma} + W^{\tilde{A}}_{\mu\sigma})))(\sigma^\mu\bar{\Theta}^A)_\alpha = 0$$

$$\tilde{D}^A_\alpha \equiv \tilde{P}^A_\alpha - i(\not{P}\bar{\theta}^A)_\alpha - i(P_\mu + \eta^A(\Pi_{\mu\sigma} + W^{\tilde{A}}_{\mu\sigma})))(\sigma^\mu\bar{\Theta}^A)_\alpha = 0$$

$$\bar{D}^A_{\bar{\alpha}} \equiv \bar{P}^A_{\bar{\alpha}} + i(\tilde{\theta}^A\not{P})_{\bar{\alpha}} + i(P_\mu + \eta^A(\Pi_{\mu\sigma} + W^{\tilde{A}}_{\mu\sigma})))(\Theta^A\sigma^\mu)_{\bar{\alpha}} = 0$$

$$\bar{\tilde{D}}^A_{\bar{\alpha}} \equiv \bar{\tilde{P}}^A_{\bar{\alpha}} - i(\theta^A\not{P})_{\bar{\alpha}} - i(P_\mu + \eta^A(\Pi_{\mu\sigma} + W^{\tilde{A}}_{\mu\sigma})))(\Theta^A\sigma^\mu)_{\bar{\alpha}} = 0$$

Canonical analysis

$$\{X^\mu(\sigma),\ P^\nu(\sigma')\}_P = \eta^{\mu\nu}\delta(\sigma-\sigma')$$

$$\{\theta^{A\alpha}(\sigma),\ P^B_\beta(\sigma')\}_P = -\delta^{AB}\delta^\alpha_\beta\delta(\sigma-\sigma') \qquad \{\tilde{\theta}^{A\alpha}(\sigma),\ \tilde{P}^B_\beta(\sigma')\}_P = -\delta^{AB}\delta^\alpha_\beta\delta(\sigma-\sigma')$$

$$\{\bar{\theta}^{A\alpha}(\sigma),\ \bar{P}^B_\beta(\sigma')\}_P = -\delta^{AB}\delta^{\bar{\alpha}}_{\bar{\beta}}\delta(\sigma-\sigma') \qquad \{\bar{\tilde{\theta}}^{A\alpha}(\sigma),\ \bar{\tilde{P}}^B_\beta(\sigma')\}_P = -\delta^{AB}\delta^{\bar{\alpha}}_{\bar{\beta}}\delta(\sigma-\sigma')$$

⬇

$$\{D^A_\alpha(\sigma),\ \bar{D}^B_\beta(\sigma')\}_P = 2i\delta^{AB}\sigma_{\mu\alpha\bar{\beta}}\delta(\sigma-\sigma')\Pi^\mu_A(\sigma')$$
$$\{\tilde{D}^A_\alpha(\sigma),\ \bar{\tilde{D}}^B_\beta(\sigma')\}_P = 2i\delta^{AB}\sigma_{\mu\alpha\bar{\beta}}\delta(\sigma-\sigma')\Pi^\mu_A(\sigma')$$
$$\{D^A_\alpha(\sigma),\ \bar{\tilde{D}}^B_\beta(\sigma')\}_P = -2i\delta^{AB}\sigma_{\mu\alpha\bar{\beta}}\delta(\sigma-\sigma')\Pi^\mu_A(\sigma')$$
$$\{\tilde{D}^A_\alpha(\sigma),\ \bar{D}^B_\beta(\sigma')\}_P = -2i\delta^{AB}\sigma_{\mu\alpha\bar{\beta}}\delta(\sigma-\sigma')\Pi^\mu_A(\sigma')$$

where

$$\Pi^\mu_A \equiv P^\mu + \eta_A(\Pi^\mu_\sigma - W^\mu_{A\sigma} + W^\mu_{\tilde{A}\sigma})$$
$$\eta^A = +1(-1) \text{ if } A = 1(2)$$
$$\tilde{A} = 2(1) \text{ if } A = 1(2)$$

➡

$$\Pi^\mu_a = \partial_a X^\mu - i\sum_{A=1,2}\Big(W^{A\mu}_a + i\partial_a(\theta^A\sigma^\mu\bar{\tilde{\theta}}^A - \tilde{\theta}^A\sigma^\mu\bar{\theta}^A)\Big)$$
$$W^{A\mu}_a = i(\Theta^A\sigma^\mu\partial_a\bar{\Theta}^A - \partial_a\Theta^A\sigma^\mu\bar{\Theta}^A)$$

Partial gauge fixing

For each A

D^A_α : 2 first class

$\tilde{D}^A_\alpha$: 1 first class +1 second class

semi-light cone gauge ➡

D^A_α : 2 first class

$\tilde{D}^A_\alpha$: 2 second class

Decompose "tilde" constraints $\tilde{D}^A_\alpha$ w.r.t. the transverse SO(2) chirality, then the upper (α =1) component is of second class, while the lower (α =2)component is gauge fixed by the semi-light cone gauge condition

$$\sigma^+\tilde{\theta}^A = 0 \;\Leftrightarrow\; \tilde{\theta}^{A2} = 0$$

Similarly for $\bar{D}^A_{\bar{\alpha}}$ and $\bar{\tilde{D}}^A_{\bar{\alpha}}$

Dirac bracket

$$
\begin{aligned}
\{A(\sigma),\ B(\sigma')\}_D \ =\ & \{A(\sigma),\ B(\sigma')\}_P \\
& + \int d\xi \{A(\sigma),\ \tilde{D}_1(\xi)\}_P \frac{i}{2\Pi^+(\xi)} \{\bar{\tilde{D}}_1(\xi),\ B(\sigma')\}_P \\
& + \int d\xi \{A(\sigma),\ \bar{\tilde{D}}_1(\xi)\}_P \frac{i}{2\Pi^+(\xi)} \{\tilde{D}_1(\xi),\ B(\sigma')\}_P \\
& + \int d\xi \{A(\sigma),\ \tilde{\theta}^2(\xi)\}_P 8i\mathcal{T}(\xi) \{\bar{\tilde{\theta}}^{\bar{2}}(\xi),\ B(\sigma')\}_P \\
& + \int d\xi \{A(\sigma),\ \bar{\tilde{\theta}}^{\bar{2}}(\xi)\}_P 8i\mathcal{T}(\xi) \{\tilde{\theta}^2(\xi),\ B(\sigma')\}_P \\
& + \int d\xi \{A(\sigma),\ \tilde{\theta}^2(\xi)\}_P \{K(\xi),\ B(\sigma')\}_P \\
& + \int d\xi \{A(\sigma),\ K(\xi)\}_P \{\tilde{\theta}^2(\xi),\ B(\sigma')\}_P \\
& + \int d\xi \{A(\sigma),\ \bar{\tilde{\theta}}^{\bar{2}}(\xi)\}_P \{\bar{K}(\xi),\ B(\sigma')\}_P
\end{aligned}
$$

where

$$
K \equiv \tilde{D}_2 - \frac{\Pi^1 + i\Pi^2}{\Pi^+}\tilde{D}_1 \qquad \bar{K} \equiv \bar{\tilde{D}}_{\bar{2}} - \frac{\Pi^1 - i\Pi^2}{\Pi^+}\bar{\tilde{D}}_{\bar{1}}
$$

$$
\{K(\sigma),\ \bar{K}(\sigma')\}_P = 8i\delta(\sigma - \sigma')\mathcal{T}(\sigma') + \text{second class}
$$

We will examine:

1. The Dirac brackets between the basic variables are shifted from the canonical values. In D=10 Aisaka-Kazama found a set of canonical variables w.r.t. the Dirac bracket which were eventually identified with the free fields in the pure spinor formalism. How about the D=4 case?

2. In quantization one replaces Dirac brackets with suitable OPEs. Then double contractions give rise to 2nd or higher order singularities and the OPEs do not close. In D=10 normal ordering ambiguities of the constraints are used to cancel the extra singularities. Does such a mechanism work in D=4?

3. If the constraint algebras close, what's the relationship between the BRST charges in this and the pure-spinor formalisms?

Answer to #1

"Free fileds" can be found

$$
\begin{aligned}
\check{P}^{\mu} &\equiv P^{\mu} + i(\theta\sigma^{\mu}\bar{\tilde{\theta}} - \tilde{\theta}\sigma^{\mu}\bar{\theta})' - i(\hat{\theta}\sigma^{\mu}\hat{\bar{\tilde{\theta}}} - \hat{\tilde{\theta}}\sigma^{\mu}\hat{\bar{\theta}})', \\
\check{P}_1^{A} &\equiv P_1^{A} + \eta^{A}\left((-iX^{+\prime} - 6\theta^{A2}\bar{\theta}^{A\bar{2}\prime})\bar{\tilde{\theta}}^{A\bar{1}} - 2\theta^{A2}\bar{\theta}^{A\bar{2}}\bar{\tilde{\theta}}^{A\bar{1}\prime}\right) \\
\check{P}_2^{A} &\equiv P_1^{A} + \eta^{A}\Big((-i(X^1 + iX^2)' - 6\theta^{A1}\bar{\theta}^{A\bar{2}\prime})\bar{\tilde{\theta}}^{A\bar{1}} - 2\theta^{A1}\bar{\theta}^{A\bar{2}}\bar{\tilde{\theta}}^{A\bar{1}\prime} \\
&\qquad + 6(\bar{\theta}^{A\bar{1}}\bar{\theta}^{A\bar{2}})'\tilde{\theta}^{A1} + 4\bar{\theta}^{A\bar{1}}\bar{\theta}^{A\bar{2}}\tilde{\theta}^{A1\prime} \\
&\qquad - 2(\tilde{\theta}^{A1}\bar{\tilde{\theta}}^{A\bar{1}})'\bar{\theta}^{A\bar{2}} + 2(\bar{\theta}^{\tilde{A}\bar{2}}\tilde{\theta}^{\tilde{A}1})'\bar{\tilde{\theta}}^{A\bar{1}}\Big) \\
\check{\bar{P}}_{\bar{1}}^{A} &= (\text{with}^{\ -} \leftrightarrow \text{without}^{\ -} \ \ \text{in} \ \ \check{P}_1^{A}) \\
\check{\bar{P}}_{\bar{2}}^{A} &= (\text{with}^{\ -} \leftrightarrow \text{without}^{\ -} \ \ \text{in} \ \ \check{P}_2^{A})
\end{aligned}
$$

$$
\{X^{\mu}(\sigma),\ \check{P}^{\nu}(\sigma')\}_D = \eta^{\mu\nu}\delta(\sigma - \sigma')
$$

➡

$$
\{\theta^{A\alpha}(\sigma),\ \check{P}_{\beta}^{B}(\sigma')\}_D = -\delta^{AB}\eta_{\beta}^{\alpha}\delta(\sigma - \sigma')
$$

$$
\{\bar{\theta}^{A\alpha}(\sigma),\ \check{\bar{P}}_{\beta}^{B}(\sigma')\}_D = -\delta^{AB}\delta_{\bar{\beta}}^{\bar{\alpha}}\delta(\sigma - \sigma')
$$

Answer to #2

In terms of new canonical variables the constraints can be written

$$D_1 = d_1 - i\sqrt{2\pi^+}\bar{S}$$
$$D_2 = d_2 - i\sqrt{\frac{2}{\pi^+}}\pi\bar{S} - 2\frac{S\bar{S}}{\pi^+}\partial_z\bar{\theta}^{\bar{2}}$$
$$\bar{D}_1 = \bar{d}_1 + i\sqrt{2\pi^+}S$$
$$\bar{D}_2 = \bar{d}_2 + i\sqrt{\frac{2}{\pi^+}}\bar{\pi}S - 2\frac{\bar{S}S}{\pi^+}\partial_z\theta^2$$
$$\mathcal{T} = -\frac{\pi^+\pi^- - \pi\bar{\pi}}{2\pi^+} - \frac{S\partial\bar{S} - \partial S\bar{S}}{2\pi^+} + \frac{\sqrt{2}i(S\partial\bar{\theta}^{\bar{2}}\bar{\pi} + \partial\theta^2\bar{S}\pi)}{\sqrt{\pi^+}^3} + \frac{\sqrt{2}i(S\partial\bar{\theta}^{\bar{1}} + \partial\theta^1\bar{S})}{\sqrt{\pi^+}} + \frac{4S\bar{S}\partial\theta^2\partial\bar{\theta}^{\bar{2}}}{\pi^+}$$
$$\check{D}_1 = D_1$$
$$\check{D}_2 = D_2 - \frac{\partial^2\bar{\theta}^{\bar{2}}}{\pi^+} - \frac{1}{2}\partial\left(\frac{1}{\pi^+}\right)\partial\bar{\theta}^{\bar{2}}$$
$$\check{\bar{D}}_1 = \bar{D}_{\bar{1}}$$
$$\check{\bar{D}}_{\bar{2}} = \bar{D}_{\bar{2}} - \frac{\partial^2\theta^2}{\pi^+} - \frac{1}{2}\partial\left(\frac{1}{\pi^+}\right)\partial\theta^2$$
$$\check{\mathcal{T}} = \mathcal{T} - \frac{\partial^2\log\pi^+}{8\pi^+} + \frac{\partial\theta^2\partial^2\bar{\theta}^{\bar{2}} - \partial^2\theta^2\partial\bar{\theta}^{\bar{2}}}{\pi^2}$$

$$d_\alpha = P_\alpha + \left(-i\partial X^\mu - \frac{1}{2}(\theta\sigma^\mu\partial\bar{\theta} - \partial\theta\sigma^\mu\bar{\theta})\right)(\sigma_\mu\bar{\theta})_\alpha$$
$$\bar{d}_\alpha = \bar{P}_{\bar{\alpha}} + \left(-i\partial X^\mu - \frac{1}{2}(\theta\sigma^\mu\partial\bar{\theta} - \partial\theta\sigma^\mu\bar{\theta})\right)(\theta\sigma_\mu)_{\bar{\alpha}}$$
$$\pi^\mu = i\partial X^\mu + \theta\sigma^\mu\partial\bar{\theta} - \partial\theta\sigma^\mu\bar{\theta}$$

$$\pi^\pm = \pi^0 \pm \pi^3$$
$$\pi = \pi^1 + i\pi^2$$
$$\bar{\pi} = \pi^1 - i\pi^2$$

$$\Rightarrow \quad \check{D}_2(z)\check{\bar{D}}_{\bar{2}}(w) \sim \frac{4\check{\mathcal{T}}(w)}{z-w}$$

OPEs close due to modifications coming from normal ordering

Answer to #3

- One can construct a nilpotent BRST charge in a standard manner, which is shown to reduce to the pure spinor BRST charge by a series of similarity transformations.

- One can also find a BRST trivial energy-momentum tensor. The central charge is automatically adjusted to zero by fixing the normal-ordering ambiguities of the constraints (so that they close).

Conclusions

- D=4 double-spinor superstring, which is classically equivalent to D=4 GS superstring, allows BRST quantization by appropriately modifying constraints. The resulting theory is equivalent to the D=4 pure-spinor superstring.
- The modification is thought of as coming from normal-ordering ambiguities of the constraints. Since this is a purely quantum effect, this mechanism is reminiscent of how conformal mode become dynamical and adjust central charge to zero in noncritical string theory. In that sense the Lorentz invariance is broken spontaneously.