Toric AdS4/CFT3 dual pairs and Crystal Lattices

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[S. Lee, hep-th/0610204]

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AdS/CFT with less SUSY

\( D = 4 \)
\( \mathcal{N} = 4 \)

\( D = 4 \)
\( \mathcal{N} = 1 \)

Simplest example: \( C(T^{1,1}) \)  
[Calabi-Yau]  
[AdS/CFT with less SUSY]

[Acharya-Figueroa O’Farill-Hull-Spence]  
[Morrison-Plesser]

Sasaki-Einstein
Toric-Quiver Duality (Dimer Model)

Faces = Gauge groups

Edges = Bi-fundamentals

Vertices = Super-potentials

Algorithm

Toric CY3 ⇔ Dimer model

complete!

[Hanany] [Feng-He-Kennaway-Vafa]
[Hanany-Benvenuti-Franco-Vegh-Wecht]
[Butti-Forcella-Zaffaroni] [Iqbal-Uranga]
[Martelli-Sparks-Yau]

[Gauntlett-Martelli-Sparks-Waldram]
[Cvetic-Lu-Page-Pope]

[Hanany]

[Gauntlett-Martelli-Sparks-Waldram]
[Cvetic-Lu-Page-Pope]
M2 on CY4

$D = 3, \mathcal{N} = 2$ SUSY

\[
\mathbb{R}^{1,2} \times C(Y_7) \rightarrow \text{AdS}_4 \times Y_7
\]

(Geometry) No qualitative difference

(Field Theory) Strongly coupled at IR. Non-abelian EM duality

3d analog of dimer model? YES!

(M5 world-volume theory)
Toric Geometry

$C^3 :$

$|z_1|^2, |z_2|^2, |z_3|^2$

$C_{Y_n} (i = 1 \sim n)$

toric fan : $v_I^i y_i \geq 0$

base of the cone : $b^i y_i = \frac{1}{2}$
M2 on toric CY4

Flavor symmetry (Isometry of CY4)

\[ F_i[S^I] \equiv NF_i^I \]
\[ Q_a[S^I] \equiv NQ_a^I \quad (a = 1 \sim d - n) \]

Baryonic symmetry

\[ v_i^j F_j^I = \delta_j^i, \quad v_i^j Q_a^I = 0. \]

\[ R = \frac{1}{2} b^i F_i \quad \Rightarrow \quad \sum_l R_l^I = 2 \]
T-duality and brane configuration

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Stack of N M2-branes

Degenerating circle fibers

Locally $\mathbb{R}^2 \times S^1$

Union of all 2-fans thickened!

Special Lagrangian (N=2 SUSY)
Crystal Lattice : Bonds and Atoms

Closed region containing vertices

A partition covering the entire toric diagram

→ Bonds

→ Atoms
Examples

$C^4$

$C(Q^{1,1,1})$

zincblende (GaAs)

NaCl
Baryon-Fundamental Transition

\[ zw = 0 \rightarrow zw = \varepsilon \]

Fundamental excitation!!!
Super-potentials

Closed spherical M2 (union of M2-discs) surrounding an atom

Bipartite (two-colored) due to the orientation of M2

\[ R = 2 \] by construction

F-term condition: 
(sum of two neighboring terms) = 0
Spectrum of chiral mesons

Holomorphic monomial in CY

\[ F_i(m) = m_i \]

\[ R(m) = \frac{1}{2} b^i F_i(m) = \frac{1}{2} (b \cdot m) = \frac{1}{2} \sum_{i=1}^{3} b^i m_i + 2m_4 \]

Closed M2-branes

F-term condition guarantees R is insensitive to the details!
Outlook

- (non-)uniqueness of graph / Seiberg duality
- Non-BPS spectrum
- Moduli space of vacua
- Marginal deformation
- Volume Minimization
- Brane Configuration for N=3?