Toric AdS4/CFT3 dual pairs and Crystal Lattices

Sangmin Lee

Seoul National University

[S. Lee, hep-th/0610204]

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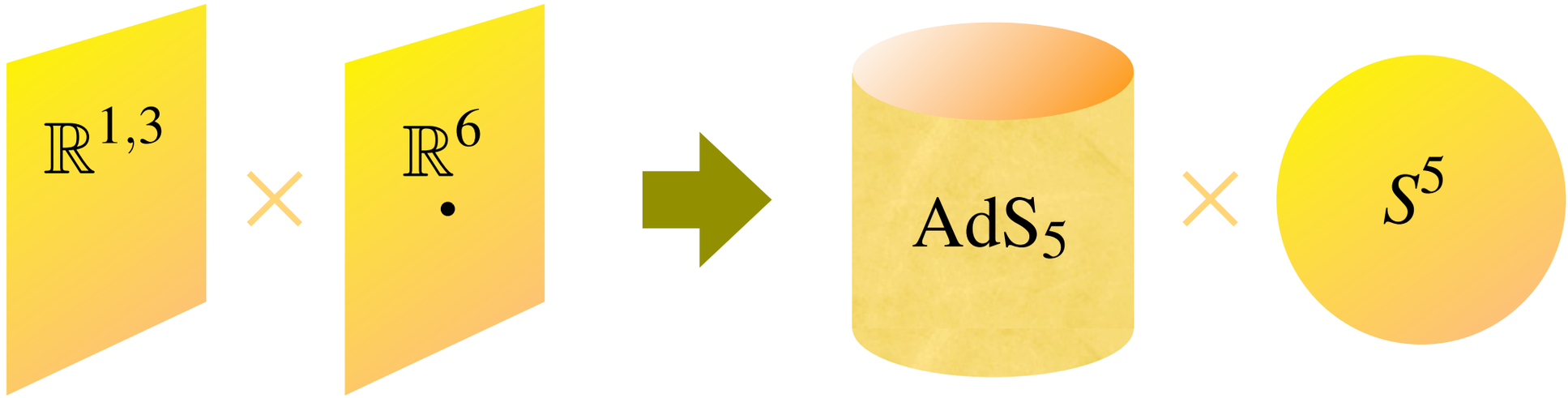
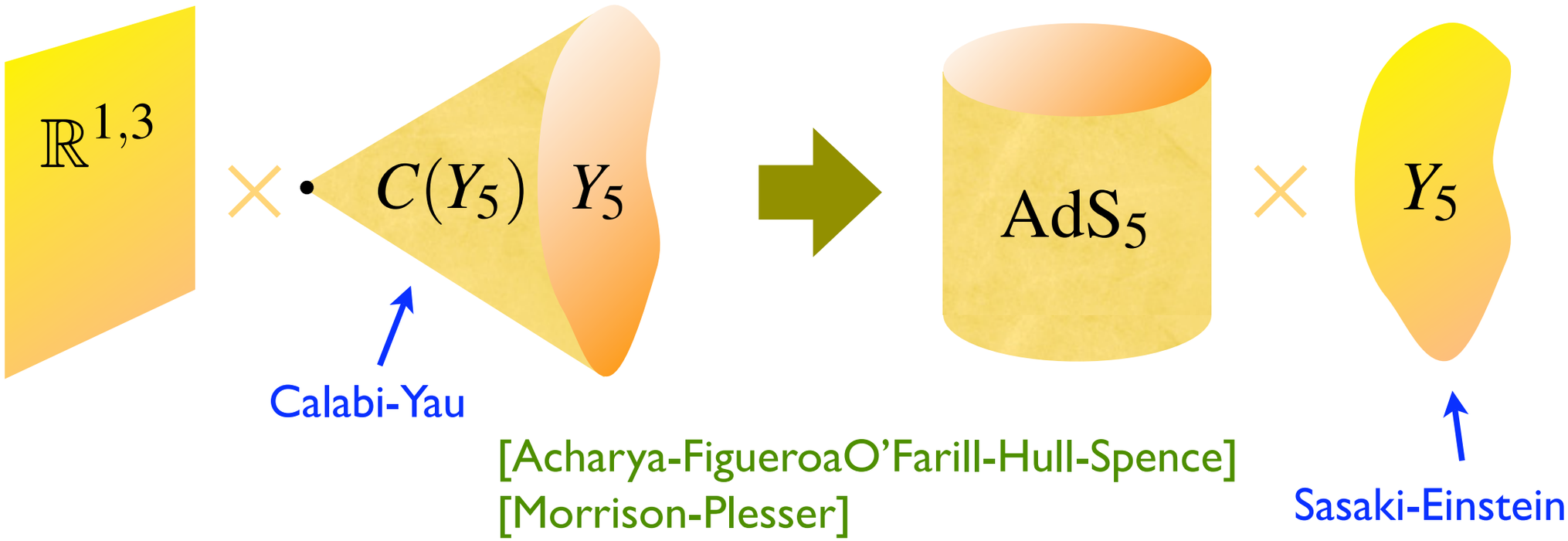
AdS/CFT with less SUSY

$D = 4$
$\mathcal{N} = 4$



$D = 4$
$\mathcal{N} = 1$



Simplest example : $C(T^{1,1})$ [Klebanov-Witten]

Toric-Quiver Duality (Dimer Model)

[Gauntlett-Martelli-Sparks-Waldram]
[Cvetic-Lu-Page-Pope]

[Hanany] [Feng-He-Kennaway-Vafa]
[Hanany-Benvenuti-Franco-Vegh-Wecht]
[Butti-Forcella-Zaffaroni] [Iqbal-Uranga]
[Martelli-Sparks-Yau]

Faces = Gauge groups

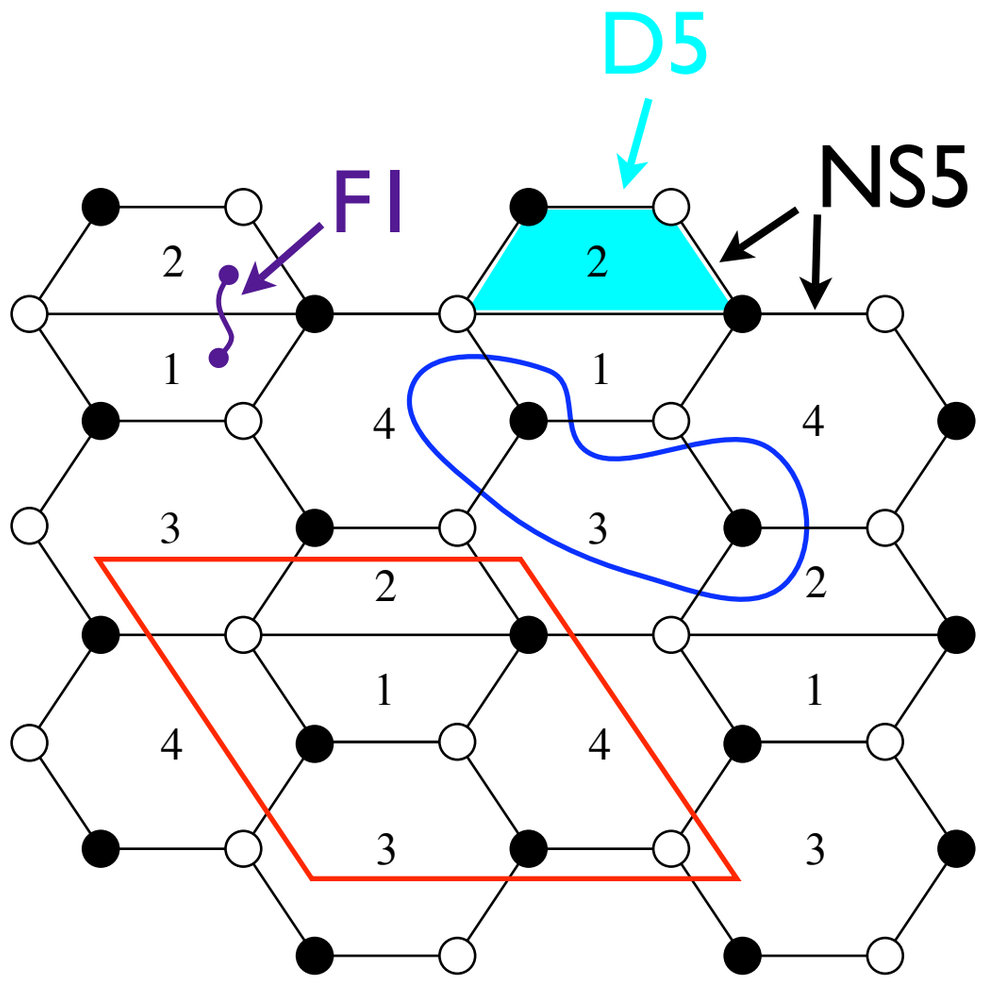
Edges = Bi-fundamentals

Vertices = Super-potentials

Algorithm

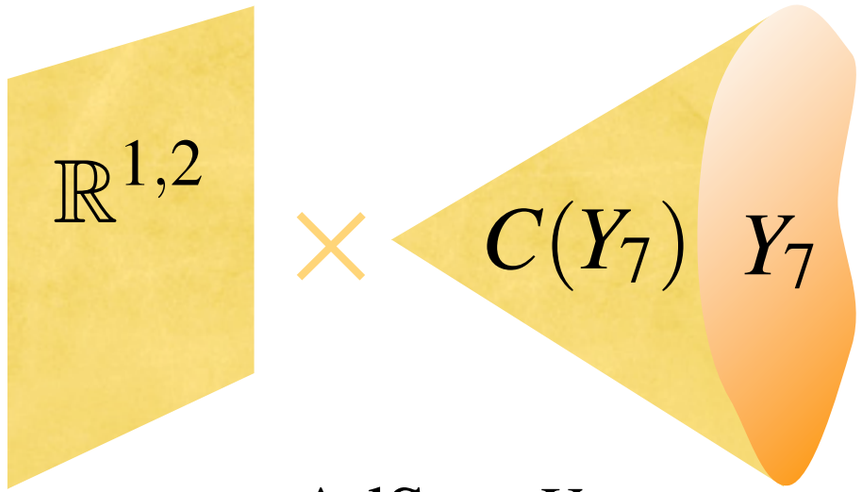
Toric CY3 ⟷ Dimer model

complete!



M2 on CY4

$D = 3, \mathcal{N} = 2$ SUSY



(Geometry) No qualitative difference

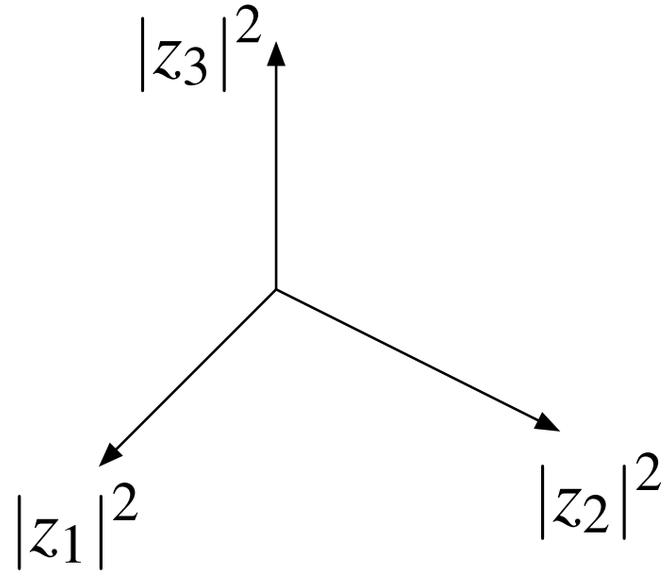
(Field Theory) Strongly coupled at IR. Non-abelian EM duality

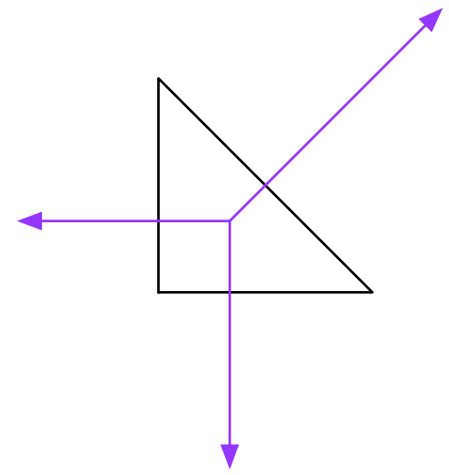
3d analog of dimer model? YES!

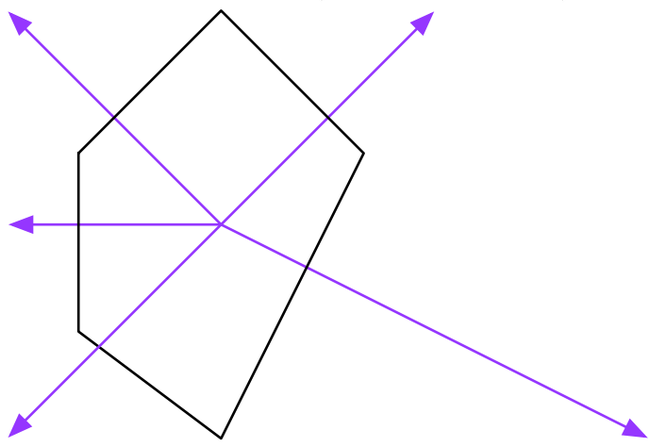
(M5 world-volume theory)

Toric Geometry

$\mathbb{C}^3$:





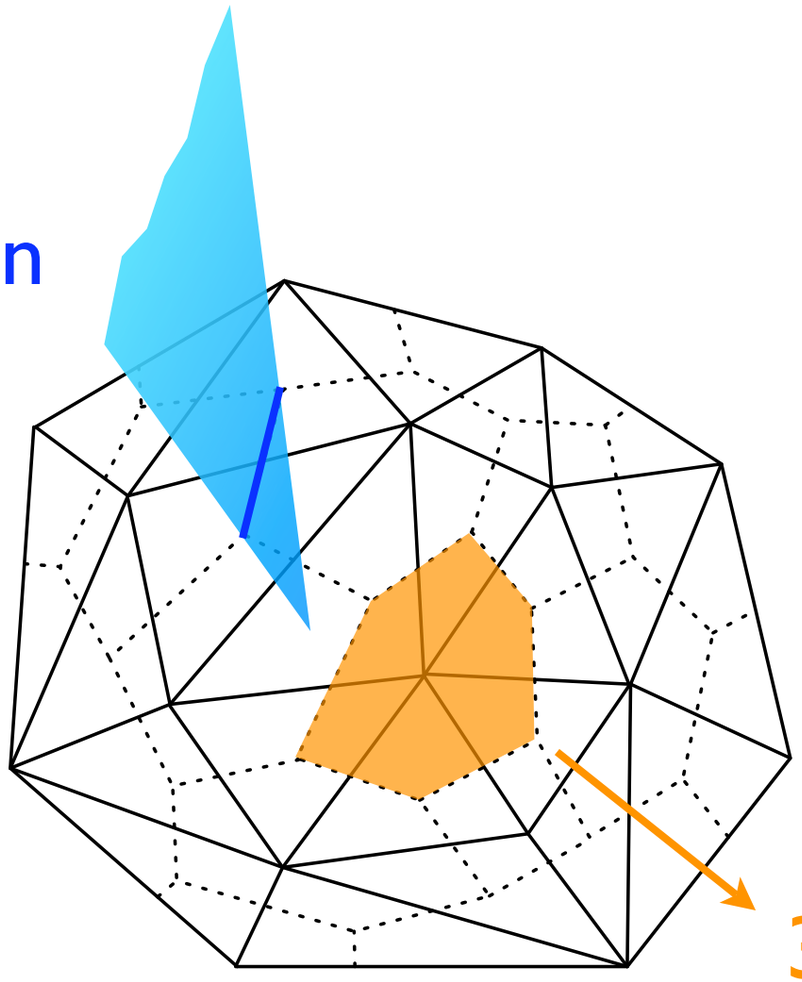


$CY_n \quad (i = 1 \sim n)$

toric fan : $v_I^i y_i \geq 0$

base of the cone : $b^i y_i = \frac{1}{2}$

M2 on toric CY4



Flavor symmetry
(Isometry of CY4)

$$F_i[S^I] \equiv NF_i^I$$

$$Q_a[S^I] \equiv NQ_a^I \qquad (a = 1 \sim d-n)$$

Baryonic symmetry

$$v_I^i F_j^I = \delta_j^i, \;\; v_I^i Q_a^I = 0.$$

$$R = \tfrac{1}{2} b^i F_i \;\; \Rightarrow \;\; \textstyle\sum_I R^I = 2$$

Baryon = M5-brane

SUSY 5-cycle

T-duality and brane configuration

Stack of N
M2-branes

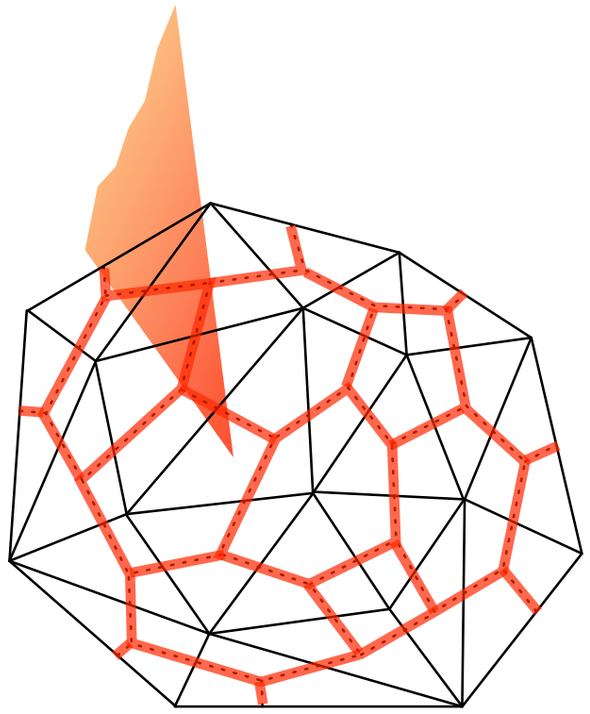
	0	1	2	3	4	5	6	7	8	9	11
				$\mathbb{R}^3$			T^3				
M5	○	○	○				○	○	○		
M5	○	○	○			Σ					

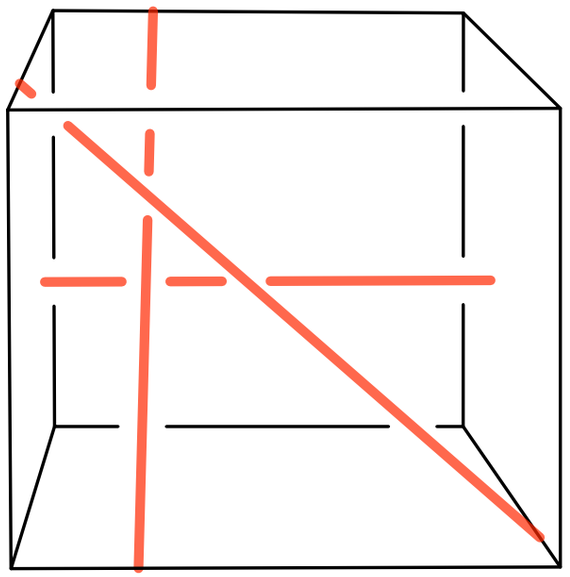
Degenerating
circle fibers

Locally $\mathbb{R}^2 \times S^1$

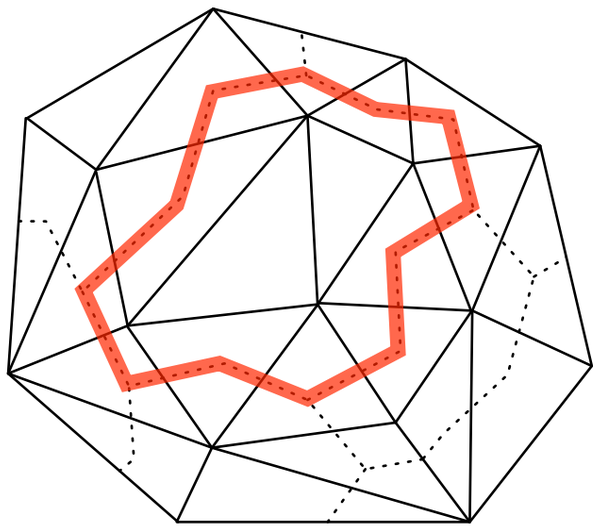
Union of all 2-fans thickened!

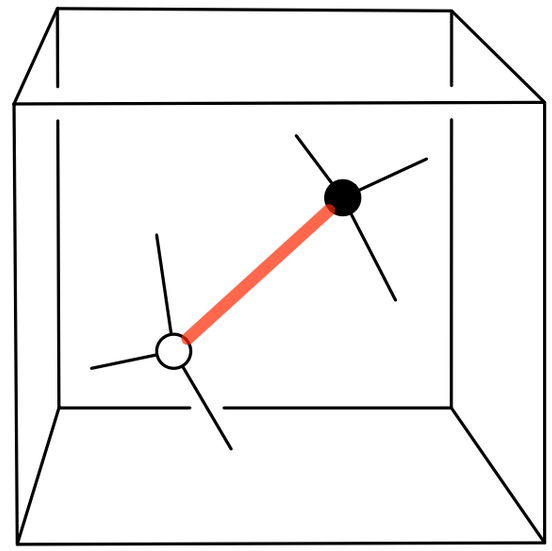
Special Lagrangian (N=2 SUSY)





Crystal Lattice : Bonds and Atoms



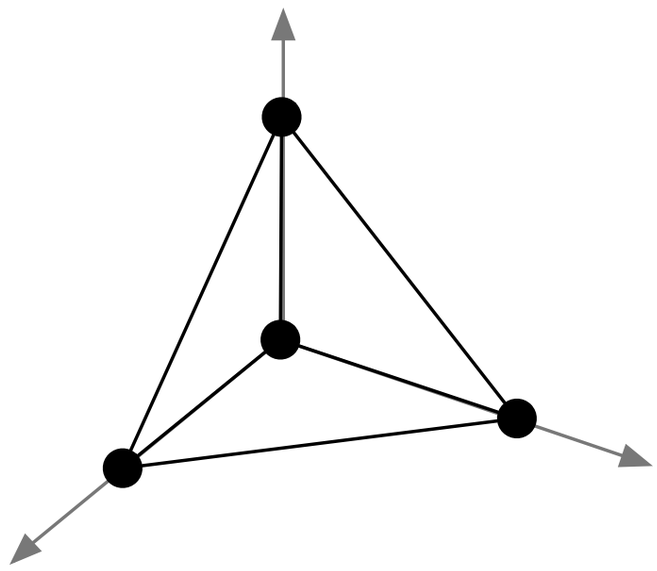


Closed region containing vertices ⟶ Bonds

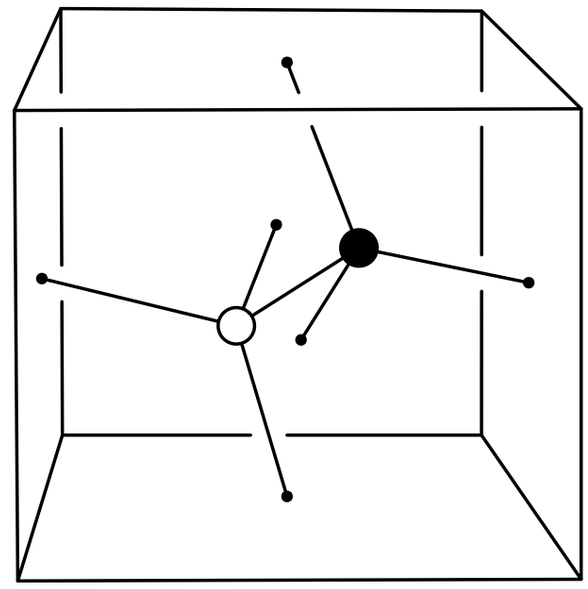
A partition covering the entire toric diagram ⟶ Atoms

Examples

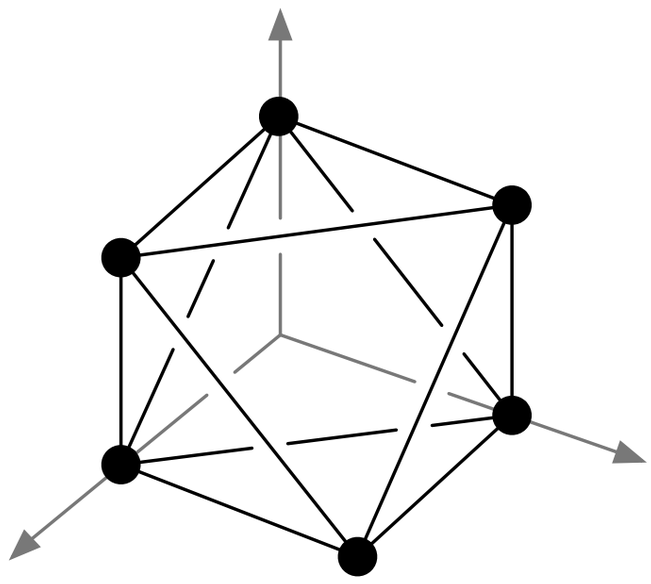
$\mathbb{C}^4$

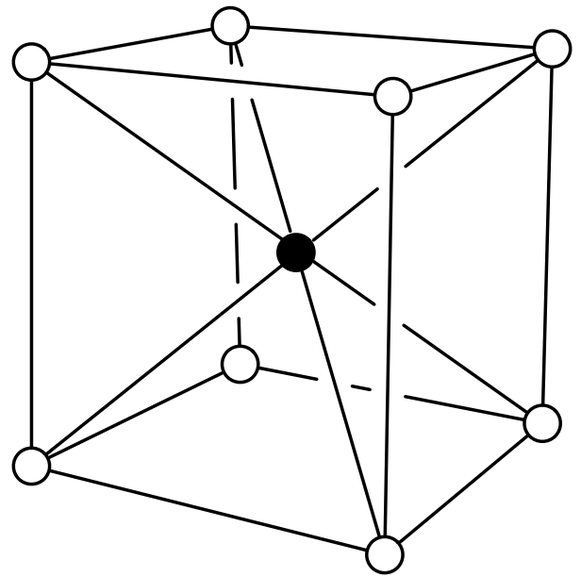


zincblende (GaAs)



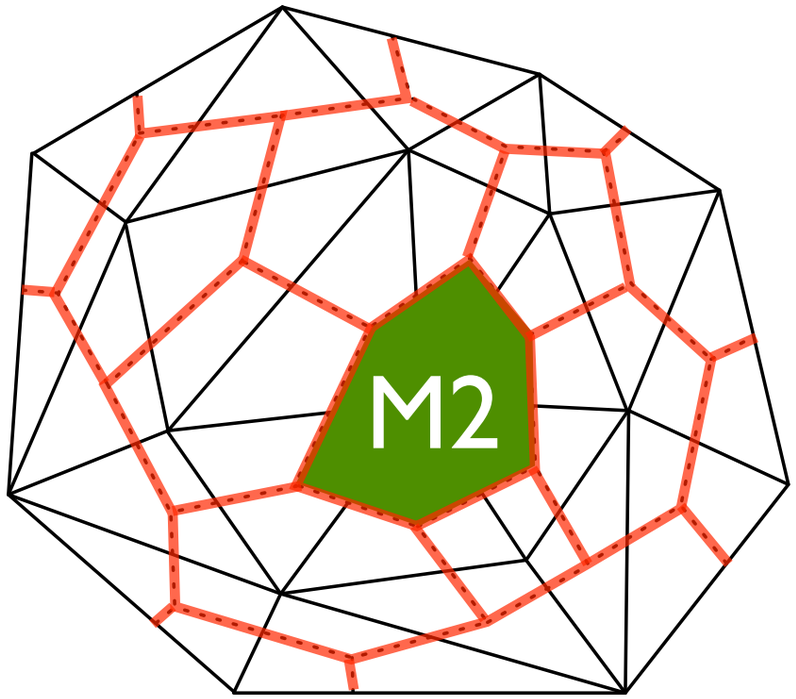
$C(Q^{1,1,1})$





NaCl

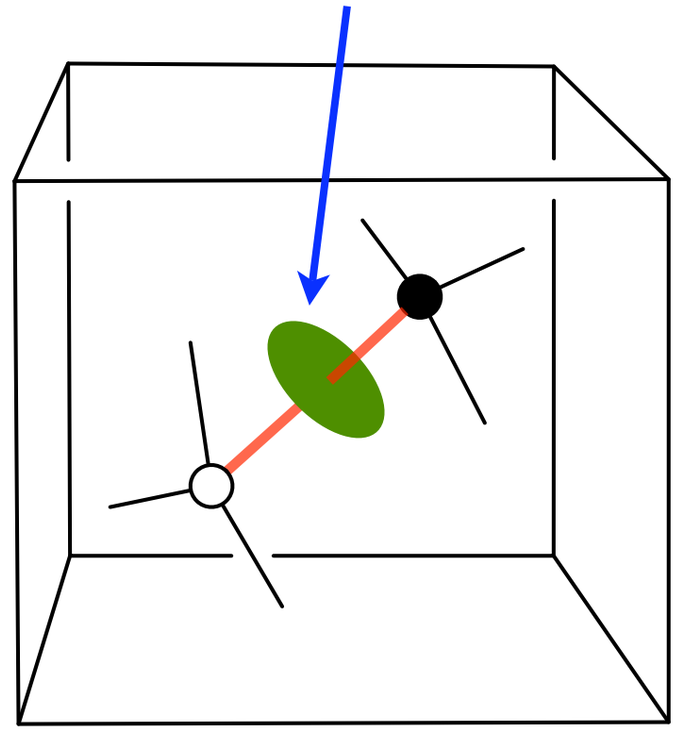
Baryon-Fundamental Transition



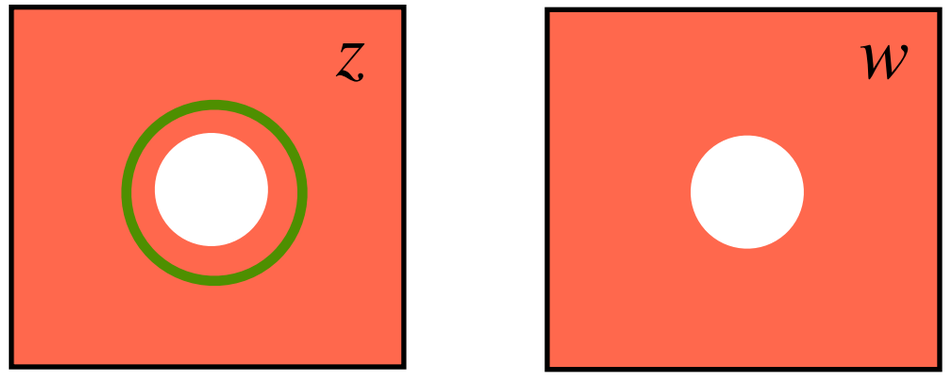
smooth
transition

Baryon ⟶ N x Fundamentals

Fundamental excitation!!!

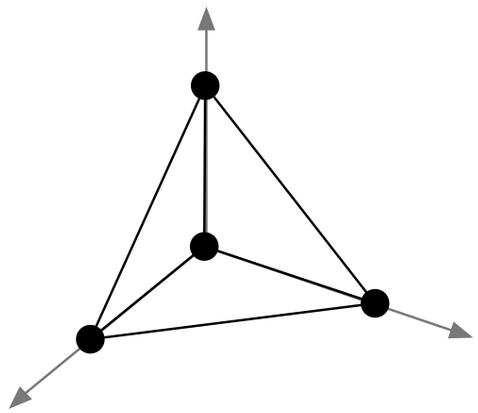


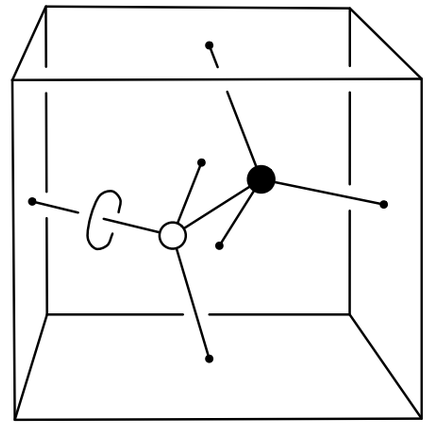
$$zw = 0 \quad \rightarrow \quad zw = \varepsilon$$



Super-potentials

Closed spherical M2
(union of M2-discs)
surrounding an atom





Bipartite (two-colored)
due to the orientation of M2

$R = 2$ by construction

F-term condition:
(sum of two neighboring terms) = 0

Spectrum of chiral mesons

Holomorphic monomial in CY

momentum along T^4

$$F_i(m) = m_i$$

$$R(m) = \frac{1}{2} b^i F_i(m) = \frac{1}{2}(b \cdot m) = \frac{1}{2}\sum_{i=1}^{3} b^i m_i + 2m_4$$

Closed M2-branes

$$(m_1, m_2, m_3, m_4)$$

winding along T^3

wrapping atoms

F-term condition guarantees
R is insensitive to the details!

Outlook

- (non-)uniqueness of graph / Seiberg duality
- Non-BPS spectrum
- Moduli space of vacua
- Marginal deformation
- Volume Minimization
- Brane Configuration for N=3?