# Connection between the Hagedorn Transision of Closed Strings and the Creation of D9-brane–$\overline{\text{D9}}$-brane pairs near the Hagedorn temperature


KEK

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## §1. Introduction

### ◇ Hagedorn Temperature $\mathcal{T}_H$

maximum temperature for perturbative strings

A single energetic string captures most of the energy.

$$d_N \sim e^{2\pi\sqrt{2N}}$$

$$\Omega(E) \sim e^{\beta_H E}$$

$$Z(\beta) = \int_0^\infty dE\ \Omega(E) e^{-\beta E}$$

$$\beta_H \equiv \frac{1}{\mathcal{T}_H} = 2\pi\sqrt{2\alpha'}$$

$Z(\beta) \to \infty$ for $\mathcal{T} > \mathcal{T}_H$

### ◇ Open String Gas on D9-brane

We need an infinite energy to reach the Hagedorn temperature.

$$\Downarrow$$

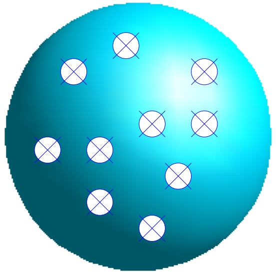
$\mathcal{T}_H$ is a 'limiting temperature'.

## ◊ Closed String Gas

We can reach the Hagedorn temperature by supplying a finite energy.

$\Downarrow$ analogy with QCD

phase transition near $\mathcal{T}_H$



Matsubara method

$F(\beta) \to \infty$ for $\mathcal{T} > \mathcal{T}_H$

$\Uparrow$

Winding modes in Euclidean time direction become tachyonic. ($\Rightarrow$ §2)

$\Downarrow$

Hagedorn transition

Sathiapalan, Kogan, Atick-Witten

A phase transition takes place due to the condensation of these tachyon fields.

## ♢ D$p$-$\overline{\text{D}p}$ Pair

unstable at zero temperature

open string tachyon ⇒ tachyon potential (⇒ §3)

Sen's conjecture potential hight = brane tension

finite temperature system of D$p$-$\overline{\text{D}p}$ pairs? (⇒ §4)

⇓ finite temperature effective potential based on BSFT

D9-$\overline{\text{D9}}$ pairs are stable near the Hagedorn temperature (⇒ Appendix A)

Hotta

## ♢ Thermodynamic Balance on D9-$\overline{\text{D9}}$ Pairs

open strings ↔ closed strings

closed strings : finite energy ⇒ $\mathcal{T} = \mathcal{T}_H$

⇕

open strings : infinite energy ⇒ $\mathcal{T} = \mathcal{T}_H$

Energy flows from closed strings to open strings.

⇒ Open strings dominate the total energy of strings. (⇒ Appendix B)

## ♢ 'Winding Tachyon' Condensation

Atick-Witten

For $\beta > \beta_H$ sphere world sheet does not contribute to free energy.

$\Updownarrow$

For $\beta < \beta_H$ sphere world sheet contributes to free energy.

$\Uparrow$

insertion of 'winding tachyon' vertex

$\Rightarrow$ creation of a tiny hole in the world sheet which wraps around Euclidean time

boundary of a hole $\Leftrightarrow$ boundary of open string on D9-$\overline{\text{D9}}$ pair?

## ♢ Relation between two phase transitions?

From above arguments we conjecture that

> D9-$\overline{\text{D9}}$ Pairs are created by the Hagedorn transition of closed strings.

## ♢ Contents

## Appendix A. Phase Transition in the case of Dp-$\overline{\text{Dp}}$ Pairs

the results of the previous work about Dp-$\overline{\text{Dp}}$ pairs at finite temperature

## Appendix B. Thermodynamic Balance on D9-$\overline{\text{D9}}$ Pairs

explanation of energy dominance of open strings by using microcanonical ensemble method

## §2. Closed String Gas

### ◇ Proper Time Form of Free Energy

$$F(\beta) = -\frac{\mathcal{V}_p}{(2\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\tau}{\tau} (4\pi\tau)^{-\frac{p+1}{2}} \sum_{{M_{boson}}^2} \sum_{r=1}^{\infty} \exp\left(-2\pi\alpha' {M_{boson}}^2 \tau - \frac{r^2\beta^2}{8\pi\alpha'\tau}\right)$$

$$+\frac{\mathcal{V}_p}{(2\pi\alpha')^{\frac{p+1}{2}}} \int_0^\infty \frac{d\tau}{\tau} (4\pi\tau)^{-\frac{p+1}{2}} \sum_{{M_{fermion}}^2} \sum_{r=1}^{\infty} (-1)^r \exp\left(-2\pi\alpha' {M_{fermion}}^2 \tau - \frac{r^2\beta^2}{8\pi\alpha'\tau}\right)$$

### ◇ Mass Spectrum of Closed Strings

$$M^2 = \frac{2}{\alpha'}(N + \tilde{N})$$

$$N = \left(N_B + N_{NS} - \frac{1}{2}\right), \qquad \text{NS sector}$$

$$N = (N_B + N_R)\,, \qquad \text{R sector}$$

oscillation mode

$$N_B = \sum_{i=1}^{8} \sum_{n=1}^{\infty} \alpha_{-n}{}^{i} \alpha_{n}{}^{i}, \qquad N_{NS} = \sum_{i=1}^{8} \sum_{r=\frac{1}{2}}^{\infty} r \; b_{-r}{}^{i} b_{r}{}^{i}, \qquad N_R = \sum_{i=1}^{8} \sum_{m=1}^{\infty} m \; d_{-m}{}^{i} d_{m}{}^{i}$$

Constraint

$$N = \tilde{N}$$

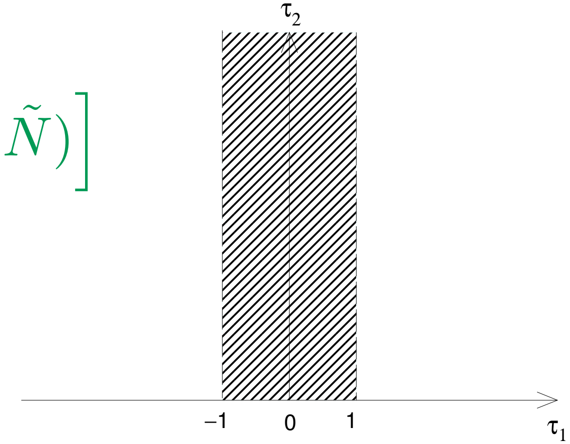
♢ **GSO Projection** $\left(1 + (-1)^F\right)$

SUSY

tachyon $\rightarrow$ project out

## ◇ Free Energy

○ E-representation




$$\delta_{N,\tilde{N}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\sigma \;\; \exp\left[ i\sigma(N-\tilde{N})\right]$$

$$F \propto \int_E \frac{d^2\tau}{\tau_2^6}\frac{1}{\left|\vartheta_1{}'(0|\tau)\right|^8}$$
$$\times\left[\left\{\left(\vartheta_3^4-\vartheta_4^4\right)\left(\bar{\vartheta}_3^4-\bar{\vartheta}_4^4\right)+\vartheta_2^4\bar{\vartheta}_2^4\right\}\left\{\vartheta_3\left(0\middle|\frac{i\beta^2}{8\pi^2\alpha'\tau}\right)-1\right\}\right.$$
$$\left.-\left\{\left(\vartheta_3^4-\vartheta_4^4\right)\bar{\vartheta}_2^4+\vartheta_2^4\left(\bar{\vartheta}_3^4-\bar{\vartheta}_4^4\right)\right\}\left\{\vartheta_4\left(0\middle|\frac{i\beta^2}{8\pi^2\alpha'\tau}\right)-1\right\}\right]$$

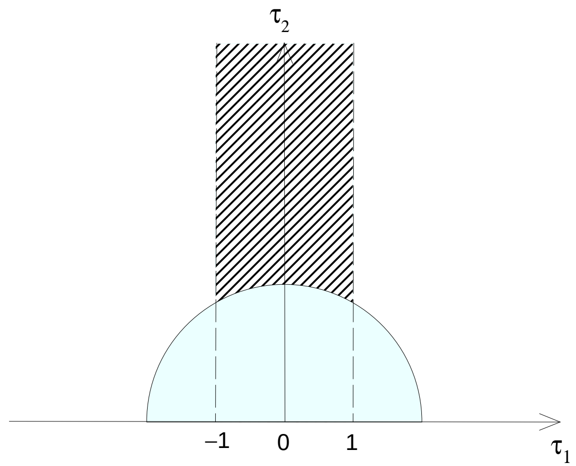
- F-ruepresentation

modular transformation

$$F \propto \int_F \frac{d^2\tau}{\tau_2^6} \frac{1}{\left|\vartheta_1{}'(0|\tau)\right|^8} \sum_{m,n} e^{-S_\beta(m,n)}$$
$$\times \Big\{ \left(\vartheta_2^4\bar\vartheta_2^4 + \vartheta_3^4\bar\vartheta_3^4 + \vartheta_4^4\bar\vartheta_4^4\right)(0|\tau) + e^{i\pi(n+m)}\left(\vartheta_2^4\bar\vartheta_4^4 + \vartheta_4^4\bar\vartheta_2^4\right)(0|\tau)$$
$$-e^{i\pi m}\left(\vartheta_2^4\bar\vartheta_3^4 + \vartheta_3^4\bar\vartheta_2^4\right)(0|\tau) - e^{i\pi n}\left(\vartheta_3^4\bar\vartheta_4^4 + \vartheta_4^4\bar\vartheta_3^4\right)(0|\tau)\Big\}$$

$$S_\beta(m,n) = \frac{\beta^2}{4\pi\alpha'\tau_2}\left(n^2 + m^2|\tau| - 2\tau_1 mn\right)$$

modular invariant




Poisson resummation formula

$$\sum_{n=-\infty}^{\infty} f(2\pi n) = \frac{1}{2\pi}\sum_{p=-\infty}^{\infty}\int_{-\infty}^{\infty} dx\ \exp(ipx) f(x)$$

- Dual-representation

$$F \propto \int_F \frac{d^2\tau}{{\tau_2}^{\frac{11}{2}}} \frac{1}{\left|{\vartheta_1}'(0|\tau)\right|^8} \sum_{w,n}$$

$$\times \Big\{ \left(\vartheta_2^4\bar\vartheta_2^4 + \vartheta_3^4\bar\vartheta_3^4 + \vartheta_4^4\bar\vartheta_4^4\right) e^{-S_\beta^D(w,n)} + \left(\vartheta_2^4\bar\vartheta_4^4 + \vartheta_4^4\bar\vartheta_2^4\right) e^{-S_\beta^D(w,n+1/2)}$$

$$- \left(\vartheta_2^4\bar\vartheta_3^4 + \vartheta_3^4\bar\vartheta_2^4\right) e^{-S_\beta^D(w,n)} - e^{i\pi w} \left(\vartheta_3^4\bar\vartheta_4^4 + \vartheta_4^4\bar\vartheta_3^4\right) e^{-S_\beta^D(w,n+1/2)} \Big\}$$

$$S_\beta^D(w,n) = 2\pi \left( w^2 \frac{\beta^2}{{\beta_H}^2} \tau_2 + iwn\tau_1 + n^2 \frac{{\beta_H}^2}{\beta^2} \tau_2 \right)$$

$(\vartheta_3^4\bar\vartheta_3^4 + \vartheta_4^4\bar\vartheta_4^4)$ term $\Rightarrow$ $\exp\left[-2\pi\left(-1 + w^2 \frac{\beta^2}{{\beta_H}^2} + n^2 \frac{{\beta_H}^2}{\beta^2}\right)\tau_2\right]$

$w = \pm 1,\ n = 0$

$$M^2 \propto \frac{\beta^2 - {\beta_H}^2}{{\beta_H}^2}$$

$\beta < \beta_H$

$\Rightarrow$ ‘winding tachyon’ in the Euclidean time direction

## ♢ $\vartheta$-functions

$$\vartheta_1{}'(0|\tau) = \pi \sum_{n=-\infty}^{\infty} (-1)^n (2n-1) q^{(n-\frac{1}{2})^2}, \qquad \vartheta_3(0|\tau) = \sum_{n=-\infty}^{\infty} q^{n^2}$$

$$\vartheta_2(0|\tau) = \sum_{n=-\infty}^{\infty} q^{(n-\frac{1}{2})^2}, \qquad \vartheta_4(0|\tau) = \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2}$$

$$q = e^{i\pi\tau}$$

$$\vartheta_1{}'\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{3}{2}} \vartheta_1{}'(0|\tau), \qquad \vartheta_3\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}} \vartheta_3(0|\tau)$$

$$\vartheta_2\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}} \vartheta_4(0|\tau), \qquad \vartheta_4\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}} \vartheta_2(0|\tau)$$

$$\vartheta_1{}'(0|\tau+1) = e^{\frac{i\pi}{4}} \vartheta_1{}'(0|\tau), \qquad \vartheta_2(0|\tau+1) = e^{\frac{i\pi}{4}} \vartheta_2(0|\tau)$$

$$\vartheta_3(0|\tau+1) = \vartheta_4(0|\tau), \qquad \vartheta_4(0|\tau+1) = \vartheta_3(0|\tau)$$

## §3. BSFT

◇ **BSFT** (Boundary String Field Theory)

solution of classical master eq. (superstring)

$$S_{eff} = Z$$

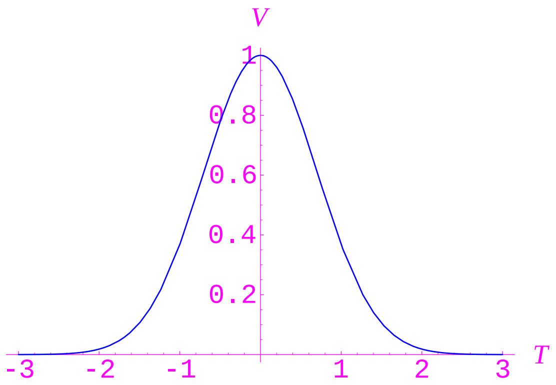
$S_{eff}$ : effective action, $Z$ : 2-dim. partition function

$$S_2 = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \; \partial_a X_\mu \partial^a X^\mu + \int_{\partial\Sigma} d\tau |T|^2 + \cdots$$

◦ tree amplitude (up to two derivative terms)

$$S_{eff} \simeq -16\tau_p \int d^{p+1}x \left[ 2\alpha' e^{-8|T|^2} \partial T \partial T^* + \frac{1}{8} e^{-8|T|^2} \right].$$

$$V(T) = 2\tau_p \mathcal{V}_p \exp\left(-2|T|^2\right)$$




$$\tau_p = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}} g_s}$$

$T$ : complex scalar  $\mathcal{V}_p$ : $p$-dim. volume

$\tau_p$ : tension of Dp-brane  $g_s = e^{\phi}$ : coupling of strings

∘ 1-loop amplitude

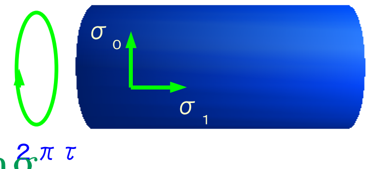
Conformal invariance is broken by the boundary terms.

$\Rightarrow$ ambiguity in choosing the Weyl factors

## ♢ Cylinder Boundary Action

Andreev-Oft

$$S_b = \int_0^{2\pi\tau} d\sigma_0 \int_0^{\pi} d\sigma_1 [|T|^2 \delta(\sigma_1) + |T|^2 \delta(\pi - \sigma_1)].$$




Both sides of the cylinder world sheet are treated on an equal footing.

2-dim. 1-loop partition function

$$Z_1 = \frac{16\pi^4 i \mathcal{V}_9}{(2\pi\alpha')^5} \int_0^\infty \frac{d\tau}{\tau} (4\pi\tau)^{-5} e^{-4\pi|T|^2\tau} \times \left[ \left( \frac{\vartheta_3(0|i\tau)}{\vartheta_1{}'(0|i\tau)} \right)^4 - \left( \frac{\vartheta_2(0|i\tau)}{\vartheta_1{}'(0|i\tau)} \right)^4 \right]$$

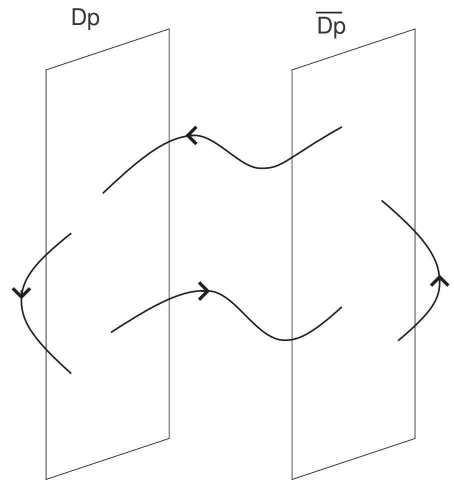
low energy part of $Z_1$ = 1-loop partition function derived from $S_{eff}$

# §4. Free Energy of Open Strings on D9-$\overline{\text{D9}}$ Pair

## ◇ Free Energy

$$F(T,\beta) = -\frac{16\pi^4 \mathcal{V}_9}{{\beta_H}^{10}} \int_0^\infty \frac{d\tau}{\tau^6}\, e^{-4\pi|T|^2\tau}$$
$$\times \left[ \left( \frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left( \vartheta_3 \left( 0 \left| \frac{i\beta^2}{\beta_H \alpha' \tau} \right. \right) - 1 \right) \right.$$
$$\left. - \left( \frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left( \vartheta_4 \left( 0 \left| \frac{i\beta^2}{\beta_H \alpha' \tau} \right. \right) - 1 \right) \right]$$

$$ {M_{NS}}^2 = \frac{1}{\alpha'} \left( N_B + N_{NS} + 2|T|^2 - \frac{1}{2} \right) $$
$$ {M_R}^2 = \frac{1}{\alpha'} \left( N_B + N_R + 2|T|^2 \right) $$



## ♢ Free Energy near the Closed String Vacuum

large $|T|$ limit $\Rightarrow$ small $\tau$ limit

$\Rightarrow$ small hole limit, high temperature limit

$$\tau = \frac{1}{t}$$

$$F(T,\beta) = -\frac{16\pi^4 \mathcal{V}_9}{{\beta_H}^{10}} \int_0^\infty dt \ \exp\left(-\frac{4\pi|T|^2}{t}\right) \times \left[\left(\frac{\vartheta_3(0|it)}{\vartheta_1{}'(0|it)}\right)^4 \left(\vartheta_3\left(0\left|\frac{i\beta^2 t}{{\beta_H}^2}\right.\right) - 1\right) - \left(\frac{\vartheta_4(0|it)}{\vartheta_1{}'(0|it)}\right)^4 \left(\vartheta_4\left(0\left|\frac{i\beta^2 t}{{\beta_H}^2}\right.\right) - 1\right)\right]$$

leading term for large $t$ (small $\tau$)

$$F(T,\beta) \simeq -\frac{4\mathcal{V}_9}{{\beta_H}^{10}} \int_0^\infty dt \ \exp\left[-\frac{4\pi|T|^2}{t} - \pi \, \frac{\beta^2 - \beta_H^2}{{\beta_H}^2} \, t\right]$$

propagator of ‘winding tachyon’ if we ignore $|T|^2$ part

momentum 0 $\Leftarrow$ momentum conservation for Neumann direction

# §5. BCFT Approach

## ♢ Boundary Action (non-BPS D9-brane)

$$S_b = \frac{\lambda}{\sqrt{2}} \int_{\partial\Sigma} d\tau \; \sigma^1 \otimes \psi \sin\left(\frac{X}{\sqrt{2\alpha'}}\right)$$

This is Euclidean version of rolling tachyon (full S-brane)

$\lambda = 0 \quad \Rightarrow \quad$ non-BPS D9-brane

$\lambda = \frac{1}{2} \quad \Rightarrow \quad$ closed string vacuum

Matsubara formalism $\quad \Rightarrow \quad X \sim X + 2\pi\sqrt{2\alpha'}\; k$

$k$ is an integer for periodicity of $S_b$

We will consider $k = 1$ case which corresponds to the Hagedorn temperature case.

## ♢ 1-loop Free Energy

$$F(\lambda) = \frac{\mathcal{V}_9}{4{\beta_H}^9} \int_0^\infty \frac{d\tau}{\tau^{\frac{11}{2}}} \sum_{r\in \boldsymbol{Z}_2}$$

$$\times \left[ \left(\frac{\vartheta_3}{{\vartheta_1}'}\right)^4 (i\tau) \cdot \left\{ \vartheta_3 \left( i\alpha\left(\lambda, \frac{r}{2}\right)\tau \middle| i\tau\right) e^{-2\pi\tau\frac{1}{2}\alpha\left(\lambda,\frac{r}{2}\right)^2} + \vartheta_3 \left( i\beta\left(\lambda, \frac{r}{2}\right)\tau \middle| i\tau\right) e^{-2\pi\tau\frac{1}{2}\beta\left(\lambda,\frac{r}{2}\right)^2} \right\} \right.$$

$$- \left(\frac{\vartheta_4}{{\vartheta_1}'}\right)^4 (i\tau) \cdot \left\{ \vartheta_3 \left( i\alpha\left(\lambda, \frac{r}{2}\right)\tau \middle| i\tau\right) e^{-2\pi\tau\frac{1}{2}\alpha\left(\lambda,\frac{r}{2}\right)^2} - \vartheta_4 \left( i\beta\left(\lambda, \frac{r}{2}\right)\tau \middle| i\tau\right) e^{-2\pi\tau\frac{1}{2}\beta\left(\lambda,\frac{r}{2}\right)^2} \right\}$$

$$\left. -2 \left(\frac{\vartheta_2}{{\vartheta_1}'}\right)^4 (i\tau) \cdot \vartheta_2 \left( i\gamma\left(\lambda, \frac{r}{2}\right)\tau \middle| i\tau\right) e^{-2\pi\tau\frac{1}{2}\gamma\left(\lambda,\frac{r}{2}\right)^2} \right]$$

$$\alpha(\lambda, z) = \frac{2}{\pi}\ \arcsin\left(\sin(\pi z)\cos(\pi\lambda)\right)$$

$$\beta(\lambda, z) = \frac{2}{\pi}\ \arccos\left(\cos(\pi z)\cos(\pi\lambda)\right)$$

$$\gamma(\lambda, z) = \frac{1}{\pi}\ \arccos\left(\cos\left(\pi\left(2z + \frac{1}{2}\right)\right)\cos^2(\pi\lambda)\right)$$

## $\diamondsuit$ **Closed String Vacuum Case** $(\lambda = \frac{1}{2})$

$$F\left(\frac{1}{2}\right) = \frac{2\mathcal{V}_9}{{\beta_H}^9}\int_0^\infty \frac{d\tau}{\tau^{\frac{11}{2}}}\left(\frac{\vartheta_4(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4 \sum_{n\in\boldsymbol{Z}} e^{-\frac{\tau}{2\pi}\left\{2\sqrt{2}\pi\left(n+\frac{1}{2}\right)\right\}^2}$$

$\tau = \frac{1}{t}$

$$F\left(\frac{1}{2}\right) = \frac{\mathcal{V}_9}{\sqrt{\pi}{\beta_H}^9}\int_0^\infty dt\ \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4 \vartheta_4\left(0\,\middle|\,\frac{2\pi^2 it}{(2\sqrt{2}\pi)^2}\right)$$

leading term for large $t$ (small $\tau$)

$$F\left(\frac{1}{2}\right) = \frac{\mathcal{V}_9}{\pi^{\frac{9}{2}}{\beta_H}^9}\int_0^\infty dt$$

propagator of 'winding tachyon'

('winding tachyon' become massless at the Hagedorn temperature.)

# §6. Conclusion and Discussion

♢ **BSFT Approach**

1-loop free energy of open strings on D9-$\overline{\text{D9}}$ pair

$\Rightarrow$ similar to propagator of 'winding tachyon' of closed strings

♢ **BCFT Approach**

1-loop free energy of open strings on non-BPS D9-brane

$\Rightarrow$ propagator of 'winding tachyon' (massless) of closed strings

♢ **Hagedorn Transition $\Rightarrow$ Creation of D9-$\overline{\text{D9}}$ Pair?**

'tree level' free energy of closed strings with 'winding tachyon' vertexes

$\leftrightarrow$ higher loop free energy of open strings

ambiguity in choosing the Weyl factors?

cf) BCFT with imaginary D-branes Gaiotto-Itzhaki-Rastelli

disk scattering amplitude with $m$ closed string insertion

$\longleftrightarrow$ sphere amplitude with $(m+1)$ closed string insertion

♢ **Dilaton?**

Atick-Witten

$\langle V_w V_{w^*} V_\phi \rangle \neq 0 \quad \Rightarrow \quad$ first order phase transition

♢ **Multiple D9-$\overline{\textbf{D9}}$ Pairs $\quad \leftrightarrow \quad$ Single Type of Closed String Tachyon?**

Chan-Paton factor?

♢ **Non-perturbative Calculation?**

Matrix model (IKKT, BFSS), K-matrix Model

♢ **Closed String Condensation $\quad \Rightarrow \quad$ "Nothing" phase**

McGreevy-Silverstein, Horowitz-Silverstein

## Appendix A. Phase Transition in the case of Dp-$\overline{\text{Dp}}$ Pairs

K.H. JHEP **0212** (2002) 072, JHEP **0309** (2003) 002

Prog. Theor. Phys. 112 (2004) 653

### ♢ Non-compact Flat Background

∘ $N$ D9-$\overline{\text{D9}}$ Pairs
$T^2$ term of finite temperature effective potential

$$\left[-16N\tau_9\mathcal{V}_9 + \frac{8\pi N^2\mathcal{V}_9}{{\beta_H}^{10}}\ln\left(\frac{\pi{\beta_H}^{10}E}{N^2\mathcal{V}_9}\right)\right]T^2.$$

The coefficient vanishes when

$$E_c \simeq \frac{2N^2\mathcal{V}_9}{\pi{\beta_H}^{10}}\exp\left(\frac{2{\beta_H}^{10}\tau_9}{\pi N}\right) \qquad \mathcal{T}_c \simeq {\beta_H}^{-1}\left[1+\exp\left(-\frac{{\beta_H}^{10}\tau_9}{\pi N}\right)\right]^{-1}$$

Above $\mathcal{T}_c$, $T=0$ becomes the potential minimum.

⇒ A phase transition occurs at $\mathcal{T}_c$ and D9-$\overline{\text{D9}}$ pairs become stable.

$E_c$ and $\mathcal{T}_c$ are decreasing functions of $N$. $(g_sN \ll 1)$

⇒ The multiple D9-$\overline{\text{D9}}$ pairs are created simultaneously.

number of D9-$\overline{\text{D9}}$ pairs? ⇒ non-perturbative calculation

- N Dp-$\overline{\text{Dp}}$ Pairs with $p \leq 8$

  No phase transition occurs.

◇ **Toroidal Flat Background** $(M_{1,9-D} \times T_D)$

- Dp-$\overline{\text{Dp}}$ pairs are extended in all the non-compact directions

  A phase transition occurs.

- Dp-$\overline{\text{Dp}}$ pairs are not extended in all the non-compact directions

  No phase transition occurrs.

◇ **non-BPS D$p$-brane**

similar to the Dp-$\overline{\text{Dp}}$ pair case

# Appendix B. Thermodynamic Balance on D9-$\overline{\text{D9}}$ Pairs

open strings $\leftrightarrow$ closed strings

thermodynamic balance condition $\Rightarrow$ $\mathcal{T}_{open} = \mathcal{T}_{closed}$

∘ open strings

$$S_{open} \simeq \beta_{H} E_{open} + 2N\sqrt{C\mathcal{V}_9 E_{open}}$$

$$\mathcal{T}_{open} \simeq \left[\beta_{H} + N\sqrt{\frac{C\mathcal{V}_9}{E_{open}}}\,\right]^{-1} < \mathcal{T}_{H}$$

$$C = 2(\pi {\beta_{H}}^{8})^{-1}$$

∘ closed strings

$$S_{closed} \simeq \beta_{H} E_{closed} - \frac{11}{2}\ln\left(\frac{\alpha'^{\frac{27}{22}} E_{closed}}{\mathcal{V}_9^{\frac{2}{11}}\,\delta E_{closed}{}^{\frac{2}{11}}}\right)$$

$$\mathcal{T}_{closed} \simeq \left[\beta_{H} - \frac{11}{2}\frac{1}{E_{closed}}\right]^{-1} > \mathcal{T}_{H}$$

cf) Hagedorn transition

Energy flows from closed strings to open strings.

$\Rightarrow$ Open strings dominate the total energy of strings.