Baryons in holographic QCD

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Work in progress with
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Refs) T. Sakai and S.S.
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1 Introduction

What is "holographic QCD"?

AdS/CFT [Maldacena 1997]

D3-brane → N=4 SYM

D3-brane → String theory in $AdS_5 \times S^5$

N=4 SYM ⟷ String theory in $AdS_5 \times S^5$: dual

holographic QCD

? (Some brane config.) → QCD

? (Some brane config.) → ? (String theory in some curved b.g.)

QCD ⟷ ?: dual

Some brane config.

String theory
in some curved b.g.

➡ "holographic QCD"

- **Recently, we proposed** [Sakai-S.S. 2004]

D4-D8-$\overline{\text{D8}}$ system

QCD with N_f massless quarks
(at low energy)

dual

String theory
in a non-SUSY background
with N_f probe D8-branes
(assuming $N_c \gg N_f$)

The meson sector is described by the 5 dim Yang-Mills-Chern-Simons theory in a curved background.

Highlights

[Sakai-S.S. 2004, 2005]

- Geometric realization of the chiral symmetry breaking

$$U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$$

- Unification of mesons

$$\pi,\ \rho,\ a_1,\ \rho',\ a_1',\cdots \quad \Rightarrow \quad \text{5 dim gauge field}$$

- Structure of interaction

  ➡ consistent with $\left\{ \begin{array}{l} \text{hidden local symmetry} \\ \text{vector meson dominance} \\ \text{GSW model} \end{array} \right.$

- Anomalies in QCD is reproduced ⟵ CS-term

  ➡ $\left\{ \begin{array}{l} \text{an easy derivation of WZW term} \\ \text{Witten-Veneziano formula} \end{array} \right.$

- Numerical estimate of the masses and couplings

  ➡ roughly agrees with the experimental data

Summary of today's talk

- We extend our analysis to baryons in this model.
- Baryons are described as (4 dim) instantons in a 5 dim gauge theory.
- We propose a new way to analyze baryons that extends Skyrme's old idea including contributions from vector mesons.
- You should not fully trust our results since they are preliminary !!

Plan

✓ 1 Introduction

2 A brief review of the model

3 Relation to Skyrmion

4 Baryons as Instantons

5 Outlook

2 A brief review of the model

- **Witten's model for pure Yang-Mills** [Witten 1998]

D4-brane
wrapped on S^1
with $\psi(\tau+2\pi) = -\psi(\tau)$

Yang-Mills (at low energy)

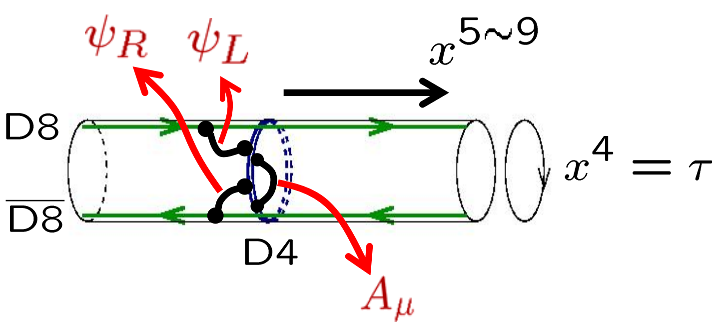
dual

String theory
in a non-SUSY background

- **Our model** [Sakai-S.S. 2004]

N_c $\quad$ N_f pairs

D4-D8-$\overline{\text{D8}}$ system



QCD with N_f massless quarks
(at low energy)

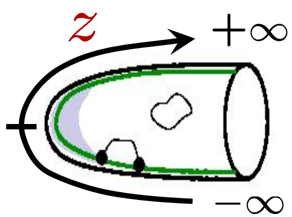
dual

String theory
in the above background
with N_f probe D8-branes
(assuming $N_c \gg N_f$)

Hadrons in the model

The topology of the background is

$$\underset{x^\mu}{\mathbf{R}^{1,3}} \times \underset{(y,z)}{\mathbf{R}^2} \times S^4$$



D8-branes are extended along $(x^\mu, z) \times S^4$

- Closed strings → glueballs
- Open strings on D8 → mesons
- D4 wrapped on S^4 → baryons

studied around 1998
[Csaki-Ooguri-Oz-Terning 1998,
Koch-Jevicki-Mihailescu-Nunes 1998,
A.Hashimoto-Oz 1998, etc etc]

Meson effective theory

We have N_f D8-branes extended along $(x^\mu, z) \times S^4$

➡ The effective theory on the D8 is a 9 dim $U(N_f)$ gauge theory

(Here we ignore KK-modes associated with the S^4 for simplicity.)

➡ The effective theory of mesons is reduced to

5 dim $U(N_f)$ gauge theory
$A_\mu(x^\mu, z), \ \ A_z(x^\mu, z)$

- **The effective action is calculated as**

$$S \simeq S_{\mathrm{YM}} + S_{\mathrm{CS}}$$

$K(z) = 1 + z^2$

$$S_{\mathrm{YM}} = \kappa \int d^4x dz \, \mathrm{Tr} \left(\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2 \right)$$

$(M_{\mathrm{KK}} = 1 \text{ unit})$

$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

(CS 5-form) $d\omega_5(A) = \mathrm{Tr} F^3$

$$\left(\frac{1}{2\pi} \int_{S^4} dC_3 = N_c \right)$$

This 5 dim YM-CS theory is considered as the effective theory of mesons.

[Cf) Son-Stephanov 2003]

mode exp.

$$A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu) \psi_n(z)$$ ➡ **vector, axial-vector mesons**

$$B^{(n)} \sim \rho, \ a_1, \ \rho', \ a_1', \ \rho'', \ \cdots$$

$$U(x^\mu) \equiv P \exp \left\{ - \int_{-\infty}^{\infty} dz A_z(x^\mu, z) \right\}$$ ➡ **pion**

3 Relation to Skyrmion [Sakai-S.S. 2004]

- **The effective action for pion is**

$$S_{\rm YM} \simeq \int d^4x \left[\frac{f_\pi^2}{4}{\rm Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e_S^2}\underbrace{{\rm Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2}_{\text{Skyrme term}} \right] + \underbrace{\cdots}_{\mathcal{O}(B_\mu^{(n)\,2})}$$

$$f_\pi^2 = \frac{4\kappa}{\pi} \qquad e_S^{-2} \simeq 2.51\cdot\kappa$$

This is the Skyrme model action.

- **Skyrme proposed** [Skyrme 1961]

$$\boxed{\textbf{Baryon} \;\simeq\; \textbf{Soliton in Skyrme model (Skyrmion)}}$$

The pion field $U(\vec{x}) : \underbrace{S^3}_{x^{1\sim3}} \to U(N_f)$ **defines**

$$\pi_3(U(N_f)) \simeq \mathbf{Z} \ni \quad n = \frac{1}{24\pi^2}\int_{S^3} {\rm Tr}(UdU^{-1})^3 \quad \Longrightarrow \quad \textbf{baryon \#}$$

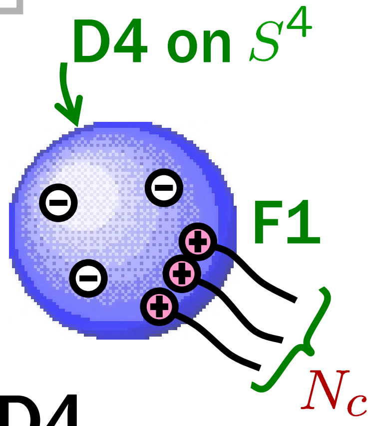
- Baryons in the AdS/CFT context are constructed by wrapped D-branes [Witten 1998, Gross-Ooguri 1998]

In our case,

Baryon $\simeq$ D4-brane wrapped on the S^4

$$\left(\frac{1}{2\pi}\int_{S^4} dC_3 = N_c \right)$$

$$S_{\text{CS}}^{\text{D4}} = \int_{\mathbf{R}\times S^4} C \wedge e^{F^{\text{D4}}/2\pi} \sim -N_c \int_{\mathbf{R}} A^{\text{D4}}$$



source of $-N_c$ electric charge on D4

➡ N_c F-strings should be attached.

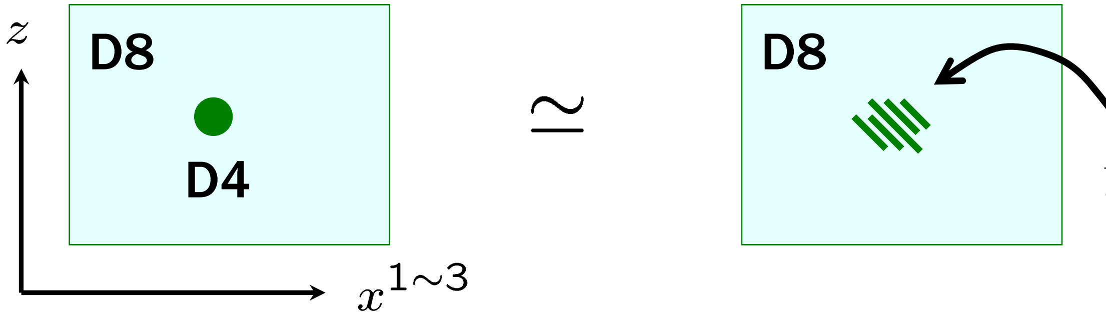
➡ Bound state of N_c quarks

➡ Baryon

- **In our model, the wrapped D4 can be embedded in D8.**

D4 within D8 $\simeq$ instanton on D8

[Douglas 1995]



$$\frac{1}{8\pi^2}\int \mathrm{Tr}F^2 = \#\mathrm{D4}$$

- **One can easily show**

$$\boxed{\frac{1}{8\pi^2}\int_{S^3\times \mathrm{R}} \mathrm{Tr}F^2 = \frac{1}{24\pi^2}\int_{S^3} \mathrm{Tr}(UdU^{-1})^3}$$

$x^{1\sim3}$ z

[Atiyah-Manton 1989, Son-Stephanov 2003, Sakai-S.S. 2004]

$\therefore$ **Wrapped D4 $\simeq$ instanton on D8 $\simeq$ Skyrmion** ■

4 Baryons as instantons

- We would like to play the game like Adkins-Nappi-Witten did for the Skyrmion. [Adkins-Nappi-Witten 1983]

➡ What we should do is

(1) Find a classical solution.

(2) Find its collective coordinates.

(3) Treat them quantum mechanically and find the baryon spectrum.

- The important point here is that the contributions from (axial-) vector mesons are included !

However, things are not as simple as they look.

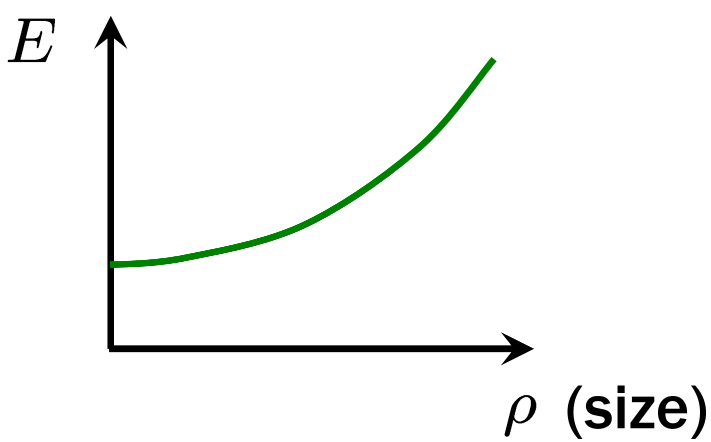
- The instanton solution for

$$S_{\rm YM} = \kappa \int d^4x dz \, {\rm Tr} \left(\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2 \right)$$

$$K(z) = 1 + z^2$$

shrinks to zero size !

(Even though the pion effective action contains the Skyrme term !)



The BPST instanton configuration with $\rho \to 0$ is the minimum energy configuration.

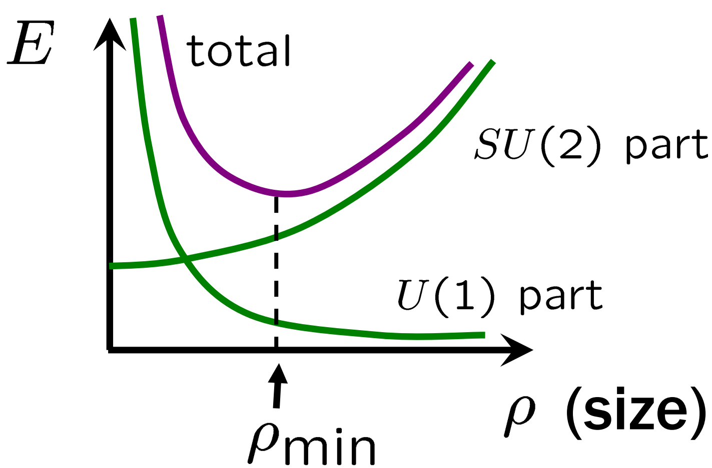
The effect of the Chern-Simons term.

$$S_{\text{CS}} = \frac{N_c}{24\pi^2}\int_5 \omega_5(A) = \frac{N_c}{16\pi^2}\int d^4x dz\, \underset{\substack{\uparrow \\ U(1)\ \text{part}}}{A_0^{U(1)}}\, \underbrace{\epsilon^{ijk}\text{Tr}F_{ij}F_{kz}}_{\text{Non-zero for instanton}} + \cdots$$

➡ **source of the U(1) charge**

➡ **point-like charge costs energy**

➡ **The size will be stabilized with a non-zero finite value.**



This is the same mechanism as the stabilization of Skyrmions via ω meson. [Adkins-Nappi 1984]

Quantization (This part is preliminary!!)

- Unfortunately, it is difficult to find an exact solution.
- To proceed, let us try the following strategy.

(1) Restrict the $SU(2)$ part of the $U(2)$ gauge field to be the BPST instanton configuration.

(2) Solve the EOM for the $U(1)$ part and insert it back into the action.

- Then, we can show $\rho_{\text{min}} \sim \mathcal{O}(\lambda^{-1/2})$ λ : 't Hooft coupling

$$\begin{aligned}\mathcal{L}_{\text{YM}} &= \kappa \text{Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2 + K(z)F_{\mu z}^2\right) \\ &\sim \kappa\left(\underbrace{\text{Tr}\left(\frac{1}{2}F_{\mu\nu}^2 + F_{\mu z}^2\right)}_{\text{YM in flat space-time}} + \mathcal{O}(\lambda^{-1})\right)\end{aligned}$$

$K(z) = 1 + z^2$

$x^i \sim z \sim \rho_{\text{min}}$

YM in flat space-time

The effect of non-trivial background is lower order in the $1/\lambda$ expansion.

- **Instanton moduli space** (for SU(2) one instanton)

$$\mathcal{M} \simeq \{(\underbrace{\vec{X}, Z,}_{\text{position}} \rho)\} \times SU(2)/\mathbf{Z_2} \simeq \mathbf{R}^4 \times \mathbf{R}^4/\mathbf{Z_2}$$

size

  - ρ & Z become massive when the $\mathcal{O}(\lambda^{-1})$ terms are turned on, but we still take these into account, since they are light compared with the other non-zero modes.

- **Treating these modes as the "collective coordinates", we obtain the spectrum of baryons.**
  - Only $I = J$ states appear. (Just as in the Skyrme model)
  - Parity odd states appear. (Unlike in the Skyrme model!)
  - Mass spectrum

$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}} (n_\rho + n_z) \right) M_{\mathrm{KK}}$$

$l = 2I = 2J = 1, 3, 5, \cdots \quad n_\rho = 0, 1, 2, \cdots \quad n_z = 0, 1, 2, \cdots \quad \text{parity} = (-1)^{n_z}$

numerical values (just for illustration)

$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}} (n_\rho + n_z) \right) M_{\mathrm{KK}}$$

$$l = 2I = 2J = 1, 3, 5, \cdots \quad n_\rho = 0, 1, 2, \cdots \quad n_z = 0, 1, 2, \cdots \quad \text{parity} = (-1)^{n_z}$$

- **If we choose** $M_{\mathrm{KK}} \simeq 500 \ \mathrm{MeV}$ **and use nucleon mass** ($\simeq 940 \ \mathrm{MeV}$) **to fix the constant** M_0 **, we obtain**

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	$[940]^+$	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-,\ 2164^-$	$2164^+, 2164^+$
$\Delta\,(l=3)$	1240^+	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

States appeared in the Skyrme model

($\pm$: parity)

- $I = J$ **states from Particle Data Group look like....**

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	1440^+	1535^-	1655^-	1710^+, ?	2090^-_*, ?	2100^+_*, ?
$\Delta\,(l=3)$	1232^+	1600^+	1700^-	1940^-_*	1920^+, ?	?, ?	?, ?

(? : not found, $*$ **: evidence of existence is poor)**

Comments

The predicted baryon spectrum looks nice,

but there are a lot of reasons that
you should NOT trust these values.

- The ansatz we used may not be a good one.
- Higher derivative terms are neglected.
- $N_c = 3$ is not large enough especially for $l \geq 3,\ n_\rho + n_z \geq 3$
- The model deviates from real QCD at high energy $\sim M_{\mathrm{KK}}$
- $M_{\mathrm{KK}} \simeq 950$ MeV is the value consistent with ρ meson mass

Comments

For $N_c \gg l$**, the mass formula becomes**

$$M \simeq \widetilde{M}_0 + \frac{1}{4}\sqrt{\frac{5}{6}}\frac{l(l+2)}{N_c}M_{\mathrm{KK}} + \sqrt{\frac{2}{3}}(n_\rho + n_z)M_{\mathrm{KK}}$$

$(\widetilde{M}_0 \sim \mathcal{O}(N_c))$

The N_c **dependence is consistent with that known in large** N_c **QCD.**

[Witten1979]

[Adkins-Nappi-Witten1983]

Cf) The mass formula in Adkins-Nappi-Witten

$$M = M_0 + \frac{l(l+2)}{8\lambda} \qquad (M_0 \sim \mathcal{O}(N_c),\ \lambda \sim \mathcal{O}(N_c))$$

5 Outlook

- Baryons are described as (4 dim) instantons in a 5 dim gauge theory.
- We proposed a new way to analyze baryons that extends Skyrme's old idea including contributions from vector mesons.
- There are a lot more to do to improve the analysis.
  (solve EOM numerically, include higher derivative terms etc.)
- It would be interesting to investigate other static properties of baryons.
  (charge radii, magnetic moments etc.)