Numerical Model of the CMB anisotropies -Simplicial Quantum Geometry-

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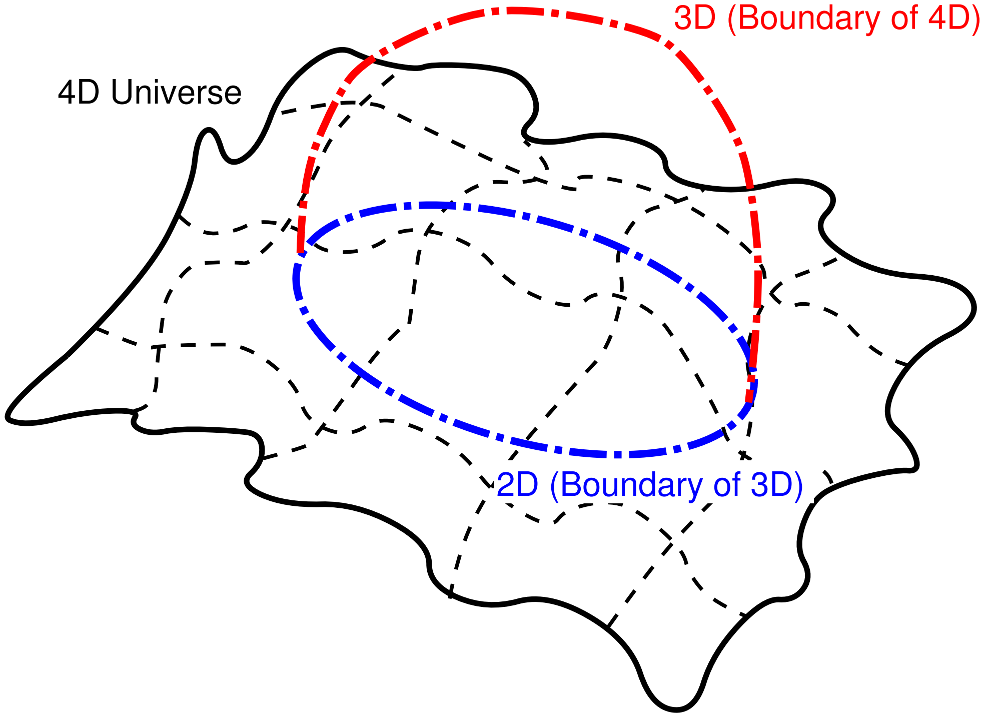
Based on HEP-TH/xxxxxxx (coming soon)



1. Introduction and motivation
2. Numerical model
3. Numerical results (2 and 4 dimensions)
4. Summary

1. Introduction

∗ Motivation (based on inflation universe)

⋆ The initial fluctuation of quantum geometry will be expanded to macroscopic scale.

↓

⋆ Effects of quantum gravity(QG) are found on the large angular scale in anisotropies of CMB.

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Numerical Simulation of QG → generates CMB anisotropies?

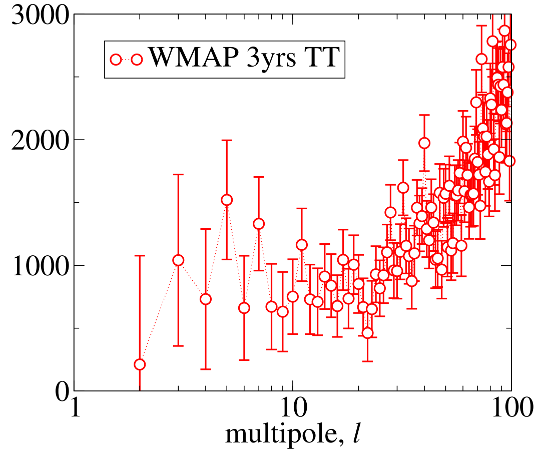
"Primordial" fluctuation of quantum field

⇕

Correlation before inflation

⇕

Large scale correlation



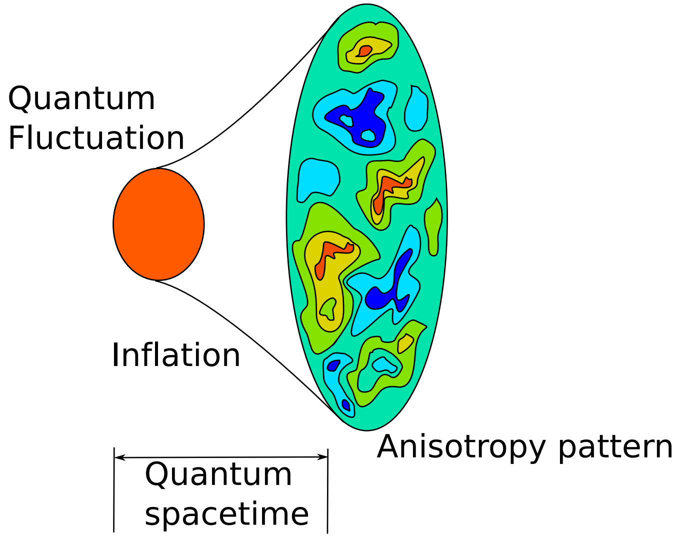
G. Hinshaw, et.al.,astro-ph/0603451.

We study properties of fluctuation of quantum geometry with numerical method, and compare to observations.

1.1. Problems on 4D QG

In order to discuss 4D QG,
we should resolve following problems,

- Renormalizable field theory
- Unitarity
- $\cdots$



The fluctuation of space-time should be discussed.

⋆ Analytical approach $\cdots$ 4D conformal gravity
(presented by K. Hamada).
▷ Background independent formulation with conformal and tensor fields.
⋆ Numerical approach $\cdots$ 4D simplicial quantum gravity (this talk).
▷ Lattice quantization

1.2. Lattice Quantization of QG

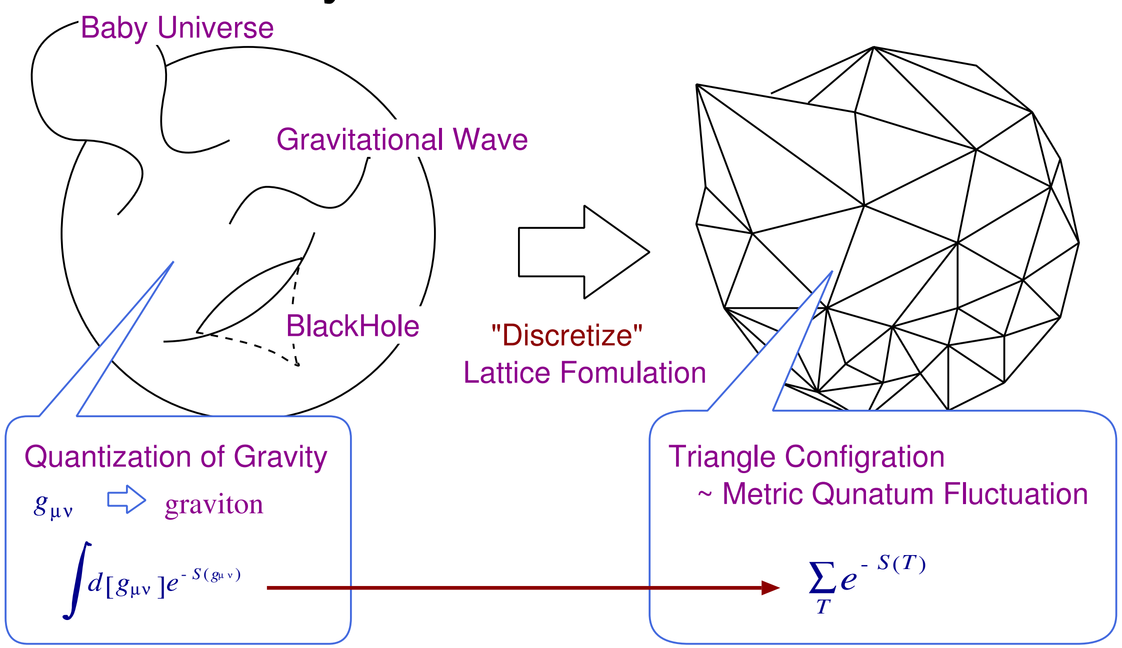
⋆ Origin of our study ··· How to quantize QG?

"Generally" quantum field theory
→ Lattice regularization on Feynman path-integral quantization.
· QCD ↔ Lattice QCD
· QG ↔ Dynamical Triangulation, Regge Calculus,...

Constructive definition of quantum field theory on discretized manifold with keeping the gauge covariance.

⋆ Dynamical Triangulation

Discretized manifold can be build with d-dim triangles (d-simplex), so-called Simplicial Quantum Gravity (SQG).



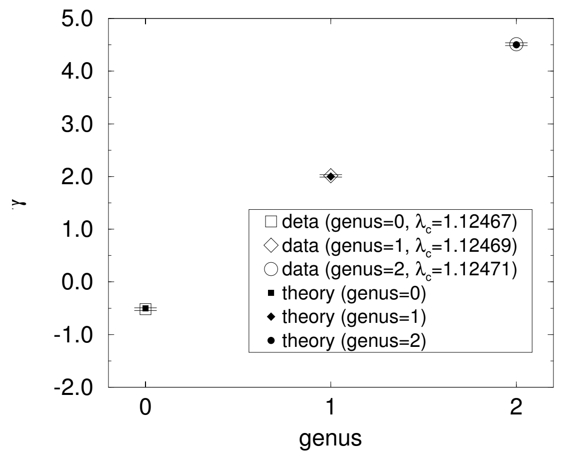
It is possible to realize Quantum Geometry in the COMPUTER.

1.3. Previous Results of SQG

Correspondence to analytical estimations,

- 2-dimensional case,

  · Correspondence to 2-dimensional conformal field theory and matrix model.

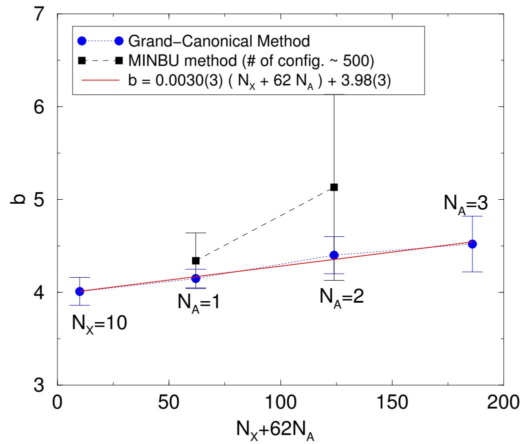


S.Oda, N.Tsuda and T.Yukawa, Nucl. Phys. B (Proc.Suppl.) 63 (1998) 733.

- 4-dimensional case,

  · Continuous phase transition (required for continuous limit),

  · Correspondence to 4-dimensional conformal gravity.



S. H., H. S. Egawa and T. Yukawa, Prog. Theor. Phys. **108** (2002).

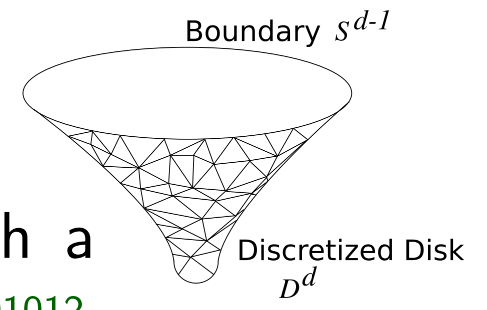
We consider ...

⋆ 4D SQG · · · Possibilities for quantization of gravity.

⋆ The primordial fluctuation on SQG can be compared with observations.

1.4. Methodology

1. Build discretized surface with open topology ($\simeq D^d$)



Compare to corresponding models,

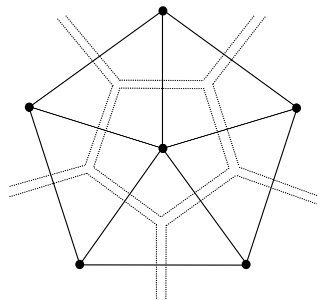
- 2-dimensional model $\leftrightarrow$ Liouville field theory with a boundary V. Fattev, A. Zamolodchikov and Al. Zamolodchikov, hep-th/0001012.

$$A_{bulk} = \frac{1}{4\pi}\int_{\Gamma}\left[\hat{g}^{ab}\partial_a\partial_b\phi + Q\hat{R}\phi + 4\pi\mu e^{2b\phi}\right]\sqrt{\hat{g}}d^2x,$$

$$A_{bound} = \frac{1}{2\pi}\int_{\partial\Gamma}\left[Q\hat{K}\phi + 2\pi\mu^B e^{b\phi}\right]\hat{g}^{1/4}d\xi,$$

$\hat{R}$: scalar curvature, $\hat{K}$: scalar curvature of the boundary

$Q(= b + 1/b)$: background charge



- Matrix model
  E. Brezin, C. Itzykson, G. Parisi and J. B. Zuber, Comm. Math. Phys. **59** (1978) 35.

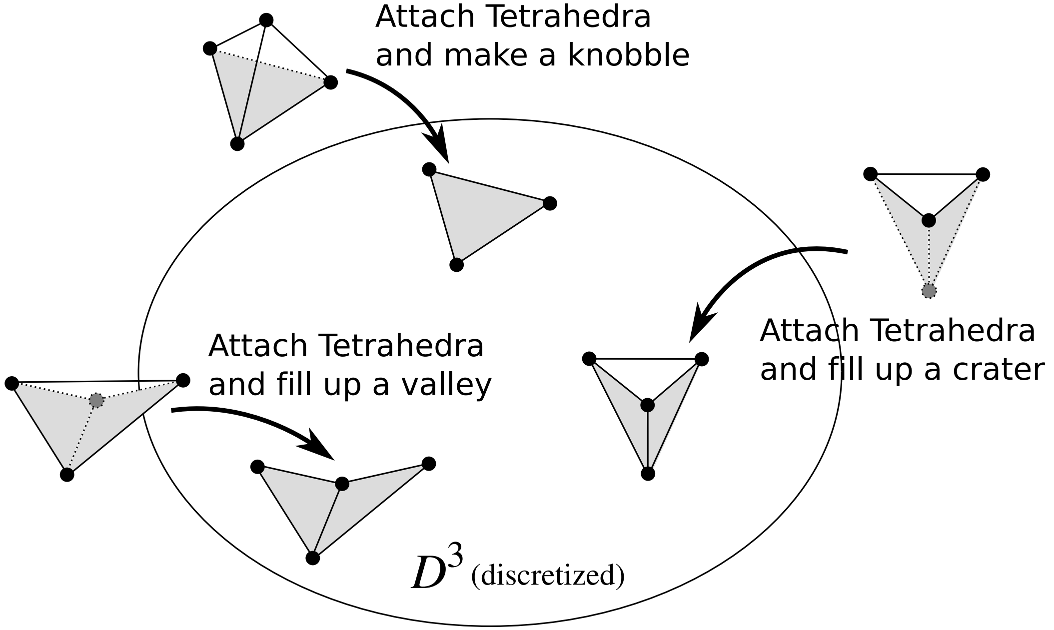
2. Grasp S^2 manifold on equal-time surface (only for 4-dimensional case)
3. Compute 2-point correlation function of scalar curvature

2. Model -How to make a universe-

Constructing d-dimensional manifold with open topology D^{d-1},

Dynamical Triangulation $\cdots$ Fix link length a and change connections between lattice sites.

(Regge Calculus $\cdots$ Change link length a and fix connections between lattice sites.)



Summing up all possible configurations (universe), the connections between sites are changed by triangulation moves.

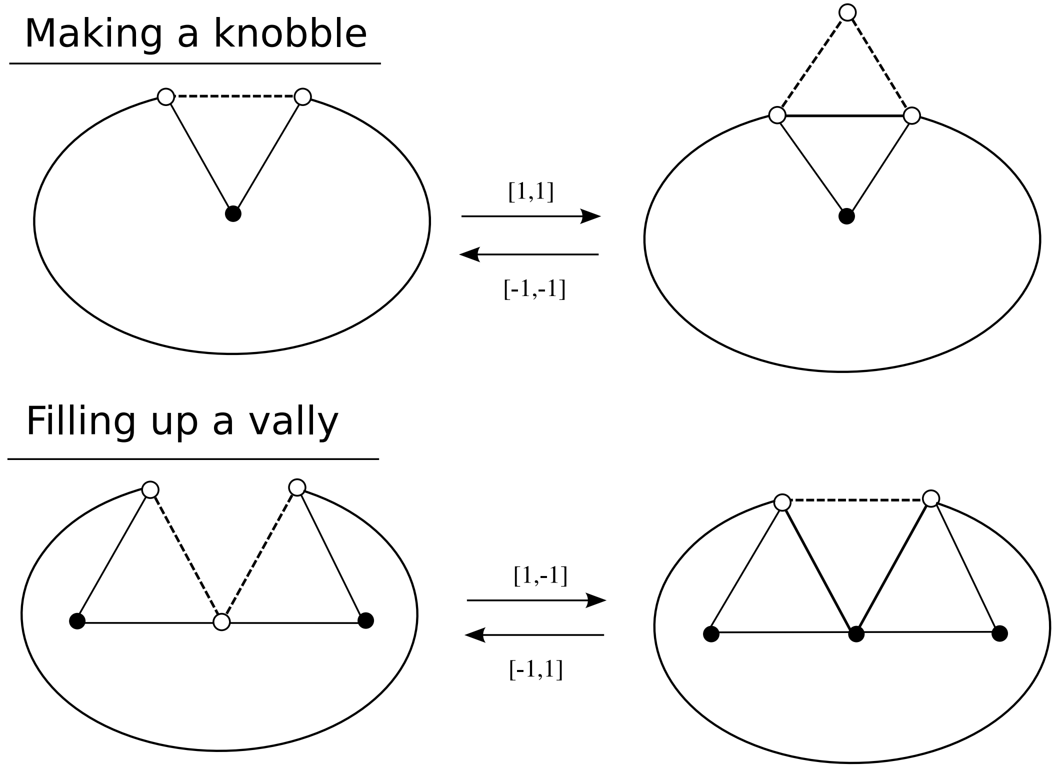
2.1. $[\Delta V, \Delta S]$-moves

$\triangle$ Triangulation moves : $(V, S) \rightarrow (V + \Delta V, S + \Delta S)$

V : Volume, number of triangles N_2.

S : Peripheral length, number of boundary link $\tilde{N}_1$.

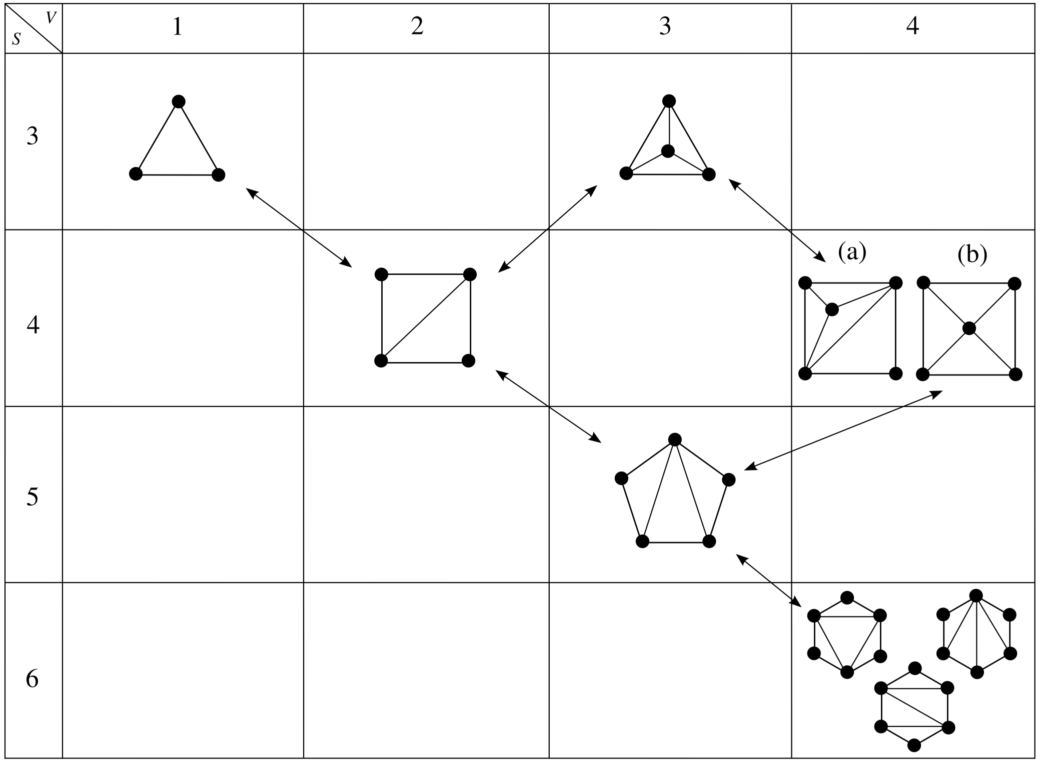
$*$ 2D case



Any possible universe $\cdots$ $\star$ Creating from simplest triangle,
$\star$ Equally probable,
$\rightarrow$ Quantization rule.

2.2. Generating Markov chain

Constructing a Markov chain under the detailed balance condition, select one of n_a possible moves at configuration a with given probability p_a.



Detailed balance condition for transition probability $w_{a\leftrightarrow b}$,

$$\frac{p_a m_a}{n_a} w_{a\to b} = \frac{p_b m_b}{n_b} w_{b\to a},$$

m_a : The rotational multiplicity of the configuration a (symmetric factor).

2.3. Action and Higher-dimensional case

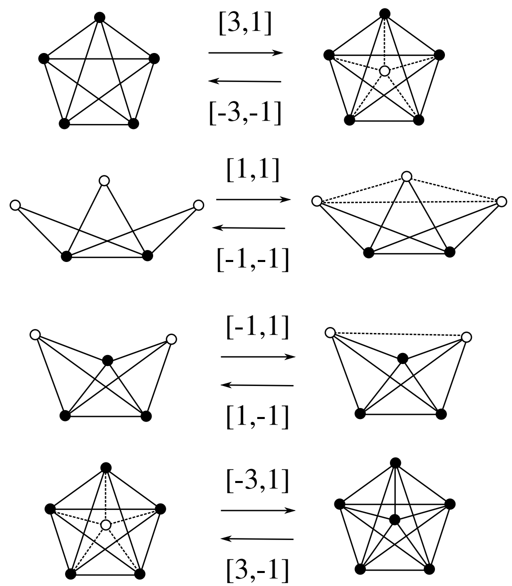
∗ 2D case

The Metropolis Monte-Carlo method is carried out with action,

$$S = \mu N_2 + \mu^B \tilde{N}_1, \quad (p_a = \exp(-S(a))),$$

μ : lattice cosmological constant, μ^B : lattice boundary cosmological constant,
N_2 : volume triangle number, $\tilde{N}_1$: boundary link number.

∗ 4D case



Starting with a 4-simplex the Monte-Carlo simulation is carried out for the partition function by constructing a Markov chain under the detailed balance condition accepting moves which fulfill the manifold conditions in exactly the same manner as the 2-dimensional simulation.

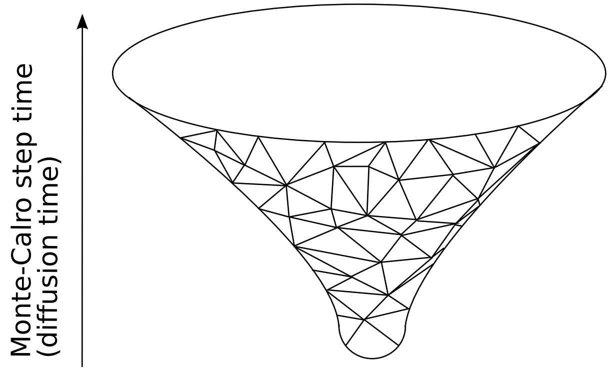
$$S = \mu N_4 + \mu^B \tilde{N}_3 + \kappa N_2 + \kappa^B \tilde{N}_1,$$

κ : lattice gravitational constant, κ^B : lattice boundary boundary gravitational constant.

2.4. Numerical Simulation

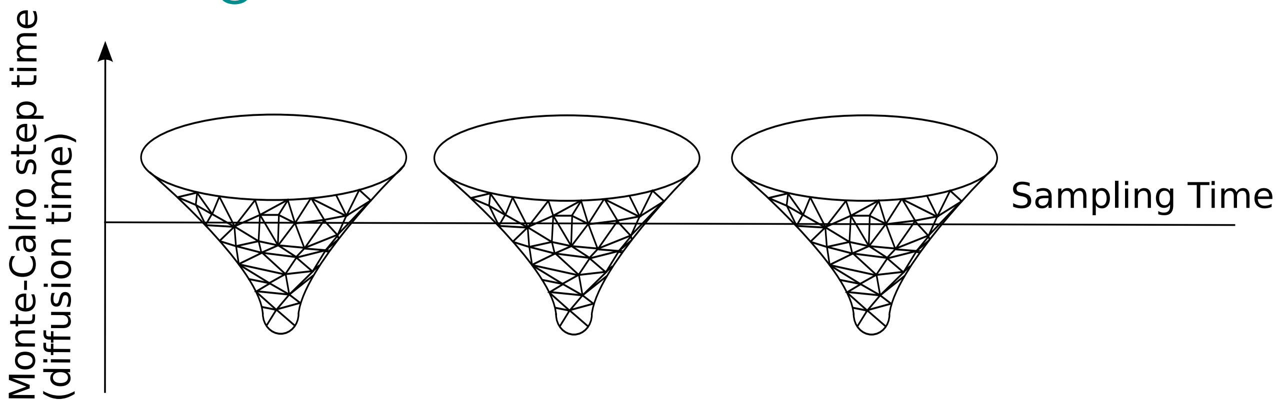
- 1-Configuration measurement,

  Trace the configuration along the Markov chain.



- Ensemble average,

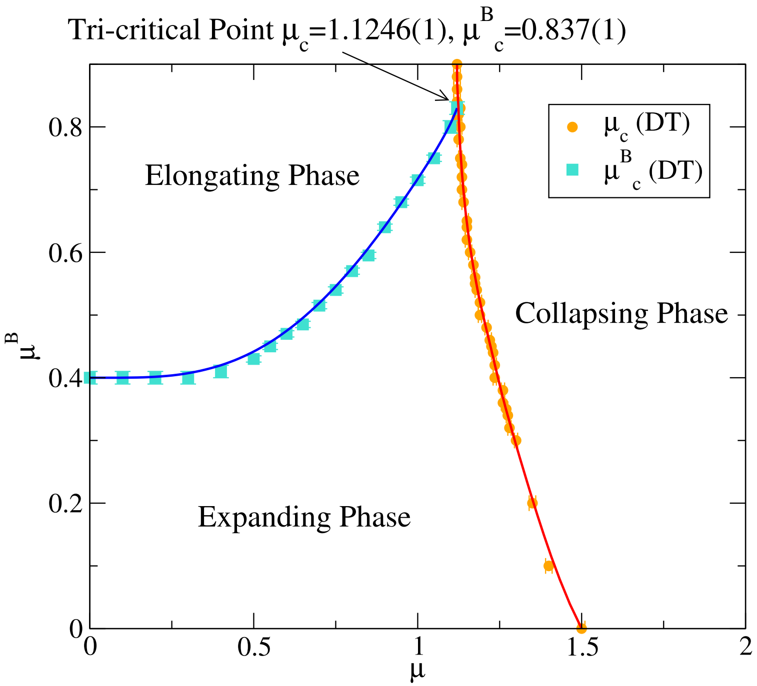
  Sum up all configurations.



3. Numerical Results

As a practical test, we perform 2-dimensional model.
2-dimensional model consists following parameters,

- The lattice cosmological constant μ,
- The lattice boundary cosmological constant μ^B.



Three phases are found as,

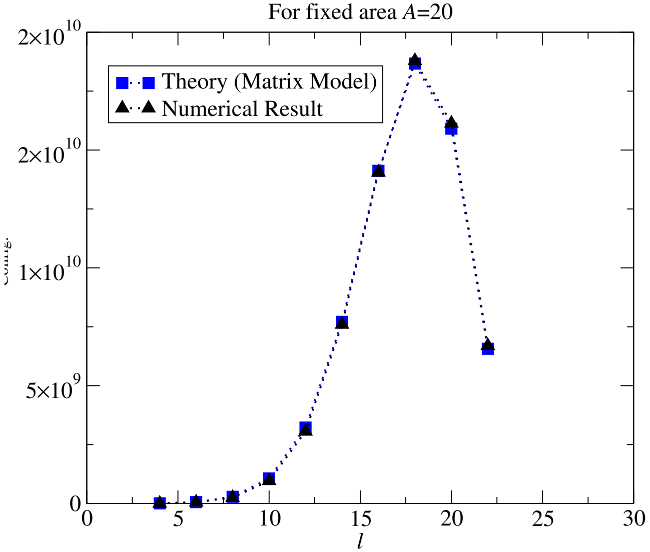
- Expanding Phase at $\mu < \mu_c, \mu^B < \mu_c^B$,
  Bulk $\cdots$ Expanding, Boundary $\cdots$ Expanding
- Elongating Phase at $\mu^B > \mu_c^B$,
  Bulk $\cdots$ Expanding, Boundary $\cdots$ Shrinking
- Collapsing Phase at $\mu > \mu_c$.
  Bulk $\cdots$ Shrinking

The value of the tri-critical point corresponds to the estimation of matrix model ($\mu_c = 1.1246, \mu_c^B = 0.8367$).

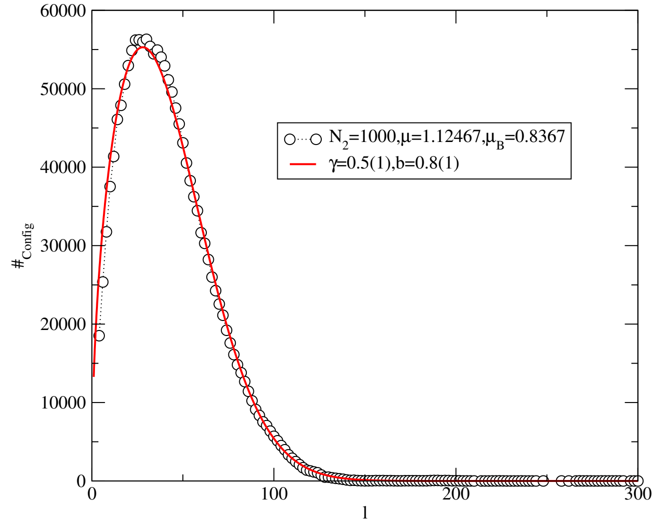
2.1. Corresponding model

Numerical results suggest the partition function of 2-dimensional dynamical triangulation model on D^2 is corresponding to the matrix model and 2-dimensional conformal field theory with a boundary.

Matrix model $\Leftrightarrow$ DT on D^2



DT on D^2 $\Leftrightarrow$ LFT



$$P(A,l) \approx A^{-2.5} l^{\pm 0.5} \exp\left[-\frac{1}{4\sin(\pi b^2)}\frac{l^2}{A}\right]\exp[\mu_c A]\exp[\mu_c^B l]$$

For pure gravity with central charge $c_L = 26$, it is known to be $b^2 = 2/3 \sim 0.67$, while the numerical estimation is $b^2 = 0.6(2)$.

2.2. Physical time -Statistical Sense-

$*$ Diffusion time $\tau \leftrightarrow$ Physical time t

A definition of the physical time can be found in the trivial relationship between the volume and the area of a universe,

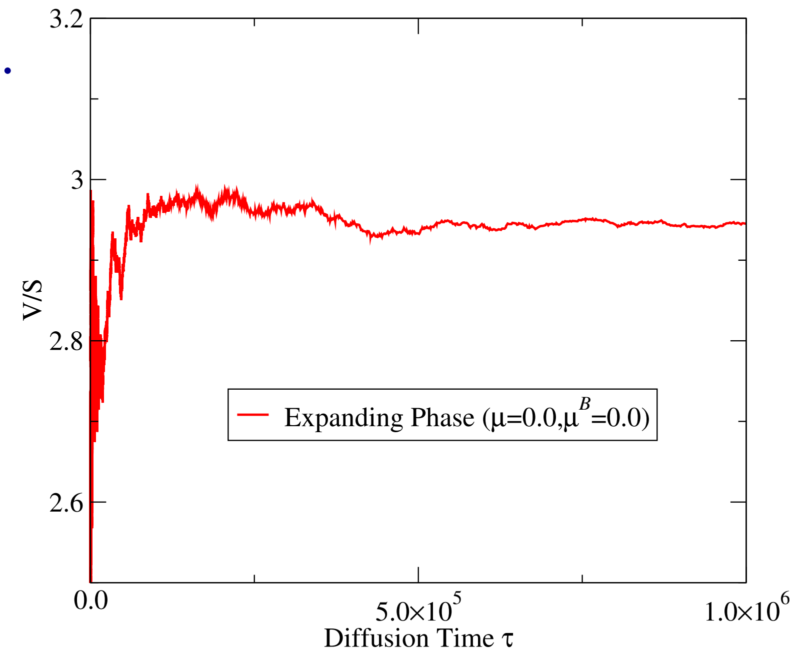
$$V(t) = c\int_0^t S(t')dt',$$

where the constant c is the ratio of length scale and time scale. Above relation can be rewritten in terms of τ as

$$ct = \int_0^\tau \frac{1}{S(\tau')}\frac{dV(\tau')}{d\tau'}d\tau'.$$

Simulations in the expanding phase show the surface area increases approximately proportional to the volume as,

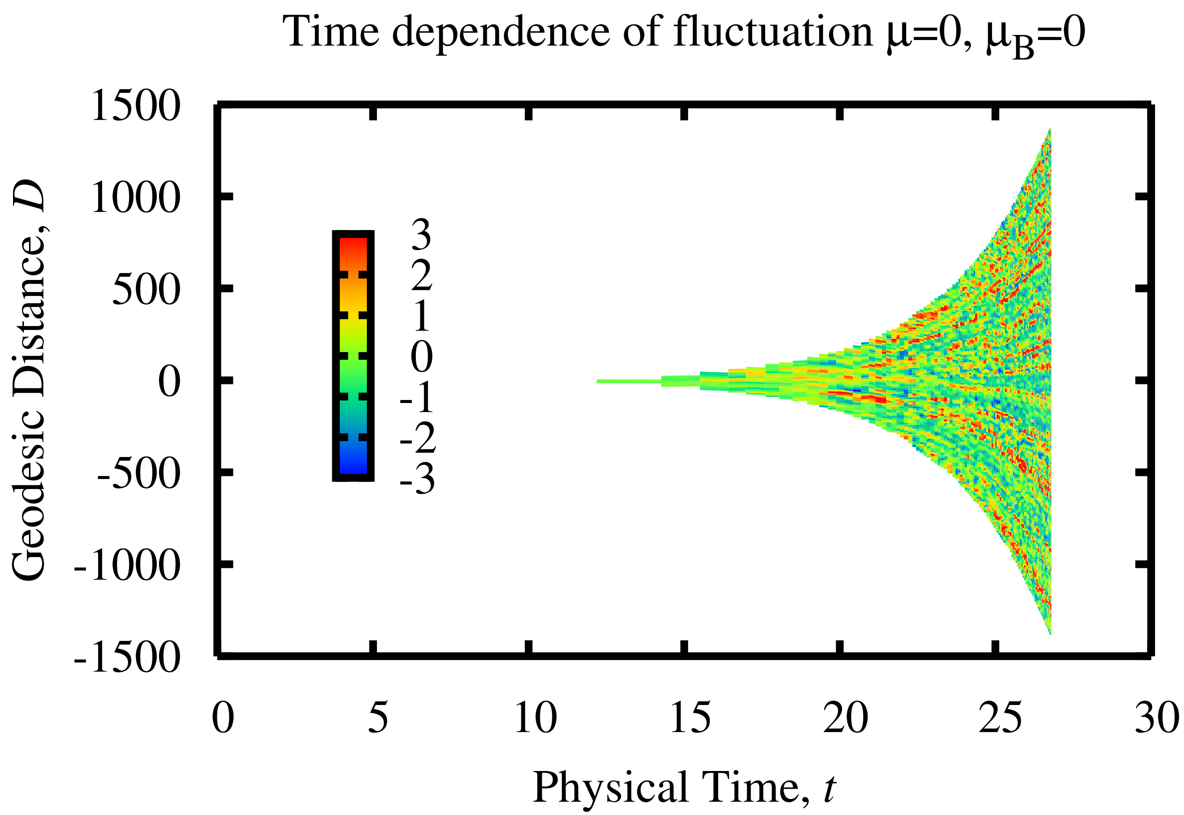
$$V(\tau) \simeq aS(\tau).$$



Thus, from the definition of the physical time, we obtain

$$t = c^{-1} a \log \{S(\tau)/S(0)\},$$

or $S(t) = S(0)e^{ct/a}$, exhibiting the exponential expansion of the universe.



We can also define the physical time on the discretized D^d under the statistical sense.

2.3. Correlation Function

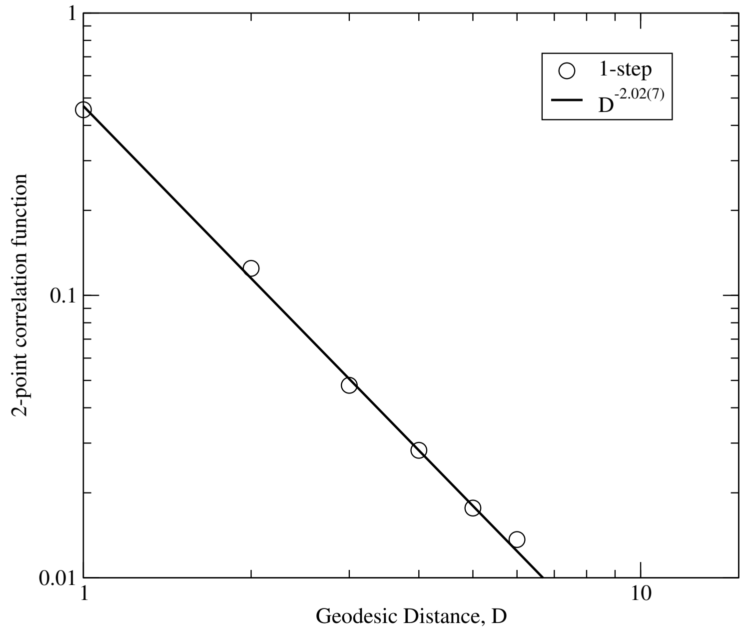
In terms of the distance, we can measure the correlation function between points on the boundary hyper-surface.

The Liouville theory predicts the two point correlation function of the boundary primary operator $B_\beta(x) = \exp\{\beta\phi(x)\}$ to be

$$\langle B_\beta(0) B_\beta(x) \rangle \sim \frac{1}{|x|^{2\Delta_\beta}},$$

where $\Delta_\beta = \beta(Q - \beta)$.

Assuming $B_\beta(x)$ for $\beta = b$ corresponded to the coordination number of a boundary vertex in the simplicial space, we measured the correlation function of coordination numbers of two vertices on the boundary.

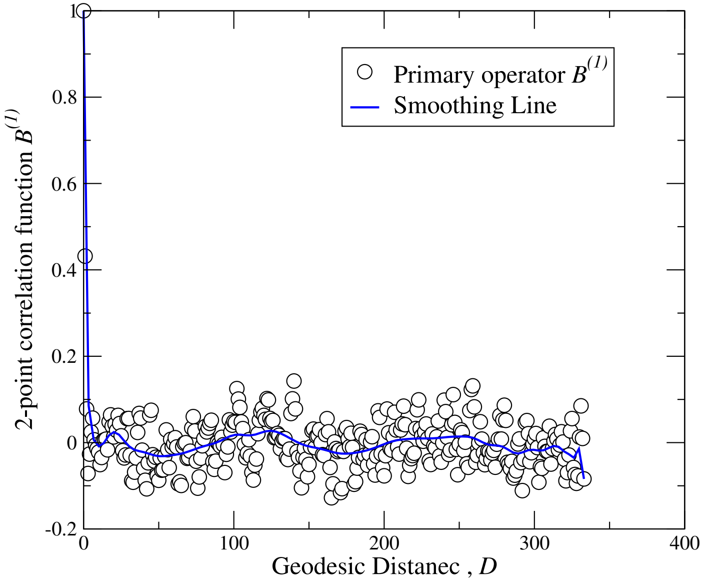


⋆ Long range correlation

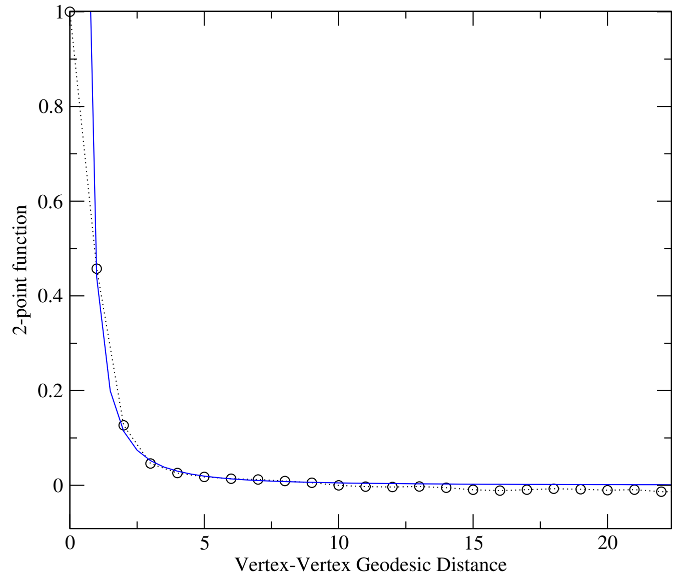
· At the 1-configuration measurement, the anisotropy fluctuation pattern of the long range correlation is found.

· The two point correlation function averaged over an ensemble of universes shows the long range correlation specific is wiped out after averaging over.

∗ 1-Configuration measurement



∗ Ensemble average

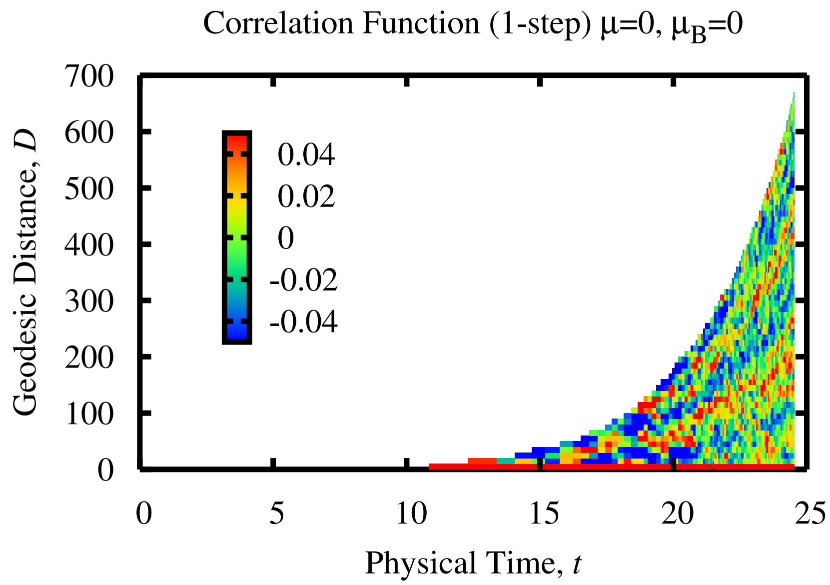


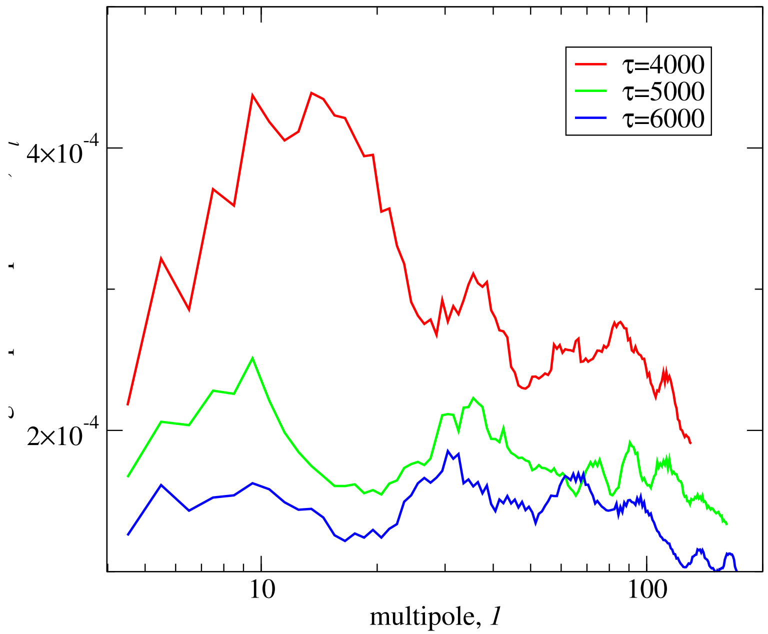
According to our model, numerical results suggest

⋆ Accidental creation of a seed universe by quantum fluctuation,

⋆ Random expansion of space by accumulations of elementary units.

The pattern of super-long distance fluctuation seems to be conserved in some degree during the expansion as we have imagined in analogy to a picture on an inflating balloon.





We also calculate the angular power spectrum of the two point correlation function defined by $|a_l|^2$ with

$$a_l = \int d\theta P_l(\cos\theta) f(\cos\theta),$$

where $\theta = x/r(\tau)$ is the ratio of geodesic distance and the peripheral length at τ.

4. 4-dimensional case

Success of 2-dimensional model $\longrightarrow$ apply for 4-dimensional model?

4-dimensional model consists following parameters,

· The lattice cosmological constant μ,

· The lattice boundary cosmological constant μ^B,

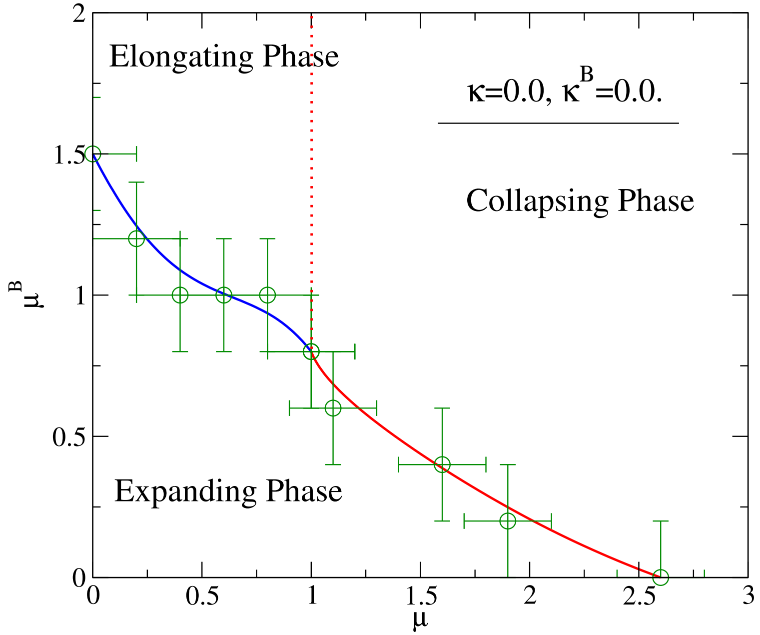
· The lattice gravitational constant κ,

· The lattice boundary gravitational constant κ^B.

For taking the continuous limit, the continuous phase transition at the critical point $\mu^c, \mu_c^B, \kappa_c, \kappa_c^B$ is required.

From the Monte-Calro simulations, we found 5-type phases as,

- Expanding Phase ($\mu < \mu_c$),
  - Expanding Phase (middle-speed-expansion) ($\kappa^B < \kappa_c^B$),
  - Expanding Phase (low-speed-expansion) ($\kappa < \kappa_c$),
- Collapsing Phase ($\mu > \mu_c$),
- Elongating Phase ($\mu^B > \mu_c^B$).



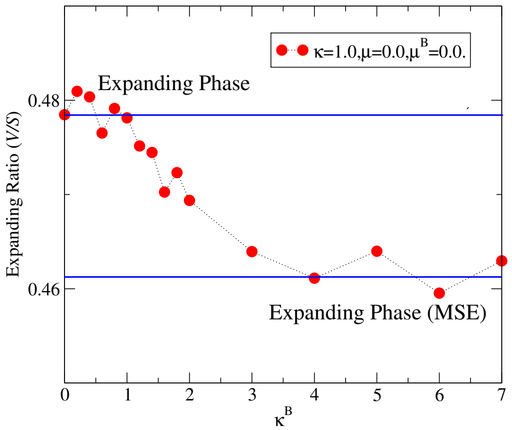
Three kinds of universes similar to the 2-dimensional case are observed by varying μ and μ^B while two parameters, κ and κ^B, are fixed to be zero.

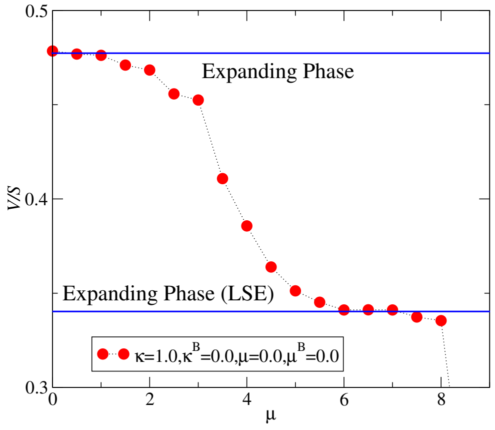
In the expanding phase, three kinds expansion ratio V/S are found,

$\cdot\ V/S \sim 0.5 \cdots$ Expanding Phase,

$\cdot\ V/S \sim 0.46 \cdots$ Expanding Phase (Middle-Speed-Expansion),

$\cdot\ V/S \sim 0.34 \cdots$ Expanding Phase. (Low-Speed-Expansion)

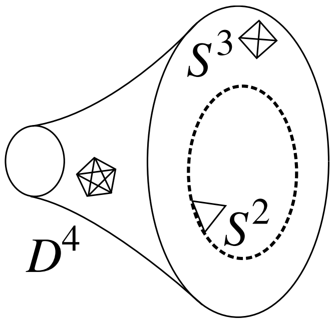


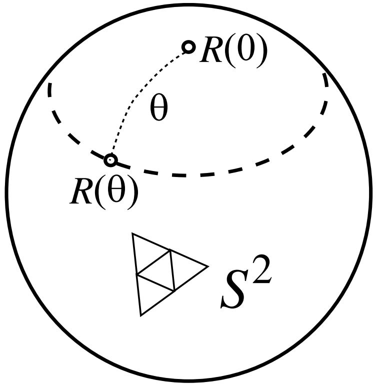


2.1. Correlation Function

As 2-dimensional model, we measure the curvature-curvature 2-point function on the boundary S^2.

In order to define the LSS on discretized manifold D^4, we extract a sectional universe in S^3.

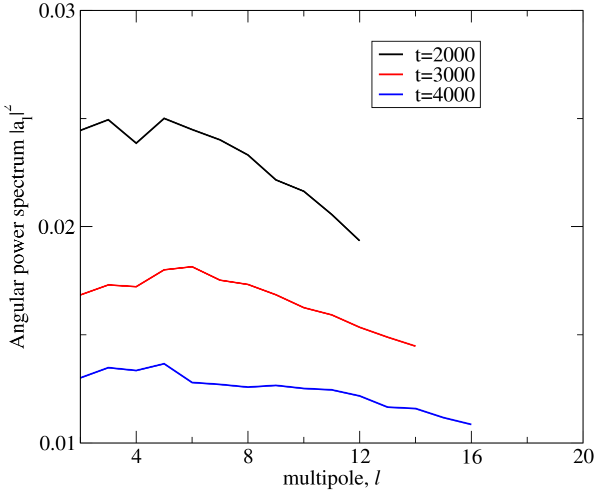




The two point correlation function is defined by

$$f(x) = \frac{1}{\tilde{N}_2} \sum_{i=1}^{\tilde{N}_2} \frac{1}{\tilde{n}_2(i,x)} \sum_{j(i,x)} \frac{(R_i - \bar{R})}{\bar{R}} \frac{(R_j - \bar{R})}{\bar{R}},$$

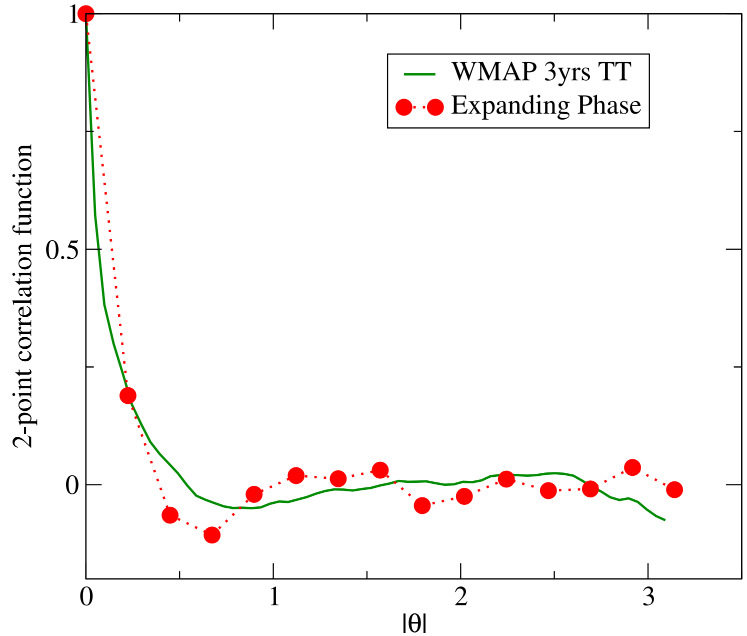
where R_i and $\bar{R}$ are the scalar curvature at a triangle i and the average over $\{j(i,x)\}$ of $\tilde{n}_2(i,x)$ triangles.



The pattern of super-long distance fluctuation seems to be conserved in some degree during the expansion as 2-dimensional model.

⋆ Comparing to WMAP observations (preliminary),

(For comparison, we compute the 2-point correlation function of the temperature on the mesh division of the WMAP observation [http://lambda.gsfc.nasa.gov/].)



When we measure the two-point correlation function (selecting one universe as COBE and WMAP observations did), it exhibits significant super-long distance correlation.
(The long range correlation specific in each universe is wiped out after averaging over the ensemble.)

5. Summary and Future problems

- Numerical Development to realize the quantum geometry with a open boundary D^d
- Monte-Calro simulation for 2D and 4D model
- Correspondence to 2-dimensional Liouville theory and matrix model
- Phase structure

  2D $\cdots$ Expanding / Collapsing / Elongating

  4D $\cdots$ 3-Expanding / Collapsing / Elongating
- 2-point correlation function

  Short distance $\cdots$ quantum correlation

  Long distance $\cdots$ expanding pattern

Future problems,

- Statistical accuracy and parameter dependence,
- Matter degrees of freedom,
- Coordinate and the ratio of length scale and time scale,
- Phase transition,
- n-point correlation function (3-point,...),
- Check with the theoretical analysis,
- $\cdots$.

Numerical Tool for QG
⇒Open the possibilities to analyze very early stage of universe with Dynamical Triangulation algorithms.

We must confess the goal is far away and high above, but worth challenging.

Thank you for listening !