Nucleon structure in lattice QCD with dynamical domain-wall fermions

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RBC and UKQCD collaborations have been producing some dynamical DWF ensembles:

- RBC 2-flavor: DBW2 and DWF, $a^{-1} = 1.7$ GeV, $m_{\rm res} = 0.00137(5)$,
  - $m_{\rm sea} = 0.04$, 0.03, and 0.02,
  - single volume, $16^3 \times 32 \times 12$,
- RBC/UKQCD (2+1)-flavor, Iwasaki and DWF, $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.00308(3)$,
  - $m_{\rm strange} = 0.04$ and $m_{\rm up,down} = 0.03$, 0.02, and 0.01,
  - two volumes, $16^3 \times 32 \times 16$ and $24^3 \times 64 \times 16$.

Here we report some nucleon form factors and moments of structure functions.

- With domain wall fermions (DWF) which preserves almost exact chiral symmetry on the lattice,

the non-perturbative renormalization of the relevant currents is simple.

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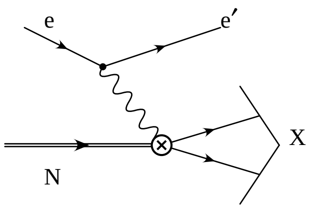
The easiest are the vector and axial charges, known from neutron β decay, $g_{_V} = G_F \cos\theta_c$ and $g_{_A}/g_{_V} = 1.2695(29)$[1]:

- $g_{_V} \propto \lim_{q^2\to 0} g_{_V}(q^2)$ with $\langle n|V_\mu^-(x)|p\rangle = i\bar{u}_n[\gamma_\mu g_{_V}(q^2) + q_\lambda \sigma_{\lambda\mu} g_{_M}(q^2)]u_p e^{-iqx}$,
- $g_{_A} \propto \lim_{q^2\to 0} g_{_A}(q^2)$ with $\langle n|A_\mu^-(x)|p\rangle = i\bar{u}_n\gamma_5[\gamma_\mu g_{_A}(q^2) + q_\mu g_{_P}(q^2)]u_p e^{-iqx}$.

Structure functions: measured in deep inelastic scatterings (and RHIC/Spin):

- DIS 

$$\left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$$
$$W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$$

$$W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu}$$
$$W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$$

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

  - Unpolarized: $F_1(x,Q^2)$, $F_2(x,Q^2)$,
  - Polarized: $g_1(x,Q^2)$, $g_2(x,Q^2)$.

- The same structure funtions appear in RHIC/Spin (which also provides $h_1(x,Q^2)$).

[1] The Particle Data Group.

Moments of the structure functions are accessible on the lattice:

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and d_n are forward nucleon matrix elements of certain local operators.

Lattice operators:

- Unpolarized (F_1/F_2):

$$
\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{trace})]
$$

$$
\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overset{\leftrightarrow}{D}_{\mu_2}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{trace})\right] q
$$

On the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

  - Higher moment operators mix with lower dimensional ones: operators belonging in irreducible representations of $O(4)$ transform reducibly under the lattice Hyper-cubic group.
  - Only $\langle x\rangle_q$ can be measured with $\vec{P}=0$.

- Polarized (g_1/g_2):

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_\sigma \overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_{[\sigma} \overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^n \gamma_5\sigma_{\rho\nu} \overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

  - On the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.
  - Only $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

On the lattice, in general, we calculate the relevant matrix elements of these currents

- with a lattice cutoff, $a^{-1} \sim$ 1-2 GeV,
- and extrapolate to the continuum, $a \to 0$,

introducing lattice renormalization, e.g.: $g_{V,A}^{\text{renormalized}} = Z_{V,A} g_{V,A}^{\text{lattice}}$.

Also, unwanted lattice artefact may result in unphysical mixing of chirally distinct operators.

DWF makes g_A/g_V particularly easy, because:

- the chiral symmetry is almost exact, and
- maintains $Z_A = Z_V$, so that $g_A^{\text{lattice}}/g_V^{\text{lattice}}$ directly yields the renormalized value.

And other non-perturbative renormalizations are feasible too, with often negligible unwanted mixings.

So DWF is a powerful tool for calculating nucleon matrix elements on the lattice: RBC had demonstrated this.

Our formulation follows the standard one,

- Two-point function: $G_{N}(t) = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}}\langle T B_1(x) B_1(0)\rangle]$, using $B_1 = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$ for proton,
- Three-point functions,
  - vector: $G_{V}^{u,d}(t,t') = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}'}\sum_{\vec{x}}\langle T B_1(x') V_t^{u,d}(x) B_1(0)\rangle]$,
  - axial: $G_{A}^{u,d}(t,t') = \frac{1}{3}\sum_{i=x,y,z}\mathrm{Tr}[(1+\gamma_t)\gamma_i\gamma_5\sum_{\vec{x}'}\sum_{\vec{x}}\langle T B_1(x') A_i^{u,d}(x) B_1(0)\rangle]$.

  with fixed $t' = t_{\mathrm{source}} - t_{\mathrm{sink}}$ and $t < t'$.
- From the lattice estimate
$$g_{\Gamma}^{\mathrm{lattice}} = \frac{G_{\Gamma}^{u}(t,t') - G_{\Gamma}^{d}(t,t')}{G_{N}(t)},$$
  with $\Gamma = V$ or A, the renormalized value
$$g_{\Gamma}^{\mathrm{ren}} = Z_{\Gamma} g_{\Gamma}^{\mathrm{lattice}},$$
  is obtained.
- Non-perturbative renormalizations, defined by
$$[\bar{u}\Gamma d]_{\mathrm{ren}} = Z_{\Gamma}[\bar{u}\Gamma d]_0,$$
  satisfies $Z_{A} = Z_{V}$ well, so that
$$\left(\frac{g_{A}}{g_{V}}\right)^{\mathrm{ren}} = \left(\frac{G_{A}^{u}(t,t') - G_{A}^{d}(t,t')}{G_{V}^{u}(t,t') - G_{V}^{d}(t,t')}\right)^{\mathrm{lattice}}.$$

g_{A} is also described as $\Delta u - \Delta d$.

Renormalization: $\mathcal{O}^{\mathrm{ren}} = Z_{\mathcal{O}}(a\mu)\mathcal{O}^{\mathrm{lat}}(a)$,

- lattice complications: operator mixing from broken Lorentz or chiral symmetry,
- NPR is required when mixing with lower dimensional operator occurs.

We calculate $Z_{\mathcal{O}}(a\mu)$ non-perturbatively in RI/MOM scheme[2] with perturbative matching to $\overline{\mathrm{MS}}$.

- compute off-shell matrix element of the operator, $\mathcal{O}$, in Landau gauge,
- impose a MOM scheme condition $\mathrm{Tr}\, V_{\mathcal{O}}(p^2)\Gamma\big|_{p^2=\mu^2} \dfrac{Z_{\mathcal{O}}}{Z_q} = 1$,
  - $V_{\mathcal{O}}(p^2)$ is the relevant amputated vertex,
  - Γ is an appropriate projector,
- extrapolate to the chiral limit, defining the RI scheme,
- in an appropriate window, $\Lambda_{\mathrm{QCD}} \ll \mu^2 \ll a^{-1}$, a scale invariant

  $$Z_{\mathrm{rgi}} = \frac{Z(\mu^2)}{C(\mu^2)}$$

  is obtained, with the operator running $C(\mu^2)$ in the continuum perturbation theory.
- Now we can perturbatively match to e.g. $\overline{\mathrm{MS}}$.

Works nicely with DWF.

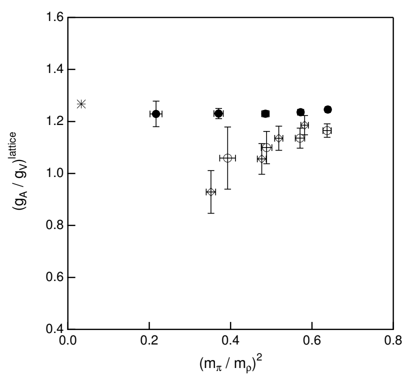
[2]Martinelli et. al, Nucl. Phys. B455, 81 (1995).

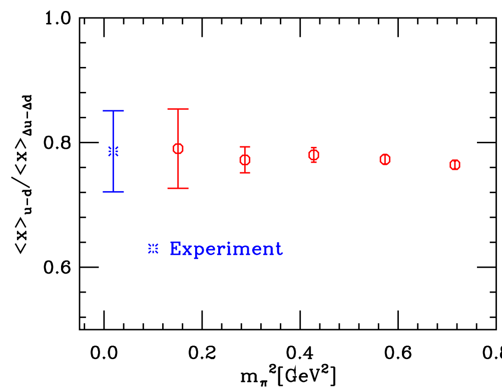
Rectangular gauge actions such as Iwasaki and DBW2, $S_G = \beta[c_0 \sum W_{1,1} + c_1 \sum W_{1,2}]$, with $c_0 + 8c_1 = 1$, help both:

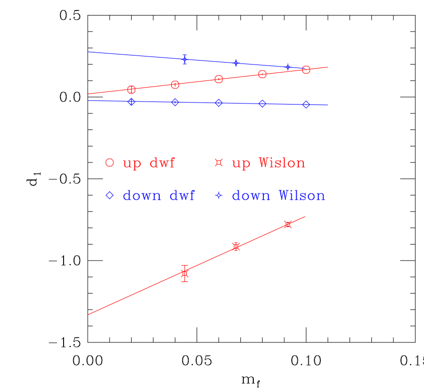
- good chiral behavior, *i.e.* close enough to the continuum, and
- sufficiently large volume to contain a nucleon.

With quenched DBW2 calculations RBC had demonstrated[3]:

- $(g_A/g_V)^{\rm lattice} = (g_A/g_V)^{\rm ren}$ is strongly volume-dependent.
  - It is not appreciably quark-mass dependent once the volume is sufficiently large,
  - $g_A/g_V = 1.212(27)_{\rm stat.}(24)_{\rm syst.}$, compared with the experiment of 1.2695(29).







- NPR for structure function moments is well-behaved.
- Both $\langle x\rangle_{u-d}$ and $\langle x\rangle_{\Delta u-\Delta d}$ overshoot the experiments by about 50%,
  - but their ratio, again naturally renormalized, agree well with the experiment
  - and again without much appreciable quark-mass dependence.
- $\langle 1\rangle_{\delta u-\delta d} = 1.193(30)$, $\overline{\rm MS}$ (2 GeV) 2-loop running.
- d_1, though not renormalized yet, appears small in the chiral limit.

[3] S. Sasaki, K. Orginos, SO, and T. Blum, Phys.Rev.D68:054509, 2003 (hep-lat/0306007); K. Orginos, T. Blum and SO, Phys.Rev.D73:094503, 2006 (hep-lat/0505024).

Here we report on two dynamical DWF ensembles.

1. RBC 2-flavor DBW2+DWF dynamical calculations:
   - $a^{-1} = 1.7$ GeV, $m_{\rm res} = 0.00137(5)$
   - $m_{\rm sea} = 0.04$, 0.03, and 0.02,
   - $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),
   - single volume, $16^3 \times 32 \times 12$,
   - about 220 configurations at each $m_{\rm sea}$ value,

   Analysis is almost complete except NPR.

2. RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations:
   - $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.00308(3)$,
   - $m_{\rm strange} = 0.04$ and $m_{\rm up,down} = 0.03$, 0.02, and 0.01,
   - $m_\pi = 620$, 520 and 390 MeV; $m_N = 1.4$, 1.3, and 1.2 GeV (10-20 % errors),
   - two volumes,
     - $16^3 \times 32 \times 16$, 2 fm across, ensemble production complete, UKQCD analyses ongoing, and
     - $24^3 \times 64 \times 16$, 3 fm across, ensemble production ongoing, RBC analyses ongoing (24-30 configurations).

   Preliminary analysis for the larger volume.

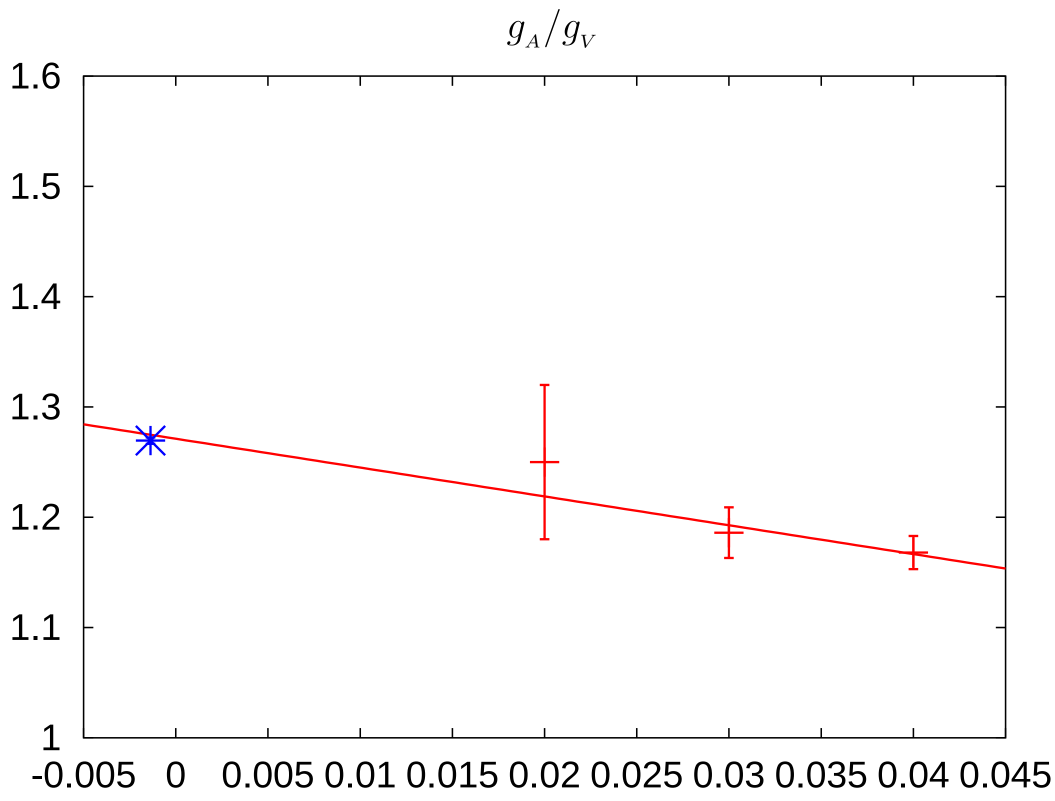
Observables: those not requiring finite momentum transfer

- ratios g_A/g_V and $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$, naturally renormalized,
- d_1, yet to be renormalized but still interesting.

bare values of the others have been calculated but are waiting for renormalization.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

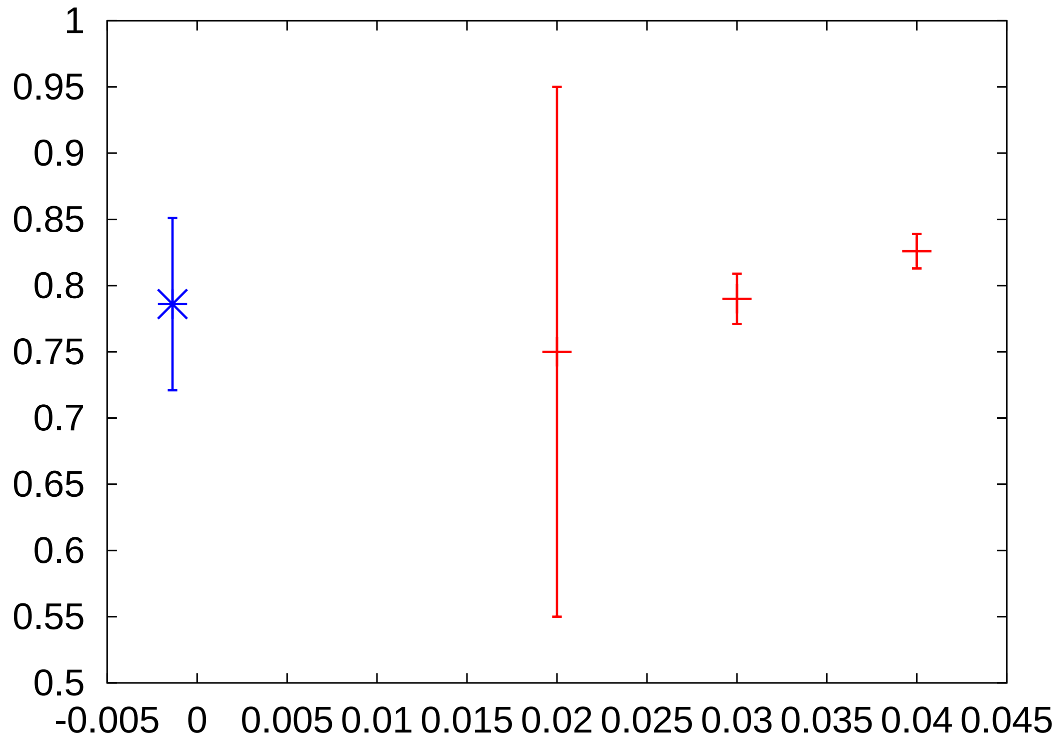


in agreement with experiment,
$g_A/g_V = 1.27(5)$, despite rather small volume,
mild quark-mass dependence.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

$\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$



in agreement with experiment,
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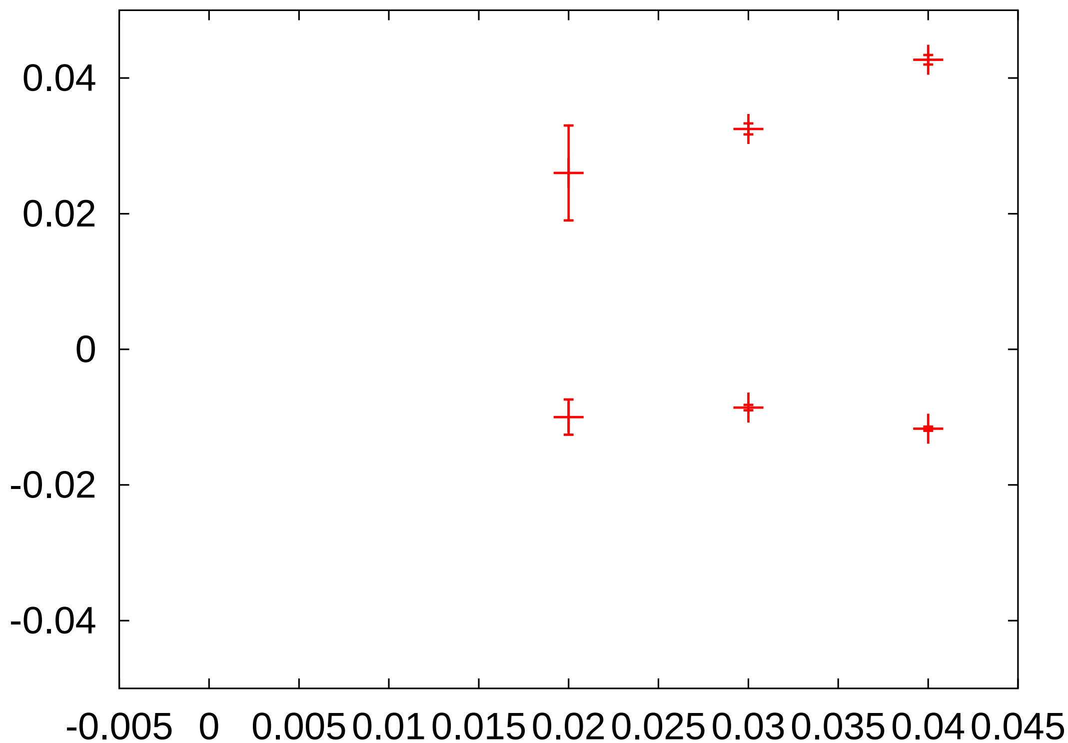
RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

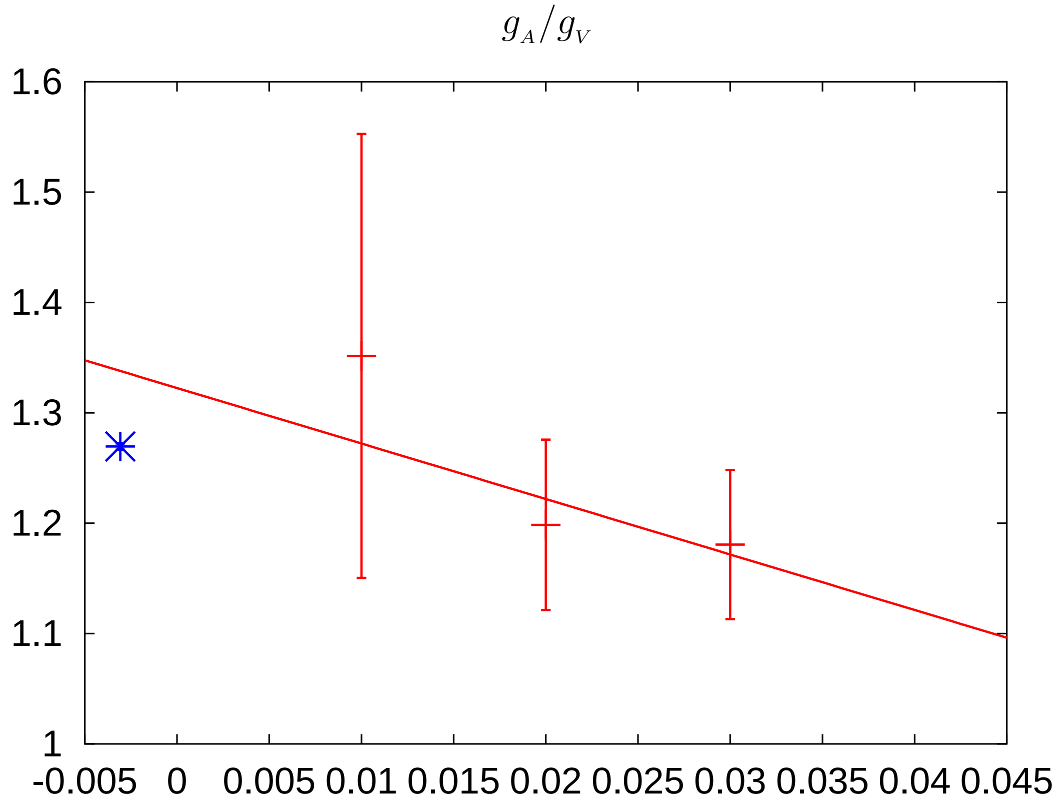
d_1



appears small, though not renormalized.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.00308(3)$, $m_{\rm strange} = 0.04$,

- $m_{\rm up,down} = 0.03$, 0.02, and 0.01; $m_\pi = 620$, 520 and 390 MeV; $m_N = 1.4$, 1.3, and 1.2 GeV (10-20 % errors),
- larger of the two volumes, $24^3 \times 64 \times 16$, 3 fm across, ongoing, preliminary analyses with 25-30 configurations,

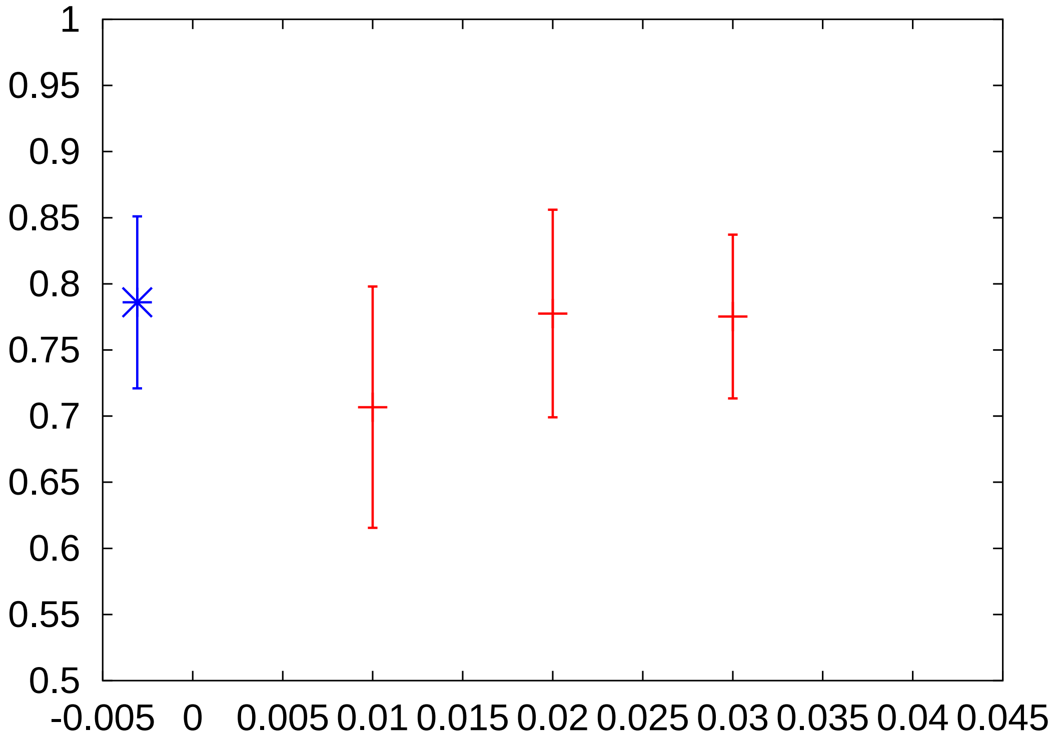


consistent with experiment,
$g_A/g_V = 1.32(11)$, despite rather small statistics,
mild quark-mass dependence.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.00308(3)$, $m_{\rm strange} = 0.04$,

- $m_{\rm up,down} = 0.03$, 0.02, and 0.01; $m_\pi = 620$, 520 and 390 MeV; $m_N = 1.4$, 1.3, and 1.2 GeV (10-20 % errors),
- larger of the two volumes, $24^3 \times 64 \times 16$, 3 fm across, ongoing, preliminary analyses with 25-30 configurations,

$\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$



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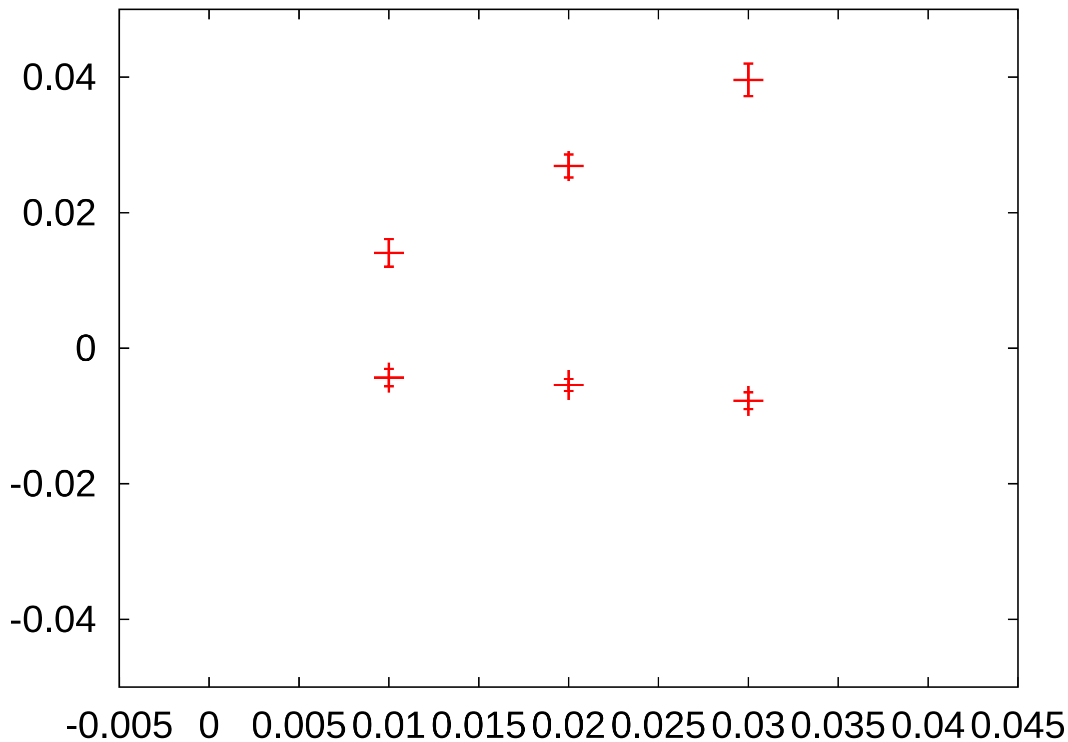
RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.00308(3)$, $m_{\rm strange} = 0.04$,

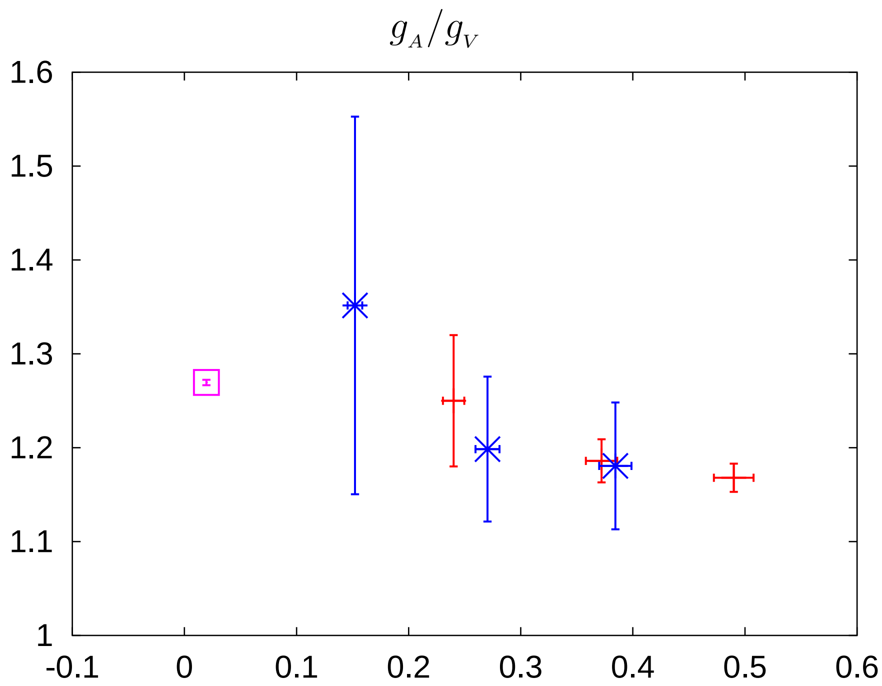
- $m_{\rm up,down} = 0.03$, 0.02, and 0.01; $m_\pi = 620$, 520 and 390 MeV; $m_N = 1.4$, 1.3, and 1.2 GeV (10-20 % errors),
- larger of the two volumes, $24^3 \times 64 \times 16$, 3 fm across, ongoing, preliminary analyses with 25-30 configurations,

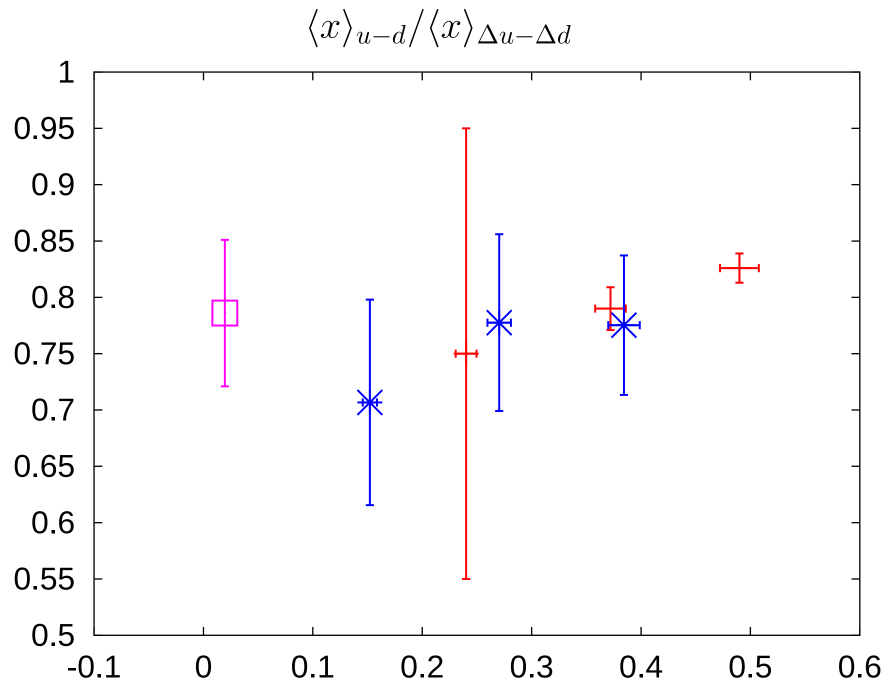
d_1



appears small, though not renormalized.

We can combine the 2-flavor and (2+1)-flavor ratios in m_π^2 (GeV^2) scale:





Conclusions:

1. RBC 2-flavor DBW2+DWF dynamical calculations are almost complete:
   - $a^{-1} = 1.7$ GeV, $m_{\rm res} = 0.00137(5)$
     - $m_{\rm sea} = 0.04$, 0.03, and 0.02,
     - NPR on the way.
   - Ratios, g_A/g_V and $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$, in agreement with experiment, despite rather small volume:
     - $g_A/g_V = 1.27(5)$,
     - mild quark-mass dependence.
   - d_1 appears small, though not renormalized.

2. RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations are ongoing,
   - $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.00308(3)$,
     - $m_{\rm strange} = 0.04$ and $m_{\rm up,down} = 0.03$, 0.02, and 0.01,
     - the larger, 3-fm across, of the two volumes reported here,
     - preliminary.
   - Ratios, g_A/g_V and $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$, consistent with experiment, despite rather small statistics:
     - $g_A/g_V = 1.32(11)$,
     - mild quark-mass dependence.
   - d_1 appears small, though not renormalized.