The NMSSM $h \longrightarrow aa$ Decay Scenario and the Elimination of Fine-Tuning

The NMSSM has no problems and indirect experimental support.

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Based largely on:
R. Dermisek and J. Gunion, hep-ph/0510322
R. Dermisek and J. Gunion, hep-ph/0502105
J. Gunion, D. Hooper and B. McElrath, hep-ph/0509024
R. Dermisek and J. Gunion, two papers in preparation.

See also: J. Gunion, D. Miller, A. Pilaftsis, in CPNSH (CP-violating and Non-Standard Higgses) CERN Yellowbook Report.

Outline

1. The LHC is coming, but is it guaranteed to find the Higgs of any SUSY model? Or might the ILC be even more critical than imagined?

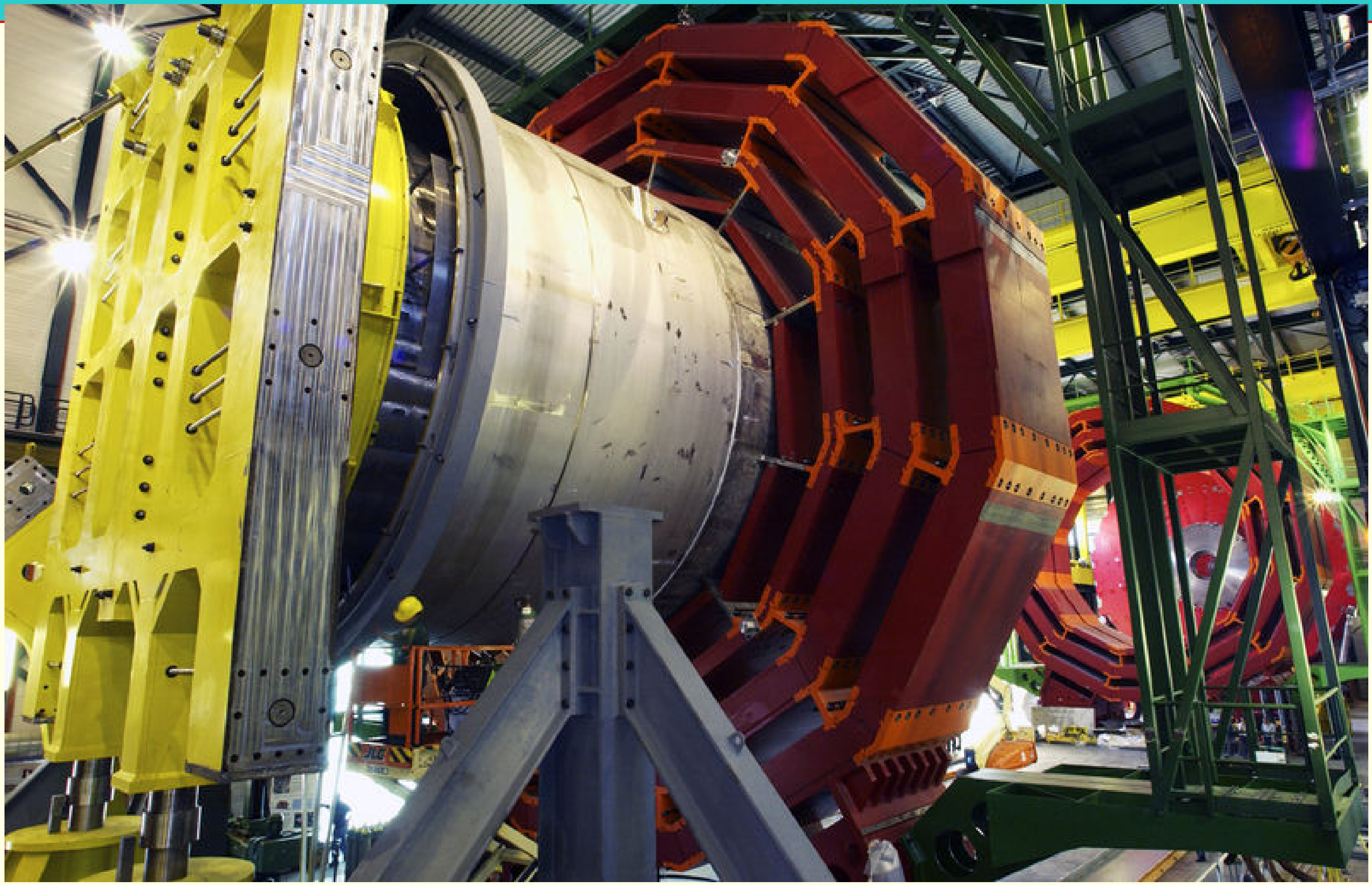
2. Why focus on SUSY?$\Rightarrow$ It remains the most attractive way to solve the hierarchy problem.

3. The Minimal SUSY Model (MSSM) is very attractive, but LEP limits on the lightest Higgs and the gluino imply that it is in a fine-tuned part of parameter space.

4. The Next to Minimal Supersymmetric Model (NMSSM) maintains all the attractive features of the MSSM while avoiding fine tuning, especially if $m_{h_1} \sim 100~\mathrm{GeV}$, as preferred by LEP data (precision and direct search).

5. Collider Implications Low-fine-tuning NMSSM models change how to search for the Higgs at the LHC and imply that one should look again at the LEP data for $h \to aa$ Higgs signals. The ILC will be essential.

The LHC is at hand



The CMS Detector

But will the LHC detectors detect the Higgs boson. The very attractive NMSSM SUSY scenario suggests we may have to work hard.

The Beauty of Supersymmetry

- **SUSY is mathematically intriguing.**
- **SUSY is naturally incorporated in string theory.**
- **Scalar fields have a natural place in SUSY, and so there are candidates for the spin-0 fields needed for electroweak symmetry breaking and Higgs bosons.**
- **SUSY cures the naturalness / hierarchy problem (quadratic divergences are largely canceled) provided the SUSY breaking scale is of order $\sim 1~\mathrm{TeV}$.**
- **The MSSM comes close to being very nice.**

  If we assume that all sparticles reside at the $\mathcal{O}(1~\mathrm{TeV})$ scale and that μ is also $\mathcal{O}(1~\mathrm{TeV})$, then, the MSSM has two particularly wonderful properties.

1. Gauge Coupling Unification

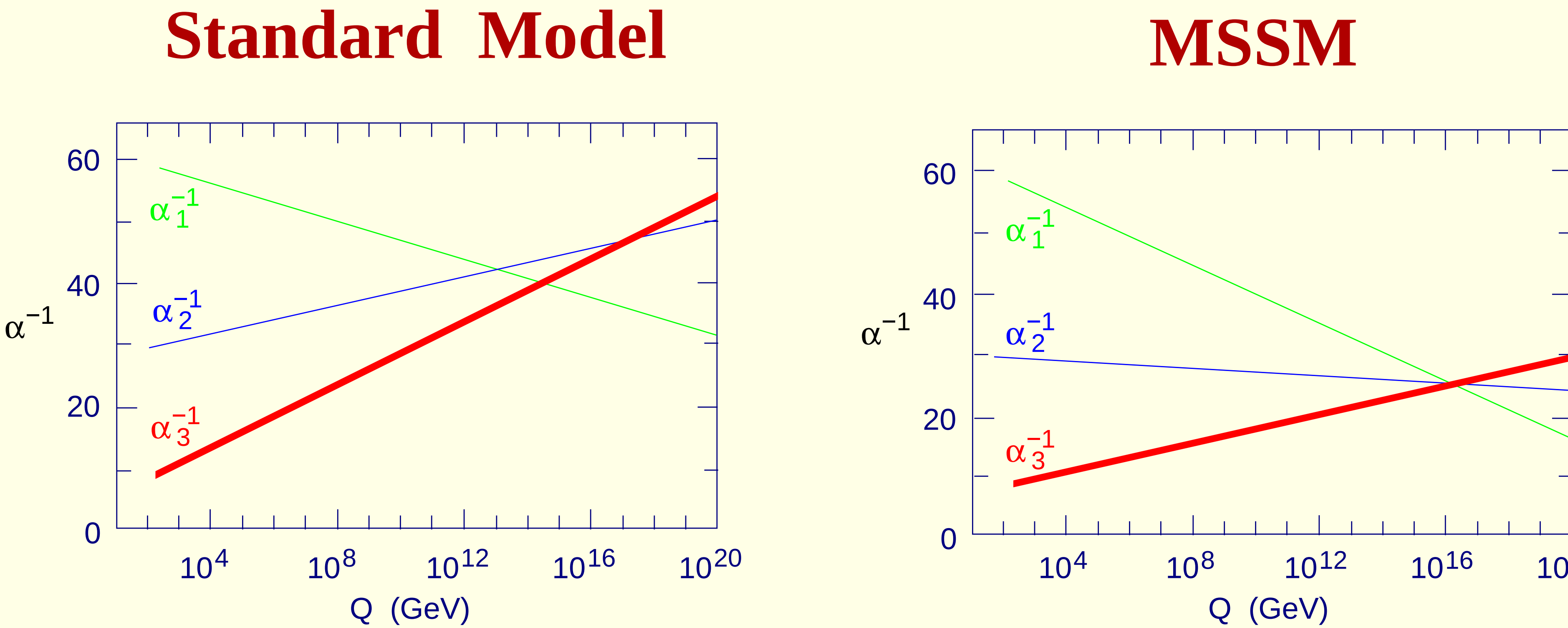


Figure 1: **Unification of couplings constants ($\alpha_i = g_i^2/(4\pi)$) in the minimal supersymmetric model (MSSM) as compared to failure without supersymmetry.**

The MSSM sparticle content + two-doublet Higgs sector ⇒ gauge coupling unification at $M_U \sim few \times 10^{16}$ GeV, close to M_P. High-scale unification correlates well with the attractive idea of gravity-mediated SUSY breaking.

2. RGE EWSB

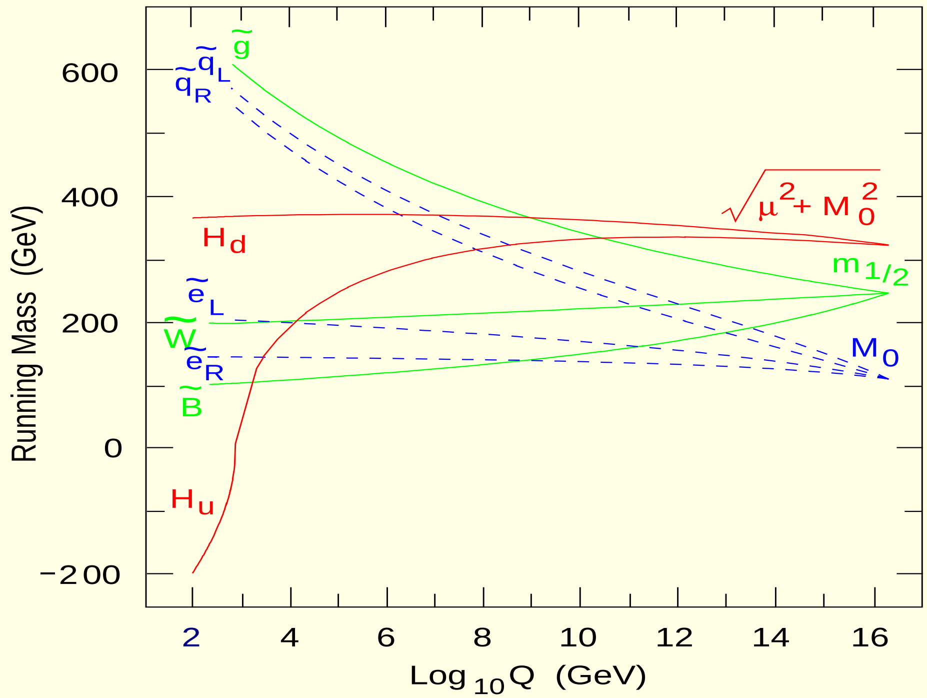


Figure 2: Evolution of SUSY-breaking masses or masses-squared, showing how $m_{H_u}^2$ is driven < 0 at low $Q \sim \mathcal{O}(m_Z)$.

Starting with universal soft-SUSY-breaking masses-squared at M_U, the RGE's predict that the top quark Yukawa coupling will drive one of the soft-SUSY-breaking Higgs masses squared ($m_{H_u}^2$) negative at a scale of order $Q \sim m_Z$, thereby automatically generating electroweak symmetry breaking ($\langle H_u \rangle = h_u, \langle H_d \rangle = h_d$), BUT MAYBE m_Z IS FINE-TUNED.

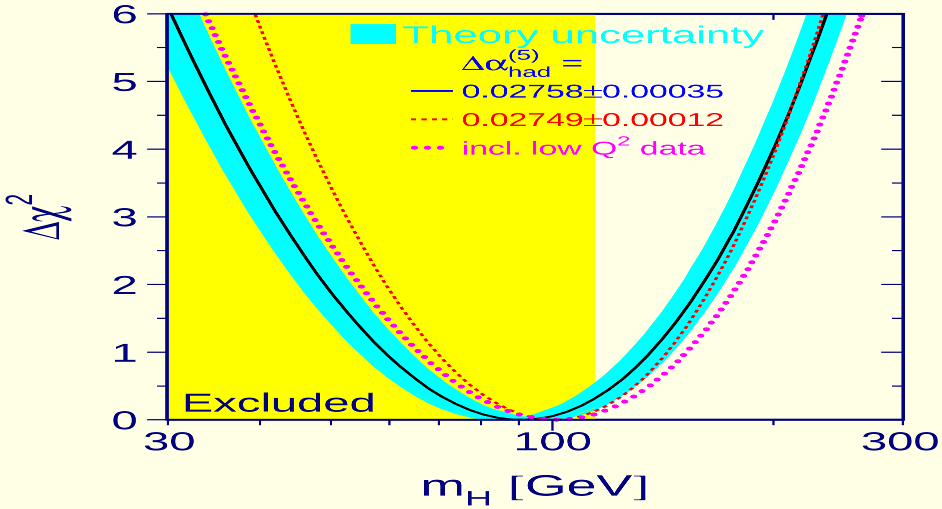
- **The Higgs Mass**

In the presence of soft-SUSY-breaking, the light Higgs has ($\tan\beta = h_u/h_d$)

$$m_h^2 \sim m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$

$$\overset{\text{large}\,\tan\beta}{\sim} (91\ \mathrm{GeV})^2 + (38\ \mathrm{GeV})^2 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right). \quad (1)$$

A Higgs mass of order $100\ \mathrm{GeV}$, as predicted for stop masses $\sim 2m_t$, is in wonderful accord with precision electroweak data.



So, why haven't we seen the Higgs? Is SUSY wrong, are stops heavy, or is the MSSM too simple?

MSSM Problems

- **The μ parameter in $W \ni \mu \widehat{H}_u \widehat{H}_d$,[1] is dimensionful, unlike all other superpotential parameters. A big question is why is it $\mathcal{O}(1~\mathrm{TeV})$ (as required for EWSB and $m_{\widetilde{\chi}_1^\pm}$ lower bound), rather than $\mathcal{O}(M_U, M_\mathrm{P})$ or 0.**

- **LEP limits:**

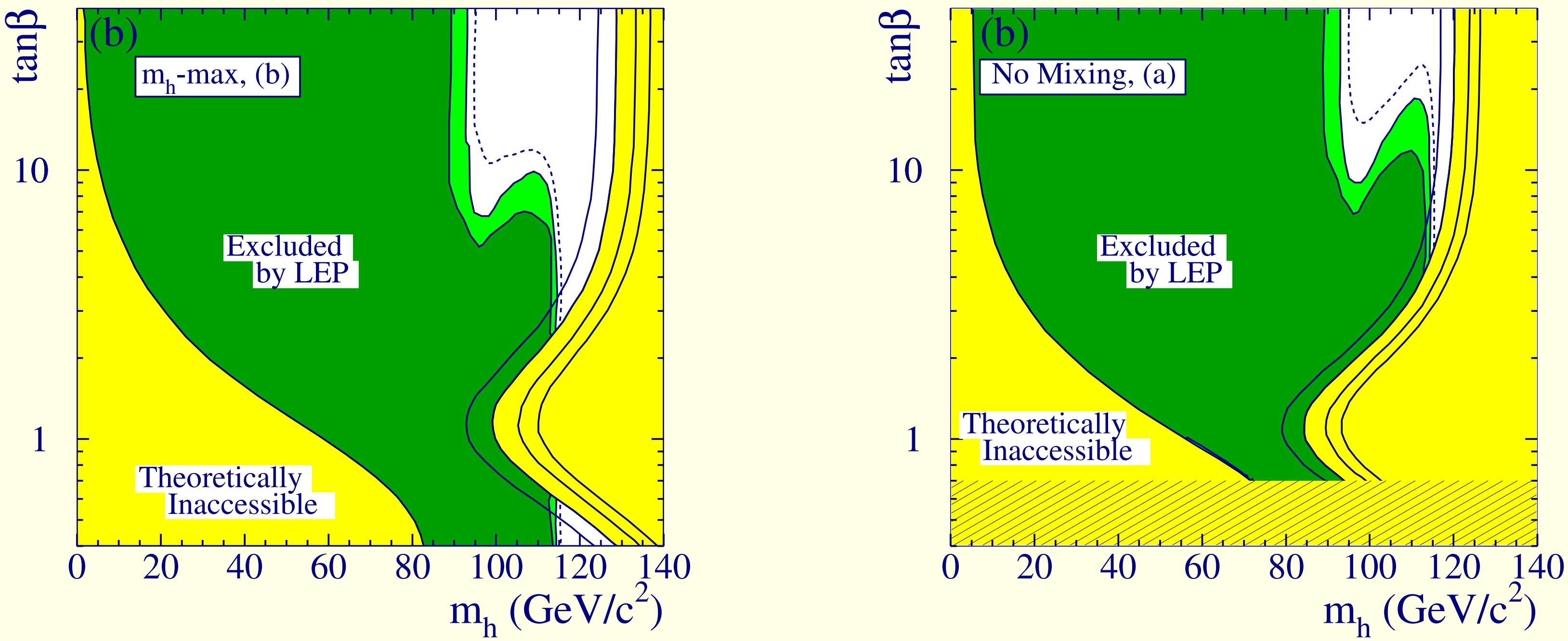


Figure 3: Maximal-mixing ($X_t = A_t - \mu \cot\beta = -2 m_{\mathrm{SUSY}} = -2~\mathrm{TeV}$, $\mu > 0$) and no-mixing (with $\mu > 0$) LEP exclusions at 90% CL. From CERN-PH-EP/2006-001.

[1] Hatted (unhatted) capital letters denote superfields (scalar superfield components).

The LEP limits on Higgs bosons have pushed the CP-conserving MSSM into an awkward corner of parameter space characterized by large $\tan\beta$ and large $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$. For $m_{\widetilde{t}_L} = m_{\widetilde{t}_R} = 1~\mathrm{TeV} \equiv m_{\mathrm{SU\!\!\!/SY}}$, we have the MSSM exclusion plots shown.

There is still room, but we need $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \gtrsim 900~\mathrm{GeV}$. This leads to ....

- Fine-tuning

Minimization of the Higgs potential gives (at scale m_Z)

$$\frac{1}{2} m_Z^2 = -\mu^2 + \frac{m_{H_d}^2 - \tan^2\beta m_{H_u}^2}{\tan^2\beta - 1} \tag{2}$$

and the m_Z-scale $\mu, m_{H_u}^2, m_{H_d}^2$ parameters are sensitive to their GUT scale values yielding at $\tan\beta = 10$ (similar to $\tan\beta = 2.5$ results in Kane and King hep-ph/9810374 and Bastero-Gil, Kane, and King hep-ph/9910506)

$$\begin{aligned} m_Z^2 &= -2.0\mu^2(M_U) + 5.9 M_3^2(M_U) + 0.8 m_Q^2(M_U) + 0.6 m_U^2(M_U) \\ &\quad -1.2 m_{H_u}^2(M_U) - 0.7 M_3(M_U) A_t(M_U) + 0.2 A_t^2(M_U) + \ldots \end{aligned}$$

One would expect that $m_Z \sim 2M_3(M_U), m_Q(M_U), m_u(M_U) \sim m_{\tilde{g}}, m_{\tilde{t}}$, $\Rightarrow$ we need a very light gluino and a rather light stop to avoid fine-tuning OR we need highly correlated cancellations and large A_t. A rigorous measure is F plotted below.

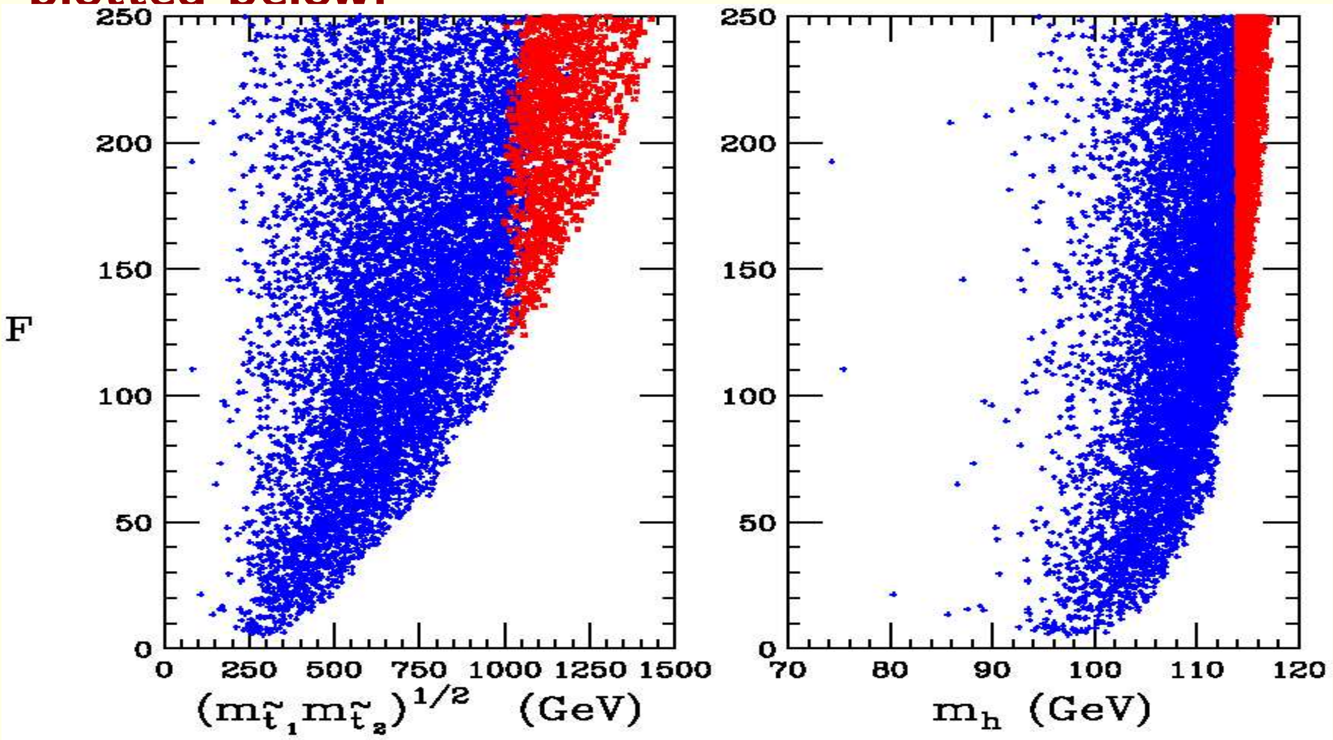


Figure 4: MSSM results for $F = \mathrm{Max}_p \left| \frac{p}{m_Z} \frac{\partial m_Z}{\partial p} \right|$, where $p \in \left\{ M_{1,2,3}, m_Q^2, m_U^2, m_D^2, m_{H_u}^2, m_{H_d}^2, \mu, A_t, B\mu, \ldots \right\}$ (all at M_U). The $+$ points have $m_h < 114~\mathrm{GeV}$, and are experimentally excluded. The $\times$ points have $m_h \geq 114~\mathrm{GeV}$. Plot is for $\tan\beta = 10$, $M_{1,2,3} = 100, 200, 300~\mathrm{GeV}$ (at scale m_Z). All parameters scanned, but $|A_t| < 500~\mathrm{GeV}$ is imposed.

Essentially all the blue points fail LEP limits due to $m_h < 114~\mathrm{GeV}$.

Note that if $m_h \sim 100~\mathrm{GeV}$ were ok, then smallest F occurs there.

One can do somewhat better by taking very large A_t values — but never as good as the NMSSM which also does not require any extreme choices.

- So, what direction should one head in?
  - CP-violating MSSM, e.g. CPX-like scenarios?
    These don't solve the μ issue, and nature has shown very little inclination for CP-violation as large as that needed to significantly alter the CP-conserving situation.
  - Large extra dimensions, little Higgs, Higgsless, ...
    All worth exploring, but these models are complicated and typically have problems of one kind or another, especially precision EW data.
  - Hints from string theory.
    In particular, it is very clear that extra singlet superfields are common in string models.
    Let's make use of singlets and let's do it in the simplest possible way (i.e. no associated gauge group and no dimensionful superpotential parameters) $\Rightarrow$ the NMSSM.

The NMSSM

- **The NMSSM introduces just one extra singlet superfield, with superpotential $\lambda \widehat{S} \widehat{H}_u \widehat{H}_d$. The μ parameter is then automatically generated by $\langle S \rangle$ leading to $\mu_{eff} \widehat{H}_u \widehat{H}_d$ with $\mu_{eff} = \lambda \langle S \rangle$. The only requirement is that $\langle S \rangle$ be of order the SUSY-breaking scale at $\sim 1 \text{ TeV}$.**

- **However, $\lambda \widehat{S} \widehat{H}_u \widehat{H}_d$ cannot be the end.**

  Including Yukawa W terms, there is a PQ symmetry that will spontaneously break when the Higgs scalars gain vevs, and a pseudo[2]-Nambu-Goldstone boson, known as the PQ axion (it is actually one of the pseudoscalar Higgs bosons), will be generated.

  For values of $\lambda \sim \mathcal{O}(1)$, this axion would have been detected in experiment and this model ruled out.

- **Gauging the $U(1)_{\rm PQ}$ (so that axion is absorbed in Z' mass) typically leads to FCNC problems.**

[2] The axion is only a "pseudo"-Nambu-Goldstone boson since the PQ symmetry is explicitly broken by the QCD triangle anomaly. The axion then acquires a small mass from its mixing with the pion.

- In the NMSSM, the PQ symmetry is explicitly broken by $W \ni \frac{1}{3}\kappa\widehat{S}^3$.

  Other possible superpotential terms with dimensionful parameters are absent if one demands that the superpotential be invariant under a Z_3 symmetry.

  If the Z_3 is applied also to soft SUSY breaking terms, only $\frac{1}{3}\kappa A_\kappa S^3$ is allowed in addition to $\lambda A_\lambda S H_u H_d$.

- However, this $\mathbb{Z}_3$ symmetry cannot be completely unbroken. If it were, a cosmological "domain wall problem" would arise.

- To avoid this problem (Panagiotakopoulos and Tamvakis), one introduces a Z_2^R symmetry that is broken by the soft-SUSY breaking terms, giving rise to harmless tadpoles of order $\frac{1}{(16\pi^2)^n} M^3_{\rm SUSY}$, with $2 \leq n \leq 4$. For example, a superpotential term of form $\widehat{S}^7/M_{\rm P}^4$ (which is ok under Z_2^R) generates at 4-loops (Abel) the tadpole form $\delta V \sim \left(\frac{1}{16\pi^2}\right)^4 m^3_{\rm SUSY}(S+S^*)$.

  Although these terms are phenomenologically irrelevant, they are entirely sufficient to break the global Z_3 symmetry and make the domain walls collapse.

- Net Result

Since the only *relevant* superpotential terms that are introduced have dimensionless couplings, the scale of the vevs (i.e. the scale of EWSB) is determined by the scale of SUSY-breaking.

- **Further**, all the good properties of the MSSM (coupling unification and RGE EWSB, in particular) are preserved under singlet addition.

- **New Particles**

  The single extra singlet superfield of the NMSSM contains an extra neutral gaugino (the singlino) ($\Rightarrow \widetilde{\chi}^0_{1,2,3,4,5}$), an extra CP-even Higgs boson ($\Rightarrow h_{1,2,3}$) and an extra CP-odd Higgs boson ($\Rightarrow a_{1,2}$).

- **The parameters of the NMSSM**

  Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda \ \widehat{S}\widehat{H_u}\widehat{H_d} + \frac{\kappa}{3} \ \widehat{S}^3 \tag{3}$$

  depending on two dimensionless couplings λ, κ beyond the MSSM. The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \,. \tag{4}$$

The final two input parameters are

$$\tan\beta = h_u/h_d\,, \quad \mu_{\rm eff} = \lambda s\,, \tag{5}$$

where $h_u \equiv \langle H_u\rangle$, $h_d \equiv \langle H_d\rangle$ and $s \equiv \langle S\rangle$. These, along with m_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S (denoted $m_{H_u}^2$, $m_{H_d}^2$ and m_S^2) through the three minimization equations of the scalar potential. (From the model building point of view, we emphasize the reverse — i.e. the SUSY-breaking scales $m_{H_u}^2$, $m_{H_d}^2$ and m_S^2, along with A_λ and A_κ determine the EWSB vevs, λ and κ being dimensionless.)

Thus, as compared to the three independent parameters needed in the MSSM context (often chosen as μ, $\tan\beta$ and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda\ ,\ \kappa\ ,\ A_\lambda\ ,\ A_\kappa,\ \tan\beta\ ,\ \mu_{\rm eff}\ . \tag{6}$$

In addition, values must be input for the gaugino masses and for the soft terms related to the (third generation) squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths.

The NMSSM is much less constrained than the MSSM, and is not necessarily forced into awkward/fine-tuned corners of parameter space either by LEP limits or by theoretical reasoning.

$\Rightarrow$ **the NMSSM should be adopted as the more likely benchmark minimal SUSY model and it should be explored in detail.**

- **To further this study, Ellwanger, Hugonie and I constructed NMHDECAY**

  `http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html`

  `http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html`

  It computes all aspects of the Higgs sector and checks against many (but, as we shall see, not all) LEP limits and various other constraints.

- **We also developed a program to examine the LHC observability of Higgs signals in the NMSSM.**

  A significant hole in the LHC no-lose theorem for Higgs discovery emerges: only if we avoid that part of parameter space for which $h \to aa$ and similar decays are present is there a guarantee for finding a Higgs boson at the LHC in one of the nine "standard" channels (e.g. $h \to \gamma\gamma$, $t\bar{t}h, a \to t\bar{t}b\bar{b}$, $t\bar{t}h, a \to t\bar{t}\gamma\gamma$, $b\bar{b}h, a \to b\bar{b}\tau^+\tau^-$, $WW \to h \to \tau^+\tau^-$, ...

A series of papers (beginning with JFG+Haber+Moroi at Snowmass 1996 and continued by JFG, Ellwanger, Hugonie, Moretti, Miller, .. .) has demonstrated the general nature of this LHC no-lose theorem "hole".

- The portion of parameter space with $h \to aa, \ldots$ is small $\Rightarrow$ one is tempted to ignore it were it not for the fact that it is where fine-tuning can be absent.

  As before, the canonical measure of fine-tuning employed is

$$F = \mathrm{Max}_p F_p \equiv \mathrm{Max}_p \left| \frac{d \log m_Z}{d \log p} \right| , \tag{7}$$

  where the parameters p comprise the GUT-scale values of λ, κ, A_λ, A_κ, and the usual soft-SUSY-breaking gaugino, squark, slepton, . . . masses.

- How do we get small fine-tuning?
  1. F is minimum for $m_{h_1} \sim 100 \div 104~\mathrm{GeV}$ (in a totally unconstrained scan of parameter space this is just what one finds for moderate $\tan\beta$). Neither lower nor higher!

For $m_{h_1} \sim 100$ GeV, $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \sim 350$ GeV.

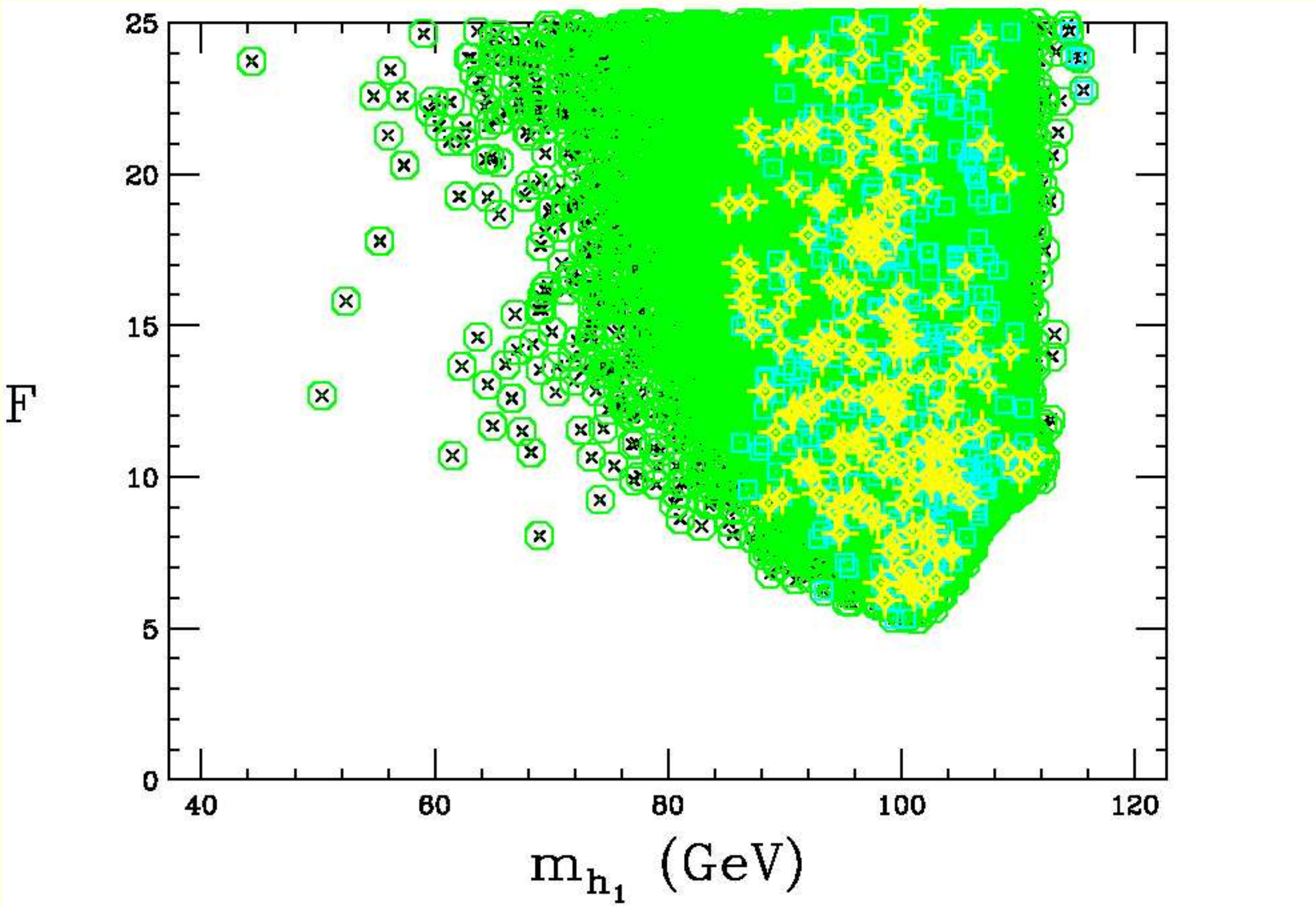


Figure 5: F vs. m_{h_1} for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan\beta = 10$. Small $\times$ = no constraints other than global and local minimum, no Landau pole before M_U and neutralino LSP. The O's = stop and chargino limits imposed, but NO Higgs limits. The □'s = all single channel Higgs limits imposed. The large FANCY CROSSES are after requiring $m_{a_1} < 2m_b$.

2. $m_{h_1} \sim 100$ GeV is only LEP-allowed if $h_1 \to a_1 a_1$ and $a_1 \to \tau^+\tau^-$ ($2m_\tau < m_{a_1} < 2m_b$) or $gg, q\bar{q}$ ($m_{a_1} < 2m_\tau$) so as to hide the h_1 in this mass range (more later).
3. How natural is a light a_1? In fact, a_1 is a pseudo-Nambu-Goldstone boson associated with a $U(1)_R$ symmetry of the superpotential, whose spontaneous breaking by the vevs of H_u, H_d and S would yield $m_{a_1} = 0$ were it not that the $U(1)_R$ is explicitly broken by the A_κ and A_λ soft-SUSY-breaking terms, implying $m_{a_1} \to 0$ for $A_\kappa, A_\lambda \to 0$ (ignoring the small one-loop contributions to $U(1)_R$ breaking from gaugino masses). (Dobrescu, Matchev) Note linearity in A_κ and A_λ.

$$m_{a_1}^2 \simeq 3s\left(\frac{3\lambda A_\lambda v^2 \sin 2\beta}{2s^2} - \kappa A_\kappa\right) \quad \text{where} \tag{8}$$

$$a_1 \equiv \cos\theta_A a_{MSSM} + \sin\theta_A a_S\,, \quad \text{with} \quad \cos\theta_A \simeq \frac{2v}{s\tan\beta}\,. \tag{9}$$

What do we expect for A_κ and A_λ?. The RGE's

$$16\pi^2 \frac{dA_\lambda}{dt} = 6A_t\lambda_t^2 + 8\lambda^2 A_\lambda + 4\kappa^2 A_\kappa + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1$$

$$16\pi^2 \frac{dA_\kappa}{dt} = 12(\lambda^2 A_\lambda + \kappa^2 A_\kappa) \tag{10}$$

imply that if $A_\kappa(M_U), A_\lambda(M_U) \sim 0$ then $A_\kappa(m_Z) \ll A_\lambda(m_Z) \sim M_2$. Since $\cos\theta_A$ is small, the contributions of the A_κ and A_λ terms to $m_{a_1}^2$ are comparable for $A_\kappa(m_Z) \ll A_\lambda(m_Z)$.

4. One finds that correlated $A_\kappa, A_\lambda \neq 0$ are needed for large $B(h_1 \to a_1 a_1)$ and small enough m_{a_1} that $a_1 \to \tau^+\tau^-$ or $jets$, both being required for h_1 with $m_{h_1} \sim 100$ GeV to escape LEP limits on $Zh_1 \to Zb\bar{b}$.

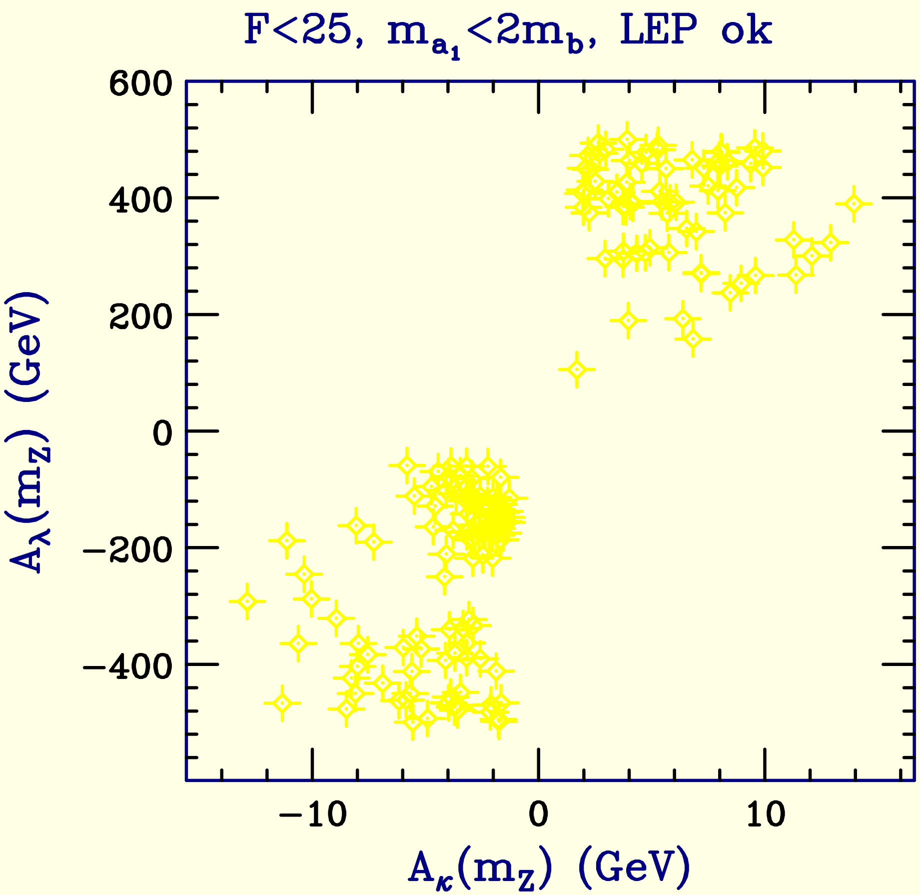


Figure 6: $M_{1,2,3} = 100, 200, 300$ GeV, $\tan\beta = 10$. **Note:** A_κ, A_λ exactly 0 is not ok.

How does $B(h_1 \to a_1 a_1)$ depend on A_κ and A_λ?

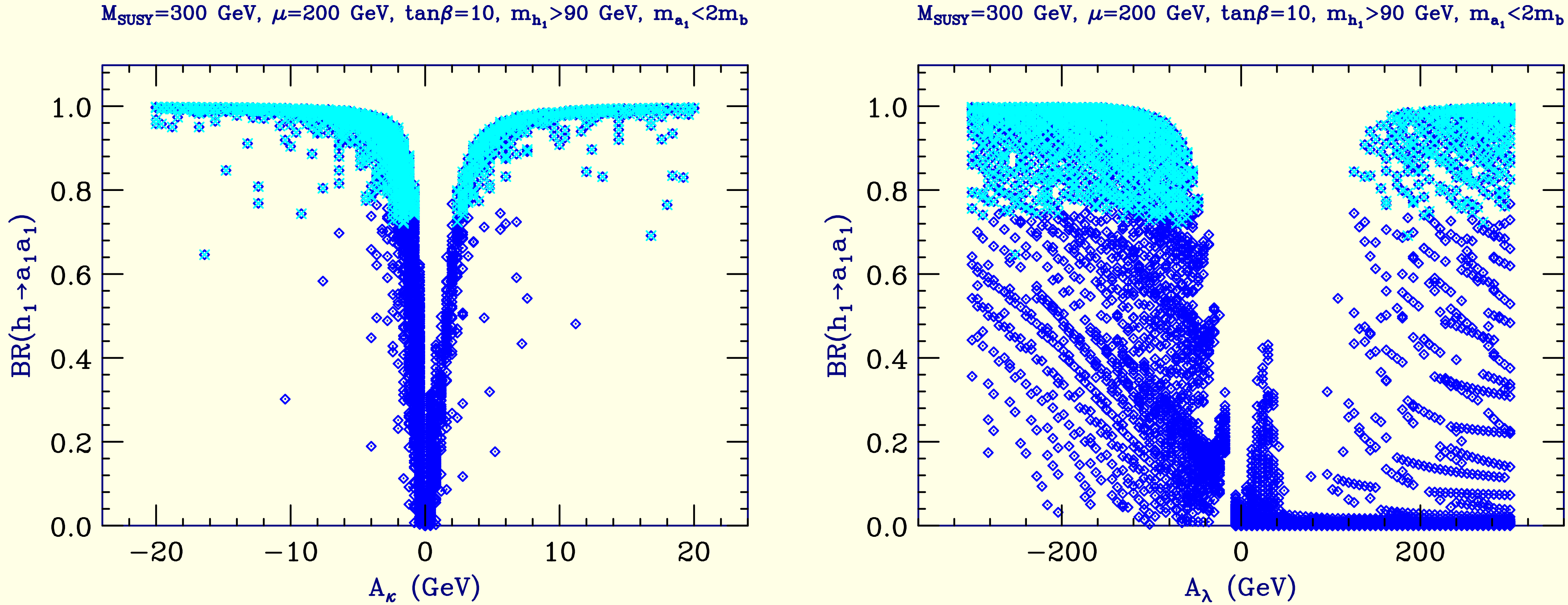


Figure 7: $B(h_1 \to a_1 a_1)$ vs. A_κ and A_λ for $M_{\rm SUSY} = 300~{\rm GeV}$, $\tan\beta = 10$, $\mu = 150~{\rm GeV}$ after scanning over λ and κ values. All points have $m_{a_1} < 2m_b$. Light cyan $\times$ points have large enough $B(h_1 \to a_1 a_1)$ to escape LEP limits on $Zh_1 \to Zb\bar{b}$.

We see again that $|A_\kappa|$ of order a few GeV is needed for large enough $B(h_1 \to a_1 a_1)$ to escape LEP limits.

Is there a new fine-tuning associated with getting both a very light m_{a_1} and large enough $B(h_1 \to a_1 a_1)$? Define

$$F_{\rm MAX} = {\rm Max}\left\{ \left| \frac{A_\lambda}{m_{a_1}^2} \frac{dm_{a_1}^2}{dA_\lambda} \right|, \left| \frac{A_\kappa}{m_{a_1}^2} \frac{dm_{a_1}^2}{dA_\kappa} \right| \right\} . \quad (11)$$

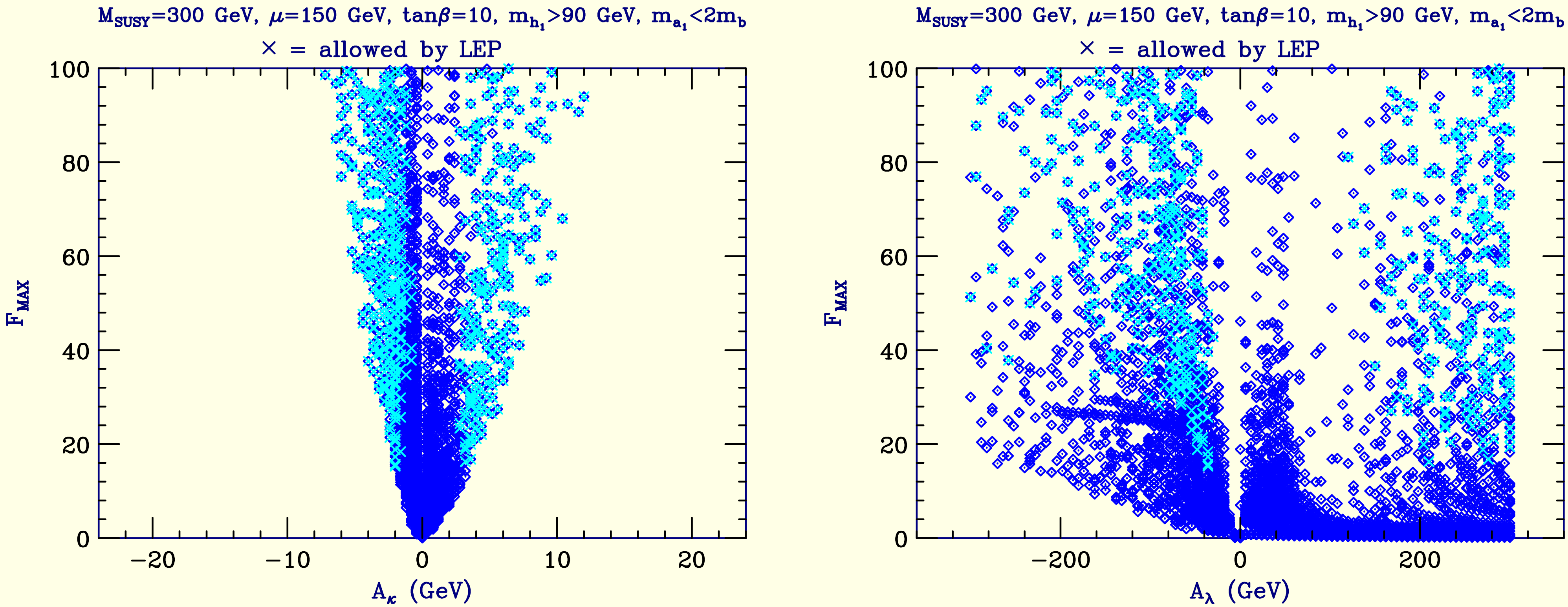


Figure 8: $F_{\rm MAX}$ **vs.** A_κ **and** A_λ **for** $M_{\rm SUSY} = 300~{\rm GeV}$**,** $\tan\beta = 10$**,** $\mu = 150~{\rm GeV}$ **after scanning over** λ **and** κ **values. All points have** $m_{a_1} < 2m_b$**. Light cyan** $\times$ **points have large enough** $B(h_1 \to a_1 a_1)$ **to escape LEP limits on** $Z h_1 \to Z b\bar b$**.**

$\Rightarrow$ at least 5% to 10% fine tuning in $A_\kappa(m_Z)$ and $A_\lambda(m_Z)$ to get $m_{a_1} < 2m_b$ and ok $B(h_1 \to a_1 a_1)$.

But, what is actually relevant is fine-tuning wrp to GUT parameters. In a large class of models this is much smaller. More later.

5. Small F is associated with small values for $m^2_{H_u}(M_U)$, $m^2_{H_d}(M_U)$ and $m^2_S(M_U)$.

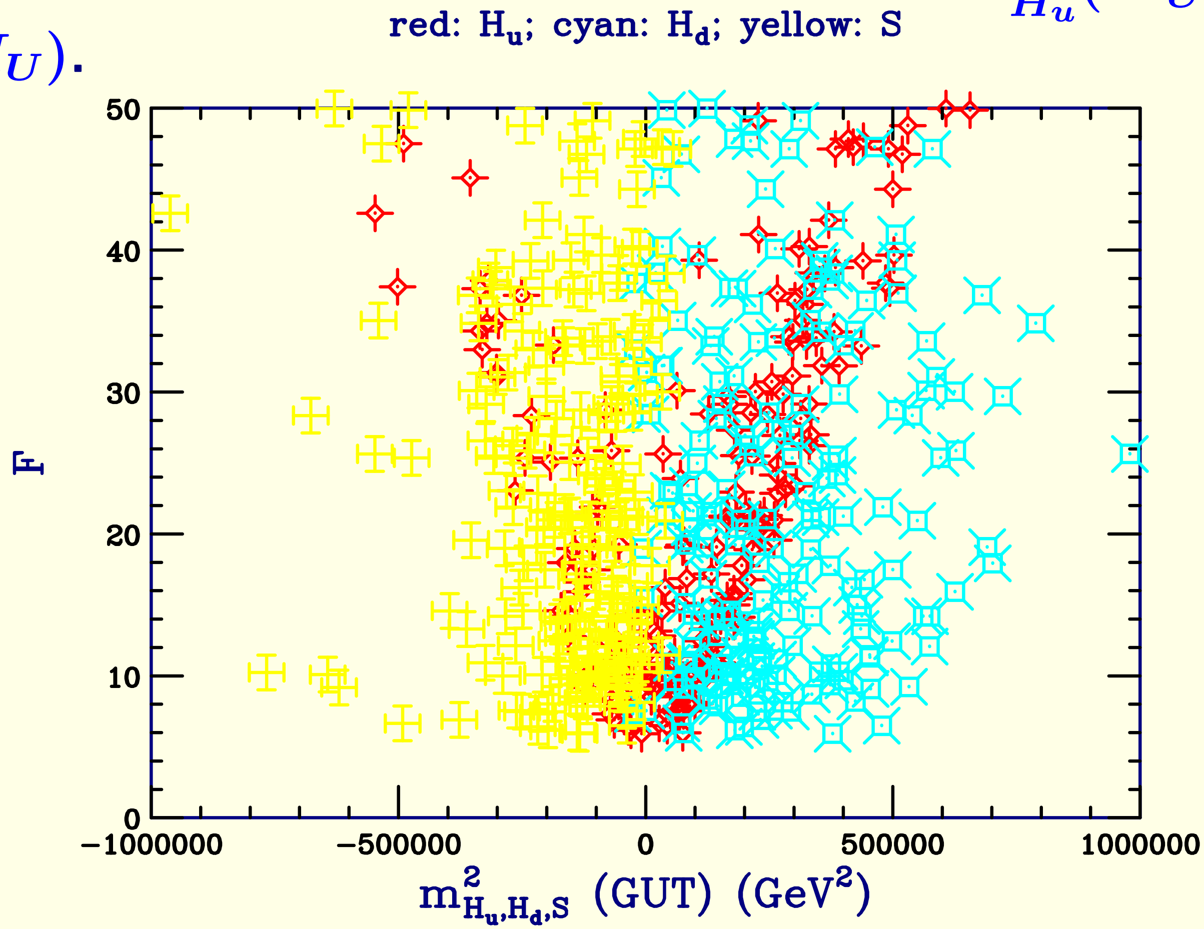


Figure 9: F vs. $m^2_{H_u,H_d,S}(M_U)$ for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan\beta = 10$ for fully ok $m_{a_1} < 2m_b$ solutions.

6. $A_\kappa(M_U)$ and $A_\lambda(M_U)$ can be very small. Even smaller than shown if allow shifts in M_1 and/or M_2 (which do not affect EWSB fine-tuning).

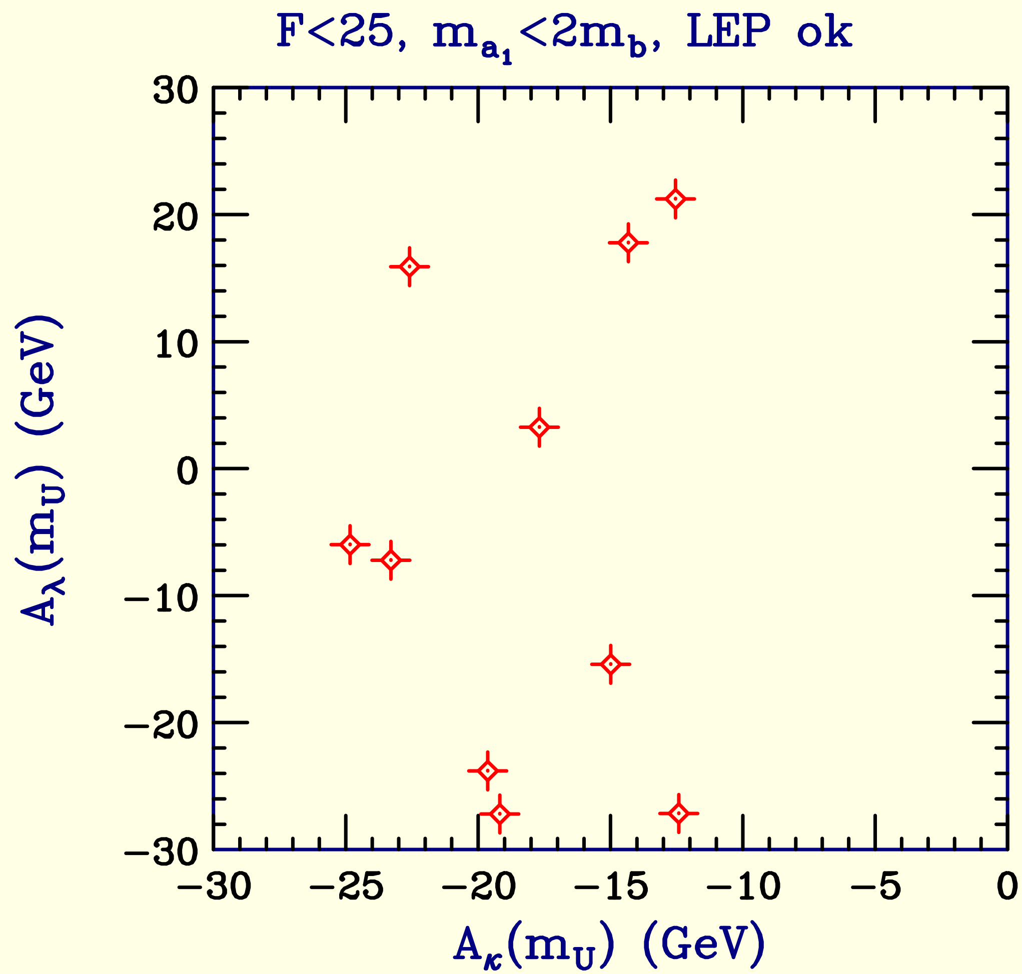


Figure 10: $A_\kappa(M_U)$ and $A_\lambda(M_U)$ for $M_{1,2,3} = 100, 200, 300~\mathrm{GeV}$ and $\tan\beta = 10$ for fully ok $m_{a_1} < 2m_b$ solutions with $F < 25$.

More Details on EWSB Fine-Tuning and LEP limits

- **Large $B(h_1 \to a_1 a_1)$ is needed to escape LEP $h_1 \to b\bar{b}$ limits (blue $+$ = $m_{h_1} < 114$ GeV, red $\times$ = $m_{h_1} > 114$ GeV)**

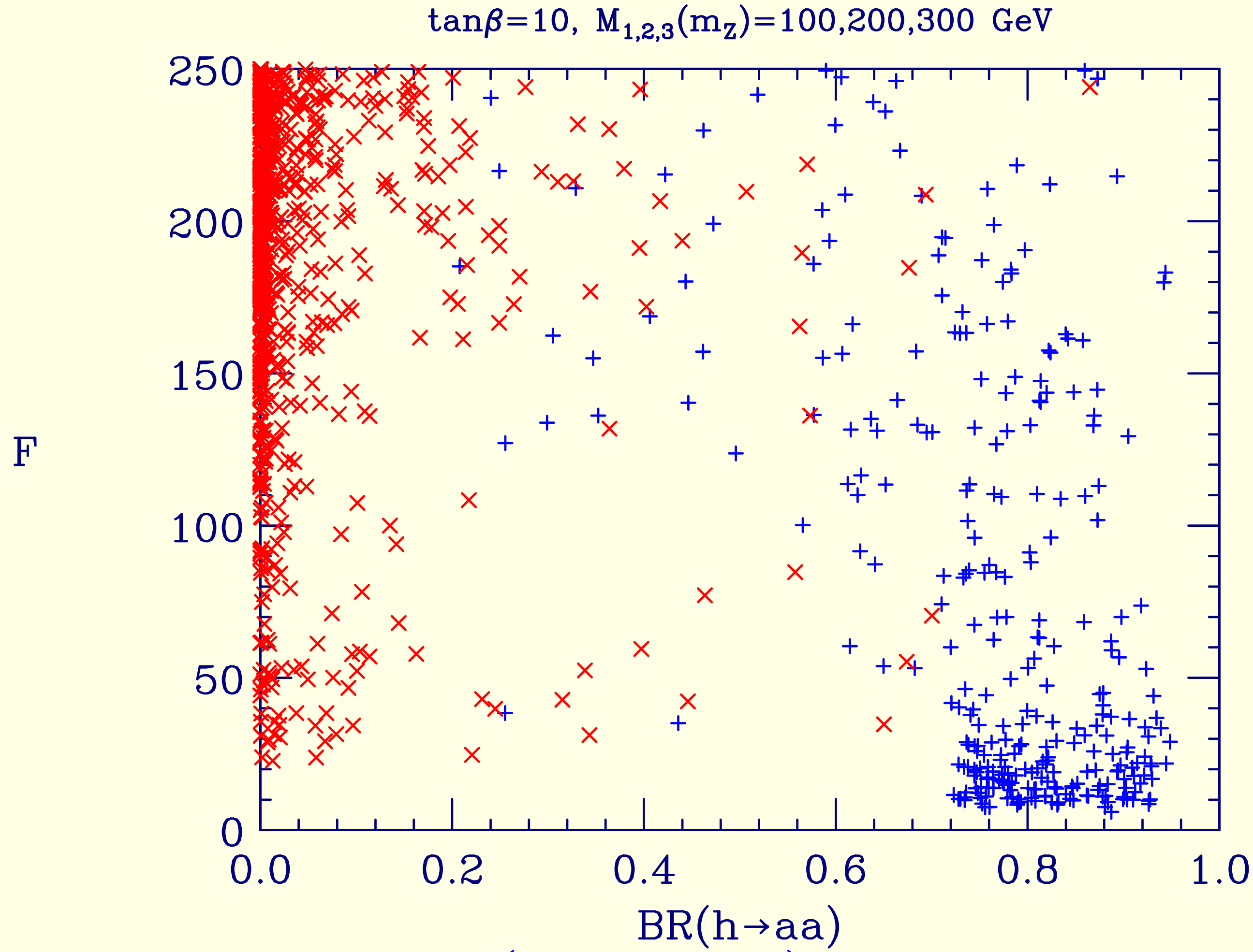


Figure 11: F as a function of $B(h_1 \to a_1 a_1)$ after *single-channel* LEP limits.

Among the very low F points with large $B(h_1 \to a_1 a_1)$ shown, there are ones with $m_{a_1} > 2m_b$ and ones with $m_{a_1} < 2m_b$. The former have problems unless $m_{h_1} \gtrsim 110~\mathrm{GeV}$.

In particular, the $Z2b$ and $Z4b$ channels are not actually independent.

- Putting the $F < 10$ scenarios with $m_{a_1} > 2m_b$ through the full LHWG analysis, one finds that all are excluded at somewhat more than the 99% CL.

  In fact, all the $m_{a_1} > 2m_b$ scenarios with $m_{h_1} \lesssim 108 \div 110~\mathrm{GeV}$ are ruled out at a similar level. What is happening is that you can change the $h_1 \to b\overline{b}$ direct decay branching ratio and you can change the $h_1 \to a_1 a_1 \to 4b$ branching ratio, but roughly speaking $B(h_1 \to b's) \gtrsim 0.85$ (a kind of sum rule). So, if the ZZh_1 coupling is full strength (as is the case in all the scenarios with any kind of reasonable F) there is no escape except high enough m_{h_1}.

- The only way to achieve really low F, which comes with low $m_{h_1} \sim 100~\mathrm{GeV}$, and remain consistent with LEP is to have $m_{a_1} < 2m_b$.

  The relevant limit from LEP is then only that from the $Z2b$ channel. (It turns out that LEP has never placed limits on the $Z4\tau$ channel for h masses larger than about $87~\mathrm{GeV}$.)

- **Note:** **Such a light to very light a_1 is not excluded by Υ, ... precision decay measurements since, as remarked earlier, the a_1 is very singlet-like for all the low-F scenarios.**

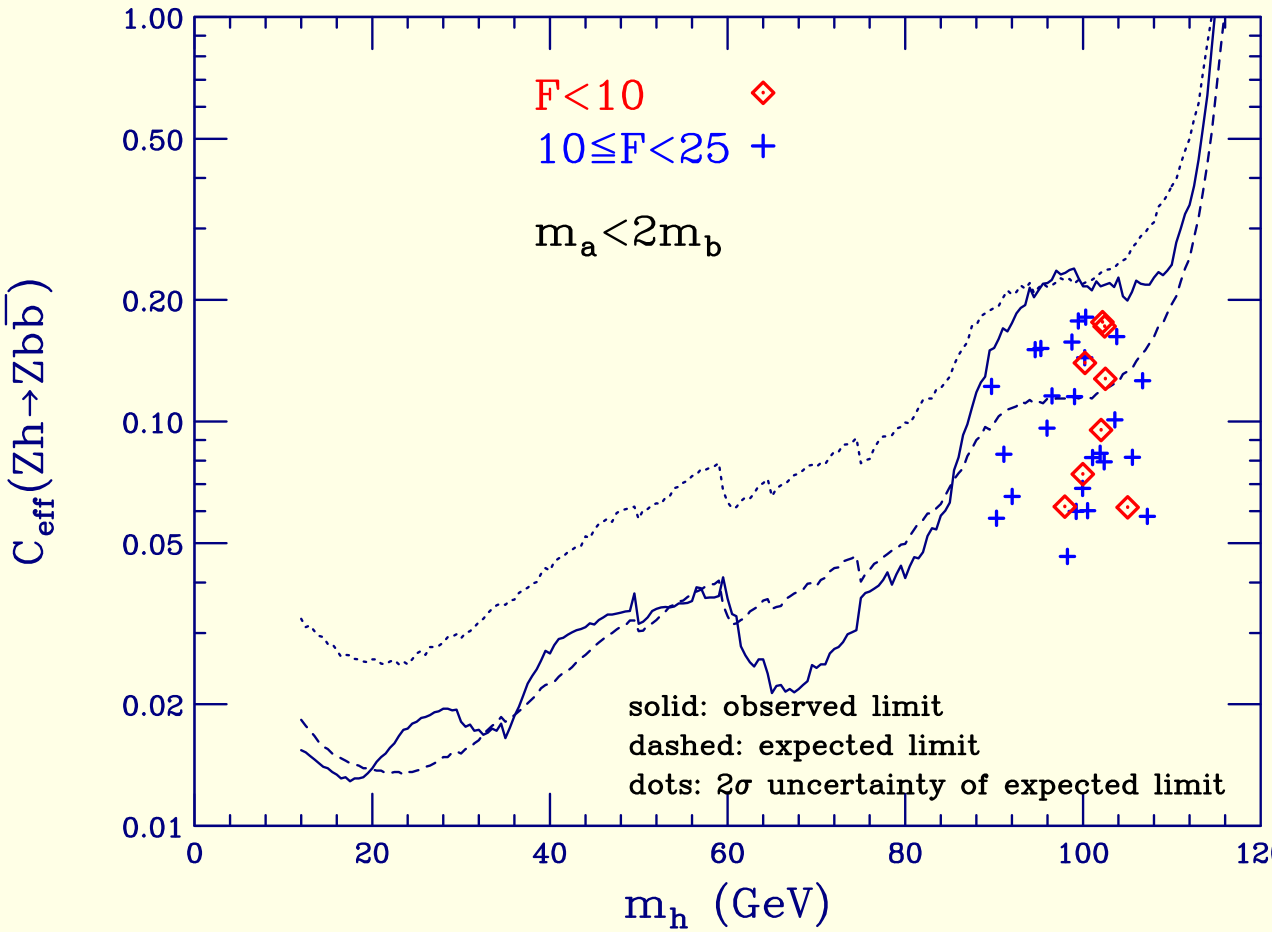


Figure 12: Observed LEP limits on C^{2b}_{eff} for the low-F points with $m_{a_1} < 2m_b$.

So just how consistent are the $F < 10$ points with the observed event excess. Although it is slightly misleading, a good place to begin is to recall the famous $1-CL_b$ plot for the $Z2b$ channel. (Recall: the smaller $1-CL_b$ the less consistent is the data with expected background only.)

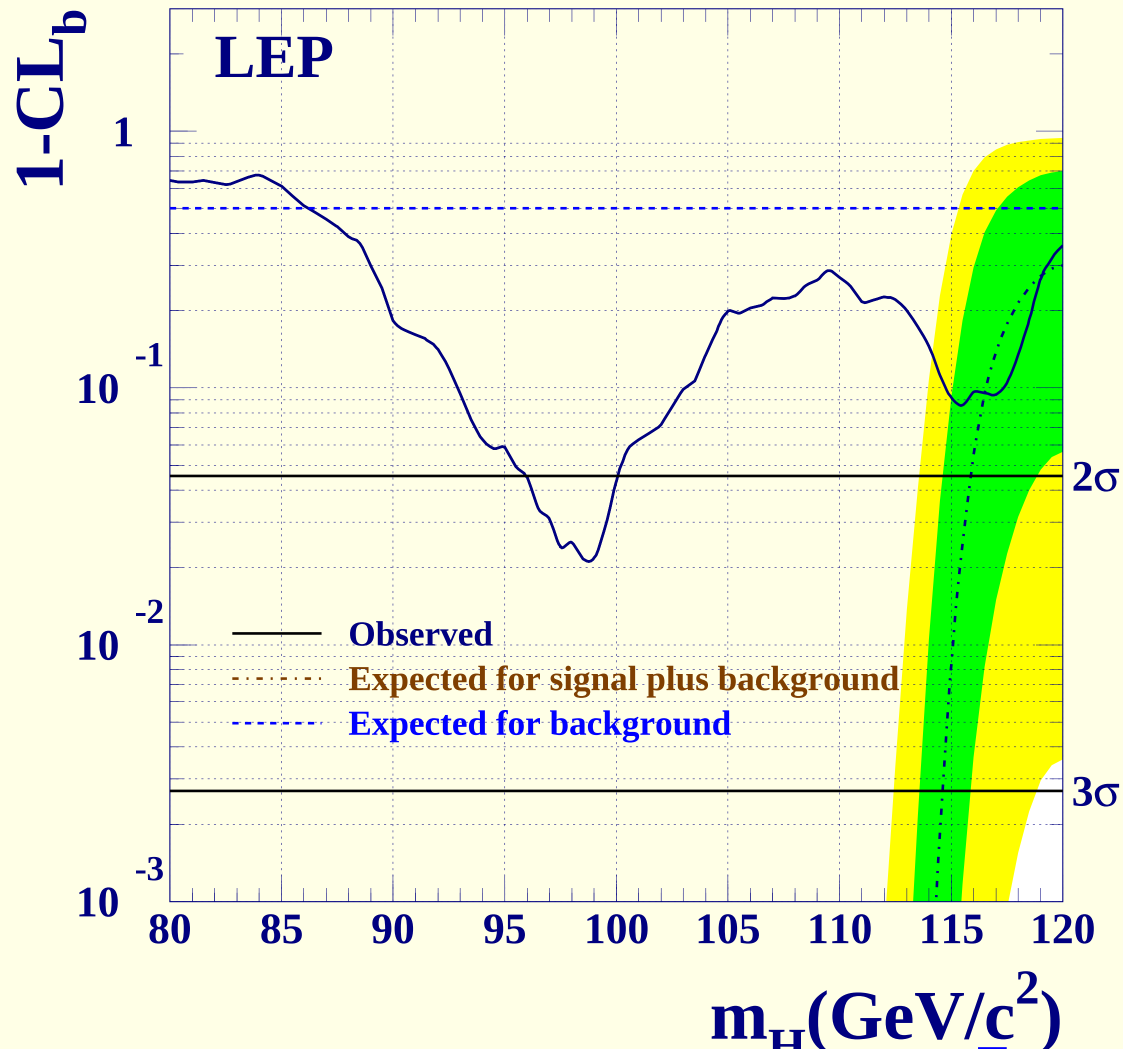


Figure 13: Plot of $1 - CL_b$ for the $Zb\bar{b}$ final state.

- **There is an observed vs. expected discrepancy exactly where we want it! And because $B(h_1 \to b\bar{b})$ is $1/10$ the SM value, the discrepancy is of about the right size.**

- **Are there other relevant limits on the kind of scenario we envision?**

  If the $a_1 a_1 \to 4\tau$ decay is the relevant scenario, the LEP limits run out for $m_h > 87$ GeV.

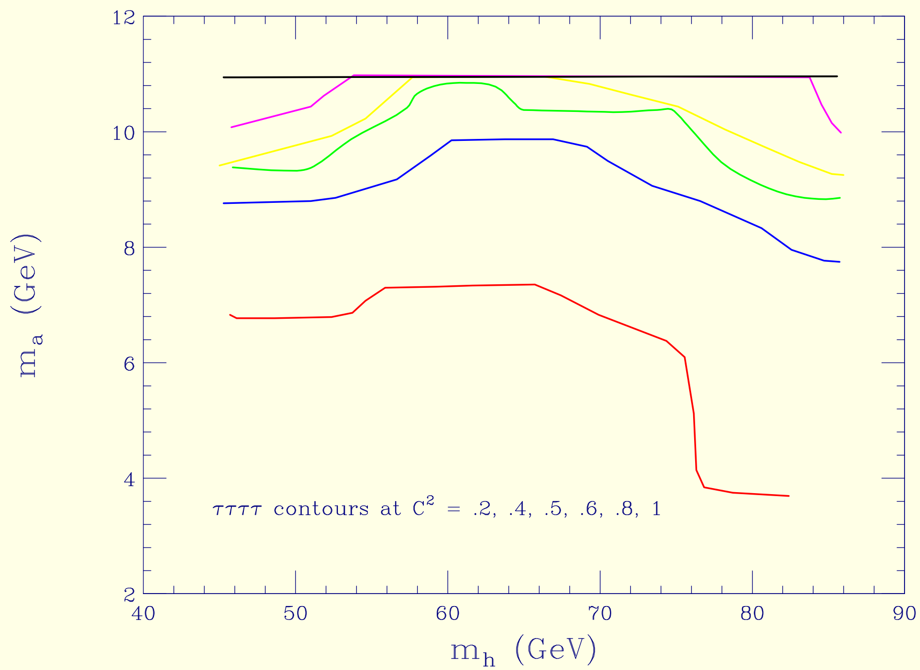


Figure 14: Contours of limits on $C^2 = [g^2_{Zh}/[g^2_{Zh}]_{SM}] \times BR(h \to aa) \times [BR(a \to \tau^+\tau^-)]^2$ at $C^2 = 0.2$, 0.4, 0.5, 0.6, 0.8 and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if $C^2 > 0.2$, then the region below the $C^2 = 0.2$ contour is excluded at 95% CL.

If the $a_1 a_1 \to (gg, q\overline{q}) + (gg, q\overline{q})$ decay is relevant, then we have the hadronic decay limits. They run out for $m_h > 80$ GeV.

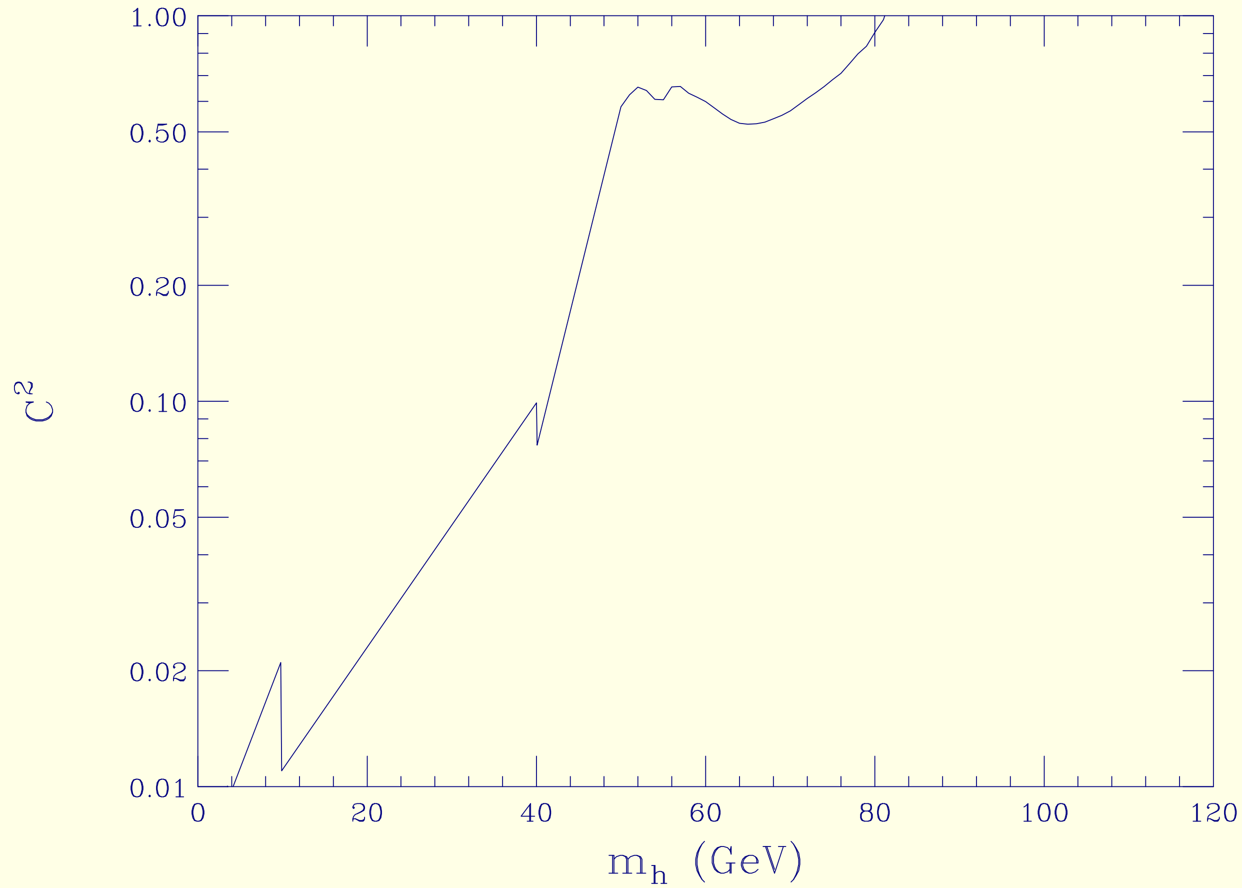


Figure 15: **Plot of the 95% CL limit on $C^2 = [g^2_{Zh}/[g^2_{Zh}]_{SM}] \times BR(h \to$ hadrons$)$, where h is only assumed to decay to hadrons, not any specific number of jets.**

Or, if we say that the gg or $q\overline{q}$ from each a_1 overlap to form a single 'jet', then we have the limits in the 'jet-jet' channel. They give $C^2 \lesssim 0.4$ for $m_h \sim 100~\mathrm{GeV}$ and might be relevant. Points mentioned later evade this limit since not enough $2-j$-like.

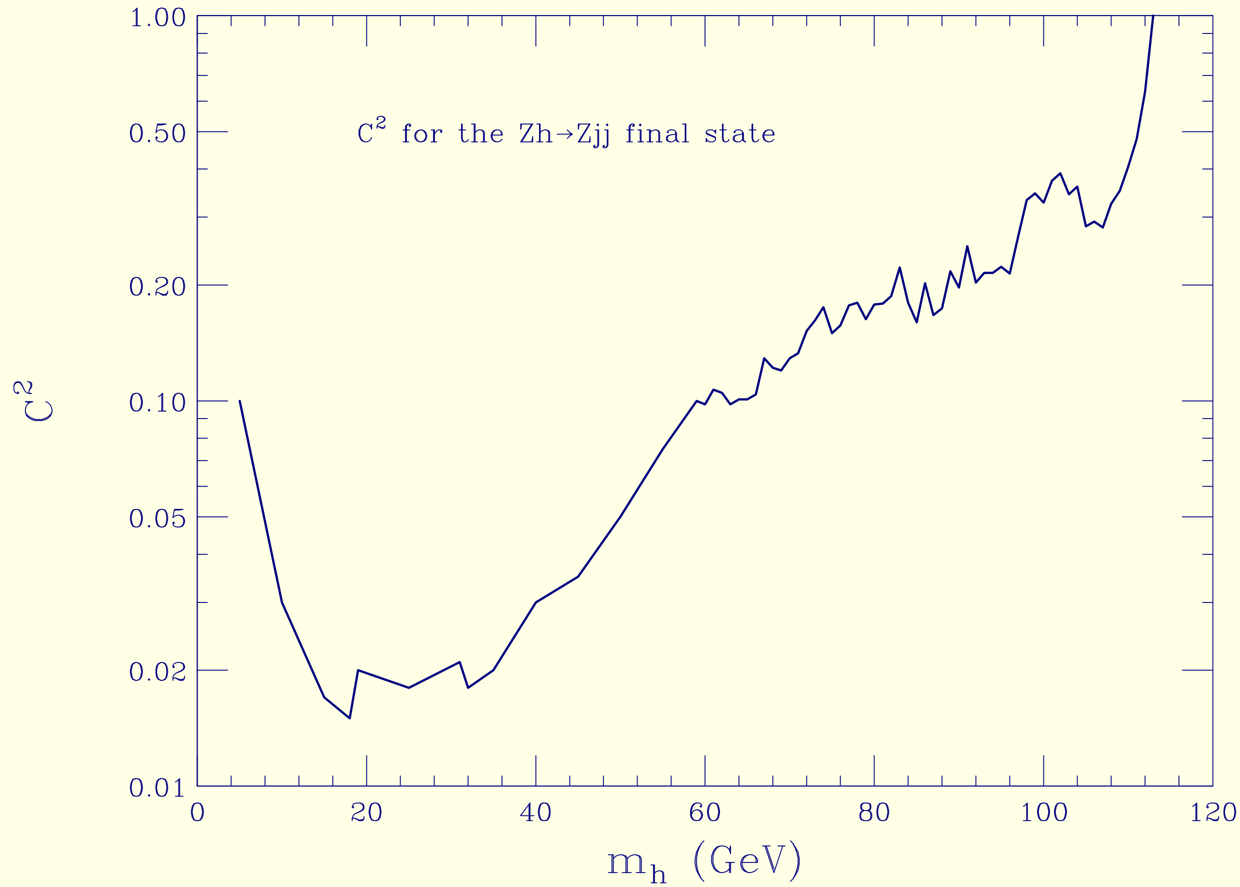


Figure 16: 95% CL upper limit on $C^2 = [g^2_{Zh}/[g^2_{Zh}]_{SM}] \times BR(h \to jj)$ from LEP analyzes.

- **To see how well the $F < 10$, $m_{a_1} < 2m_b$ points describe the LEP excesses we have to run them through the full LHWG code. Well, we didn't do it, but Philip Bechtle did it for us.**

  In Table 1, we give the precise masses and branching ratios of the h_1 and a_1 for all the $F < 10$ points in our early scans ($=$ those analyzed by Bechtle).

  We also give the number of standard deviations, $n_{\rm obs}$ ($n_{\rm exp}$) by which the observed rate (expected rate obtained for the predicted signal+background) exceeds the predicted background. The numbers are obtained after full processing of all Zh final states using the preliminary LHWG analysis code (thanks to P. Bechtle). They are derived from $(1 - CL_b)_{\rm observed}$ and $(1 - CL_b)_{\rm expected}$ using the usual tables: e.g. $(1 - CL_b) = 0.32$, 0.045, 0.0027 correspond to 1σ, 2σ, 3σ excesses, respectively.

  The quantity $s95$ is the factor by which the signal predicted in a given case would have to be multiplied in order to exceed the 95% CL. All these quantities are obtained by processing each scenario through the full preliminary LHWG confidence level/likelihood analysis.

m_{h_1}/m_{a_1} (GeV)	Branching Ratios $h_1 \to b\bar{b}$	$h_1 \to a_1 a_1$	$a_1 \to \tau\bar{\tau}$	$n_{\rm obs}/n_{\rm exp}$ units of 1σ	$s95$	N_{SD}^{LHC}
98.0/2.6	0.062	0.926	0.000	2.25/1.72	2.79	1.2
100.0/9.3	0.075	0.910	0.852	1.98/1.88	2.40	1.5
100.2/3.1	0.141	0.832	0.000	2.26/2.78	1.31	2.5
102.0/7.3	0.095	0.887	0.923	1.44/2.08	1.58	1.6
102.2/3.6	0.177	0.789	0.814	1.80/3.12	1.03	3.3
102.4/9.0	0.173	0.793	0.875	1.79/3.03	1.07	3.6
102.5/5.4	0.128	0.848	0.938	1.64/2.46	1.24	2.4
105.0/5.3	0.062	0.926	0.938	1.11/1.52	2.74	1.2

Table 1: Some properties of the h_1 and a_1 for the eight allowed points with $F < 10$ and $m_{a_1} < 2m_b$ from our $\tan\beta = 10$, $M_{1,2,3}(m_Z) = 100, 200, 300~{\rm GeV}$ NMSSM scan. N_{SD}^{LHC} is the statistical significance of the best "standard" LHC Higgs detection channel for integrated luminosity of $L = 300~{\rm fb}^{-1}$.

Comments

- **If $n_{\rm exp}$ is larger than $n_{\rm obs}$ then the excess predicted by the signal plus background Monte Carlo is larger than the excess actually observed and vice versa.**
- **The points with $m_{h_1} \lesssim 100~{\rm GeV}$ have the largest $n_{\rm obs}$.**

Two of these have $m_{a_1} < 2m_\tau$ (i.e. $a_1 \to jets$) and one has $a_1 \to \tau\overline{\tau}$. This is typical of the still larger scans performed since this table was constructed.

- Point 2 gives the best consistency between $n_{\rm obs}$ and $n_{\rm exp}$, with a predicted excess only slightly smaller than that observed.
- Points 1 and 3 also show substantial consistency.
- For the 4th and 7th points, the predicted excess is only modestly larger (roughly within 1σ) compared to that observed.
- The 5th and 6th points are very close to the 95% CL borderline and have a predicted signal that is significantly larger than the excess observed.
- LEP is not very sensitive to point 8.

Thus, a significant fraction of the $F < 10$ points are very consistent with the observed event excess.

- In our scan there are many, many points that satisfy all constraints and have $m_{a_1} < 2m_b$. The remarkable result is that those with $F < 10$ have a substantial probability that they predict the Higgs boson properties that would imply a LEP $Zh \to Z + b$'s excess of the sort seen.

More Details on A_κ–A_λ Possible Fine-Tuning

- The fine-tuning measure for $m_{a_1}^2$ relative to GUT scale parameters that is completely analogous to that employed for EWSB is

$$F_{m_{a_1}} \equiv \max_p f_p \,, \quad \text{with} \quad f_p \equiv \frac{d \log m_{a_1}^2}{d \log p} \,, \tag{12}$$

where p is any GUT-scale parameter,

$$p = M_i \; (i = 1, 2, 3), \; m_{H_u}^2, \; m_{H_d}^2, \; m_S^2, \; A_t, \; A_\kappa, \; A_\lambda, \; m_Q^2, \; m_U^2, \; m_D^2, \; \ldots \tag{13}$$

(parameters without an argument are those defined at the GUT scale).

- Simplification: only interested for cases where

$$m_Z^2 = 2\left[-\lambda^2(m_Z)s^2(m_Z) + \frac{m_{H_d}^2(m_Z) - \tan^2\beta(m_Z) m_{H_u}^2(m_Z)}{\tan^2\beta(m_Z) - 1}\right] \tag{14}$$

is insensitive to the parameters p.

Small fine tuning for m_Z^2 means that $v(m_Z)$ (which sets m_Z), $s(m_Z)$, $\tan\beta(m_Z)$, $m_{H_u}^2(m_Z)$ and $m_{H_d}^2(m_Z)$ are not fine-tuned with respect to the various p.

$\Rightarrow$ The only additional parameters upon which $m_{a_1}^2$ depends that could still be sensitive to the GUT-scale parameters p when m_Z is not are $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$.

$\Rightarrow$ For any of the p, we can then approximate

$$f_p \sim \frac{p}{A_\lambda(m_Z)} F_{A_\lambda} \frac{dA_\lambda(m_Z)}{dp} + \frac{p}{A_\kappa(m_Z)} F_{A_\kappa} \frac{dA_\kappa(m_Z)}{dp}\,. \qquad (15)$$

- $\Rightarrow$ Must solve the RG equations and express $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$ in terms of the GUT-scale values of all the soft-SUSY-breaking parameters.

  The solution depends on λ, κ and $\tan\beta$. Example: $\lambda = 0.2$, $\kappa = \pm 0.2$ and $\tan\beta = 10$.

$$A_\lambda(m_Z) = -0.03 A_\kappa + 0.93 A_\lambda - 0.35 A_t - 0.03 M_1 - 0.37 M_2 + 0.66 M_3, \qquad (16)$$

$$A_\kappa(m_Z) = 0.90 A_\kappa - 0.11 A_\lambda + 0.02 A_t + 0.003 M_1 + 0.025 M_2 - 0.017 M_3, \qquad (17)$$

If there is a p_λ that dominates $A_\lambda(m_Z)$ and a p_κ that dominates $A_\kappa(m_Z)$, and these are different, then

$$\frac{p_\lambda}{A_\lambda(m_Z)}\frac{dA_\lambda(m_Z)}{dp_\lambda} \sim \mathcal{O}(1) , \quad \text{and} \quad \frac{p_\kappa}{A_\kappa(m_Z)}\frac{dA_\kappa(m_Z)}{dp_\kappa} \sim \mathcal{O}(1) , \tag{18}$$

and roughly $F_{m_{a_1}} \sim F_{MAX}$.

If the same p dominates both $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$ then

$$F_{m_{a_1}} \sim f_p \sim F_{A_\lambda} + F_{A_\kappa} . \tag{19}$$

This result also holds if the GUT-scale parameters are correlated.

For example, universal gaugino masses and zero trilinear couplings at the GUT scale, for which $A_\lambda(m_Z) = 0.26 M_{1/2}$ and $A_\kappa(m_Z) = 0.01 M_{1/2}$. Then,

$$\frac{M_{1/2}}{A_\lambda(m_Z)}\frac{dA_\lambda(m_Z)}{dM_{1/2}} = \frac{M_{1/2}}{A_\kappa(m_Z)}\frac{dA_\kappa(m_Z)}{dM_{1/2}} = 1 \tag{20}$$

and it is quite precisely the case that

$$F_{m_{a_1}} = f_{M_{1/2}} \sim F_{A_\lambda} + F_{A_\kappa} \tag{21}$$

- This is good since F_{A_λ} and F_{A_κ} can be opposite in sign and of similar magnitude.

  In particular, this is the case when the approximate formula of Eq. (8) applies.

  When it does, $m_{a_1}^2$ is approximately linear in $A_\lambda(m_Z)$ and $A_\kappa(m_Z) \Rightarrow$

$$F_{A_\lambda} + F_{A_\kappa} \sim 1\,. \tag{22}$$

  Then, the measure of fine-tuning wrp to GUT scale parameters, $F_{m_{a_1}}$ will be small,

$$F_{m_{a_1}} \sim F_{A_\lambda} + F_{A_\kappa}\,, \tag{23}$$

  whenever the gaugino masses are correlated (in any way, $e.g.$ through $M_{1/2}$) and trilinears (at the GUT scale) are small.

  The same applies whenever a single term dominates in Eqs. (16) and (17).

  Many models fall into one or the other of these categories.

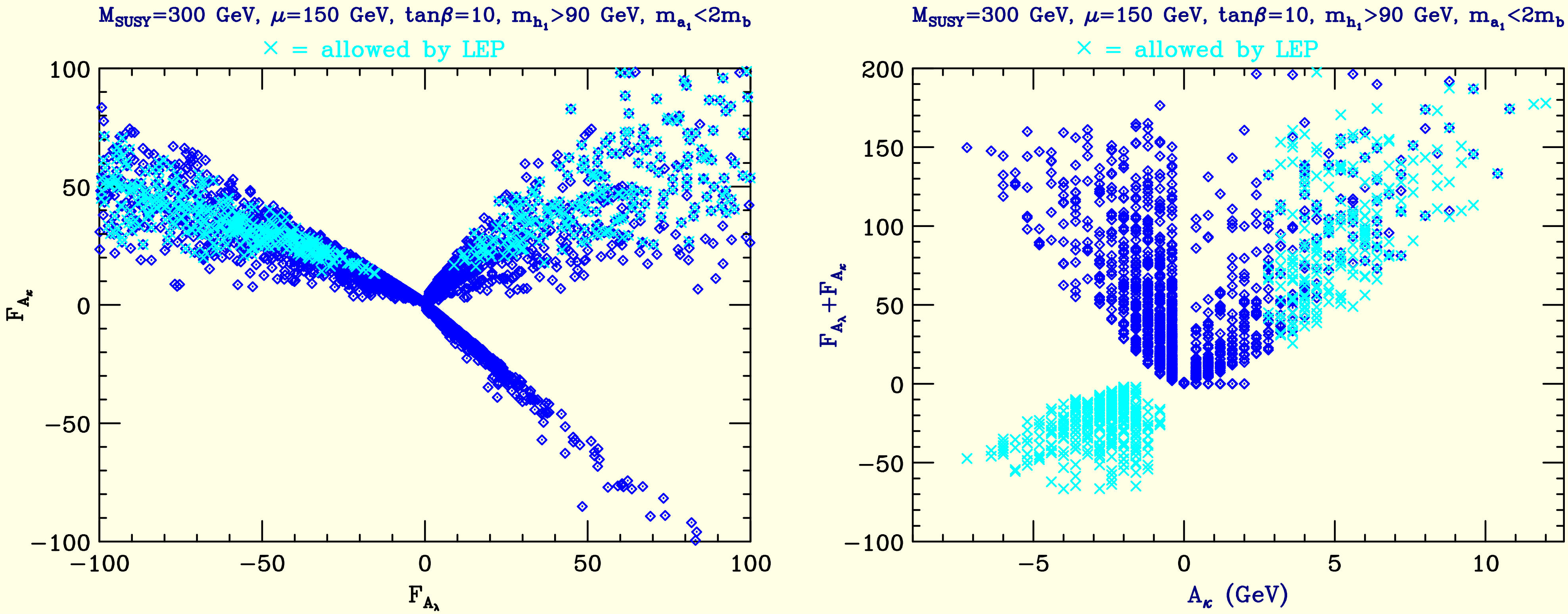


Figure 17: In the left-hand frame, we plot F_{A_κ} vs. F_{A_λ} for the points with $F_{MAX} < 100$. In the right-hand frame, we plot $F_{A_\lambda} + F_{A_\kappa}$ vs. A_κ for these same points.

Note the region of LEP-OK points with $A_\kappa < 0$ for which $F_{A_\kappa} \sim -F_{A_\lambda}$ yielding $F_{A_\kappa} + F_{A_\lambda} \sim 0$.

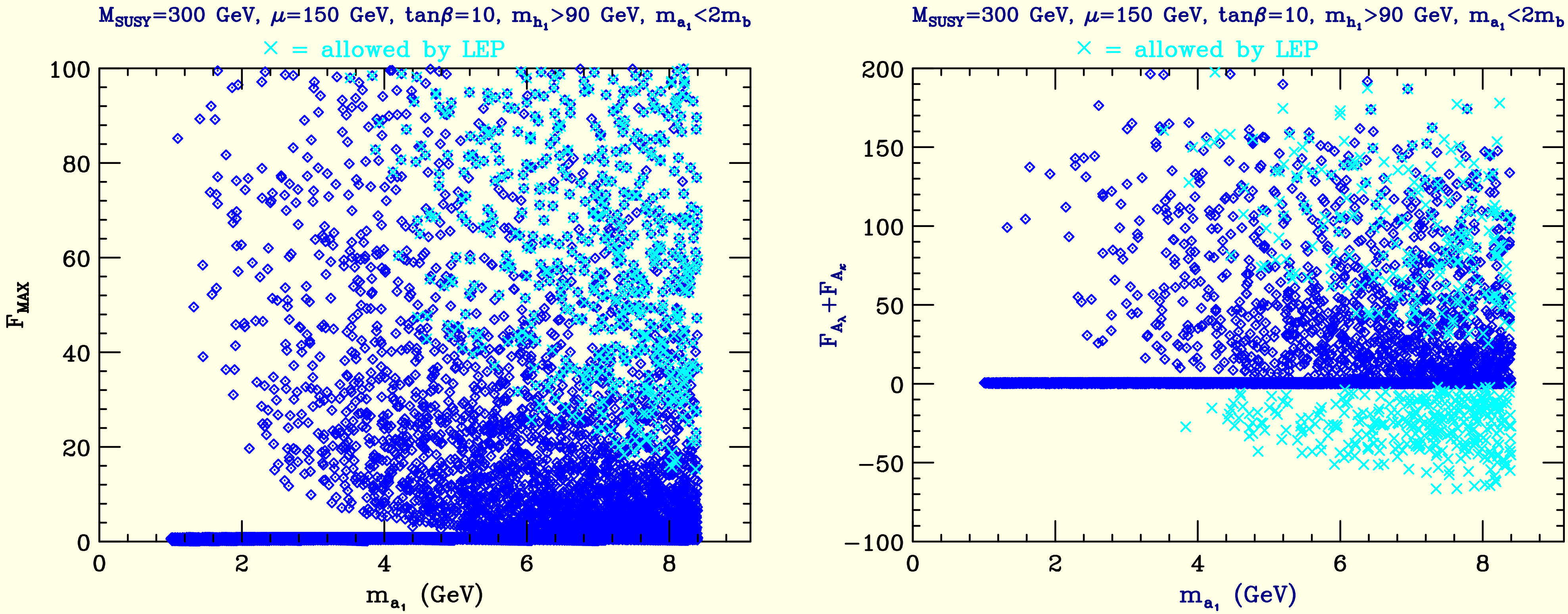


Figure 18: In the left-hand frame, we plot F_{MAX} vs. m_{a_1} for the points with $F_{MAX} < 100$. In the right-hand frame, we plot $F_{A_\lambda} + F_{A_\kappa}$ vs. m_{a_1} for these same points.

Note that $m_{a_1} > 2m_\tau$ is preferred to get small $F_{A_\kappa} + F_{A_\lambda}$.

- **Even more generically, tuning in $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$ is completely unnecessary to achieve a light CP-odd Higgs boson in models with specific relations among GUT-scale parameters.**

  Any SUSY breaking scenario that determines the other soft trilinear couplings (in particular, A_t) and gaugino masses from a SUSY breaking scale will automatically give $A_\lambda(m_Z) = c_\lambda M_{SUSY}$ and $A_\kappa(m_Z) = c_\kappa M_{SUSY}$, where c_λ and c_κ depend (given the known values of g_1 and g_2) only on the couplings λ, κ and λ_t (equivalently, $\tan\beta$, given the known value of m_W).

  One gets

  $$m_{a_1}^2 = f(\lambda, \kappa, \tan\beta) M_{SUSY}^2 \tag{24}$$

  and m_{a_1} will either be small or not.

  This holds even if there are large cancellations among the RG contributions to $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$. Whether or not a light CP odd Higgs boson is possible simply depends on the above couplings.

- **Let us now turn our attention to $B(h_1 \to a_1 a_1)$.**

  It is important to understand the limit in which $A_\lambda, A_\kappa \to 0$. This is nicely illustrated in the case of $v/s \ll 1$.

One finds (in terms of m_Z-scale parameters):

$$\begin{aligned} &\Gamma(h_1 \to a_1 a_1) \\ \sim \quad & \frac{m_W^2}{32\pi g_2^2 m_{h_1}} \left[2\lambda^2 + 2\lambda\kappa \sin 2\beta - \frac{4\lambda\kappa s}{v} \cos\theta_A + \frac{4\kappa^2 s}{v} \sin\theta_S + \mathcal{O}\left(v^2/s^2\right) \right]^2 \\ \sim \quad & \frac{m_W^2}{32\pi g_2^2 m_{h_1}} \left[\mathcal{O}\left(v^2/s^2\right) \right]^2 \end{aligned} \quad (25)$$

i.e. it does not actually vanish for $A_\lambda(m_Z) = A_\kappa(m_Z) = 0$ (there is no exact symmetry argument).

Nonetheless, the above v^2/s^2 suppression $\Rightarrow$ for $A_\lambda(m_Z) = A_\kappa(m_Z) = 0$, $B(h_1 \to a_1 a_1)$ is always quite small.

- Thus, having an adequate size for $B(h_1 \to a_1 a_1)$ for escaping the LEP constraints depends critically upon having non-zero values for $A_\kappa(m_Z)$ and, in particular, $A_\lambda(m_Z)$.

  Don't forget: Magnitudes for $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$ of order those developed from RGE running beginning with $A_\lambda(M_U) = A_\kappa(M_U) = 0$ are sufficient to give large $B(h_1 \to a_1 a_1)$.

Collider Implications

- An important question is the extent to which the type of $h \to aa$ Higgs scenario (whether NMSSM or other) described here can be explored at the Tevatron, the LHC and a future e^+e^- linear collider.

  At the first level of thought, the $h_1 \to a_1 a_1$ decay mode renders inadequate the usual Higgs search modes that might allow h_1 discovery at the LHC.

  Since the other NMSSM Higgs bosons are rather heavy and have couplings to b quarks that are not greatly enhanced, they too cannot be detected at the LHC. The last column of Table 1 shows the statistical significance of the most significant signal for any of the NMSSM Higgs bosons in the "standard" SM/MSSM search channels for the eight $F < 10$ NMSSM parameter choices.

  For the h_1 and a_1, the most important detection channels are $h_1 \to \gamma\gamma$, $Wh_1 + t\bar{t}h_1 \to \gamma\gamma\ell^{\pm}X$, $t\bar{t}h_1/a_1 \to t\bar{t}b\bar{b}$, $b\bar{b}h_1/a_1 \to b\bar{b}\tau^+\tau^-$ and $WW \to h_1 \to \tau^+\tau^-$.

  Even after $L = 300~\mathrm{fb}^{-1}$ of accumulated luminosity, the typical maximal signal strength is at best 3.5σ. For the eight points of Table 1, this largest signal derives from the $Wh_1 + t\bar{t}h_1 \to \gamma\gamma\ell^{\pm}X$ channel.

There is a clear need to develop detection modes sensitive to the $h_1 \to a_1a_1 \to \tau^+\tau^-\tau^+\tau^-$ and (unfortunately) $4j$ decay channels.

I will focus on 4τ in my discussion of possibilities below, but keep in mind the $4j$ case.

Hadron Colliders

The LHC

1. An obvious possibility is $WW \to h_1 \to a_1a_1 \to 4\tau$.

   Study under way with Schumacher. Looks moderately promising but far from definitive results at this time.

2. Another mode is $t\bar{t}h_1 \to t\bar{t}a_1a_1 \to t\bar{t}\tau^+\tau^-\tau^+\tau^-$.

   Study begun.

3. A third possibility: $\widetilde{\chi}_2^0 \to h_1\widetilde{\chi}_1^0$ with $h_1 \to a_1a_1 \to 4\tau$.

(Recall that the $\widetilde\chi_2^0 \to h_1\widetilde\chi_1^0$ channel provides a signal in the MSSM when $h_1 \to b\overline{b}$ decays are dominant.)

4. **Last, but definitely not least: diffractive production** $pp \to pph_1 \to ppX$.

   The mass M_X can be reconstructed with roughly a $1-2$ GeV resolution, potentially revealing a Higgs peak, independent of the decay of the Higgs.

   Preliminary results are that one expects about 3 clean, i.e. reconstructed and tagged, events per $30\ \mathrm{fb}^{-1}$ of luminosity. $\Rightarrow$ clearly a high luminosity game.

Tevatron

1. It is possible that Zh_1 and Wh_1 production, with $h_1 \to a_1a_1 \to 4\tau$, will provide a small signal. (Wacker et.al; JG, McElrath, Conway).

   Backgrounds can be made small, but efficiencies are low and one must simply accumulate enough events.

2. Wacker et. al. and JG+McElrath have considered $gg \to h_1 \to a_1a_1 \to 4\tau$ which would have substantially larger rate. But cuts etc. imply low efficiencies.

Wacker et. al. suggest hints are possible in the all lepton channel with $6\ \mathrm{fb}^{-1}$. We estimated $15\ \mathrm{fb}^{-1}$ would be needed for believable signal.

Further points

- If supersymmetry is detected at the Tevatron, but no Higgs is seen, and if LHC discovery of the h_1 remains uncertain, the question will arise of whether Tevatron running should be extended so as to allow eventual discovery of $h_1 \to 4\tau$.

  However, rates imply that the h_1 signal could only be seen if Tevatron running is extended until $L > 10 - 15\ \mathrm{fb}^{-1}$ (our estimates) has been accumulated.

  And, there is the risk that $m_{a_1} < 2m_\tau$, in which case Tevatron backgrounds to $a_1 a_1 \to 4 - jet$ would be impossibly large regardless of how the h_1 is produced.

- Of course, even if the LHC is unable to see any of the NMSSM Higgs bosons, it *would* observe numerous supersymmetry signals and *would* confirm that $WW \to WW$ scattering is perturbative, implying that something like a light Higgs boson must be present.

Lepton Colliders

- Of course, discovery of the h_1 will be straightforward at an e^+e^- linear collider via the inclusive $Zh \to \ell^+\ell^- X$ reconstructed M_X approach (which allows Higgs discovery independent of the Higgs decay mode).

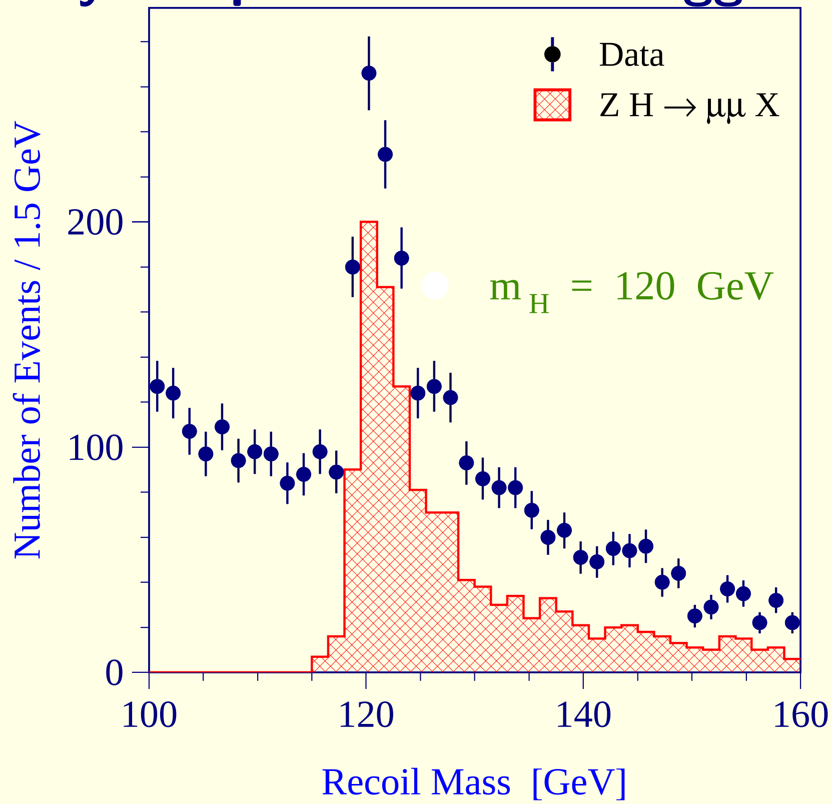


Figure 19: Decay-mode-independent Higgs M_X peak in the $Zh \to \mu^+\mu^- X$ mode for $L = 500~\mathrm{fb}^{-1}$ at $\sqrt{s} = 350~\mathrm{GeV}$, taking $m_h = 120~\mathrm{GeV}$..

There are lots of events in just the $\mu^+\mu^-$ channel (which you may want to restrict to since it has the best mass resolution).

- Although the $h \to b\bar{b}$ and $h \to \tau^+\tau^-$ rates are $1/10$ of the normal, the number of Higgs produced will be such that you can certainly see $Zh \to Zb\bar{b}$ and $Zh \to Z\tau^+\tau^-$ in a variety of Z decay modes.

  This is quite important, as it will allow you to subtract these modes off and get a determination of $B(h_1 \to a_1a_1)$, which will provide unique information about $\lambda, \kappa, A_\lambda, A_\kappa$.

- Presumably direct detection in the $Zh \to Za_1a_1 \to Z4\tau$ mode will also be possible although I am unaware of any actual studies.

  This would give a direct measurement of $B(h_1 \to a_1a_1 \to \tau^+\tau^-\tau^+\tau^-)$. Error?

- Coupled with the indirect measurement of $B(h_1 \to a_1a_1)$ from subtracting the direct $b\bar{b}$ and $\tau^+\tau^-$ modes would give a measurement of $B(a_1 \to \tau^+\tau^-)$.

  This would allow a first unfolding of information about the a_1 itself.

Of course, the above assumes we have accounted for all modes.

- Maybe, given the large event rate, one could even get a handle on modes such as $h_1 \to a_1 a_1 \to \tau^+ \tau^- jj$ ($j = c, g$), thereby getting still more cross checks.

- At a $\gamma\gamma$ collider, the $\gamma\gamma \to h_1 \to 4\tau$ signal will be easily seen (Gunion, Szleper).

  This could help provide still more information about the h.

- In contrast, since (as already noted) the a_1 in these low-F NMSSM scenarios is fairly singlet in nature, its *direct* (i.e. not in h_1 decays) detection will be very challenging even at the ILC.

- Further, the low-F points are all such that the other Higgs bosons are fairly heavy, typically above $400~\mathrm{GeV}$ in mass, and essentially inaccessible at both the LHC and all but a $\gtrsim 1~\mathrm{TeV}$ ILC.

A few notes on $m_{a_1} > 2m_b$.

- We should perhaps also not take describing the LEP excess and achieving extremely low fine tuning overly seriously.

  Indeed, scenarios with $m_{h_1} > 114~\mathrm{GeV}$ (automatically out of the reach of LEP) begin at a still modest (relative to the MSSM) $F \gtrsim 25$.

  In fact, one can probably push down to as low as $m_{h_1} \gtrsim 108 \div 110~\mathrm{GeV}$ when $m_{a_1} > 2m_b$.

  $\Rightarrow$ must be on the lookout for the $4b$ and $2b2\tau$ final states from h_1 decay, with $h_1 \to 4b$ being the largest when $m_{a_1} > 2m_b$.

- At the LHC, the modes that seem to hold some promise are:
  1. $WW \to h_1 \to a_1 a_1 \to b\bar{b}\tau^+\tau^-$.
     Our (JFG, Ellwanger, Hugonie, Moretti) work suggested some hope. Experimentalists (esp. D. Zerwas) are working on a fully realistic evaluation but are not that optimistic.
  2. $t\bar{t}h_1 \to t\bar{t}a_1 a_1 \to t\bar{t}4b$.
     This I imagine will be viable.

3. Gluino cascades containing $\widetilde{\chi}_2^0 \to h_1 \widetilde{\chi}_1^0$.
4. Doubly diffractive $pp \to pph_1$ followed by $h_1 \to a_1 a_1 \to 4b$ or $2b2\tau$.
   These modes are also under consideration by JFG, Khoze, ...

- At the Tevatron, perhaps the lack of overlapping events and lower background rates might allow some sign of a signal in modes such as Wh_1 and Zh_1 production with $h_1 \to a_1 a_1 \to 4b$ or $2b2\tau$. There is a study underway by G. Huang, Tao Han and collaborators.

General Considerations

- We should note that much of the discussion above regarding Higgs discovery is quite generic. Whether the a is truly the NMSSM CP-odd a_1 or just a lighter Higgs boson into which the SM-like h pair-decays, hadron collider detection of the h in its $h \to aa$ decay mode will be very challenging — only an e^+e^- linear collider can currently guarantee its discovery.

Conclusions

- The NMSSM naturally has small fine-tuning of all types, *i.e.* for:

  1) EWSB, *i.e.* m_Z^2

  2) small $m_{a_1}^2$, as needed for 1), and (simultaneously) large $B(h_1 \to a_1 a_1)$.

- If low fine-tuning is imposed for an acceptable SUSY model, we should expect:

  – a $m_{h_1} \sim 100~\mathrm{GeV}$ Higgs decaying via $h_1 \to a_1 a_1$.
  Higgs detection will be quite challenging at a hadron collider.
  Higgs detection at the ILC is easy using the missing mass $e^+e^- \to ZX$ method of looking for a peak in M_X.
  Higgs detection in $\gamma\gamma \to h_1 \to a_1 a_1$ will be easy.

  – The very smallest F values are attained when:

    * h_2 and h_3 have "moderate" mass, i.e. in the $300~\mathrm{GeV}$ to $700~\mathrm{GeV}$ mass range;
    * the a_1 mass is $< 2m_b$ (with $m_{a_1} > 2m_\tau$ preferred for lowest A_κ–A_λ tuning wrp to GUT parameters) and the a_1 has a substantial singlet component.

- * the stops and other squarks are light;
  * the gluino, and, by implication assuming conventional mass orderings, the wino and bino all have modest mass;

- Detailed studies of the $WW \to h_1 \to a_1 a_1$, $t\bar{t}h_1 \to t\bar{t}a_1 a_1$, diffractive $pp \to pph_1$ and $\widetilde{g}$ cascades with $\widetilde{\chi}_2^0 \to h_1 \widetilde{\chi}_1^0$ channels (with $h_1 \to 4b$ or 4τ) by the experimental groups at both the Tevatron and the LHC should receive significant priority.

- Even if the LHC sees the Higgs $h_1 \to a_1 a_1$ directly, it will not be able to get much detail. Only the ILC can do the detailed measurements needed to verify the model in detail.

- It is likely that other models in which the MSSM μ parameter is generated using additional scalar fields can achieve small fine-tuning in a manner similar to the NMSSM.

- In general, very natural solutions to the fine-tuning and little hierarchy problems are possible in relatively simple extensions of the MSSM.

  One does not have to employ more radical approaches or give up on small fine-tuning!

  Further, small fine-tuning probably requires a light SUSY spectrum in all such models and SUSY should be easily explored at both the LHC (and very possibly the Tevatron) and the ILC and $\gamma\gamma$ colliders.

Only Higgs detection at the LHC will be a real challenge.

Ability to check perturbativity of $WW \to WW$ at the LHC might prove to be very crucial to make sure that there really is a light Higgs accompanying light SUSY.

- A light a_1 allows for a light $\widetilde{\chi}_1^0$ to be responsible for dark matter of correct relic density: annihilation would be via $\widetilde{\chi}_1^0\widetilde{\chi}_1^0 \to a_1$. Only the ILC could have any chance of measuring the properties of the very light $\widetilde{\chi}_1^0$ and of the a_1 in sufficient detail to verify that it all fits together.