UV Finite Approach to Quantum Gravity

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Introduction

- New approach to quantum gravity
- Based on resummation of Feynman graphs
- See papers by BFLW, S. Jadach et al.: hep-ph/0503189; JCAP 0402(2004)011; MPL A19(2004)143; A17 (2002)2371; CPC 130(2000)260; and references therein

Topics of Discussion

- Preliminaries
- Review of Feynman's Formulation of Einstein's Theory
- Resummed Quantum Gravity
- Massive Elementary Particles and Black Holes: Hawking Radiation to Planck Scale Remnants
- Conclusions

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Preliminaries

- Newton's Law: Most Basic – Taught To All Beginning Students
- Albert Einstein: Special Case of Classical General Relativity

$g_{00}=1+2\varphi \Rightarrow \nabla^2\varphi = 4\pi G_N \rho$

from

$R^{\alpha\gamma} - \frac{1}{2} g^{\alpha\gamma} R = -8\pi G_N T^{\alpha\gamma}$

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Quantum Mechanics

- Heisenberg & Schrödinger, following Bohr
- Tremendous progress: Quantum Field Theory, Superstrings, Loop Quantum Gravity, etc.
- NO SATISFACTORY QUANTUM TREATMENT OF NEWTON'S LAW IS KNOWN TO BE PHENOMENOLOGICALLY CORRECT

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Today’s Talk

- New Approach: Building on work by Feynman (Acta Phys. Pol.**24**(1963)697; *Feynman Lectures on Gravitation*, eds. Moringo and Wagner, 1971)
- BASIC IDEA: QG is a point particle field theory – Bad UV due to our NAIVETE.
- Union of Bohr & Einstein Possible

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Four Approaches to QG

Weinberg (in *Gen. Rel.*, p.790)

- Extended Theories: Susy – Superstrings; Loop QG, etc.
- Resummation ⇐ Today's Talk – New Version
- Composite Gravitons
- Asymptotic Safety: Fixed Pt. Theory (See Lautscher & Reuter ;Bonnanno & Reuter, hep-th/0205062, PRD62(2000)043008; I. Shapiro et al., JCAP0501(2005)012; D.Litim,PRL92(2004)201301; Percaci&Perini,PRD68(2003)044018, and refs. therein)

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BASIC STRATEGY:

- INSIDE FEYNMAN LOOP INTEGRAL – LARGE CONTRIBUTION REGIMES

  1. ULTRA-VIOLET (UV)

  2. INFRARED (IR)

  3. COLLINEAR (CL)

- RESUM IR

  1. UV ⇔ RENORMALIZATION

  2. CL ⇔ BOTH ABELIAN & NON-ABELIAN -- LL

  3. IR ⇔ DOMINANT TERMS RESUMMABLE

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New Approach: Resummed QG

- Resummation Cures QG BAD UV
- Consequences & Tests:

(1) Quantum corrections to Newton's Law

(2) SM pt. particle of mass $m \neq 0 \Rightarrow$
$r_S = 2(m/M_{Pl}^2) \Rightarrow$ Black Hole in Einstein's classical theory – is this true in QG?

(3) Fate of Hawking Radiation for a Massive Classical Black Hole – Planck Scale Remnants?

⋮

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Review of Feynman's Formulation of Einstein's Theory

- Known world,

$$\mathcal{L}(x) = -\frac{1}{2\kappa^2}\sqrt{-g}\,R + \sqrt{-g}\,L^{G}_{SM}$$

- R is the curvature scalar
- $-g = -\det g_{\mu\nu}$
- $\kappa = \sqrt{8\pi G_N} \equiv \sqrt{8\pi / M^2_{Pl}}$
- SM Lagrangian density $= L^{G}_{SM}$

One gets L^{G}_{SM} from usual L_{SM} as follows:

- $\partial_\mu \phi$ generally covariant for scalars
- $\partial_\mu A^J_\nu - \partial_\nu A^J_\mu$ generally covariant for vectors and these are all derivatives of vectors we have in SM.
- For fermions, use standard differentiable structure approach.

$L^{G}_{SM} \Rightarrow L^{G}_{SM}$(scalar), show relevant dynamics.

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$$
\begin{aligned}
\mathcal{L}(x) &= -\frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g} \\
&= \quad \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'}\right\} \\
&\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right] \\
&\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots
\end{aligned}
\tag{2}
$$

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where $\varphi_{,\mu} \equiv \partial_\mu \varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = diag\{1,-1,-1,-1\}$
- $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho\right)$ for any tensor $y_{\mu\nu}$
- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge, $\partial^\mu \bar{h}_{\nu\mu} = 0$

$\Leftrightarrow$ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in Fig. 1.

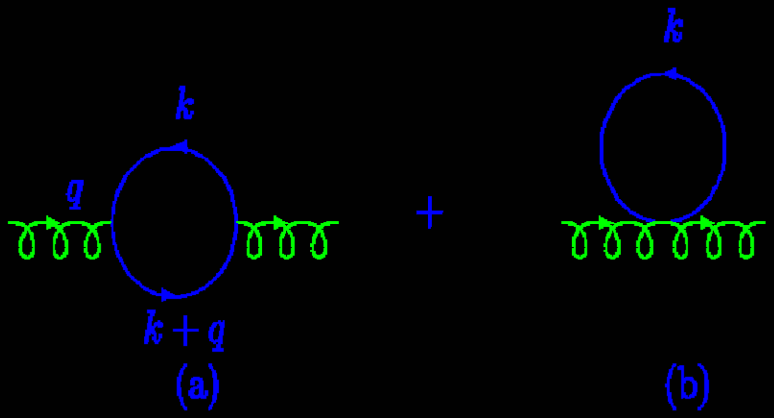

Figure 1: The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

These graphs already illustrate the QG's BAD UV behavior.

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UV DIVERGENCE DEGREES

- (a) AND (b) HAVE SUPERFICIAL $D = +4$
- AFTER TAKING GAUGE INVARIANCE INTO ACCOUNT, WE STILL EXPECT $D_{eff} \geq 0$
- HIGHER LOOPS GIVE HIGHER VALUES OF D_{eff}
- CONCLUSION: QG IS NONRENORMALIZABLE

WE SHOW THESE ESTIMATES EXPLICITLY SHORTLY.

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QG04

PHYSICAL EFFECT

DEEP UV EUCLIDEAN REGIME OF FEYNMAN INTEGRAL

- GRAVITATIONAL FORCE IS ATTRACTIVE AND $\propto$ TO MASS^2
- DEEP UV EUCLIDEAN REGIME $\Leftrightarrow$ LARGE NEGATIVE MASS^2
- IN THIS REGIME, GRAVITY IS REPULSIVE
- PROPAGATION BETWEEN TWO DEEP EUCLIDEAN POINTS SEVERELY SUPPRESSED IN EXACT SOLUTIONS OF THE THEORY $\Rightarrow$
- RESUM LARGE SOFT GRAVITON EFFECTS TO GET MORE PHYSICALLY CORRECT RESULTS

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Resumed Quantum Gravity

- YFS RESUM PROPAGATORS: EW

$$\Sigma_F(p) = e^{\alpha B''_\gamma}\left[\Sigma'_F(p) - S_F^{-1}(p)\right] + S_F^{-1}(p) \qquad (3)$$

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'_F(p)} \qquad (4)$$

$$\Sigma'_F(p) = \sum_{n=0}^{\infty} \Sigma'_{Fn}(p) \qquad (5)$$

QG, WE NEED ANALOG OF

$$\alpha B''_\gamma = \frac{\int d^4\ell\, S''(k,k,\ell)}{(2\pi)^4(\ell^2 - \lambda^2 + i\varepsilon)} \qquad (6)$$

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WHERE $\lambda \equiv$ IR CUT-OFF AND

$$S''(k,k,\ell) = \frac{-i8\alpha}{(2\pi)^3}\frac{kk'}{(\ell^2-2\ell k+\Delta+i\epsilon)(\ell^2-2\ell k'+\Delta'+i\epsilon)}\Big|_{k=k'}, \quad (7)$$

$\Delta = k^2 - m^2, \Delta' = k'^2 - m^2.$

TO THIS END, NOTE ALSO

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4}\frac{-i\eta^{\mu\nu}}{(\ell^2-\lambda^2+i\epsilon)}\frac{-ie(2ik_\mu)}{(\ell^2-2\ell k+\Delta+i\epsilon)}\frac{-ie(2ik'_\nu)}{(\ell^2-2\ell k'+\Delta'+i\epsilon)}\Big|_{k=k'}. \quad (8)$$

WE FOLLOW WEINBERG/THE FEYNMAN RULES

AND IDENTIFY THE CONSERVED GRAVITON CHARGES AS

$e \to \kappa k_\rho$ FOR SOFT EMISSION FROM k

$\Rightarrow$ WE GET THE ANALOGUE , $-B''_g(k)$, OF $\alpha B''_\gamma$ BY

- REPLACING THE γ PROPAGATOR IN (8) BY THE GRAVITON

PROPAGATOR,

$$\frac{i\frac{1}{2}(\eta^{\mu\nu}\eta^{\bar\mu\bar\nu}+\eta^{\mu\bar\nu}\eta^{\bar\mu\nu}-\eta^{\mu\bar\mu}\eta^{\nu\bar\nu})}{\ell^2-\lambda^2+i\epsilon}$$

,

- BY REPLACING THE QED CHARGES BY THE CORRESPONDING GRAVITY CHARGES $\kappa k_{\bar\mu}$, $\kappa k'_{\bar\nu}$

$\Rightarrow$

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4}\frac{1}{\ell^2-\lambda^2+i\epsilon}\frac{1}{(\ell^2+2\ell k+\Delta+i\epsilon)^2} \quad (9)$$

AND

$$i\Delta'_F(k)|_{Resummed} = \frac{ie^{B''_g(k)}}{(k^2-m^2-\Sigma'_s+i\epsilon)} \quad (10)$$

THIS IS THE BASIC RESULT.

NOTE THE FOLLOWING:

- Σ'_s STARTS IN $\mathcal{O}(\kappa^2)$, SO WE MAY DROP IT IN CALCULATING ONE-LOOP EFFECTS.

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- EXPLICIT EVALUATION GIVES,

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \qquad (11)$$

$\Rightarrow$ THE RESUMMED PROPAGATOR FALLS FASTER THAN ANY POWER OF $|k^2|$!

- IF m VANISHES, USING THE USUAL $-\mu^2$ NORMALIZATION POINT WE GET $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|k^2|}\right)$ WHICH AGAIN VANISHES FASTER THAN ANY POWER OF $|k^2|$!

THIS MEANS THAT ONE-LOOP CORRECTIONS ARE FINITE!

INDEED, ALL QUANTUM GRAVITY LOOPS ARE UV FINITE!

All Orders Proof in MPLA**17**, 2371 (2002)

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Newton's Law

- Consider the one-loop corrections to Newton's law from Fig. 1 – they directly show the effects of resummation. Our resummed propagators ⇒ (k → $(ik^0, \vec{k})$)

$$\Sigma^{1a}_{\bar{\mu}\bar{\nu};\mu\nu} = \kappa^2 \frac{\int d^4k}{2(2\pi)^4} \frac{(k'_{\bar{\mu}} k_{\bar{\nu}} + k'_{\bar{\nu}} k_{\bar{\mu}}) e^{\frac{\kappa^2 |k'^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k'^2|}\right)}}{k'^2 - m^2 + i\varepsilon}$$

$$\frac{(k'_{\mu} k_{\nu} + k'_{\nu} k_{\mu}) e^{\frac{\kappa^2 |k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right)}}{k^2 - m^2 + i\varepsilon}$$

⇔ CONVERGENT; SO IS FIG. 1b.

In transv.-traceless space, graviton propagator denominator is

$$q^2 + \frac{1}{2} q^4 \Sigma^{T(2)} + i\varepsilon$$

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with the result

$$-\frac{1}{2}\Sigma^{T(2)} \cong \frac{c_2}{360\pi M_{Pl}^2} \quad \text{for}$$

$$c_2 = \int_0^\infty dx\, x^3 (1+x)^{-4-\lambda_c x} \cong 72.1,$$

$$\lambda_c = \frac{2m^2}{\pi M_{Pl}^2}.$$

$$\Rightarrow$$

$$\Phi_{Newton}(r) = -\frac{G_N M_1 M_2}{r}(1-e^{-ar}),$$

$$a = \frac{1}{\sqrt{-\frac{1}{2}\Sigma^{T(2)}}} \cong 3.96 M_{Pl}, \quad \text{when}$$

$$m \cong 120 \text{ GeV}.$$

$$\text{Sum over SM particles} \Rightarrow a_{eff} = 0.210 M_{Pl}$$

Many consequences, hep-ph/0602025 and refs. therein

CONTACT WITH AYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G_N(k) = G_N/(1+\frac{k^2}{a_{eff}^2})$$

⇒ FIXED POINT BEHAVIOR FOR
$k^2 \to \infty$,
IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONNANNO & REUTER IN PRD62(2000) 043008.

- OUR RESULT THAT AN ELEMENTARY PARTICLE HAS NO HORIZON ALSO AGREES WITH BONNANNO'S & REUTER'S RESULT THAT A BLACKHOLE WITH A MASS LESS THAN
$M_{cr} \sim M_{Pl}$
HAS NO HORIZON. BASIC PHYSICS: $G_N(k)$ VANISHES FOR $k^2 \to \infty$.

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QG04

- A FURTHER AGREEMENT: FINAL STATE OF HAWKING RADIATIO
  ORIGINALLY VERY MASSIVE BLACKHOLE
  BECAUSE OUR VALUE OF THE COEFFICIENT,
  $$\frac{1}{a_{eff}^2},$$
  OF k^2 IN THE DENOMINATOR OF $G_N(k)$
  AGREES WITH THAT FOUND BY BONNANNO & REUTER,
  IF WE USE THEIR PRESCRIPTION FOR THE
  RELATIONSHIP BETWEEN k AND r
  IN THE REGIME WHERE THE LAPSE FUNCTION VANISHES,
  WE GET THE SAME HAWKING RADIATION PHENOMEMNOLOGY
  THE BLACK HOLE EVAPORATES UNTIL IT REACHES A MASS
  $$M_{cr} \sim M_{Pl}$$
  AT WHICH THE BEKENSTEIN-HAWKING TEMPERATURE VANISH
  LEAVING A PLANCK SCALE REMNANT.
- FATE OF REMNANT? IN hep-ph/0503189 $\Rightarrow$ OUR QUANTUM LOO
  COMBINED WITH THE $G_N(r)$ OF B-R ACTUALLY THE HORIZON
  SCALE REMNANTIS OBVIATED SO THAT THERE ARE THEN NO
  REMNANTS AT ALL! – CONSISTENT WITH RECENT RESULTS OF

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TO WIT, IN THE METRIC CLASS

$$ds^2 = f(r)dt^2 - f(r)^{\;1} dr^2 - r^2 d\Omega^2 \qquad (26)$$

THE LAPSE FUNCTION IS, FROM B-R,

$$f(r) = 1 - \frac{2G_N(r)M}{r} = \frac{B(x)}{B(x)+2x^2}\Big|_{x=\frac{r}{G_N M}}, \qquad (27)$$

WHERE

$$B(x) = x^3 - 2x^2 + \Omega x + \gamma\Omega \qquad (28)$$

FOR

$$\Omega = \frac{\tilde{\omega}}{G_N M^2} = \frac{\tilde{\omega} M_{Pl}^2}{M^2}. \qquad (29)$$

AFTER H-RADIATING TO $M_{cr} \sim M_{Pl}$, QUANTUM LOOPS CHANGE THE $-2x^2$ IN $B(x)$ TO $-2\xi x^2$ WITH $\xi = \xi(x) = 1 - e^{-aG_N M_{cr} x} < 1$, REMOVING THE DOUBLE ZERO AT x_{cr}. MONTONICITY $\Rightarrow$ HORIZON OBVIATED.

NOTE: M. BOJOWALD *et al.*, gr-qc/0503041, – LOOP QG CONCURS.

- Continuous Transition:

  $1-2G(r)M/r=1-2G_N(1-e^{-ar})M/r$

Outermost solution: $\Omega=.2$, $\gamma=0$,

$\Rightarrow r_>=27.1/M_{Pl}$, $x_+=1.15$, $x_- <0$

$\Rightarrow x_+ \rightarrow 0$ for $M \rightarrow 2.4\ M_{Pl}\ =M'_{cr}$

with $T_{BH} > 0$, as above.

- **Remnant M'_{cr} :**

  Decays: 2-body,…,n-body

  $\Rightarrow$ Planck Scale Cosmic Rays, etc.

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Conclusions

- RESUMMATION RENDERS QGR UV FINITE
- SUB-PLANCK SCALE PHYSICS ACCESSIBLE TO QFT (TUT)
- MINIMAL UNION OF BOHR & EINSTEIN
- BLACK HOLES WITH $M<M_{cr} \sim M_{Pl}$ HAVE NO HORIZON
- FINAL STATE OF HAWKING RADIATION ⇒ PLANCK SCALE REMNANT ⇒ PLANCK SCALE COSMIC RAYS, …

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