Axiomatic Bounds on New Physics
or
What we WONT see at the LHC

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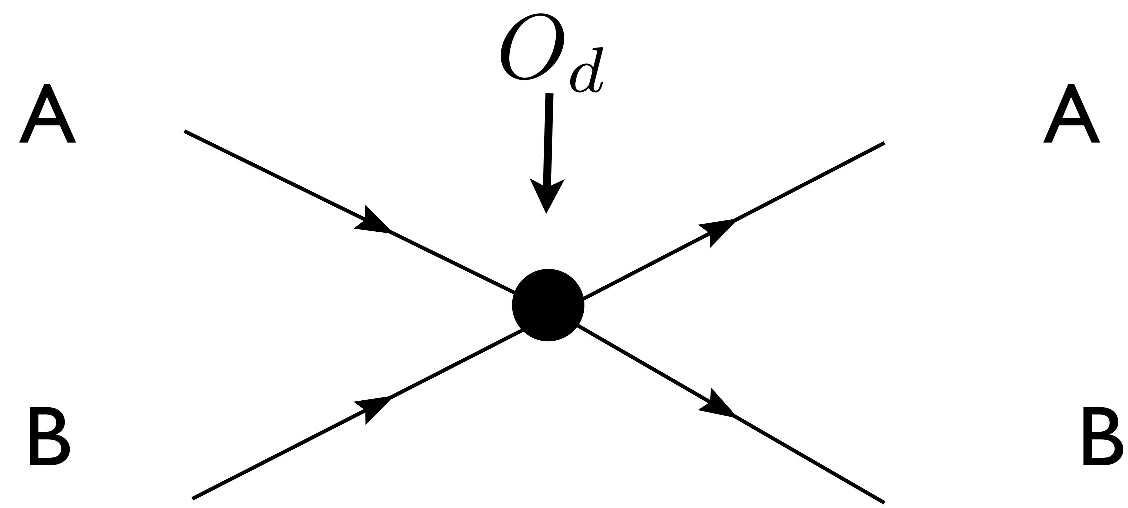
EFT Generalities

$$L = L_0 + \sum_n C_n \frac{O_d}{\Lambda^{d-4}}$$

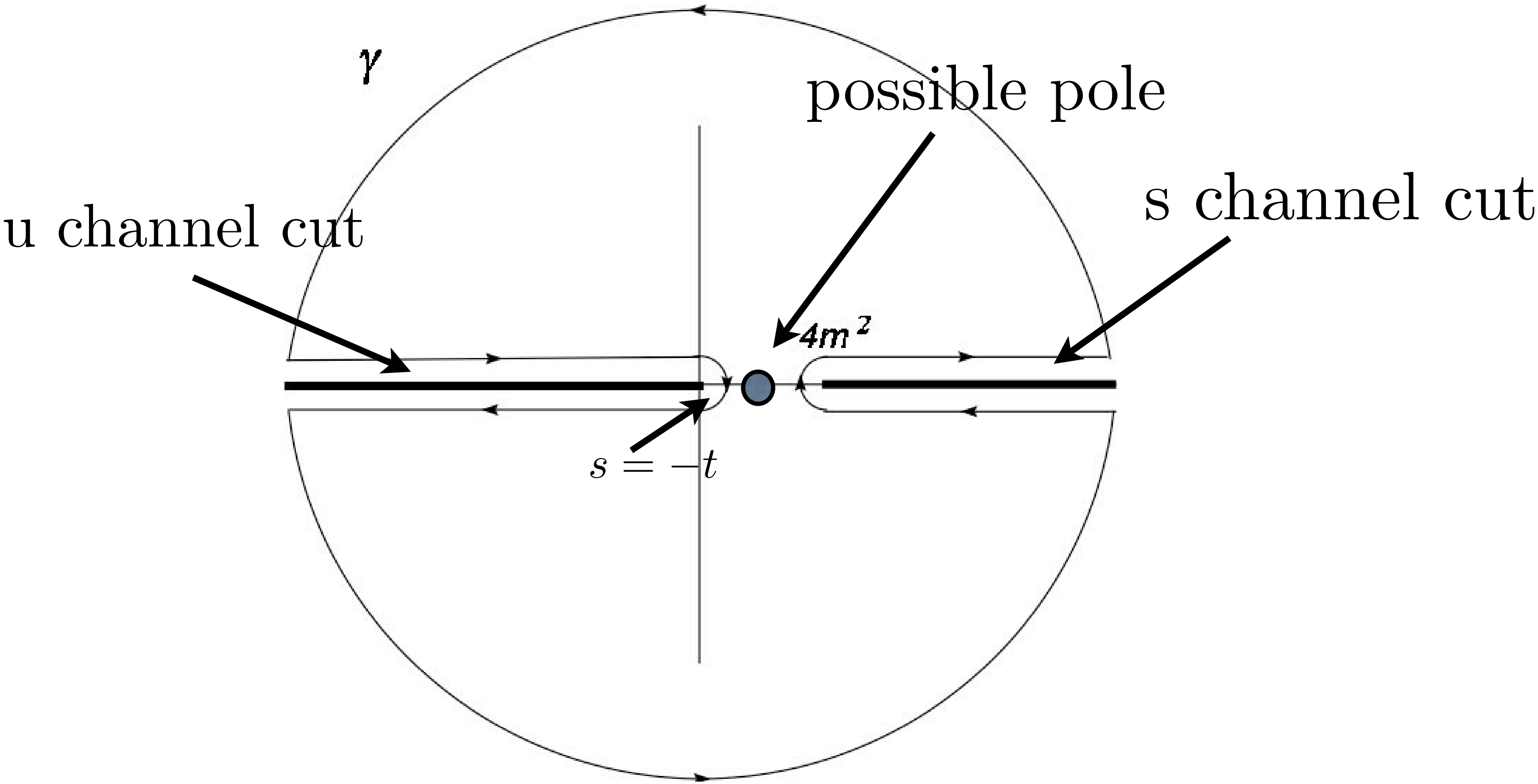
- C's are fixed by a matching procedure
- Can they take on arbitrary values?
- Naturalness: Given scale of new physics, C's should be order one.

Consider an elastic scattering process to which the operator of interest contributes
Defining the s-channel as:



$$iM \propto C_o \frac{F(s,t,u)}{\Lambda^a}$$

Fixed t:Dispersion Relation



- Assuming cut structure dictated by unitarity

Assumes unitarity and Lorentz invariance at all scales

$$\frac{\partial^2 T(s,t)}{\partial s^2} = \frac{2}{\pi}\int_t^\infty \frac{dx\ ImT(-x+i\epsilon,t)}{(x+s)^3} + \frac{2}{\pi}\int_{4M_\pi^2}^\infty \frac{dx\ ImT(x+i\epsilon,t)}{(x-s)^3}$$

$$+\sum_{s_0^i}\frac{res(s_0^i)}{(s-s_0^i)^2}$$

- Twice subtracted for convergence at infinity.
- Froissart Bound follows from unitarity

$$\lim_{s\rightarrow\infty}\sigma(s) < s\ln^2 s$$

Also no long range forces

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$$+\sum_{s_0^i} \frac{res(s_0^i)}{(s-s_0^i)^2}$$

If residue contribution is pos. def. and if we choose $s > 4m^2$, $t = 0$

Then RHS is positive definate:

LHS= $iM \propto C_o \frac{F(s,t,u)}{\Lambda^a} + \text{low energy known physics cont.}$

Leads to a bound on the coupling: only assuming: Unitarity L.I. and Analyticity

Not all operators are boundable: Naively seems to need AT LEAST two derivatives.

Gauged Non-linear sigma model (Heavy Higgs)

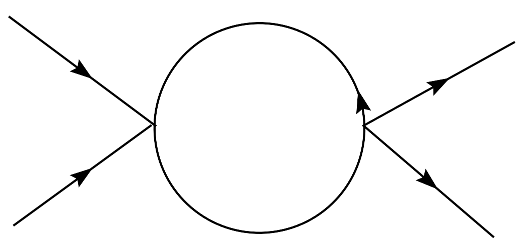
$$L = -\frac{v^2}{4} Tr(V_\mu V^\mu) + l_1 (Tr(V_\mu V^\mu))^2 + l_2 (Tr(V_\mu V^\mu))^2 \quad + \ldots$$

$$V_\mu = (D_\mu U) U^\dagger$$

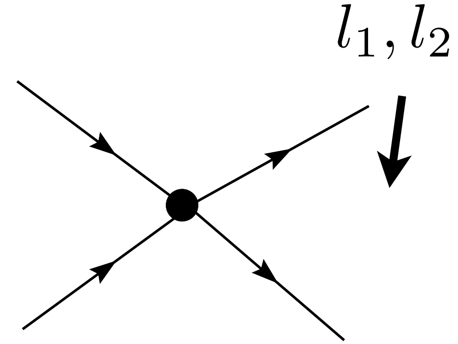
Other ops well constrained by EWPO's

Consider pi-pi scattering: $(\pi_0 \pi_0, \pi^+ \pi_0)$

$iM =$



$+$



$+$ Tree level Poles

Poles are not an obstruction: $$\frac{(s, m^2, s^2/m^2)}{s - m^2}$$

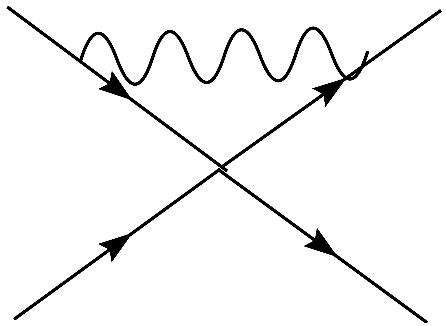
General structure:

$$\frac{\partial^2 T(s,t)}{\partial s^2} = \frac{2}{\pi}\int_t^\infty \frac{dx \; ImT(-x+i\epsilon,t)}{(x+s)^3} + \frac{2}{\pi}\int_{4M_\pi^2}^\infty \frac{dx \; ImT(x+i\epsilon,t)}{(x-s)^3}$$
$$+\sum_{s_0^i} \frac{res(s_0^i)}{(s-s_0^i)^2}$$

Pole contributions cancel: Would not be true for s^3 terms. Which can not be there.

Proof: higgs unitarizes the theory and its contributions go like s^2

Power counting:



$$\frac{\partial^2 T}{\partial s^2} \propto \frac{g^2}{16\pi^2}\left(\frac{1}{v^4}, \frac{1}{sv^2}\right)$$

Compare to the contribution from our operators

$$\frac{l_i}{v^4}$$

$$l_i \propto \frac{1}{16\pi^2} \;\; (NDA)$$

Working below threshold:

$$s < 4m^2 \propto g^2 v^2$$

To get bounds would need to include EW corrections

Working Above Threshold: Pion Loops dominate

$$\frac{d^2\hat{T}(s)}{ds^2} = 2! \int_{4m^2}^{\infty} \frac{dx}{\pi} \sqrt{x(x-4m^2)} \times \left(\frac{\sigma(x)}{(x-s)^3} + \frac{\sigma_u(x)}{(x-4m^2+s)^3} \right)$$

S-channel contribution no longer positive definate

However, for $s < 4\pi v^2$ we can calculate using the EFT

At t=0: split integral up into to intervals

$$4m^2 < s < kv^2 \qquad\qquad kv^2 < s < \infty$$

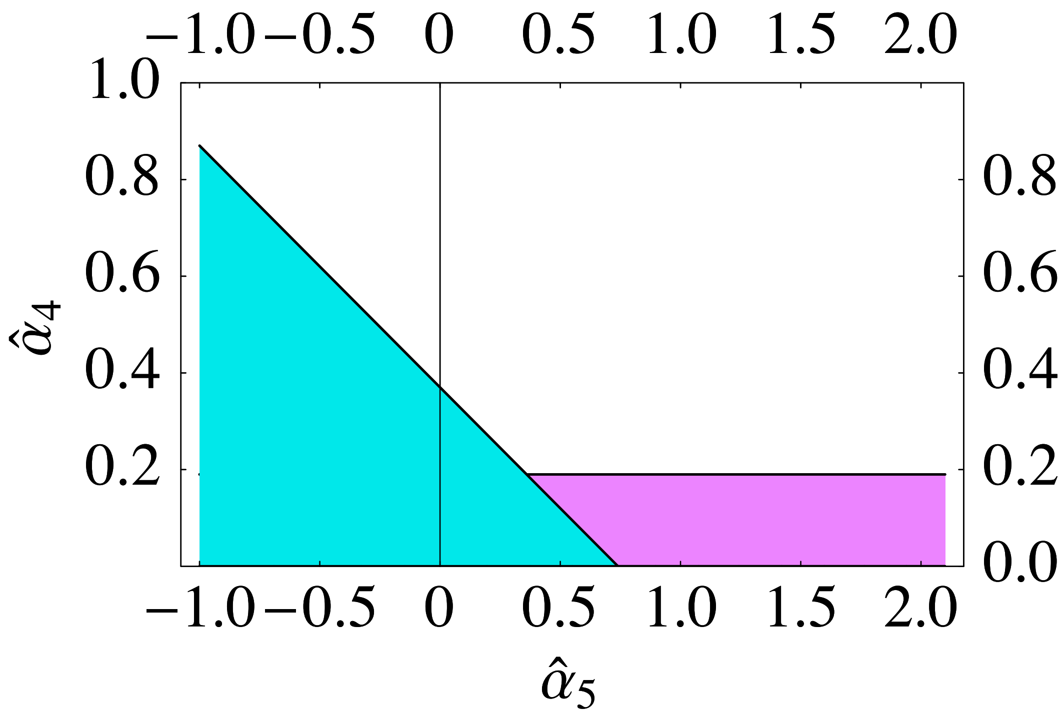
Take $s \sim v^2$ and use Eq. Thm

S matrix elements of long. GB's can be reproduced by S matrix elements of GB's up to corrections of order m^2/v^2.

Use ET to calculate LHS and the first integral on the RHS

$$Re\left(\frac{d^2\hat{T}(s)}{ds^2}\right) = 2!(\int_{4m^2}^{kv^2} + \int_{kv^2}^{\infty})\frac{dx}{\pi}\sqrt{x(x-4m^2)}\times\left(\frac{\sigma(x)}{(x-s)^3} + \frac{\sigma_u(x)}{(x-4m^2+s)^3}\right)$$

Since RHS grows with s (near threshold), choose s to be as large as possible within errors (chPT should still converge)



Dominant source of errors on bound come from LHS

$$\delta l \sim \frac{g^2}{k} \sim \%20$$

- Note: can't use this technique (reliably in chiral Lagrangian (QCD) since in that case

$$f_\pi < m_\pi$$

So there is very little window to work with

$$4m_\pi^2 < s < 4\pi f_\pi$$

Suppose bounds were violated:

1) Underlying theory does not obey usual axioms of QFT. NOT string theory (at least in form we build models with).

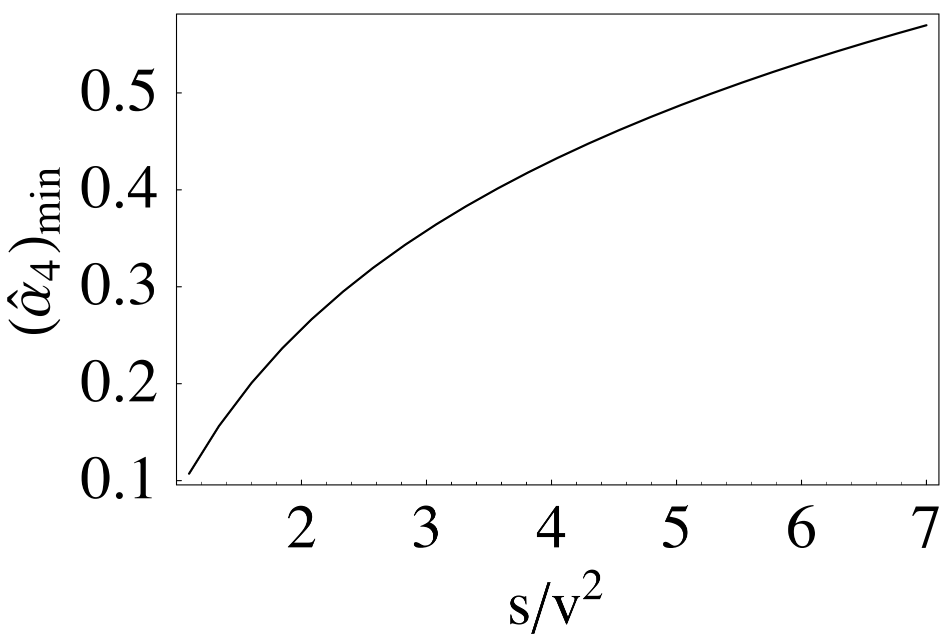
2) There exists light resonances below $4\pi v$

e.g. 5-d theory in Ads dual to large N

ERRORS to RHS

$$\delta_{ew} T \propto O\left(\frac{g^2 s}{(4\pi v)^2} \ln(s/\mu^2)\right)$$ EW loops

$$\delta_{\chi} T \propto O\left(\frac{s^3}{(4\pi)^4 v^6} \ln(s/\mu^2)\right).$$ Chiral loops+CT's



Choose s large while keeping these errors order %20