Exploring dark energy models with perturbation dynamics

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Beautiful ocean view from my laboratory
in Henoko, Okinawa
Futenma Air Base will move here...

1. Introduction

Recent cosmological observations have revealed two main features of the universe:

- High-z type Ia supernovae
  → late-time acceleration of cosmic expansion

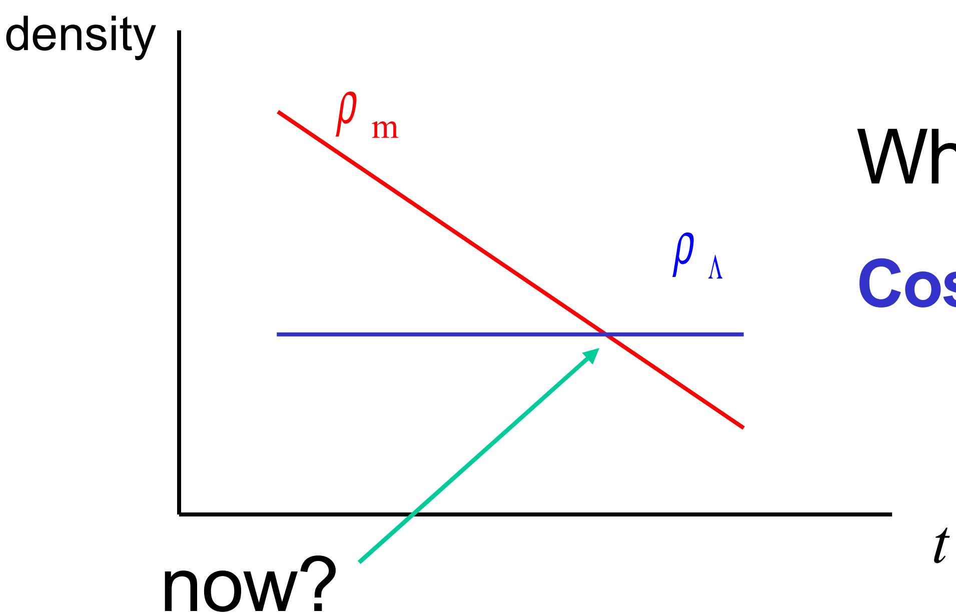
- Cosmic Microwave Background by WMAP Satellite
  → the universe is spatially flat

Dark Energy (DE) – 70 %
Dark Matter (DM) – 30 %

Almost all components of the universe is unknown

Lambda CDM cosmology gives a best fit

Cosmological constant (DE)
+ Cold Dark Matter (CDM)



Why now?

Cosmic coincidence problem

- Physical substance of DE and DM
- Cosmic coincidence problem

Many possible models of DE

- Dynamical DE
  with a standard scalar field (quintessence)
- K-essence with a non-canonical scalar field
- Phantom field with a negative kinetic term
- **(Generalized) Chaplygin gas** $P = -A\rho^{-\alpha}$
  a fluid model unifying DE and DM
- $f(R)$-gravity

Generalized Chaplygin Gas

Kamenshchik et al (2001), Bento et al (2002), ...

$$P = -A\rho^{-\alpha}$$

$$\dot{\rho} + 3H(\rho + P) = 0 \quad \rightarrow \quad \rho = \left(A + B a^{-3(1+\alpha)}\right)^{1/(1+\alpha)}$$

Quartessence

early time $P \approx 0$ late time $P \approx -\rho$

Fluid models unifying DE and DM

interesting possibility to solve the coincidence problem

A generalized Chaplygin gas model is *equivalent* to a type of quintessence models:

fluid

$$P = -A\rho^{-\alpha}$$

⟺

scalar field

$$V(\phi) = (A^{1/2}/2)(\cosh\alpha\phi + (\cosh\alpha\phi)^{-1})$$

These two give the same cosmic expansion.

zeroth-order dynamics

But how is the **perturbation dynamics**?

The difference of the perturbation dynamics may give a clue to distinguish the physical substance of DE.

2. Perturbations of quartessence models

Conformal longitudinal gauge:

$$ds^2 = a^2[-(1+2\Phi)d\eta^2 + (1-2\Phi)\gamma_{ij}dx^i dx^j]$$

Einstein gravity + perfect fluid (Kodama & Sasaki 1984)

$$\Phi_{\eta\eta} + 3H(1+c_s^2)\Phi_\eta + [3H^2(c_s^2 - w) + k^2 c_s^2]\Phi = 0$$

$$H := a_\eta / a\ ,\quad w := P/\rho\ ,\quad c_s^2 := dP/d\rho$$

$$(\,)' := \frac{d}{d\ln a}$$

$$\Phi'' + \left(\frac{5}{2} + 3c_s^2 - \frac{3}{2}w\right)\Phi' + \left[3(c_s^2 - w) + \left(\frac{k\,c_s}{a\,H}\right)^2\right]\Phi = 0$$

Einstein gravity + standard scalar field

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi-V(\varphi)\right]$$

$$\Phi_{\eta\eta}+2\left(H-\frac{\varphi_{\eta\eta}}{\varphi_\eta}\right)\Phi_\eta+\left[2\left(H_\eta-H\frac{\varphi_{\eta\eta}}{\varphi_\eta}\right)+k^2\right]\Phi=0$$

$$(\,)':=\frac{d}{d\ln a}$$

φ : scalar field

(Mukhanov et al 1992)

$$\Phi''+\left(1-\frac{H'}{H}-2\frac{\varphi''}{\varphi'}\right)\Phi'+\left[-2\frac{\varphi''}{\varphi'}+\left(\frac{k}{aH}\right)^2\right]\Phi=0$$

Imposing the same background dynamics

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \quad , \qquad P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

⇒ $$\dot{\varphi}^2 = \rho(1+w)$$

⇒ $$\varphi''/\varphi' = (H'/H)(c_s^2 - w)/(1+w)$$

we obtain for the corresponding quintessence model,

$$\Phi'' + \left(5/2 + 3c_s^2 - 3w/2\right)\Phi' + \left[3(c_s^2 - w) + (k/aH)^2\right]\Phi = 0$$

Fluid case:

Coincide in the large-scale limit

$$\Phi'' + \left(5/2 + 3c_s^2 - 3w/2\right)\Phi' + \left[3(c_s^2 - w) + (kc_s/aH)^2\right]\Phi = 0$$

3. Perturbations of general scalar-field models

General scalar field model (including k-essence)

$$S=\int d^4x\sqrt{-g}\;p(\varphi,X)\quad;\qquad X=-\frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi$$

$$\Phi''+\left(4+\frac{H'}{H}+\frac{Z}{H}+3c_X^2\right)\Phi'+\left[3+2\frac{H'}{H}+\frac{Z}{H}+3c_X^2+\left(\frac{kc_X}{aH}\right)^2\right]\Phi=0$$

$$c_X^2:=p_X/(p_X+2Xp_{XX})=p_X/\rho_X$$

$$Z:=\frac{-2(p_X p_\varphi+Xp_\varphi p_{XX}-Xp_X p_{X\varphi})}{p_X\dot{\varphi}(p_X+2Xp_{XX})}$$

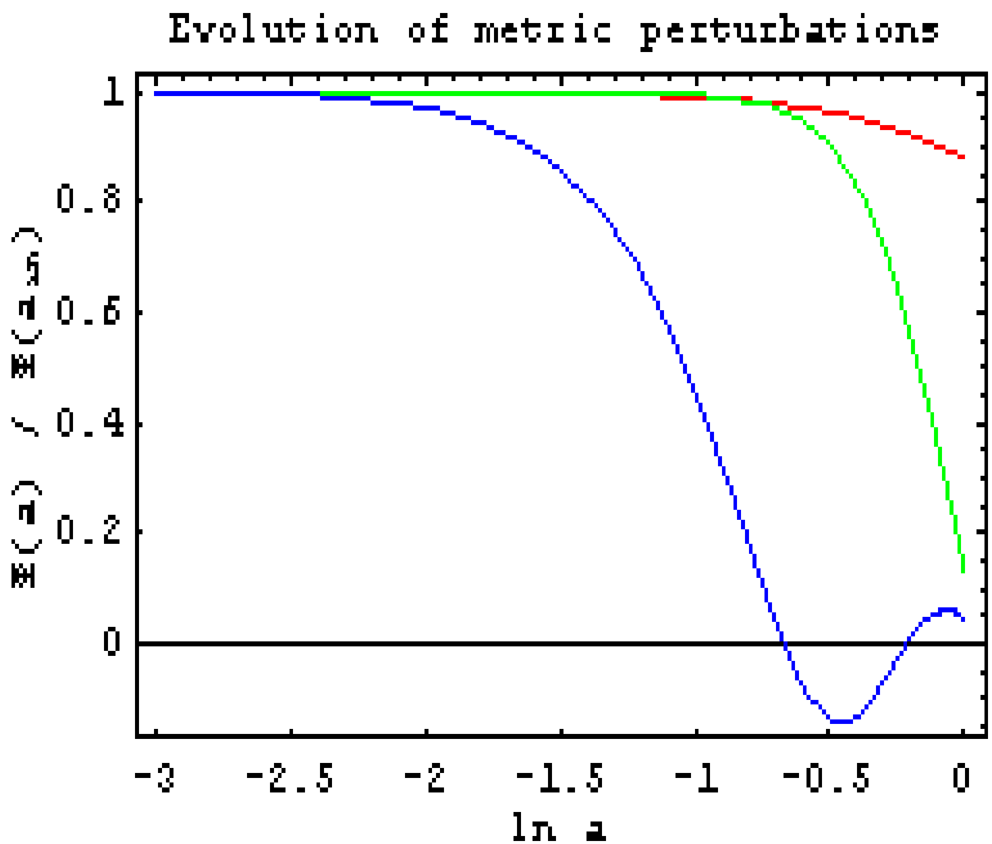
$$\Phi'' + \left(\frac{5}{2} + 3c_X^2 - \frac{3}{2}w + \frac{Z}{H}\right)\Phi' + \left[3(c_X^2 - w) + \frac{Z}{H} + \left(\frac{kc_X}{aH}\right)^2\right]\Phi = 0$$

- **Does not coincide with a fluid case generally even in the large-scale limit**

- **In purely kinetic k-essence** $P=P(X)$, **completely coincides with a fluid case because** $Z=0,\ c_X^2=c_s^2$ (i.e. the dynamics are degenerated)

- May coincide with a fluid case generally in uniform-field gauge $\delta\varphi=0$
  If so, ``fluid analogy'' is a gauge-dependent notion in cosmological perturbations

4. Illustrations of the perturbation dynamics



LCDM

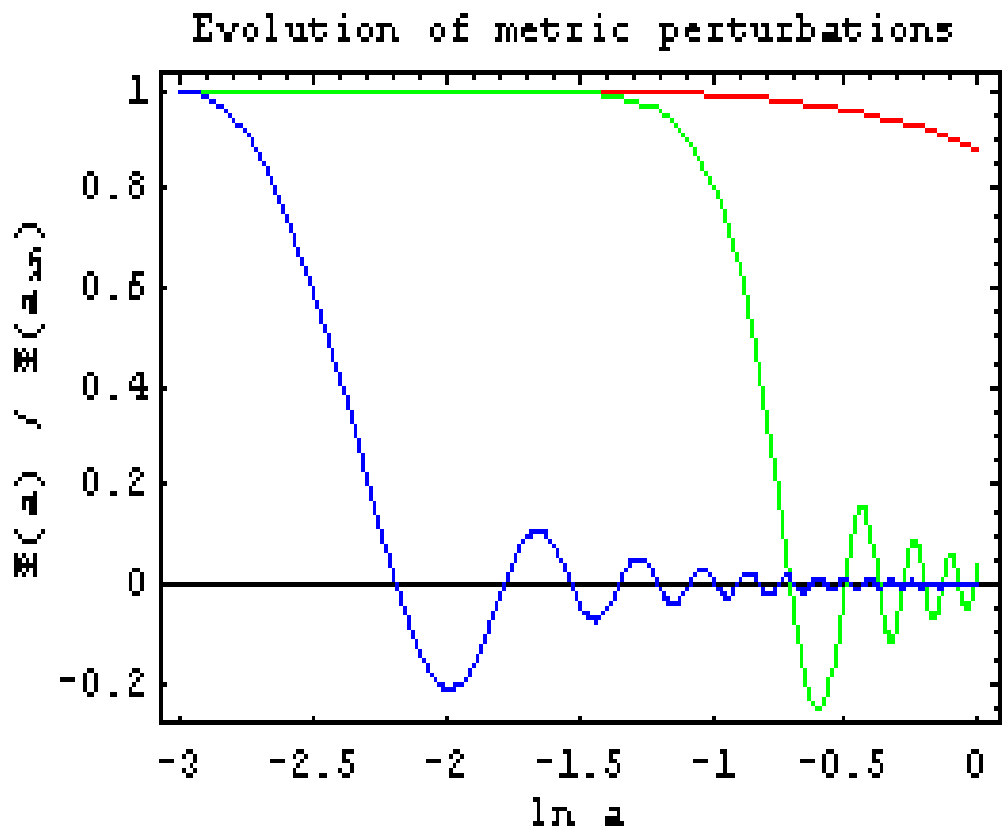
$(\Omega_m = 0.3,\ \Omega_\Lambda = 0.7)$

Chaplygin fluid

$(\alpha = 0.5,\ \xi = 10)$

Corresponding quintessence

$\xi = k / H_0$



LCDM

$(\Omega_m = 0.3,\ \Omega_\Lambda = 0.7)$

Chaplygin fluid

$(\alpha = 0.5,\ \xi = 100)$

Corresponding quintessence

5. Summary & Discussions

- Dynamics of linear perturbations can be a tool of exploring unified dark energy models

- Dynamics of linear perturbations coincides between a fluid quartessence
 and the corresponding scalar-field quintessence
 in the large-scale limit but are quite different
 in small scales.

- Does not coincide between a fluid model and a general scalar-field model generally even in the large-scale limit.

- Damping of metric perturbations will cause the Integrated SW effect on CMB anisotropy and is strongly constrained by observations. (Amendola et al 2003) $0 \leq \alpha < 2$

- To construct a viable model of DE-DM unification, the sound velocity should be designed to be small.

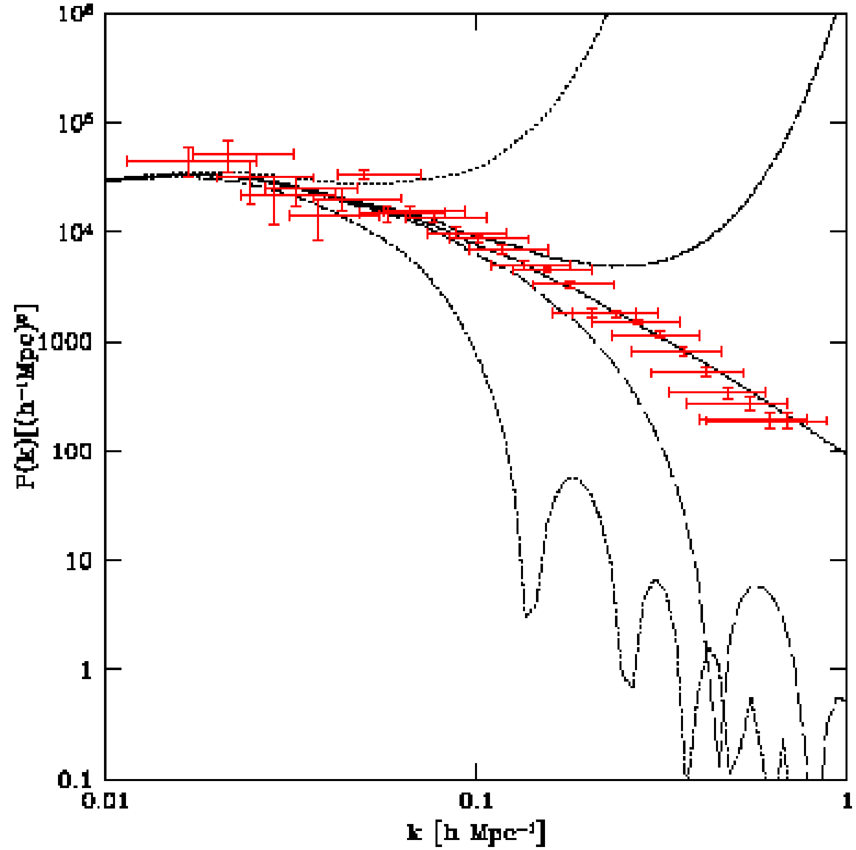


FIG. 1. UDM solution for perturbations as function of wavenumber, k. From top to bottom, the curves are GCG models with $\alpha = -10^{-4}$, -10^{-5}, 0 (ΛCDM), 10^{-5} and 10^{-4}, respectively. The data points are the power spectrum of the 2df galaxy redshift survey.

Linear power spectrum in Chaplygin gas models

Sandvik et al, PRD 69 (2004) 123524