A Heterotic Flux Background and Calibrated Five-Branes

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Motivation

• One of the most fruitful setup of flux compactification: type IIB compactification on warped Calabi-Yau orientifold / F-theory.

• The progress of heterotic flux compactification, despite its phenomenological interest, has been relatively slow since [Strominger 1986].

• Simpler examples: supersymmetric heterotic flux vacua dual to type IIB ones, e.g., on $K3 \times T^2/\mathbb{Z}_2$ [Dasgupta et.al., Becker et.al., etc.]

• Branes have been playing interesting roles: space-time filling branes ($D3$, $D7$) [Gidding-Kachru-Polchinski, etc.], or domain walls ($D5$, $NS5$) [Gukov-Vafa-Witten].

• In the heterotic side, at least for examples dual to type II, these roles should played mostly by the heterotic 5-branes.

Plan

• Overview of the general structure of supersymmetric heterotic flux backgrounds

• Investigate supersymmetric 5-branes in the above backgrounds $\longrightarrow$ generalized calibration conditions [Gutowski et.al., Gauntlett et.al.]

• Apply the results to the heterotic dual of the type IIB flux vacua on $K3 \times T^2/\mathbb{Z}_2$ orientifold.

• Heterotic 5-branes wrapping 'non-topological' internal cycles play the role dual to (1) space-filling $D3$-branes and (2) $D5$ domain walls of type IIB flux vacua in an interesting and unified way.

Heterotic flux compactification

• Geometry: With $\mathcal{N}=1$ SUSY, geometry in the string frame is a direct product of 4d Minkowski space with an internal 6-manifold $\mathcal{M}_6$

$$ds_{10}^{\ 2} = \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n \ ,$$

without a warp factor.

• One can turn on internal 3-form flux:

$$\frac{1}{2\pi\alpha'}\int_{\Sigma_3} H \in 2\pi\mathbb{Z} \qquad (\ \Sigma_3 \in H_3(\mathcal{M}_6,\mathbb{Z})\ )\ .$$

• Supersymmetry requires the internal space $\mathcal{M}_6$ to be a complex manifold:

$$0 = N_{mn}{}^p = J_m{}^q \nabla_{[q} J_{n]}{}^p - J_n{}^q \nabla_{[q} J_{m]}{}^p \ ,$$

where J is a complex structure.

• Supersymmetry further relates the dilaton $\Phi(y)$ and flux $H(y)$ to the gradient of J:

$$H_{mnp} = -3 J_m{}^q J_n{}^r J_p{}^s \nabla_{[q} J_{rs]} \;, \;\; \nabla_m \Phi = \frac{3}{4} J^{np} \nabla_{[m} J_{np]} \;.$$

With nonzero flux, $\mathcal{M}_6$ cannot even be Kähler : $dJ \neq 0$.

• Using the above relations, one obtains an important equation

$$e^{-2\Phi} *_6 H = d\left(e^{-2\Phi} J\right) \equiv (vol)_{3+1} \lrcorner d\tilde{B}_6 \quad \text{(generalized calibration)}$$

This lets us identify

$$\tilde{B}_6 \equiv (vol)_{3+1} \wedge \left(e^{-2\Phi} J\right) \quad \text{(+ pure gauge)}$$

as the magnetic 6-form potential, coupling minimally to five-branes.

Calibrated five-branes

• Consider a space-filling fivebrane in the above background: it should wrap a 2-cycle Σ_2 in $\mathcal{M}_6$.

• The worldvolume energy density consists of volume plus Coulomb energy coming from the dual 6-form potential

$$\mathcal{E} = \int_{\Sigma_2} \left(e^{-2\Phi} (vol)_{5+1} - \tilde{B}_6 \right) \sim \int_{\Sigma_2} e^{-2\Phi} \left((vol)_{\Sigma_2} - J \right) \geq 0 \ .$$

Since $(vol)_{\Sigma_2} \geq *_2 J$ holds locally, a configuration with $\mathcal{E} = 0$ is a classical solution of the equation of motion.

• The 'BPS' condition $(vol)_{\Sigma_2} = *_2 J$ requires Σ_2 to be a holomorphic embedding : generalized calibrated cycles.

• The 5-branes wrapping these cycles are indeed supersymmetric configurations [Gutowski-Papadopoulos-Townsend].

Heterotic dual of type IIB flux vacua on $K3 \times T^2/\mathbb{Z}_2$

• As a concrete example, let us take $\mathcal{M}_6$ to be a T^2 fibration over a $K3$ surface: dual to type IIB flux backgrounds.

• Basic ingredients of dual type IIB configurations are: (1) Internal geometry: direct product $K3 \times T^2$ (overall warp factor). (2) Internal RR and NSNS 3-form fluxes. (3) Space-filling $D3$ branes. (4) $D7$-branes and $O7$-planes wrapping $K3$ and localized in T^2 ($\rightarrow T^2/\mathbb{Z}_2$).

• T-dualizing along two directions of torus, we expect type I vacua compactified on T^2 fibration over $K3$: NS 3-form flux $\rightarrow$ metric.

• A further S-duality would map the above to (SO(32)-)heterotic flux vacua: RR 3-form $\rightarrow$ heterotic 3-form flux.

• The class of internal geometry that we will consider has $U(1)^2$ isometry [Fu-Yau, Becker et.al.]:

$$ds_6{}^2 = e^{2(\Phi-\Phi_0)} \; R_B^2 \; ds_{K3}^2 + \ell_F^2 \, |dz + \alpha|^2$$

$$z \sim z + 2\pi\sqrt{\alpha'} \sim z + 2\pi\sqrt{\alpha'} \; i \; .$$

$\alpha_1 = Re(\alpha)$, $\alpha_2 = Im(\alpha)$ are 1-forms in $K3$, providing nontrivial fibrations: $\omega_{1,2} \equiv d\alpha_{1,2} \in 2\pi\sqrt{\alpha'}H^2(K3,\mathbb{Z})$. They are 'T-duals' of type IIB NSNS internal flux along $K3 \times T^2/\mathbb{Z}_2$.

• Due to the nontrivial fibration, T^2 is *not* a topological cycle.

• Analogue: $S^3/\mathbb{Z}_k$ is a Hopf fibration of S^1 over S^2. $\pi_1(S^3/\mathbb{Z}_k) = \mathbb{Z}_k$, which is not extensive. A loop winding S^1 k times is contractible .

• α' corrections are crucial for consistent flux compactification (a way to avoid the Malcacena-Nunez no-go theorem)

$$\mathcal{L} = \frac{\sqrt{-g}}{2\kappa_{10}^2} e^{-2\Phi} \left(R(\omega) - \frac{1}{12} H^2 + 4(\nabla\Phi)^2 + \frac{\alpha'}{8}(\mathrm{tr} F_{MN} F^{MN} - \mathrm{tr} R^{-}_{MN} R^{-MN}) \right)$$

$$dH = \frac{\alpha'}{4}\left(\mathrm{tr} R' \wedge R' - \mathrm{tr} F \wedge F - 16\pi^2 \delta_{5-brane} \right) \ .$$

• For instance, integrating $J \wedge dH$ over $\mathcal{M}_6$, one obtains

$$N_5 + \int_{K3} |\tilde{\omega}_1|^2 + |\tilde{\omega}_2|^2 = \int_{K3} \mathrm{tr} R_{K3} \wedge R_{K3} - \mathrm{tr} F \wedge F = 24 + \frac{p_1(F)}{2}$$

where $2\pi\sqrt{\alpha'}\ \tilde{\omega}_{1,2} = d\alpha_{1,2} \in H^2(K3, \mathbb{Z})$. It is the heterotic dual of $D3$ tadpole cancellation condition for type IIB flux vacua:

$$N_{D3} + \frac{1}{(4\pi^2\alpha')^2} \int_{K3\times T^2} H_3 \wedge F_3 = \frac{\chi(K3)^2}{24} = 24 \ .$$

Five-branes wrapping the T^2 fiber

• The space-filling $D3$-branes in type IIB are supposed to map to heterotic 5-branes wrapping non-topological T^2 fiber and localized in $K3$ base.

• Recalling the structure of generalized calibration, we recognize that Coulomb force $\tilde{B}_6$ holds the 5-brane to be stable against small fluctuations, keeping it not to contract.

• For a 5-brane wrapping T^2 fiber,

$$e^{-2\Phi}\left((vol)_2 - J\right) = e^{-2\Phi}\left((vol)_2 - \frac{i}{2}dz \wedge d\bar{z}\right) \ .$$

An obvious holomorphic embedding saturates the energy bound.

• Since the above configuration is supersymmetric, it should be stable and is U-dual to type IIB $D3$-branes.

• However, one can imagine a process of contracting this supersymmetric 5-brane so that it would shrink and disappear. This unwrapping process requires an infinite energy cost to cross an energy barrier.

• Furthermore, the tadpole cancellation condition should hold after this process:

$$N_5 + \int_{K3} |\tilde{\omega}_1|^2 + |\tilde{\omega}_2|^2 = 24 + \frac{p_1(F)}{2} \ .$$

A process $\Delta N_5 < 0$ should be accompanied with $\Delta \int_{K3} |\tilde{\omega}|^2 > 0$: ends up with a different vacuum.

• Imagining that this contraction proceeds as one moves along a spatial direction of R^{3+1}, it can be identified as a domain wall.

More on heterotic/type IIB domain walls

• Consider a $D5$ domain wall wrapping a 3-cycle $\Sigma_3 \equiv S^1 \times \Sigma_2 \subset T^2 \times K3$, where Σ_3 contains N_3 NSNS flux ($T^2 = S^1 \times \tilde{S}^1$).

• N_3 space-filling $D3$-branes should end on $D5$. Suppose n $D3$'s:

$$S_{WZ} = -\int_{D5} B \wedge C_4 - n \int_{D3} C_4 \ .$$

The gauge invariance under $\delta C_4 = d\Lambda_3$

$$\delta S_{WZ} \sim \left(\int_{\Sigma_3} dB - n \right) \int_{R^{2+1}} \Lambda_3 \stackrel{!}{=} 0$$

requires $n = N_3$.

• One can also notice this fact from the $D3$ tadpole condition:

$$0 = \Delta N_{D3} + \frac{1}{(4\pi^2\alpha')^2} \int H_3 \wedge (\Delta F_3) = -n + N_3 \ .$$

• This structure is same as the unwinding process of T^2 wrapping 5-branes: tadpole cancellation condition requires

$$0 = \Delta N_5 + \Delta \int_{K3} |\tilde{\omega}_1|^2 + |\tilde{\omega}_2|^2 \ .$$

• For simplicity, consider a simple toy example where $S^1 \subset T^2$ is fibred over $\Sigma_2 (\approx S^2) \subset K3$ with $[\omega_1] \approx k[\Sigma_2]$: one may regard Σ_3 as $S^3/\mathbb{Z}_k$ and $S^1 \sim$ Hopf fiber.

• 5-brane wrapping S^1 k times, and $\tilde{S}^1 \subset T^2$ (trivial over Σ_2) once.

• As we try to contract $kS^1 (\subset T^2)$, the locus of the brane in $S^2 (\subset K3)$ cannot maintain to be a point, due to the nontrivial fibration: it becomes a closed loop in $\Sigma_2 \approx S^2$.

• As kS^1 shrinks to a point, the closed loop completely sweeps the S^2 base once and shrinks back to a point: a domain wall 'wrapping' $\Sigma_2 \times \tilde{S}^1 \subset \mathcal{M}_6$, beyond which k space-filling 5-branes disappear.

• k ($\sim$number of 'twists' of the S^1 fiber) is T-dual to the NSNS 3-form flux piercing $\Sigma_2 \times S^1$.

$\therefore$ Combined system of domain wall and k space-filling branes (dual to $D5 + k\ D3$) would be described as a single smooth heterotic 5-brane.

• It will be interesting to study these domain wall configurations more quantitatively (and compare it with the type IIB ones).

Conclusion

• In the heterotic flux vacua with $\mathcal{N} = 1$ SUSY, space-filling supersymmetric 5-branes are described by a generalized calibration condition.

• For the heterotic dual of $K3 \times T^2/\mathbb{Z}_2$ vacua, we found space-filling 5-branes whose stability is guaranteed *not* by topology, but dynamically by the flux.

• Winding number of 5-brane on T^2 is non-extensive, and there is a process of unwinding it by crossing an infinite energy barrier: dual to the space-filling $D3$ branes ending on a $D5$ domain wall.

• The patterns of heterotic 5-brane domain walls are in good agreement with the type IIB ones. It will be worthwhile to study the heterotic version in more detail.