Heavy Quark-Diquark Symmetry and χPT for Doubly Heavy Baryons

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- S. Fleming & T.M., PRD 73, 034502 (2006)
- J. Hu & T.M., PRD 73, 054003 (2006)
- T.M. & B. Tiburzi, PRD 74, 054505 (2006)

Outline

- Motivation

Experiment: SELEX $\Xi_{cc}^{+}, \Xi_{cc}^{++}$ candidates

Theory: Heavy Quark-Diquark Symmetry $\bar{Q}\bar{Q}\bar{q} \leftrightarrow Q\bar{q}$

Savage, Wise

- Diquark Effective actions from vNRQCD

S. Fleming, T.M., PRD 73, 034502 (2006)

$Q\bar{Q}, QQ$ bound states characterized by several scales:

$$m_Q, \quad p \sim m_Q v, \quad E \sim m_Q v^2, \quad \Lambda_{\rm QCD}$$

HQET: expansion in $\Lambda_{\rm QCD}/m_Q$ $m_Q v, m_Q v^2$ missing

Heavy $Q\bar{Q}, QQ$ systems require NonRelativistic QCD

- Chiral Lagrangians with Quark-Diquark Symmetry

J. Hu, T.M., PRD 73, 054003 (2006)

NEW symmetry predictions for $\Xi_{cc}^* \to \Xi_{cc} + \gamma$

em, strong decays of excited doubly heavy baryons

- Quenched and Partially Quenched Theories

T.M., B. Tiburzi, PRD 74, 054505 (2006)

nonanalytic chiral corrections for lattice simulations

doubly heavy masses, hyperfine splittings

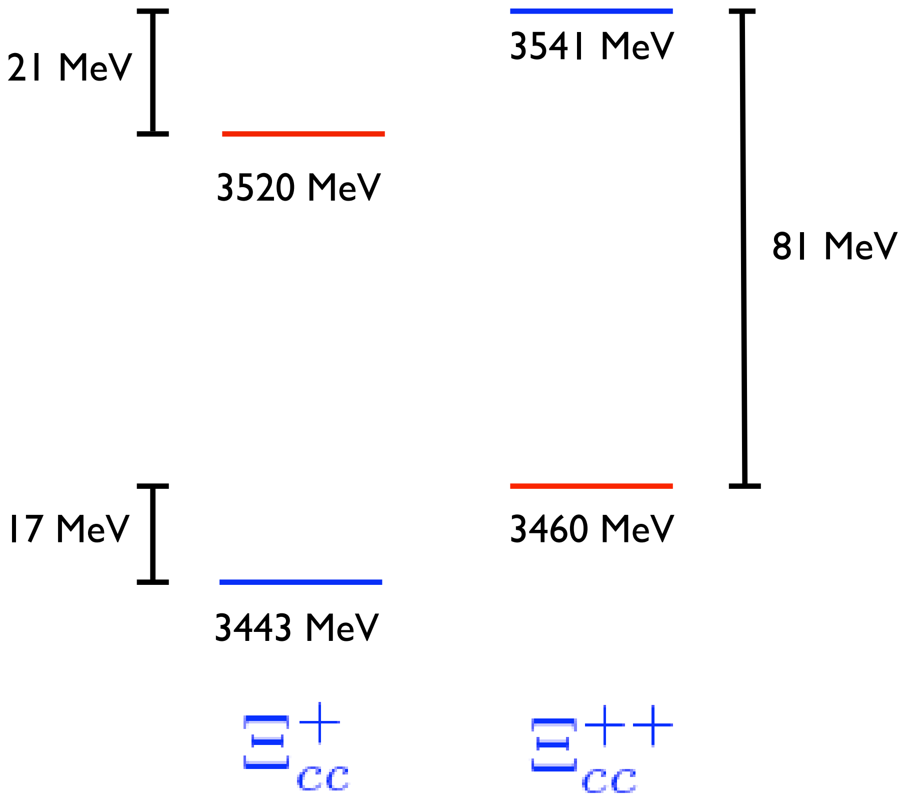
Q-DQ hyperfine prediction: small corrections!

em decays, including $1/m_Q$ corrections

- Outlook

SELEX Doubly Charm Baryons?

PRL 89, 112001(2002), PLB 628,18 (2004)



- Observed decays

$$\Xi_{cc}^{+}(3520) \to \Lambda_c^{+} K^{-} \pi^{+}$$

$$\to p D^{+} K^{-}$$

$$\Xi_{cc}^{++}(3460) \to \Lambda_c^{+} K^{-} \pi^{+} \pi^{+}$$

$$\Xi_{cc}^{++}(3780) \to \Xi_{cc}^{+}(3520)\pi^{+} \text{ (?)}$$

(not shown)

$$\to \Lambda_c^{+} K^{-} \pi^{+} \pi^{+}$$

- $\Xi_{cc}^{+}(3443), \Xi_{cc}^{++}(3541)$ unpublished www-selex.fnal.gov/Welcome.html

- high statistical significance, few events (~10-30)

- search by BELLE fails to confirm $\Xi_{cc}^{+}(3520)$ (hep-ex/0606051)

- Masses, hyperfine consistent with quark model, lattice

  hyperfine splitting ~ 80 MeV   Lewis, Mathur, Woloshyn; Flynn, Mescia, Tariq

Puzzling Aspects of SELEX Observation

- Isospin splitting ~ 20 MeV ?   larger than expected

- radiative decays should dominate weak decay

  $\Xi_{cc}^{+}(3520)$ excited state? $\Xi_{cc}^{*} \to \Xi_{cc} + \gamma$?

- Weak lifetimes   Guberina, et. al. EPJ C9, 213 (1999)

  experiment: < 33 fs   theory (HQET+OPE): ~100 fs

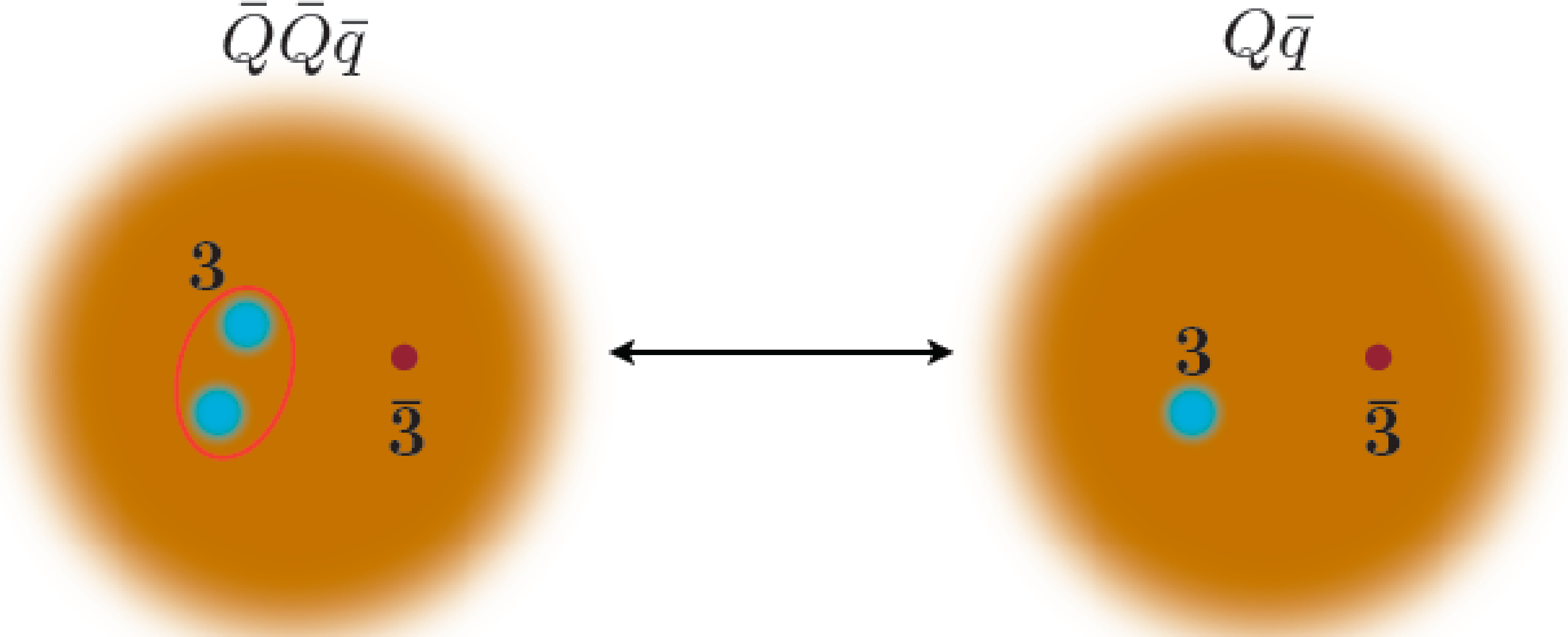
- Production cross sections   (Kiselev, Likhoded, hep-ph/0208231)

  significantly larger than expected from LO pQCD

  forward production $\langle x_F \rangle \sim 0.3$ seen in p, Σ not π beams

  nonperturbative mechanisms (e.g. intrinsic charm, leading particle effect) ?

Heavy Quark-Diquark Symmetry



- HQET-like Lagrangian for Heavy Diquarks (Savage, Wise)

$$\mathcal{L} = h^\dagger\, iD_0\, h + \vec{\mathbf{T}}^\dagger \cdot iD_0\, \vec{\mathbf{T}} + \frac{g_s}{2m_Q} h^\dagger\, \vec{\boldsymbol{\sigma}} \cdot \vec{\mathbf{B}}\, h + \frac{ig_s}{2m_Q} \vec{\mathbf{T}}^\dagger \cdot \vec{\mathbf{B}} \times \vec{\mathbf{T}} + \ldots$$

- At lowest order, U(5) symmetry acting on $(h, \vec{T})$

Hyperfine splittings: $m_{\Xi^*} - m_{\Xi} = \frac{3}{4}(m_{H^*} - m_H)$

- Hyperfine splittings in quark model

$$H_{Q\bar{q}} = \ldots + \frac{\lambda}{m_Q m_q} \vec{S}_Q \cdot \vec{S}_{\bar{q}}$$

$$H_{\bar{Q}\bar{Q}\bar{q}} = \ldots + \frac{1}{2} \frac{\lambda}{m_Q m_q} \sum_{\bar{Q}} \vec{S}_{\bar{Q}} \cdot \vec{S}_{\bar{q}}$$

- $\frac{1}{2}$ - color SU(3) factor: $\bar{Q}\bar{q}$ in $\bar{3}$ vs. $Q\bar{q}$ in 1

- Prediction for doubly charm baryons

$$m_{\Xi_{cc}^*} - m_{\Xi_{cc}} \approx \frac{3}{4}(m_{D^*} - m_D) = 106\,\mathrm{MeV}$$

- Error? $O\left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right) \sim \frac{1}{3}$ 106 MeV vs. 80 MeV ??

Heavy $Q\bar{Q}$, $\bar{Q}\bar{Q}$ Systems: NonRelativistic QCD (NRQCD)

- vNRQCD: separate k into $O(m_Q v)$ and $O(m_Q v^2)$ parts

$$\vec{k} = \vec{p} + \vec{k}' \qquad h_v = \sum_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} \psi_{\vec{p}} \qquad D_\mu \psi_{\vec{p}} = O(m_Q v^2)\psi_{\vec{p}}$$

- vNRQCD Lagrangian (Luke, Manohar, Rothstein)

$$\mathcal{L} = \sum_{\mathbf{p}} \chi_{\mathbf{p}}^\dagger \left(iD^0 - \frac{(\mathbf{p} - i\mathrm{D})^2}{2m_Q} + \frac{g}{2m_Q} \boldsymbol{\sigma} \cdot \mathrm{B} \right) \chi_{\mathbf{p}}$$
$$- \frac{1}{2} \sum_{\mathbf{p},\mathbf{q}} \frac{g_s^2}{(\mathbf{p} - \mathbf{q})^2} \chi_{\mathbf{q}}^\dagger \bar{T}^A \chi_{\mathbf{p}} \chi_{-\mathbf{q}}^\dagger \bar{T}^A \chi_{-\mathbf{p}} + \ldots .$$

- Power counting: $\vec{p} \sim m_Q v \quad \psi_{\vec{p}}, \chi_{\vec{p}} \sim (m_Q v)^{3/2}$

$D_0, \vec{D} \sim m_Q v^2$ (usoft gluons) $A^\mu \sim m_Q v^2$ (soft) $A^\mu_{\vec{p}} \sim m_Q v$

- HQET + $\bar{Q}\bar{Q}\bar{Q}\bar{Q}$ Operators, different power counting

- Effective Action for Heavy Diquarks

S. Fleming, T.M., PRD 73, 034502 (2006)

- composite diquark field $\mathbf{T}^i_{\mathbf{r}} = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{1}{2}\epsilon^{ijk}(\chi_{-\mathbf{p}})_j \epsilon\sigma(\chi_{\mathbf{p}})_k$

- Hubbard-Stratonovich trans., integrate out χ_p

$$\mathcal{L}_{\mathbf{T}} = \int d^3\mathbf{r}\, \mathbf{T}^\dagger_{\mathbf{r}} \left(iD_0 + \frac{\boldsymbol{\nabla}^2_{\mathbf{r}}}{m_Q} - V^{(3)}(r) \right) \mathbf{T}_{\mathbf{r}} + \frac{g}{2m_Q} \int d^3\mathbf{r}\, i\, \mathbf{T}^\dagger_{\mathbf{r}} \cdot \mathbf{B} \times \mathbf{T}_{\mathbf{r}}$$

$$= \sum_n \mathbf{T}^\dagger_n (iD_0 + \delta_n)\mathbf{T}_n + \frac{g}{2m_Q} i \sum_n \mathbf{T}^\dagger_n \cdot \mathbf{B} \times \mathbf{T}_n$$

$$\mathbf{T}^i_{\mathbf{r}} = \sum_n \mathbf{T}^i_n \phi_n(\mathbf{r}) \qquad \left(-\frac{\boldsymbol{\nabla}^2_{\mathbf{r}}}{m_Q} + V^{(3)}(r) \right) \phi_n(\mathbf{r}) = -\delta_n \phi_n(\mathbf{r})$$

- Savage-Wise Lagrangian obtained after integrating out all excited diquark fields

Chiral Lagrangian with Heavy Quark-Diquark Symmetry

J. Hu, T.M., PRD 73, 054003 (2006)

- Heavy Hadron Chiral Perturbation Theory

(Wise; Burdman, Donoghue, Yan, et. al.)

- Heavy Meson Fields combine 0^- and 1^- heavy mesons in single field

$$H_a = (Q\bar{q}_a) = \left(\frac{1+\not{v}}{2}\right)(P_a^{*\mu}\gamma_\mu - \gamma_5 P_a)$$

- Goldstone Bosons (π, K, η) $SU(3)_L \times SU(3)_R \to SU_{L+R}(3)$

$$\xi = e^{i\Pi/f} \qquad \Sigma = \xi^2$$

$$D^\mu_{ab} = \delta_{ab}\,\partial^\mu - V^\mu_{ab} \quad V^\mu_{ab} = \frac{1}{2}(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger) \quad A^\mu_{ab} = \frac{i}{2}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger)$$

- $$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\,\partial^\mu\Sigma\,\partial_\mu\Sigma^\dagger + \frac{f^2 B_0}{4}\mathrm{Tr}(m_q\Sigma + m_q\Sigma^\dagger)$$

$$-\mathrm{Tr}\,\bar{H}_a iv\cdot D_{ba}H_b + g\,\mathrm{Tr}\,\bar{H}_a H_b\gamma_\mu\gamma_5 A^\mu_{ba} + \ldots$$

- In rest frame, $v = (1, \vec{0})$

$$H_v = \begin{pmatrix} 0 & -\vec{P}_v \cdot \vec{\sigma} - P_v \\ 0 & 0 \end{pmatrix}$$

For processes where four velocity is conserved we can work in heavy meson rest frame and H_v can be represented by 2 x 2 field

$$H_a = \vec{P}_a \cdot \vec{\sigma} + P_a$$

$$\mathcal{L} = \mathrm{Tr}[H_a^\dagger (iD_0)_{ba} H_b] - g\,\mathrm{Tr}[H_a^\dagger H_b\, \vec{\sigma} \cdot \vec{A}_{ba}] + \frac{\Delta_H}{4} \mathrm{Tr}[H_a^\dagger\, \sigma^i\, H_a\, \sigma^i]$$

- Including Doubly Heavy Baryons

2 x 2 field $\rightarrow$ 5 x 2 field

$$H_{a,\alpha\beta} \rightarrow \mathcal{H}_{a,\mu\beta} = H_{a,\alpha\beta} + T_{a,i\beta}$$

$$T_{a,i\beta} = \sqrt{2} \left(\Xi^*_{a,i\beta} + \frac{1}{\sqrt{3}} \Xi_{a,\gamma}\, \sigma^i_{\gamma\beta} \right) \qquad \Xi^*_{a,i\beta}\, \sigma^i_{\beta\gamma} = 0$$

- Transformations

rotations	$\mathcal{H}'_a = \mathcal{R}\mathcal{H}_a U^\dagger$	$\mathcal{R} = \begin{pmatrix} U & 0 \\ 0 & R \end{pmatrix}$
heavy quark spin	$\mathcal{H}'_a = S\mathcal{H}_a$	
parity	$\mathcal{H}'_a = -\mathcal{H}_a$	
SU(3) chiral	$\mathcal{H}'_a = \mathcal{H}_b V^\dagger_{ba}$	

U - 2 x 2

R - 3 x 3

$S \subset U(5) \qquad V \subset SU(3)$

- Chiral Lagrangian

$$\mathcal{L} = \mathrm{Tr}[\mathcal{H}^\dagger_a (iD_0)_{ba} \mathcal{H}_b] - g\mathrm{Tr}[\mathcal{H}^\dagger_a \mathcal{H}_b \vec{\sigma} \cdot \vec{A}_{ba}] + \frac{\Delta_H}{4} \mathrm{Tr}[\mathcal{H}^\dagger_a \Sigma^i \mathcal{H}_a \sigma^i]$$

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\mathcal{T}} \end{pmatrix} \qquad (\mathcal{T}^i)_{jk} = -i\epsilon_{ijk}$$

- hyperfine splitting $\quad m_{\Xi^*} - m_{\Xi} = \frac{3}{4}(m_{P^*} - m_P)$

- Including electromagnetic interactions

$$\frac{e\beta}{2}\mathrm{Tr}[\mathcal{H}_a^\dagger \mathcal{H}_b \vec{\sigma}\cdot\vec{B}\,Q_{ab}] + \frac{e}{2m_Q}Q'\mathrm{Tr}[\mathcal{H}_a^\dagger \vec{\Sigma}'\cdot\vec{B}\,\mathcal{H}_b] \qquad \vec{\Sigma}' = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -2\vec{T} \end{pmatrix}$$

- Radiative Decay Rates $\beta = \frac{1}{m_q}$ in quark model

$$\Gamma[P_a^* \to P_a\gamma] = \frac{\alpha}{3}\left(\beta Q_{aa} + \frac{Q'}{m_Q}\right)^2 \frac{m_P}{m_{P^*}}E_\gamma^3$$

$$\Gamma[\Xi_a^* \to \Xi_a\gamma] = \frac{4\alpha}{9}\left(\beta Q_{aa} - \frac{Q'}{m_Q}\right)^2 \frac{m_\Xi}{m_{\Xi^*}}E_\gamma^3$$

- Include $O(\sqrt{m_q})$ chiral corrections (Amundsen, et. al.)

$$\beta Q_{11} \to \frac{2}{3}\beta - \frac{g^2 m_K}{4\pi f_K^2} - \frac{g^2 m_\pi}{4\pi f_\pi^2}$$

$$\beta Q_{22} \to -\frac{1}{3}\beta + \frac{g^2 m_\pi}{4\pi f_\pi^2} \qquad \beta Q_{33} \to -\frac{1}{3}\beta + \frac{g^2 m_K}{4\pi f_K^2}$$

Fit	β^{-1}(MeV)	m_c(MeV)	$\Gamma[\Xi_{cc}^{*++}]$ (keV)	$\Gamma[\Xi_{cc}^{*+}]$ (keV)
QM 1	379	1863	$3.3\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$	$2.6\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$
QM 2	356	1500	$3.4\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$	$3.2\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$
χPT 1	272	1432	$2.3\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$	$3.5\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$
χPT 2	276	1500	$2.3\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$	$3.3\left(\frac{E_\gamma}{80\,\mathrm{MeV}}\right)^3$

- QM: no chiral correction   1 - fit m_c 2- fix m_c = 1500 MeV
- $\chi\mathrm{PT}: \;\; f_\pi = 130 \;\mathrm{MeV} \quad f_K = 159 \;\mathrm{MeV}$
- $\Gamma[\Xi^*] \sim 3\,\mathrm{keV}$

$$\mathrm{QM}: \Gamma[\Xi^{*++}] \approx \Gamma[\Xi^{*+}]$$

$$\chi\mathrm{PT}: \Gamma[\Xi^{*++}] < \Gamma[\Xi^{*+}]$$

- One loop mass corrections

- nonanalytic corrections to hyperfine splitting

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} = \begin{cases} -7.0\,\mathrm{MeV} & \mu = 500\,\mathrm{MeV} \\ 8.1\,\mathrm{MeV} & \mu = 1000\,\mathrm{MeV} \\ 16.9\,\mathrm{MeV} & \mu = 1500\,\mathrm{MeV} \end{cases}$$

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} - \frac{3}{4}(\delta m_{D^*} - \delta m_D) = \begin{cases} 3.9\,\mathrm{MeV} & \mu = 500\,\mathrm{MeV} \\ 5.3\,\mathrm{MeV} & \mu = 1000\,\mathrm{MeV} \\ 6.1\,\mathrm{MeV} & \mu = 1500\,\mathrm{MeV} \end{cases}$$

- μ dependence cancelled by counterterms (not included)
- nonanalytic chiral corrections small

Quenched and Partially Quenched Theories

- Existing calculations of bbq, ccq baryons are quenched

  Lewis, Mathur, Woloshyn; Flynn, Mescia, Tariq

- Lattice can potentially answer important questions:

  how heavy quarks have to be for Quark-Diquark symmetry to work?

  predict masses, hyperfine splittings

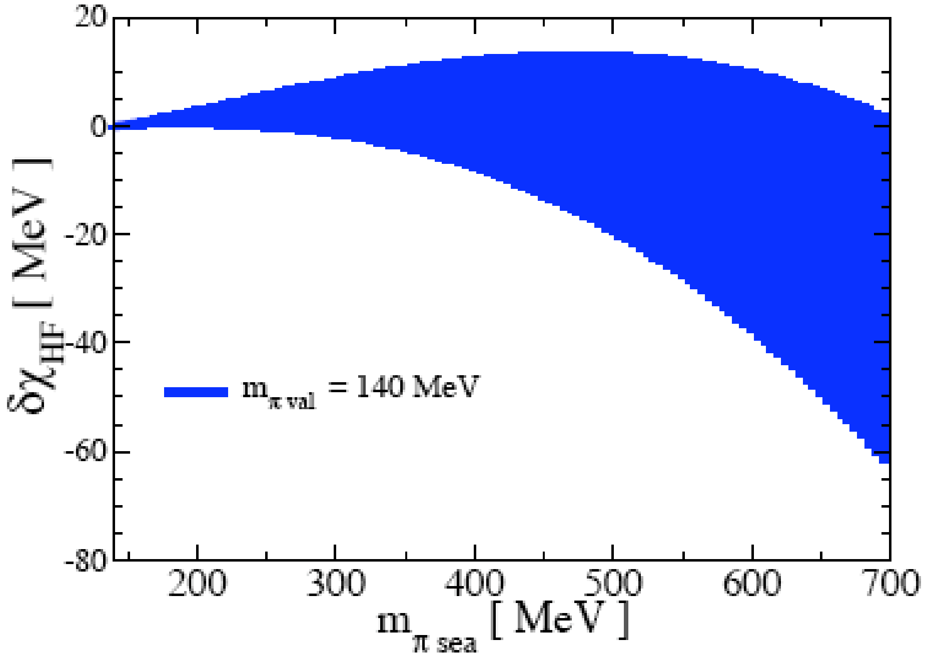
  matrix elements for em, weak decays

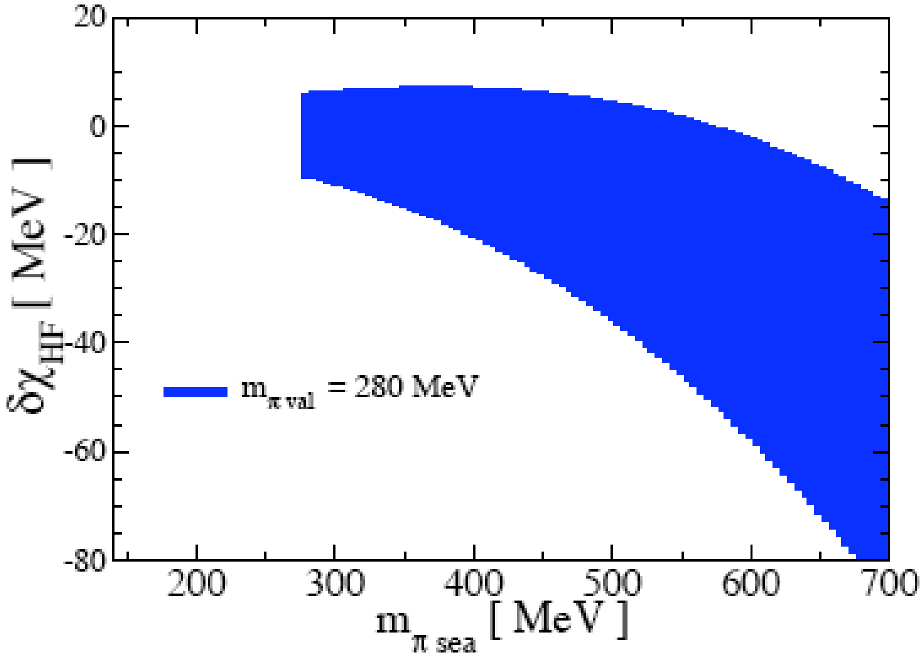
- Simulations w/ dynamical sea quarks needed (and are being planned)

- Motivates construction of quenched and partially quenched versions of Quark-Diquark symmetric χPT

T.M., B. Tiburzi, PRD 74, 054505 (2006)

- PQQχPT nonanalytic corrections to hyperfine splittings

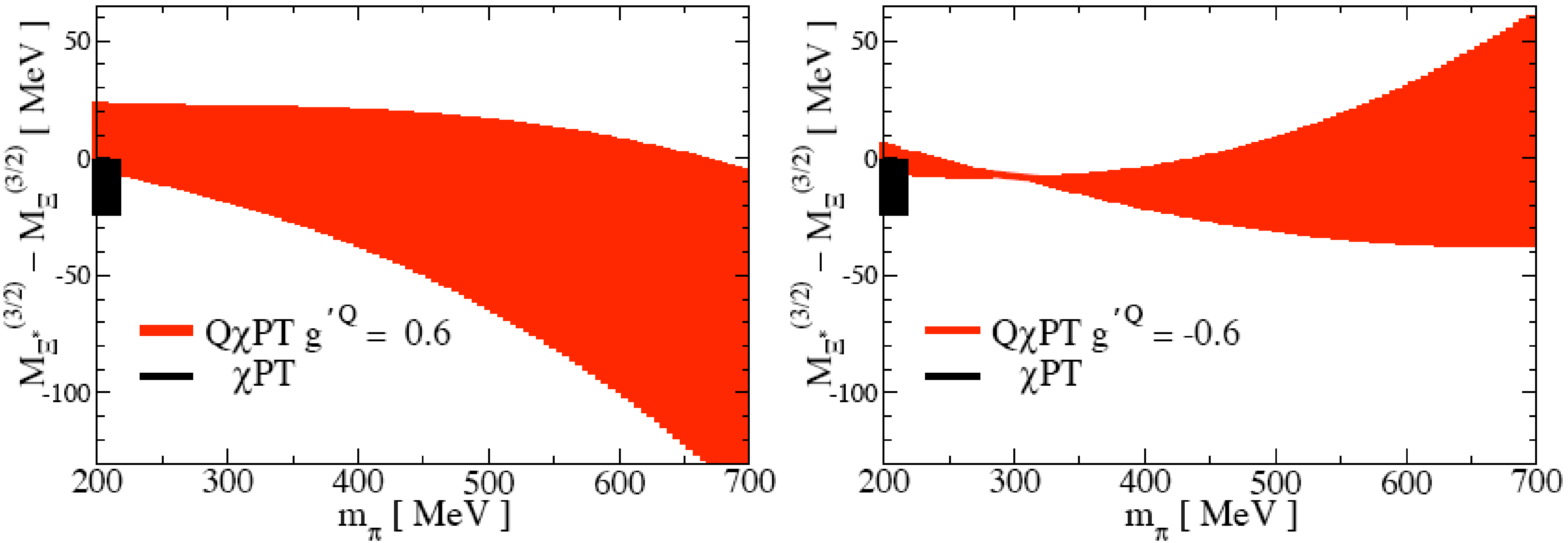




$$\delta\chi_{HF} = \left(M_{\Xi^*}^{(3/2)} - M_{\Xi}^{(3/2)}\right)_{\mathrm{PQ}\chi\mathrm{PT}} - \left(M_{\Xi^*}^{(3/2)} - M_{\Xi}^{(3/2)}\right)_{\chi\mathrm{PT}}$$

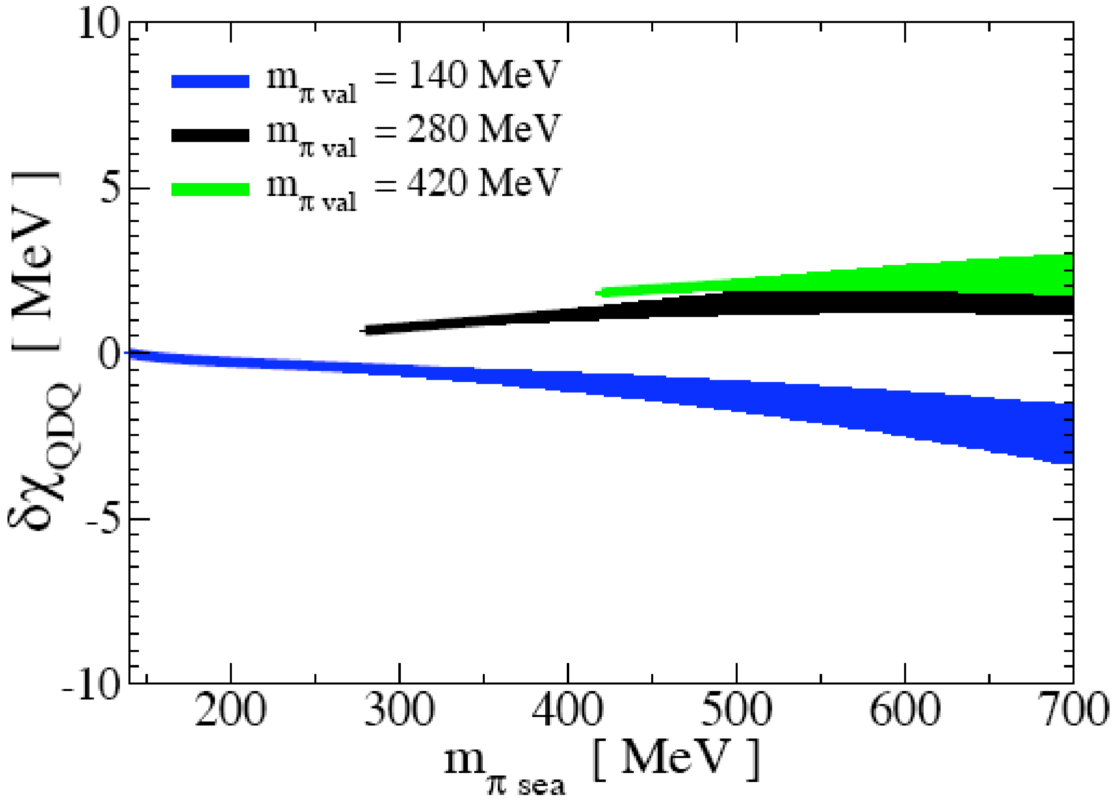
- nonanalytic corrections large: +15 MeV, -60 MeV

- qualitatively similar results in quenched theory



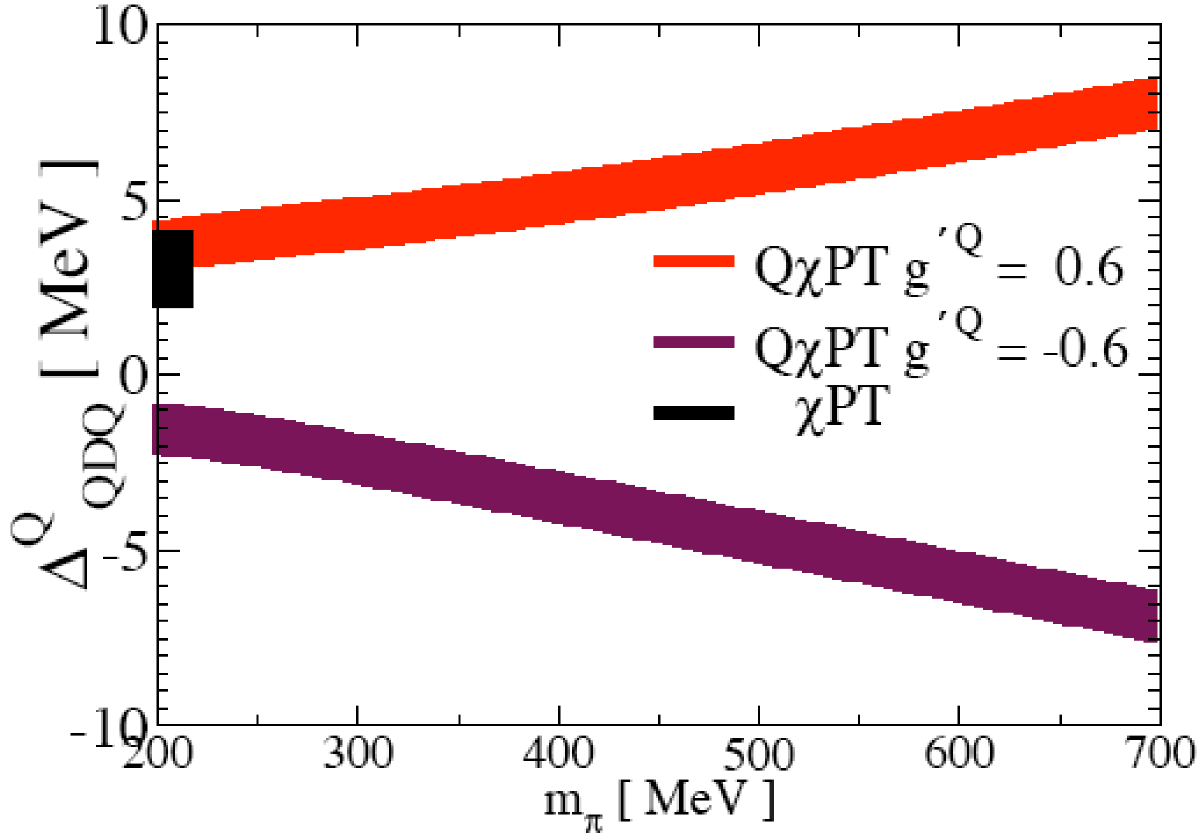
- $$\mathcal{L}^Q = \left(\mathcal{H}^\dagger(\mathcal{H} i \overleftarrow{D}_0)\right) - g^Q\left(\mathcal{H}^\dagger\mathcal{H}\boldsymbol{A}\cdot\boldsymbol{\sigma}\right) - g'^Q\left(\mathcal{H}^\dagger\mathcal{H}\boldsymbol{\sigma}\right)\cdot \mathrm{str}(\boldsymbol{A})$$
$$+\frac{\Delta_H^Q}{4}\left(\mathcal{H}^\dagger\boldsymbol{\Sigma}\cdot\mathcal{H}\boldsymbol{\sigma}\right) + \sigma^Q\left(\mathcal{H}^\dagger\mathcal{H}\mathcal{M}\right) + \sigma'^Q\left(\mathcal{H}^\dagger\mathcal{H}\right)\mathrm{str}(\mathcal{M})$$

- Small Corrections (< 10 MeV) to Quark-Diquark Symmetry Prediction for hyperfine splittings



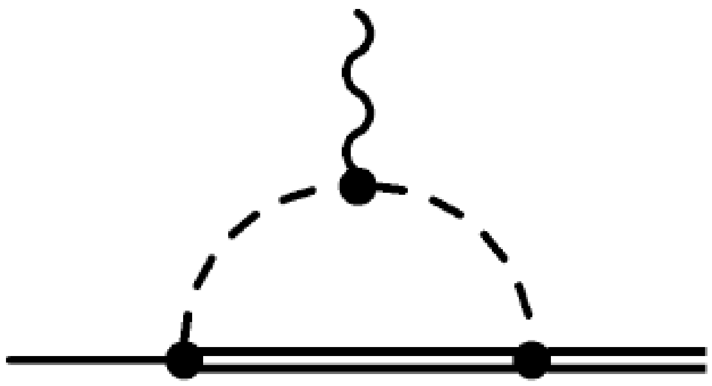
$$\delta\chi_{QDQ} = \left[M_{\Xi^*}^{(3/2)} - M_{\Xi}^{(3/2)} - \frac{3}{4}\left(M_{P^*}^{(3/2)} - M_{P}^{(3/2)} \right) \right]_{PQ\chi PT} - \left[M_{\Xi^*}^{(3/2)} - M_{\Xi}^{(3/2)} - \frac{3}{4}\left(M_{P^*}^{(3/2)} - M_{P}^{(3/2)} \right) \right]_{\chi PT}$$

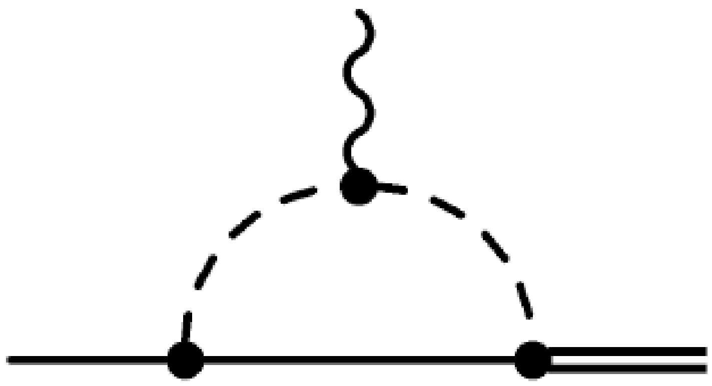
- Small Corrections in Quenched Theory as well



- Chiral Extrapolation For EM decays (PQQχPT)

$$\Gamma(\Xi_a^* \to \Xi_a \gamma) = \frac{4\alpha}{9}\left[\left(\beta Q_a - \frac{Q'}{2m_Q} + \delta\beta_a\right)^2 + \frac{3}{4}E_\gamma^2\left(\frac{\beta_{E2}Q_a}{m_Q\Lambda_\chi} + \delta\beta_{E2a}\right)^2\right]\frac{M_\Xi}{M_{\Xi^*}}E_\gamma^3$$





- Expanding in Δ/m_{GB} and keeping $O(\sqrt{m_q})$ correction only, gives <~10% error for physical meson masses

Summary/Outlook

- Heavy Quark-Diquark Symmetry: useful for doubly heavy baryons

$$\bar{Q}\bar{Q}\bar{q} \leftrightarrow Q\bar{q}$$

- NRQCD required for doubly heavy baryons

  Quark-Diquark symmetry at lowest order in v expansion

- Chiral Lagrangians with Quark-Diquark Symmetry

  predictions for $\Xi_{cc}^{*} \rightarrow \Xi_{cc} + \gamma$ in conflict with SELEX

- Partially Quenched/Quenched Generalizations

  light quark mass dependence of hyperfine splittings, em decays

- Future Work

ChPT: develop covariant formalism, apply to weak decays

NRQCD: v^2, α_s corrections

lattice studies: how heavy must quarks be for Q-DQ symmetry to hold?

Extra Slides

- # One loop mass corrections

$$\delta m_{\Xi_a^*} = \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f^2}\left(\frac{5}{9}K(m_{\Xi_b^*} - m_{\Xi_a^*}, m_i, \mu) + \frac{4}{9}K(m_{\Xi_b} - m_{\Xi_a^*}, m_i, \mu)\right)$$

$$\delta m_{\Xi_a} = \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f^2}\left(\frac{1}{9}K(m_{\Xi_b} - m_{\Xi_a}, m_i, \mu) + \frac{8}{9}K(m_{\Xi_b^*} - m_{\Xi_a}, m_i, \mu)\right)$$

$$\delta m_{H_a} = \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f^2} K(m_{H_b^*} - m_{H_a}, m_i, \mu)$$

$$\delta m_{H_a^*} = \sum_{i,b} C_{ab}^i \frac{g^2}{16\pi^2 f^2}\left(\frac{1}{3}K(m_{H_b} - m_{H_a^*}, m_i, \mu) + \frac{2}{3}K(m_{H_b^*} - m_{H_a^*}, m_i, \mu)\right)$$

$$K(\delta, m) = (-2\,\delta^3 + 3\,m^2\,\delta)\,\ln\left(\frac{m^2}{\mu^2}\right) + 2\,\delta\,(\delta^2 - m^2)\,F\left(\frac{\delta}{m}\right) + 4\,\delta^3 - 5\,\delta\,m^2$$ + cntrtm.

$$\begin{aligned} F(x) &= 2\frac{\sqrt{1-x^2}}{x}\left[\frac{\pi}{2} - \mathrm{Tan}^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right] & |x| < 1 \\ &= -2\frac{\sqrt{x^2-1}}{x}\ln\left(x + \sqrt{x^2-1}\right) & |x| > 1 \end{aligned}$$

- $g = 0.6,\ \Delta_H = 140\ \mathrm{MeV},$ double charm SU(3) splitting of $\delta_s = 100\,\mathrm{MeV}$
- nonanalytic corrections to hyperfine splitting

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} = \begin{cases} -7.0\,\mathrm{MeV} & \mu = 500\,\mathrm{MeV} \\ 8.1\,\mathrm{MeV} & \mu = 1000\,\mathrm{MeV} \\ 16.9\,\mathrm{MeV} & \mu = 1500\,\mathrm{MeV} \end{cases}$$

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} - \frac{3}{4}(\delta m_{D^*} - \delta m_D) = \begin{cases} 3.9\,\mathrm{MeV} & \mu = 500\,\mathrm{MeV} \\ 5.3\,\mathrm{MeV} & \mu = 1000\,\mathrm{MeV} \\ 6.1\,\mathrm{MeV} & \mu = 1500\,\mathrm{MeV} \end{cases}$$

- μ dependence cancelled by counterterms (not included)
- nonanalytic chiral corrections small

Excited Doubly Heavy Baryons

- Light quark excitation energies

$$m_{D'} - m_D \sim 425\,\mathrm{MeV} \qquad m_{D'_s} - m_{D_s} \approx 350\,\mathrm{MeV}$$

- P-wave $\bar{c}\bar{c}$ diquark excitations

$$\Xi_{cc}^{\mathcal{P}*},\ \Xi_{cc}^{\mathcal{P}},\quad J^P = \frac{3}{2}^+, \frac{1}{2}^+ \qquad \vec{S}_{cc} = 0$$ heavy quark singlets

$$V_{cc} = \frac{1}{2} V_{c\bar{c}} \qquad m_{\Xi^{\mathcal{P}}} - m_{\Xi} = 225\,\mathrm{MeV} \approx \frac{m_{h_c} - m_{J/\psi}}{2}$$

(numerical estimate from relativistic quark models)

- Lowest mass excited double charm baryons are diquark excitations

- ## S-wave Strong Decays

$$\Gamma[\Xi_{cc}^{\mathcal{P}*} \to \Xi_{cc}^{*}\,\pi] = \frac{\lambda_{3/2}^2}{2\pi f^2}\left(\frac{1}{2}E_{\pi^0}^2 p_{\pi^0} + E_{\pi^+}^2 p_{\pi^+}\right)\frac{m_{\Xi^*}}{m_{\Xi^{\mathcal{P}*}}} = \lambda_{3/2}^2\,111\,\mathrm{MeV}$$

$$\Gamma[\Xi_{cc}^{\mathcal{P}} \to \Xi_{cc}\,\pi] = \frac{\lambda_{1/2}^2}{2\pi f^2}\left(\frac{1}{2}E_{\pi^0}^2 p_{\pi^0} + E_{\pi^+}^2 p_{\pi^+}\right)\frac{m_{\Xi}}{m_{\Xi^{\mathcal{P}}}} = \lambda_{1/2}^2\,111\,\mathrm{MeV}\,.$$

- $\lambda_{1/2}, \lambda_{3/2} \sim \dfrac{\Lambda_{\mathrm{QCD}}}{m_c}$ Relatively narrow states ?

- radial $\overline{cc}$ diquark excitations

$$\Xi'^{*}_{cc}\,,\,\Xi'_{cc} \qquad J^P = \frac{3}{2}^-,\frac{1}{2}^-$$

heavy quark doublet

$$m_{\Xi'} - m_{\Xi} = 300\,\mathrm{MeV} \approx \frac{m_{\psi'} - m_{J/\psi}}{2}$$

(numerical estimate from relativistic quark models)

- P-wave Strong Decays $\quad \tilde{g} \sim 1 \quad$ unknown coupling constant

Assume $m_{\Xi} = 3440\,\mathrm{MeV}$, $m_{\Xi^*} = 3520\,\mathrm{MeV}$

$$\Gamma[\Xi'_{cc}] = \tilde{g}^2\,336\,\mathrm{MeV} \qquad \Gamma[\Xi^{*\prime}_{cc}] = \tilde{g}^2\,78\,\mathrm{MeV}$$

$$\frac{\Gamma[\Xi'^{*}_{cc} \to \Xi^{*}_{cc}\,\pi]}{\Gamma[\Xi'^{*}_{cc} \to \Xi_{cc}\,\pi]} = 0.56 \qquad \frac{\Gamma[\Xi'_{cc} \to \Xi^{*}_{cc}\,\pi]}{\Gamma[\Xi'_{cc} \to \Xi_{cc}\,\pi]} = 2.3$$

Summary

- Heavy Quark-Diquark Symmetry: useful handle on doubly heavy baryons

$$\bar{Q}\bar{Q}\bar{q} \leftrightarrow Q\bar{q}$$

- NRQCD required for doubly heavy baryons

  Quark-Diquark symmetry at lowest order in v expansion

$$m_{\Xi^*} - m_{\Xi} = \frac{3}{4}(m_{P^*} - m_P)$$

  pNRQCD from vNRQCD

- Chiral Lagrangian with Quark-Diquark symmetry

  new symmetry predictions:

  em decays, chiral mass corrections

  strong decays of excited states

- More experimental results in near future ...?