Curvature squared terms in five-dimensional supergravity

based on hep-th/0611nnn

Keisuke Ohashi
(TITech)

in collaboration with

Kentaro Hanaki and Yuji Tachikawa

§1. Introduction & motivation

- Recently, roles of higher-derivative corrections in supergravity actions become important for examining the validity of string theory.

cf. entropy of black hole, AdS/CFT

- So far, a supersymmetric action in 5D containing

$$R^{abcd}R_{abcd}, \quad R_{ab}R^{ab}, \quad R^2, \quad \epsilon^{abcde}A_a R_{bc}{}^{fg}R_{defg}$$

as a leading correction of 5D SUGRA, though desired, have not been constructed.

$$\Downarrow$$

- We construct this in terms of
  superconformal tensor calculus in 5D

(off-shell formalism for Supergravity)

§2. Superconformal Tensor Calculus in 5D

Fujita-Ohashi hep-th/0104130

Bergshoeff et.al. hep-th/0104113

- generators of the gauge transformations

$$\left.\begin{array}{ll} P_a & : \text{translation} \\ Q^i & : \text{supersymmetry trf.} \\ M_{ab} & : \text{local Lorentz trf.} \end{array}\right\} \text{super Poincaré trf.}$$

$$\left.\begin{array}{ll} D & : \text{dilatation} \\ U^{ij} & : SU(2)_R \text{ trf.} \\ S^i & : \text{conformal supersymmetry trf.} \\ K_a & : \text{special conformal boost trf.} \end{array}\right\} \text{extra trf.}$$

$a, b = 0, \ldots, 4, \quad i = 1, 2$

$D, U^{ij}, S^i, K_a \Longrightarrow$ fix these so that Einstein-Hilbert term and Rarita-Schwinger term are canonical

Weyl multiplet

fields			Weyl-weight
$e_\mu{}^a$	boson	fünfbein	-1
ψ_μ^i	fermion	SU(2)-Majorana	$-\frac{1}{2}$
b_μ	boson	real	0
V_μ^{ij}	boson	$V_\mu^{ij} = V_\mu^{ji} = (V_{\mu ij})^*$	0
v_{ab}	boson	real, antisymmetric	1
χ^i	fermion	SU(2)-Majorana	$\frac{3}{2}$
D	boson	real	2

•Embedding formulas

We can construct a composite multiplet using a set of multiplets.

$L(V)$: vector multiplet $V = \left(W_\mu,\ M,\ \Omega^i,\ Y^{ij}\right)$

$\longrightarrow$ linear multiplet $L = \left(L^{ij}, \varphi^i, E_a, N\right)$

Weyl weight: $w(L^{ij}) = 3$, $w(M) = 1$, $w(Y^{ij}) = 2$

$$L_{ij}(V^I V^J) = M^I Y^J_{ij} - i\bar{\Omega}^I_i \Omega^J_j,$$

$$E_a(V^I V^J) = \frac{1}{8}\epsilon_{abcde} F^{bcI} F^{cdJ} + \cdots, \quad N(V^I V^J) = \cdots .$$

• Gauge-invariant action formula (the VL formula)

a gauge-invariant coupling between
the vector multiplet V and the linear multiplet L:

$$e^{-1}\mathcal{L}_{VL}(VL) \equiv Y^{ij} L_{ij} - \frac{1}{2} W_a E^a + \frac{1}{2} M N + (\text{fermionic terms}),$$

cf. D-term formula, F-term formula in 4D

- Ordinary off-shell Poincaré Supergravity in 5D

A system for the supergravity coupled generally to Vector multiplets and a compensator H^α, $(\alpha = 1, 2)$:

$$\mathcal{L}_0 = -\mathcal{L}_{VL}\left(V^I L(V^J V^K) c_{IJK}\right) - 2\mathcal{L}_{VL}\left(V^0 L(H^\alpha d_{\alpha\beta} Z H^\beta)\right)$$

With certain gauge choices of the dilatation and $SU(2)_{\mathrm{R}}$,

$$e^{-1}\mathcal{L}_0|_{\mathrm{boson}} = -\frac{1}{2}\mathcal{N}R - \frac{1}{4}a_{IJ}F^I_{ab}F^{abJ} - \frac{1}{4}\mathcal{N}_{IJ}\mathcal{D}^a M^I \mathcal{D}_a M^J - V_{\mathrm{pot}} + e^{-1}\frac{1}{8}\epsilon^{\lambda\mu\nu\rho\sigma} c_{IJK} W^I_\lambda F^J_{\mu\nu} F^K_{\rho\sigma} + e^{-1}\mathcal{L}_{\mathrm{aux}}$$

where $\mathcal{N}$ is a cubic polynomial of M^I

$$\mathcal{N} \equiv c_{IJK} M^I M^J M^K, \quad \mathcal{N}_I = \frac{\partial \mathcal{N}}{\partial M^I}, \cdots, a_{IJ} = -\frac{1}{2}\left(\mathcal{N}_{IJ} - \frac{\mathcal{N}_I \mathcal{N}_J}{\mathcal{N}}\right)$$

The potential V_{pot} is

$$V_{\mathrm{pot}} = -4(\mathcal{N}^{-1})^{IJ} P_I P_J - 2(P_I M^I)^2$$

where P_I are charges of the compensator $\delta_G(\Lambda)H^\alpha = i\Lambda^I P_I (i\sigma_3)^\alpha{}_\beta H^\beta$ and a combination $P_I W^I_\mu$ are, so called, gravi-photon.

The rest of the Lagrangian is

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\text{aux}} = \; & 2\left(\mathcal{N}-1\right)\left(\frac{1}{8}D+\frac{3}{16}R-\frac{1}{4}v^2\right) \\
& -\left(V_{a\,j}^{\,i}-\langle V_{a\,j}^{\,i}\rangle_0\right)^*\left(V^{aj}{}_i-\langle V^{aj}{}_i\rangle_0\right) \\
& -\frac{1}{2}\mathcal{N}_{IJ}\left(Y_{ij}^I-\langle Y_{ij}^I\rangle_0\right)\left(Y^{Jij}-\langle Y^{Jij}\rangle_0\right) \\
& +2\mathcal{N}\left(v_{ab}-\langle v_{ab}\rangle_0\right)\left(v^{ab}-\langle v^{ab}\rangle_0\right)
\end{aligned}
$$

D is a Lagrange multiplier leading the condition $\mathcal{N}=1$
$\longrightarrow$ the canonical form of Einstein-Hilbert term:

$V_\mu^{ij}, v_{ab}, Y_{ij}^I$ are auxiliary fields. The square completed terms gives leading solution of auxiliary fields,

$$
\langle V_\mu^{ij}\rangle_0=(P_I W_\mu^I)(i\sigma_3)^{ij}, \quad \langle v_{ab}\rangle_0=-\frac{\mathcal{N}_I}{4\mathcal{N}}F_{ab}^I, \quad \langle Y_{ij}^I\rangle_0=2(\mathcal{N}^{-1})^{IJ}P_J(i\sigma_3)_{ij}.
$$

Next, let us consider a supersymmetric R^2 term $\mathcal{L}_1$ as an α' correction of this ordinary 5D SUGRA ($\mathcal{L}_1 \ll \mathcal{L}_0$).

§3. R^2 terms in 5d SUGRA with the gravitational CS term

Embedding formulae for R^2 terms ($W^2 \to L$) must contains

$$N,\ E_a \ni R(M)^2 \quad \overset{Q}{\leftarrow} \quad \varphi \ni R(M)R(Q) \quad \overset{Q}{\leftarrow} \quad L_{ij} \ni R(Q)^2.$$

Such emmbedding is unique because of $\delta_S L_{ij} = 0$

$$\begin{aligned}
L^{ij}[\mathrm{W}^2] &= i\bar{\hat{R}}_{ab}{}^{(i}(Q)\hat{R}^{abj)}(Q) + \frac{1}{12}i\bar{\chi}^{(i}\chi^{j)} - \frac{4}{3}v^{ab}\hat{R}_{ab}{}^{ij}(U), \\
\varphi^i[\mathrm{W}^2] &= \frac{1}{12}\chi^i D + \frac{1}{4}\gamma_{ab}\hat{R}_{cd}{}^i(Q)\hat{R}^{abcd}(M) - \hat{R}^{abi}{}_j(U)\left(\hat{R}^j_{ab}(Q) + \frac{1}{12}\gamma_{ab}\chi^j\right) \\
&\quad + 8\gamma_{[c}\hat{\mathcal{D}}^c\hat{R}_{a]b}{}^i(Q)v^{ab} - 2\gamma_c\hat{R}_{ab}{}^i(Q)\hat{\mathcal{D}}^a v^{bc} \\
&\quad - \frac{1}{3}\gamma_{[a}\hat{\mathcal{D}}_{b]}\chi^i v^{ab} + \frac{1}{6}\gamma^{ab}\gamma^c\chi^i\hat{\mathcal{D}}_a v_{bc} - \frac{2}{3}\gamma_{ab}\hat{R}_{cd}{}^i(Q)v^{ac}v^{bd}, \\
E_a[\mathrm{W}^2] &= -\frac{1}{8}\epsilon_{abcde}\hat{R}^{bcfg}(M)\hat{R}^{de}{}_{fg}(M) + \frac{1}{6}\epsilon_{abcde}\hat{R}^{bc}{}_{jk}(U)\hat{R}^{dejk}(U) + \cdots \\
N[\mathrm{W}^2] &= \frac{1}{4}\hat{R}^{abcd}(M)\hat{R}_{abcd}(M) - \frac{2}{3}\hat{R}_{abjk}(U)\hat{R}^{abjk}(U) + \cdots
\end{aligned}$$

where $\hat{R}(M), \hat{R}(Q), \hat{R}(U)$ are fully covariant field strengthes (curvatures) for M^{ab}, Q^i, U^{ij}

$$\hat{R}_{\mu\nu}{}^{ab}(M) = R_{\mu\nu}{}^{ab} - \frac{4}{3}R^{[a}_{[\mu}e^{b]}_{\nu]} + \frac{1}{6}e^{[a}_{[\mu}e^{b]}_{\nu]}R + \text{fermionic terms}$$

An gauge-invariant term $\mathcal{L}_1$ containes R^2 terms is given by

$$e^{-1}\mathcal{L}_1 = \; e^{-1}\mathcal{L}_{VL}(c_I V^I, L[W^2]),$$

Under the assumption of $c_L/c_{IJK} = \mathcal{O}(\alpha') \ll 1$, the next leading terms are obtained by substituting the leading solutions of axuliary feilds to $\mathcal{L}_1$,

$$\begin{aligned} e^{-1}\mathcal{L}_1\Big|_{\rm sol} = \; & -\epsilon^{abcde}\left(\frac{1}{16}c_I W_a^I R_{bc}{}^{fg}R_{defg} + \frac{1}{6}c_I P_J P_K W_a^I F_{bc}^J F_{de}^K\right) \\ & +\frac{1}{8}c_I M^I \left(R^{abcd}R_{abcd} - \frac{4}{3}R_{ab}R^{ab} + \frac{1}{6}R^2\right) \\ & + \cdots , \end{aligned}$$

The other terms which have not been calculated yet do not disturb R^2 terms.

We determined ratios of coefficients R^2 terms including the gravitational Chern-Simons term.

We find a modification of the CS term, $\quad c_{IJK} \to c_{IJK} - \frac{4}{3}c_{(I}P_J P_{K)}$

Summary

- We constructed (partially) a supersymmetric R^2 term in five-dimentional supergravity, by use of superconformal tensor calculus.
- By use of this result we can completely confirm the duality supersymmetric AdS condition $\leftrightarrow$ a-maximaization (CFT)

$$a = \frac{27\pi^2}{8}\frac{\mathcal{N}}{(P_I M^I)^3} \quad \leftrightarrow \quad \text{central charge} : a = \frac{3}{32}\left(3\text{tr}R_{SC}^3 - \text{tr}R_{SC}\right)$$

ref. Y.Tachikawa hep-th/0507057

Future problems are

- Completion of the calculation
- Corrections to the entropy of the five-dimensional black rings and black hole
- $\cdots$